1 Схема Кранка – Николсон

$$\frac{U^{n+1} - U^n}{\tau} = \frac{1}{2} L_1 U^{n+1} + \frac{1}{2} L_1 U^n \tag{1}$$

Проведём разделение известных и неизвестных.

$$\left(I - \frac{\tau}{2} L_1\right) U^{n+1} = \left(I + \frac{\tau}{2} L_1\right) U^n \tag{2}$$

Оператор имеет вид

$$L_1 U = \frac{D_x^+ \frac{U_{i+1} - U_i}{h} - D_x^- \frac{U_i - U_{i-1}}{h}}{h} - qU_i = \frac{D_x^+}{h^2} U_{i+1} - \left(\frac{D_x^+}{h^2} + \frac{D_x^-}{h^2} + q\right) U_i + \frac{D_x^-}{h^2} U_{i-1}$$

Левая часть имеет вид

$$\left(I - \frac{\tau}{2} L_1\right) U^{n+1} = U_i^{n+1} - \frac{\tau}{2} \left[\frac{D_x^+}{h^2} U_{i+1} - \left(\frac{D_x^+}{h^2} + \frac{D_x^-}{h^2} + q\right) U_i^{n+1} + \frac{D_x^-}{h^2} U_{i-1}^{n+1}\right] =$$
$$- \frac{\tau D_x^+}{2h^2} U_{i+1}^{n+1} + \left(1 + \frac{\tau D_x^+}{2h^2} + \frac{\tau D_x^-}{2h^2} + \frac{\tau q}{2}\right) U_i^{n+1} - \frac{\tau D_x^-}{2h^2} U_{i-1}^{n+1}$$

Правая часть имеет вид

$$\left(I + \frac{\tau}{2} L_1\right) U^n = U_i^n + \frac{\tau}{2} \left[\frac{D_x^+}{h^2} U_{i+1}^n - \left(\frac{D_x^+}{h^2} + \frac{D_x^-}{h^2} + q\right) U_i^n + \frac{D_x^-}{h^2} U_{i-1}^n\right] =$$
$$\frac{\tau D_x^+}{2h^2} U_{i+1}^n + \left(1 - \left[\frac{\tau D_x^+}{2h^2} + \frac{\tau D_x^-}{2h^2} + \frac{\tau q}{2}\right]\right) U_i^n + \frac{\tau D_x^-}{2h^2} U_{i-1}^n$$

Окончательно имеем

$$- \frac{\tau D_x^+}{2h^2} U_{i+1}^{n+1} + \left(1 + \frac{\tau D_x^+}{2h^2} + \frac{\tau D_x^-}{2h^2} + \frac{\tau q}{2}\right) U_i^{n+1} - \frac{\tau D_x^-}{2h^2} U_{i-1}^{n+1} =$$
$$= \frac{\tau D_x^+}{2h^2} U_{i+1}^n + \left(1 - \left[\frac{\tau D_x^+}{2h^2} + \frac{\tau D_x^-}{2h^2} + \frac{\tau q}{2}\right]\right) U_i^n + \frac{\tau D_x^-}{2h^2} U_{i-1}^n$$

2 Схема Дугласа – Ганна для двумерной области

$$\begin{cases} \frac{U^* - U^n}{\tau} = \frac{1}{2} L_1 U^* + \frac{1}{2} L_1 U^n + L_2 U^n, \\ \frac{U^{n+1} - U^n}{\tau} = \frac{1}{2} L_1 U^* + \frac{1}{2} L_1 U^n + \frac{1}{2} L_2 U^{n+1} + \frac{1}{2} L_2 U^n \end{cases} \tag{3}$$

Проведём разделение известных и неизвестных переменных для каждого из уравнений.

$$\begin{cases} \frac{U^*}{\tau} - \frac{1}{2} L_1 U^* = \frac{U^n}{\tau} + \frac{1}{2} L_1 U^n + L_2 U^n, \\ \frac{U^{n+1}}{\tau} - \frac{1}{2} L_2 U^{n+1} = \frac{U^n}{\tau} + \frac{1}{2} L_1 U^* + \frac{1}{2} L_1 U^n + \frac{1}{2} L_2 U^n \end{cases} \tag{4}$$

Сгруппируем операторы

$$\begin{cases} \left(\frac{1}{\tau} I - \frac{1}{2} L_1\right) U^* = \left(\frac{1}{\tau} I + \frac{1}{2} L_1 + L_2\right) U^n, \\ \left(\frac{1}{\tau} I - \frac{1}{2} L_2\right) U^{n+1} = \left(\frac{1}{\tau} I + \frac{1}{2} L_1 + \frac{1}{2} L_2\right) U^n + \frac{1}{2} L_1 U^* \end{cases} \tag{5}$$

Вычтем первое уравнение из второго:

$$\begin{cases} \left(\frac{1}{\tau}I - \frac{1}{2}L_1\right)U^* = \left(\frac{I}{\tau} + \frac{1}{2}L_1 + L_2\right)U^n, \\ \left(\frac{1}{\tau}I - \frac{1}{2}L_2\right)U^{n+1} - \left(\frac{1}{\tau}I - \frac{1}{2}L_1\right)U^* = -\frac{1}{2}L_2U^n + \frac{1}{2}L_1U^* \end{cases} \tag{6}$$

Отсюда

$$\begin{cases} \left(\frac{1}{\tau}I - \frac{1}{2}L_1\right)U^* = \left(\frac{I}{\tau} + \frac{1}{2}L_1 + L_2\right)U^n, \\ \left(\frac{1}{\tau}I - \frac{1}{2}L_2\right)U^{n+1} = \frac{1}{\tau}U^* - \frac{1}{2}L_2U^n \end{cases} \tag{7}$$

Домножим на τ

$$\begin{cases} \left(I - \frac{\tau}{2}L_1\right)U^* = \left(I + \frac{\tau}{2}L_1 + \tau L_2\right)U^n, \\ \left(I - \frac{\tau}{2}L_2\right)U^{n+1} = U^* - \frac{\tau}{2}L_2U^n, \end{cases} \tag{8}$$

Операторы имеют вид:

$$\begin{aligned} L_1U &= \frac{D_x^+\frac{U_{i+1,j}-U_{i,j}}{h} - D_x^-\frac{U_{i,j}-U_{i-1,j}}{h}}{h} - qU_{i,j} = \\ &= \frac{D_x^+U_{i+1,j} + D_x^-U_{i-1,j}}{h^2} - \frac{D_x^+}{h^2}U_{i,j} - \frac{D_x^-}{h^2}U_{i,j} - qU_{i,j} = \\ &= \frac{D_x^+U_{i+1,j} + D_x^-U_{i-1,j}}{h^2} - \left(\frac{D_x^+ + D_x^-}{h^2} + q\right)U_{i,j}, \quad \forall i,j; \\ L_2U &= \frac{D_y^+U_{i,j+1} + D_y^-U_{i,j-1}}{h^2} - \left(\frac{D_y^+ + D_y^-}{h^2} + q\right)U_{i,j} \end{aligned} \tag{9}$$

Преобразуем левые части уравнений

$$\left(I - \frac{\tau}{2}L_1\right)U^* = U^*_{i,j} - \frac{\tau}{2}\left[\frac{D_x^+U^*_{i+1,j} + D_x^-U^*_{i-1,j}}{h^2} - \left(\frac{D_x^+ + D_x^-}{h^2} + q\right)U^*_{i,j}\right] =$$
$$- \frac{\tau D_x^+}{2h^2}U^*_{i+1,j} + \left(1 + \frac{\tau D_x^+}{2h^2} + \frac{\tau D_x^-}{2h^2} + \frac{\tau q}{2}\right)U^*_{i,j} - \frac{\tau D_x^-}{h^2}U^*_{i-1,j}$$

$$\left(I - \frac{\tau}{2}L_2\right)U^{n+1} = U^{n+1}_{i,j} - \frac{\tau}{2}\left[\frac{D_y^+U^{n+1}_{i,j+1} + D_y^-U^{n+1}_{i,j-1}}{h^2} - \left(\frac{D_y^+ + D_y^-}{h^2} + q\right)U^{n+1}_{i,j}\right] =$$
$$- \frac{\tau D_y^+}{2h^2}U^{n+1}_{i,j+1} + \left(1 + \frac{\tau D_y^+}{2h^2} + \frac{\tau D_y^-}{2h^2} + \frac{\tau q}{2}\right)U^{n+1}_{i,j} - \frac{\tau D_y^-}{h^2}U^{n+1}_{i,j-1}$$

Преобразуем правые части:

$$\left(I + \frac{\tau}{2}L_1 + \tau L_2\right)U^n = U^n_{i,j} + \frac{\tau}{2}\left[\frac{D_x^+U^n_{i+1,j} + D_x^-U^n_{i-1,j}}{h^2} - \left(\frac{D_x^+ + D_x^-}{h^2} + q\right)U^n_{i,j}\right] +$$
$$+ \tau\left[\frac{D_y^+U^n_{i,j+1} + D_y^-U^n_{i,j-1}}{h^2} - \left(\frac{D_y^+ + D_y^-}{h^2} + q\right)U^n_{i,j}\right] = \frac{\tau D_x^+}{2h^2}U^n_{i+1,j} + \frac{\tau D_x^-}{2h^2}U^n_{i-1,j} +$$
$$+ \frac{\tau D_y^+}{h^2}U^n_{i,j+1} + \frac{\tau D_y^-}{h^2}U^n_{i,j-1} + \left[1 - \frac{\tau D_x^+}{2h^2} - \frac{\tau D_x^-}{2h^2} - \frac{\tau D_y^+}{h^2} - \frac{\tau D_y^-}{h^2} - \frac{3\tau q}{2}\right]U^n_{i,j}$$

$$U^* - \frac{\tau}{2}L_2U^n = U^*_{i,j} - \frac{\tau}{2}\left[\frac{D_y^+U_{i,j+1} + D_y^-U_{i,j-1}}{h^2} - \left(\frac{D_y^+ + D_y^-}{h^2} + q\right)U_{i,j}\right] =$$
$$- \frac{\tau D_y^+}{2h^2}U_{i,j+1} - \frac{\tau D_y^-}{2h^2}U_{i,j-1} + \left(1 + \frac{\tau D_y^+}{2h^2} + \frac{\tau D_y^-}{2h^2} + \frac{\tau q}{2}\right)U_{i,j}$$

В итоге

3 Схема Дугласа – Ганна

Многошаговая реализация метода Дугласа – Ганна имеет вид:

$$\begin{cases} \frac{U^*-U^n}{\tau} & = \frac{1}{2}L_1U^* + \frac{1}{2}L_1U^n + L_2U^n + L_3U^n \\ \frac{U^{**}-U^n}{\tau} & = \frac{1}{2}L_1U^* + \frac{1}{2}L_1U^n + \frac{1}{2}L_2U^{**} + \frac{1}{2}L_2U^n + L_3U^n \\ \frac{U^{n+1}-U^n}{\tau} & = \frac{1}{2}L_1U^* + \frac{1}{2}L_1U^n + \frac{1}{2}L_2U^{**} + \frac{1}{2}L_2U^n + \frac{1}{2}L_3U^{n+1} + \frac{1}{2}L_3U^n \end{cases} \tag{10}$$

Каждое из уравнений системы (10) — аппроксимация полного уравнения диффузии => не нужно модификаций для граничных условий. Проведём разделение известных и неизвестных переменных для каждого из уравнений.

$$\begin{cases} \frac{U^*}{\tau} - \frac{1}{2}L_1U^* & = \frac{U^n}{\tau} + \frac{1}{2}L_1U^n + L_2U^n + L_3U^n \\ \frac{U^{**}}{\tau} - \frac{1}{2}L_2U^{**} & = \frac{U^n}{\tau} + \frac{1}{2}L_1U^* + \frac{1}{2}L_1U^n + \frac{1}{2}L_2U^n + L_3U^n \\ \frac{U^{n+1}}{\tau} - \frac{1}{2}L_3U^{n+1} & = \frac{U^n}{\tau} + \frac{1}{2}L_1U^* + \frac{1}{2}L_1U^n + \frac{1}{2}L_2U^{**} + \frac{1}{2}L_2U^n + \frac{1}{2}L_3U^n \end{cases}$$

Сгруппируем операторы

$$\begin{cases} \left(\frac{1}{\tau}I - \frac{1}{2}L_1\right)U^* & = \left(\frac{1}{\tau}I + \frac{1}{2}L_1 + L_2 + L_3\right)U^n \\ \left(\frac{1}{\tau}I - \frac{1}{2}L_2\right)U^{**} & = \left(\frac{1}{\tau}I + \frac{1}{2}L_1 + \frac{1}{2}L_2 + L_3\right)U^n + \frac{1}{2}L_1U^* \\ \left(\frac{1}{\tau}I - \frac{1}{2}L_3\right)U^{n+1} & = \left(\frac{1}{\tau}I + \frac{1}{2}L_1 + \frac{1}{2}L_2 + \frac{1}{2}L_3\right)U^n + \frac{1}{2}L_1U^* + \frac{1}{2}L_2U^{**} \end{cases} \tag{11}$$

В системе (11) вычтем первое уравнение из второго и второе из третьего:

$$\begin{cases} \left(\frac{1}{\tau}I - \frac{1}{2}L_1\right)U^* = \left(\frac{1}{\tau}I + \frac{1}{2}L_1 + L_2 + L_3\right)U^n \\ \left(\frac{1}{\tau}I - \frac{1}{2}L_2\right)U^{**} - \left(\frac{1}{\tau}I - \frac{1}{2}L_1\right)U^* = \\ \quad = \left(\frac{1}{\tau}I + \frac{1}{2}L_1 + \frac{1}{2}L_2 + L_3\right)U^n + \frac{1}{2}L_1U^* - \\ \quad - \left(\frac{1}{\tau}I + \frac{1}{2}L_1 + L_2 + L_3\right)U^n \\ \left(\frac{1}{\tau}I - \frac{1}{2}L_3\right)U^{n+1} - \left(\frac{1}{\tau}I - \frac{1}{2}L_2\right)U^{**} = \\ \quad = \left(\frac{1}{\tau}I + \frac{1}{2}L_1 + \frac{1}{2}L_2 + \frac{1}{2}L_3\right)U^n + \frac{1}{2}L_1U^* + \frac{1}{2}L_2U^{**} - \\ \quad - \left(\frac{1}{\tau}I + \frac{1}{2}L_1 + \frac{1}{2}L_2 + L_3\right)U^n - \frac{1}{2}L_1U^* \end{cases}$$

Отсюда

$$\begin{cases} \left(\frac{1}{\tau}I - \frac{1}{2}L_1\right)U^* & = \left(\frac{1}{\tau}I + \frac{1}{2}L_1 + L_2 + L_3\right)U^n \\ \left(\frac{1}{\tau}I - \frac{1}{2}L_2\right)U^{**} - \left(\frac{1}{\tau}I - \frac{1}{2}L_1\right)U^* & = -\frac{1}{2}L_2U^n + \frac{1}{2}L_1U^* \\ \left(\frac{1}{\tau}I - \frac{1}{2}L_3\right)U^{n+1} - \left(\frac{1}{\tau}I - \frac{1}{2}L_2\right)U^{**} & = -\frac{1}{2}L_3U^n + \frac{1}{2}L_2U^{**} \end{cases}$$

Переносим вычитаемые в правую часть и сокращаем

$$\begin{cases} \left(\frac{1}{\tau}I - \frac{1}{2}L_1\right)U^* & = \left(\frac{1}{\tau}I + \frac{1}{2}L_1 + L_2 + L_3\right)U^n \\ \left(\frac{1}{\tau}I - \frac{1}{2}L_2\right)U^{**} & = \frac{1}{\tau}U^* - \frac{1}{2}L_2U^n \\ \left(\frac{1}{\tau}I - \frac{1}{2}L_3\right)U^{n+1} & = \frac{1}{\tau}U^{**} - \frac{1}{2}L_3U^n \end{cases}$$

Домножим все части на τ:

$$\begin{cases} \left(I - \frac{\tau}{2}L_1\right)U^* & = \left(I + \frac{\tau}{2}L_1 + \tau L_2 + \tau L_3\right)U^n \\ \left(I - \frac{\tau}{2}L_2\right)U^{**} & = U^* - \frac{\tau}{2}L_2 U^n \\ \left(I - \frac{\tau}{2}L_3\right)U^{n+1} & = U^{**} - \frac{\tau}{2}L_3 U^n \end{cases}$$

Операторы L_1, L_2 и L_3 имеют вид:

$$\begin{aligned} L_1 U = \frac{D_x^+ \frac{U_{i+1,j,k} - U_{i,j,k}}{h} - D_x^- \frac{U_{i,j,k} - U_{i-1,j,k}}{h}}{h} - qU_{i,j,k} = \\ = \frac{D_x^+ U_{i+1,j,k} + D_x^- U_{i-1,j,k}}{h^2} - \frac{D_x^+}{h^2}U_{i,j,k} - \frac{D_x^-}{h^2}U_{i,j,k} - qU_{i,j,k} = \\ = \frac{D_x^+ U_{i+1,j,k} + D_x^- U_{i-1,j,k}}{h^2} - \left(\frac{D_x^+ + D_x^-}{h^2} + q\right)U_{i,j,k}, \quad \forall i,j,k; \\ L_2 U = \frac{D_y^+ U_{i,j+1,k} + D_y^- U_{i,j-1,k}}{h^2} - \left(\frac{D_y^+ + D_y^-}{h^2} + q\right)U_{i,j,k} \\ L_3 U = \frac{D_z^+ U_{i,j,k+1} + D_z^- U_{i,j,k-1}}{h^2} - \left(\frac{D_z^+ + D_z^-}{h^2} + q\right)U_{i,j,k} \end{aligned} \tag{12}$$

Перепишем левые части уравнений системы (3) в виде, удобном для заполнения матриц

$$\begin{aligned} \left(I - \frac{\tau}{2}L_1\right)U^* = U^*_{i,j,k} - \frac{\tau}{2}\left[\frac{D_x^+ U^*_{i+1,j,k} + D_x^- U^*_{i-1,j,k}}{h^2} - \left(\frac{D_x^+ + D_x^-}{h^2} + q\right)U^*_{i,j,k}\right] = \\ = -\frac{\tau D_x^+}{2h^2}U^*_{i+1,j,k} + \left(1 + \frac{\tau}{2}\left[\frac{D_x^+ + D_x^-}{h^2} + q\right]\right)U^*_{i,j,k} - \frac{\tau D_x^-}{2h^2}U^*_{i-1,j,k} = \\ = -\frac{\tau D_x^+}{2h^2}U^*_{i+1,j,k} + \left(1 + \frac{\tau D_x^+}{2h^2} + \frac{\tau D_x^-}{2h^2} + \frac{\tau q}{2}\right)U^*_{i,j,k} - \frac{\tau D_x^-}{2h^2}U^*_{i-1,j,k} \\ \left(I - \frac{\tau}{2}L_2\right)U^{**} = -\frac{\tau D_y^+}{2h^2}U^{**}_{i,j+1,k} + \left(1 + \frac{\tau D_y^+}{2h^2} + \frac{\tau D_y^-}{2h^2} + \frac{\tau q}{2}\right)U^{**}_{i,j,k} - \frac{\tau D_y^-}{2h^2}U^{**}_{i,j-1,k} \\ \left(I - \frac{\tau}{2}L_3\right)U^{n+1} = -\frac{\tau D_z^+}{2h^2}U^{n+1}_{i,j,k+1} + \left(1 + \frac{\tau D_z^+}{2h^2} + \frac{\tau D_z^-}{2h^2} + \frac{\tau q}{2}\right)U^{n+1}_{i,j,k} - \frac{\tau D_z^-}{2h^2}U^{n+1}_{i,j,k-1} \end{aligned} \tag{13}$$

Распишем правую часть первого уравнения системы (3)

$$\begin{aligned} \left(I + \frac{\tau}{2}L_1 + \tau L_2 + \tau L_3\right)U = U_{i,j,k} + \frac{\tau}{2}\left[\frac{D_x^+ U_{i+1,j,k} + D_x^- U_{i-1,j,k}}{h^2} - \left(\frac{D_x^+ + D_x^-}{h^2} + q\right)U_{i,j,k}\right] + \\ + \tau\left[\frac{D_y^+ U_{i,j+1,k} + D_y^- U_{i,j-1,k}}{h^2} - \left(\frac{D_y^+ + D_y^-}{h^2} + q\right)U_{i,j,k}\right] + \\ + \tau\left[\frac{D_z^+ U_{i,j,k+1} + D_z^- U_{i,j,k-1}}{h^2} - \left(\frac{D_z^+ + D_z^-}{h^2} + q\right)U_{i,j,k}\right] = \\ = \frac{\tau}{2}\left[\frac{D_x^+ U_{i+1,j,k} + D_x^- U_{i-1,j,k}}{h^2}\right] + \tau\left[\frac{D_y^+ U_{i,j+1,k} + D_y^- U_{i,j-1,k}}{h^2}\right] + \tau\left[\frac{D_z^+ U_{i,j,k+1} + D_z^- U_{i,j,k-1}}{h^2}\right] + \\ + \left[1 - \frac{\tau}{2}\left(\frac{D_x^+ + D_x^-}{h^2} + q\right) - \tau\left(\frac{D_y^+ + D_y^-}{h^2} + q\right) - \tau\left(\frac{D_z^+ + D_z^-}{h^2} + q\right)\right]U_{i,j,k} = \\ = \frac{\tau D_x^+}{2h^2}U_{i+1,j,k} + \frac{\tau D_x^-}{2h^2}U_{i-1,j,k} + \frac{\tau D_y^+}{h^2}U_{i,j+1,k} + \frac{\tau D_y^-}{h^2}U_{i,j-1,k} + \frac{\tau D_z^+}{h^2}U_{i,j,k+1} + \frac{\tau D_z^-}{h^2}U_{i,j,k-1} + \\ + \left[1 - \left(\frac{\tau D_x^+}{2h^2} + \frac{\tau D_x^-}{2h^2} + \frac{\tau D_y^+}{h^2} + \frac{\tau D_y^-}{h^2} + \frac{\tau D_z^+}{h^2} + \frac{\tau D_z^-}{h^2} + \frac{5\tau q}{2}\right)\right]U_{i,j,k}, \quad \forall i,j,k; \end{aligned} \tag{14}$$

То же для остальных уравнений:

$$U^* - \frac{\tau}{2}L_2U = U^*_{i,j,k} - \frac{\tau}{2}\left[\frac{D_y^+U_{i,j+1,k} + D_y^-U_{i,j-1,k}}{h^2} - \left(\frac{D_y^+ + D_y^-}{h^2} + q\right)U_{i,j,k}\right] =$$
$$= U^*_{i,j,k} - \frac{\tau D_y^+}{2h^2}U_{i,j+1,k} - \frac{\tau D_y^-}{2h^2}U_{i,j-1,k} + \left(\frac{\tau D_y^+}{2h^2} + \frac{\tau D_y^-}{2h^2} + \frac{\tau q}{2}\right)U_{i,j,k}$$
$$U^{**} - \frac{\tau}{2}L_3U = U^{**}_{i,j,k} - \frac{\tau}{2}\left[\frac{D_z^+U_{i,j,k+1} + D_z^-U_{i,j,k-1}}{h^2} - \left(\frac{D_z^+ + D_z^-}{h^2} + q\right)U_{i,j,k}\right] =$$
$$= U^{**}_{i,j,k} - \frac{\tau D_z^+}{2h^2}U_{i,j,k+1} - \frac{\tau D_z^-}{2h^2}U_{i,j,k-1} + \left(\frac{\tau D_z^+}{2h^2} + \frac{\tau D_z^-}{2h^2} + \frac{\tau q}{2}\right)U_{i,j,k}$$

Окончательно на n-ом шаге имеем:

$$\begin{aligned}
&-\frac{\tau D_x^+}{2h^2}U^*_{i+1,j,k} + \left(1 + \frac{\tau D_x^+}{2h^2} + \frac{\tau D_x^-}{2h^2} + \frac{\tau q}{2}\right)U^*_{i,j,k} - \frac{\tau D_x^-}{2h^2}U^*_{i-1,j,k} = \\
&= \frac{\tau D_x^+}{2h^2}U^n_{i+1,j,k} + \frac{\tau D_x^-}{2h^2}U^n_{i-1,j,k} + \frac{\tau D_y^+}{h^2}U^n_{i,j+1,k} + \frac{\tau D_y^-}{h^2}U^n_{i,j-1,k} + \frac{\tau D_z^+}{h^2}U^n_{i,j,k+1} + \frac{\tau D_z^-}{h^2}U^n_{i,j,k-1} + \\
&\quad + \left[1 - \left(\frac{\tau D_x^+}{2h^2} + \frac{\tau D_x^-}{2h^2} + \frac{\tau D_y^+}{h^2} + \frac{\tau D_y^-}{h^2} + \frac{\tau D_z^+}{h^2} + \frac{\tau D_z^-}{h^2} + \frac{5\tau q}{2}\right)\right]U^n_{i,j,k}
\end{aligned} \tag{15}$$

$$\begin{aligned}
&-\frac{\tau D_y^+}{2h^2}U^{**}_{i,j+1,k} + \left(1 + \frac{\tau D_y^+}{2h^2} + \frac{\tau D_y^-}{2h^2} + \frac{\tau q}{2}\right)U^{**}_{i,j,k} - \frac{\tau D_y^-}{2h^2}U^{**}_{i,j-1,k} = \\
&= U^*_{i,j,k} - \frac{\tau D_y^+}{2h^2}U^n_{i,j+1,k} - \frac{\tau D_y^-}{2h^2}U^n_{i,j-1,k} + \left(\frac{\tau D_y^+}{2h^2} + \frac{\tau D_y^-}{2h^2} + \frac{\tau q}{2}\right)U^n_{i,j,k}
\end{aligned} \tag{16}$$

$$\begin{aligned}
&-\frac{\tau D_z^+}{2h^2}U^{n+1}_{i,j,k+1} + \left(1 + \frac{\tau D_z^+}{2h^2} + \frac{\tau D_z^-}{2h^2} + \frac{\tau q}{2}\right)U^{n+1}_{i,j,k} - \frac{\tau D_z^-}{2h^2}U^{n+1}_{i,j,k-1} = \\
&= U^{**}_{i,j,k} - \frac{\tau D_z^+}{2h^2}U^n_{i,j,k+1} - \frac{\tau D_z^-}{2h^2}U^n_{i,j,k-1} + \left(\frac{\tau D_z^+}{2h^2} + \frac{\tau D_z^-}{2h^2} + \frac{\tau q}{2}\right)U^n_{i,j,k}
\end{aligned} \tag{17}$$