02427 Advanced Time Series Analysis

Computer exercise 1

Niklas Christoffer Petersen, niklch@dtu.dk (niklch@dtu.dk)

October 10, 2018

In [ ]:

```
import numpy as np
import numpy.polynomial.polynomial as poly
import pandas as pd
import statsmodels.api as sm
import statsmodels.tsa.arima_model
from statsmodels.tsa.stattools import acf, pacf

from IPython.display import Markdown as md
import matplotlib.pyplot as plt

%matplotlib inline
```

Part 1

We define number of simulations, and time steps

In [2]:

```
N = 1000
ts = np.arange(N)
```

SETAR

We define a $\text{SETAR}(2, 1, 1)$ model, i.e. two regimes, delay 1 and auto-regressive order 1 (in both regimes):

$$y_t = \begin{cases} 0.2 + y_{t-1} + \epsilon_t & \text{if } y_{t-1} < 100 \\ 10 + 0.95 y_{t-1} + \epsilon_t & \text{if } y_{t-1} \geq 100 \end{cases}$$

This could be some systam that has a threashhold around 100 at which the process will increase faster than before (times 8).

In [3]:

```
def setar(N):
    r = np.random.randn(N)
    y = np.empty(N)
    y[0] = r[0]
    for t in range(1, N):
        if y[t-1] < 100:
            y[t] = .2 + y[t-1] + r[t]
        else:
            y[t] = 10 + 0.95*y[t-1] + r[t]
    return y, r
```

In [4]:

```
# Ensure reproducability
np.random.seed(seed=42)
# Run N simulations
y_setar, r_setar = setar(N)
```

Save the results for later:

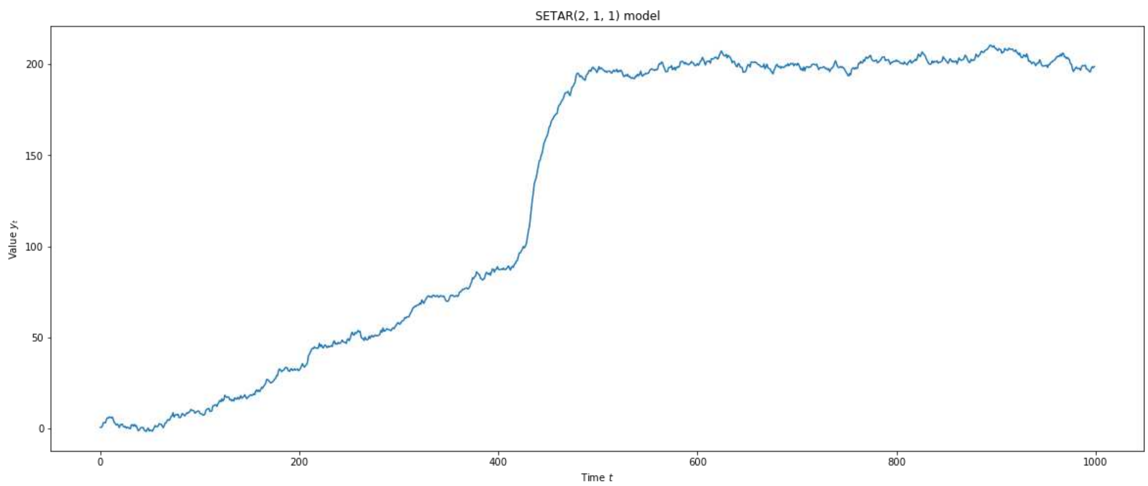
In [5]:

```
np.save('setar_2_1_1.npy', np.stack([r_setar, y_setar], axis = 1))
```

We plot the time series. The model parameters define a ragime change when y_{t-1} gets above 100, triggering from a slowly increasing state to a more rapid increasing state, that stabilizes quickly around 200.

In [6]:

```
fig, ax = plt.subplots(figsize = (20, 8))
ax.set_title('SETAR(2, 1, 1) model')
ax.set_xlabel(r'Time $t$')
ax.set_ylabel(r'Value $y_t$')
ax.plot(ts, y_setar, marker = '', linestyle = '-')
None
```



IGAR

Next we try with a $\mathrm{IGAR}(2, 1)$ model:

$$\begin{cases} 0.2 + y_{t-1} & \text{with } p = 99\% \\ -8 + 0.8 y_{t-1} & \text{with } p = 1\% \end{cases}$$

This could be simulation of earnings of stocks. That is, in general there is a good chance (99%) that you will earn a little bit (0.2) every day. But on some more rare occasions the stock will drop, costing your 20% of your earnings plus a fixed loss (8).

In [7]:

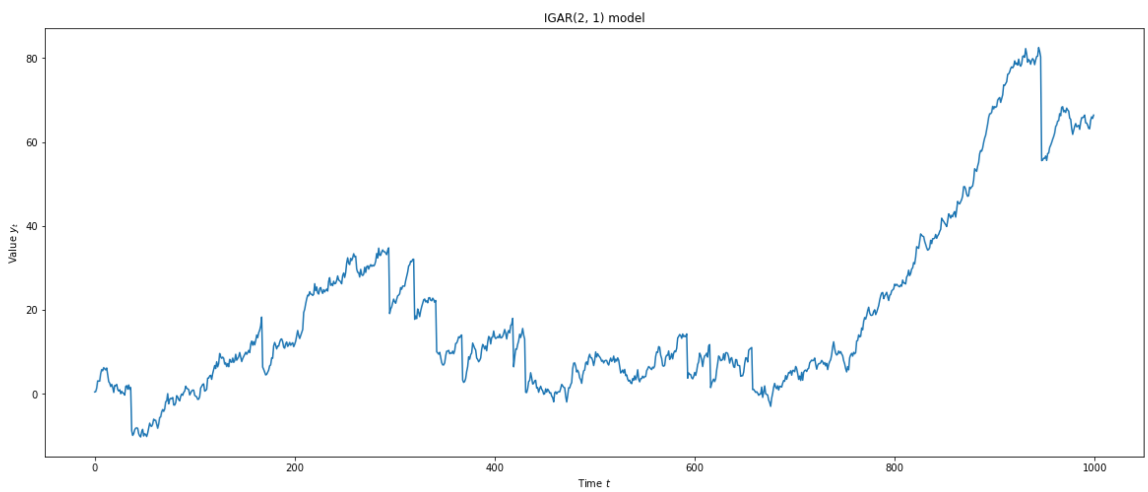
```
def igar(N):
    r = np.random.randn(N)
    y = np.empty(N)
    y[0] = r[0]
    j = np.random.rand(N)
    for t in range(1, N):
        if j[t] < .99:
            y[t] = .2 + y[t-1] + r[t]
        else:
            y[t] = -8 + 0.8 * y[t-1] + r[t]
    return y, r
```

In [8]:

```
# Ensure reproducability
np.random.seed(seed=42)
# Run N simulations
y_igar, r_igar = igar(N)
```

In [9]:

```
fig, ax = plt.subplots(figsize = (20, 8))
ax.set_title('IGAR(2, 1) model')
ax.set_xlabel(r'Time $t$')
ax.set_ylabel(r'Value $y_t$')
ax.plot(ts, y_igar, marker = '', linestyle = '-')
None
```



MMAR

We try a MMAR(2, 1) model with the transition probabilities given as:

$$P = \begin{bmatrix} 0.99 & 0.01 \\ 0.10 & 0.90 \end{bmatrix}$$

In [10]:

```
P = np.array([
    [.99, .01],
    [.1, .9]
])
```

Again, this could be simulation of earnings of stocks, but now when a loosing regime is entered ($\text{State } 1$), there is a higher probability that one will stay in the loosing regime for a while before returning to the earnings regime ($\text{State } 0$).

In [11]:

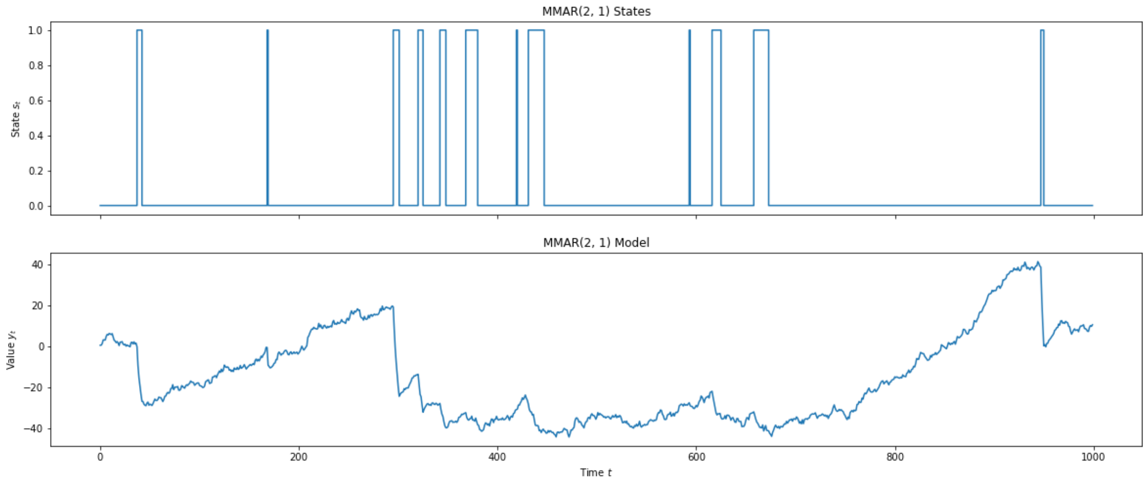
```
def mmar(N,P):
    r = np.random.randn(N)
    y = np.empty(N)
    y[0] = r[0]
    state = np.zeros(N).astype(int)
    for t in range(1, N):
        p_trans = P[state[t - 1]]
        state[t] = np.random.choice(len(P), p = p_trans)
        if state[t] == 0:
            y[t] = .2 + y[t-1] + r[t]
        else:
            y[t] = -8 + 0.8 * y[t-1] + r[t]
    return y, r, state
```

In [12]:

```
# Ensure reproducability
np.random.seed(seed=42)
# Run N simulations
y_mmar, r_mmar, s_mmar = mmar(N,P)
```

In [13]:

```
fig, ax = plt.subplots(2, 1, figsize = (20, 8), sharex = True)
ax[0].set_title('MMAR(2, 1) States')
ax[0].set_ylabel(r'State $s_t$')
ax[0].step(ts, s_mmar, marker = '', linestyle = '-')

ax[1].set_title('MMAR(2, 1) Model')
ax[1].set_xlabel(r'Time $t$')
ax[1].set_ylabel(r'Value $y_t$')
ax[1].plot(ts, y_mmar, marker = '', linestyle = '-')
None
```



Part 2

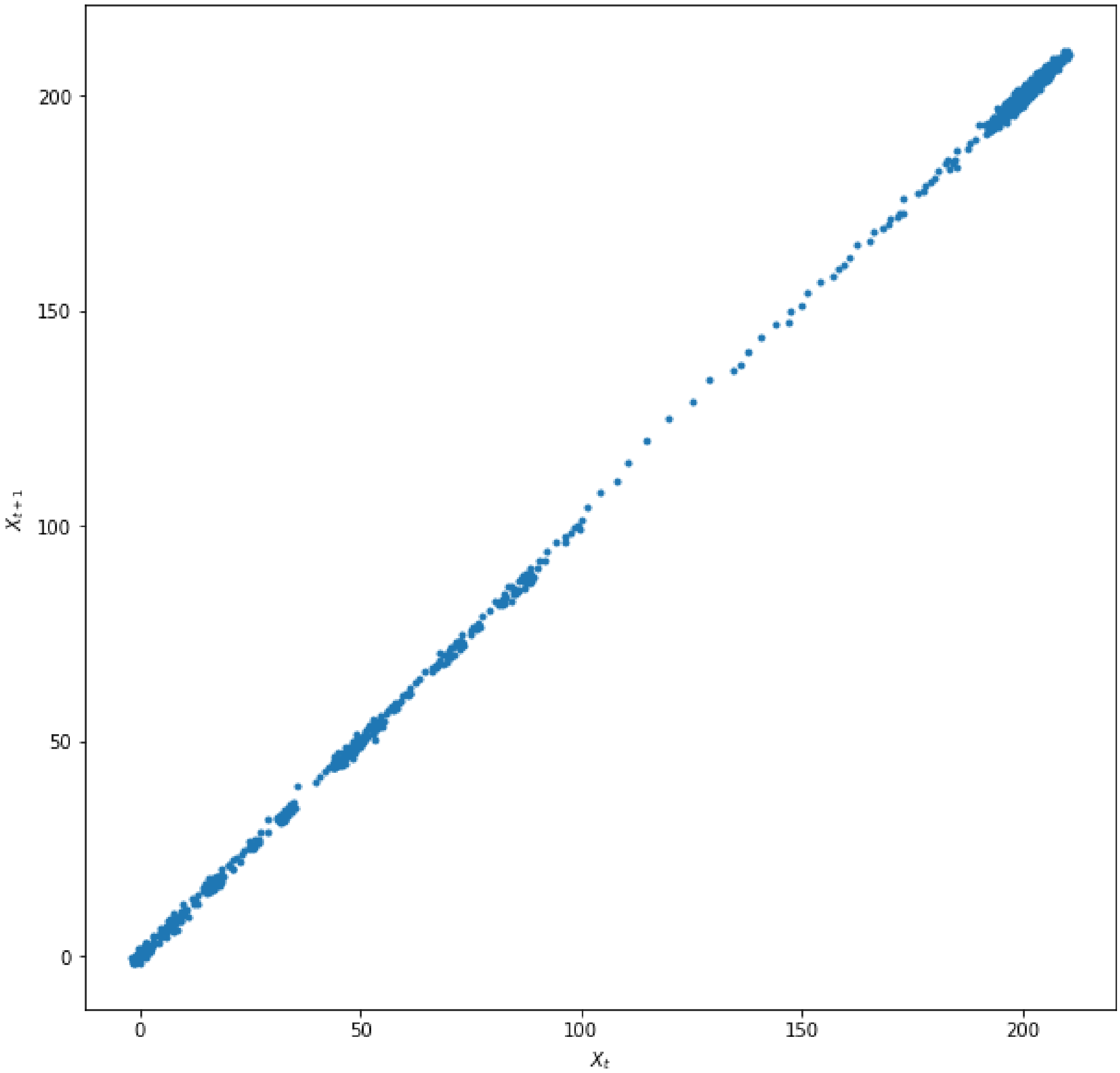
We compute the theoretical conditional mean, $M(x) = E\left\{X_{t+1} \mid X_t = x\right\}$, for the $\mathrm{SETAR}(2, 1, 1)$ model provided above.

$$M(x) = \begin{cases} 0.2 + x & \text{if } x < 100 \\ 10 + 0.95x & \text{if } x \geq 100 \end{cases}$$

We simulate 1000 values from the chosen SETAR model. First we plot the expected value $E[X_{t+11} \mid X_t = x]$ as a function of the condition $X_t = x$:

In [14]:

```
fig, ax = plt.subplots(figsize = (10, 10))
ax.plot(y_setar[:-1], y_setar[1:], linestyle = '', marker = '.')
ax.set_xlabel(r'$X_t$')
ax.set_ylabel(r'$X_{t + 1}$')
None
```



Then, we use the simulated data and a local regression model to estimate the $\hat{M}(x) = E\left\{X_{t+1} \mid X_t = x\right\}$:

In [15]:

```
bws = np.arange(.1, .6, .1)
md("We use the folloing bandwidths: " + ', '.join(['**%.1f**' % bw for bw in bws]))
```

Out[15]:

We use the folloing bandwidths: **0.1**, **0.2**, **0.3**, **0.4**, **0.5**

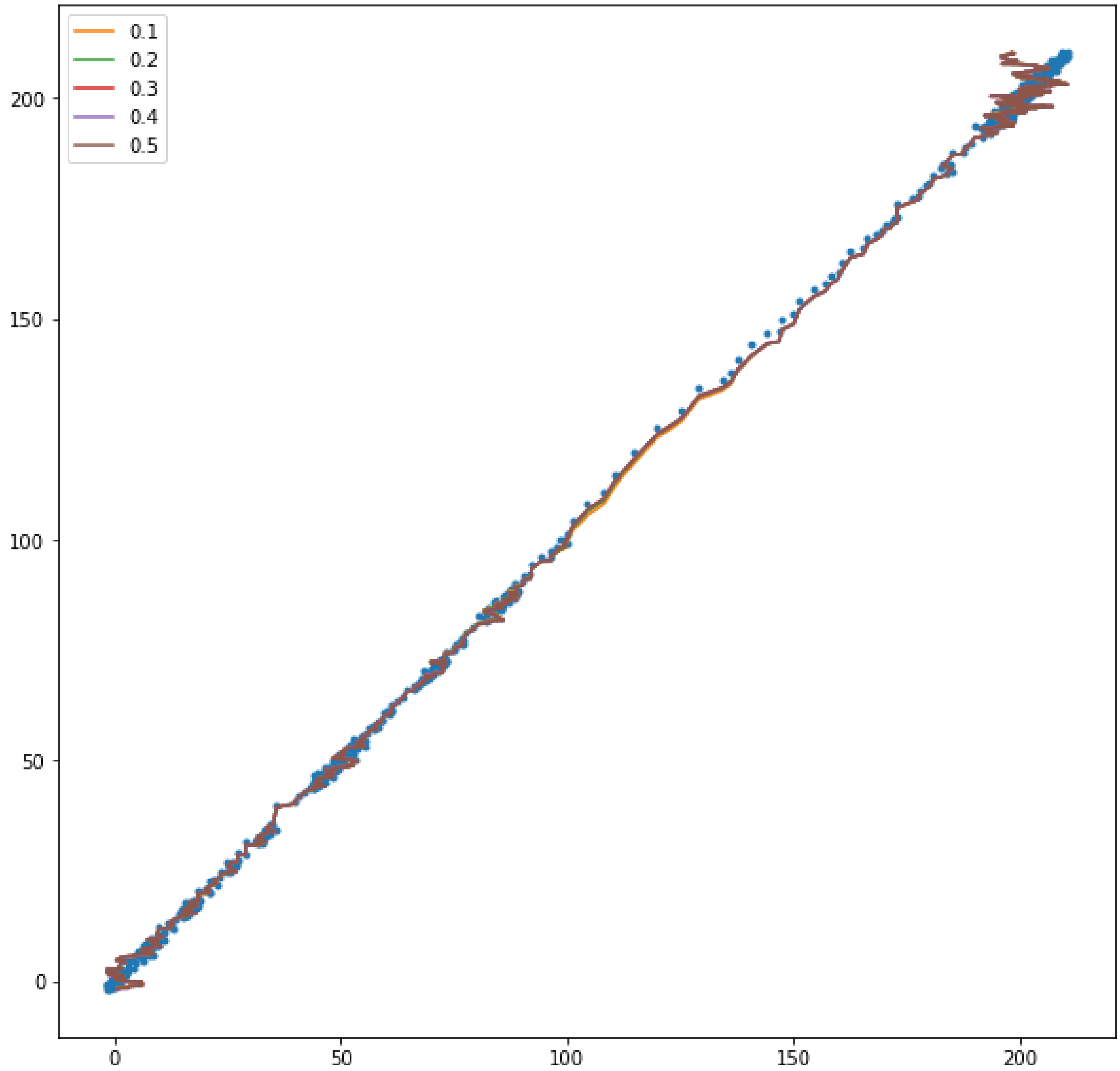
We calculate local linear estimates given the different bandwidths:

In [16]:

```
lowess = [sm.nonparametric.lowess(y_setar[:-1], y_setar[1:], frac = bw, it = 0) for bw
in bws]
```

In [17]:

```
fig, ax = plt.subplots(figsize = (10, 10))
ax.plot(y_setar[:-1], y_setar[1:], marker = '.', linestyle = '')
for i in range(len(bws)):
    ax.plot(y_setar[:-1], lowess[i][:, 1], marker = '', linestyle = '-', label = str(bw
s[i]))
ax.legend()
None
```



Wee see that the 0.5 bandwoth seems to be too averaging in the ends, but otherwise they all seem to capture the relationship with the lag 1 input.

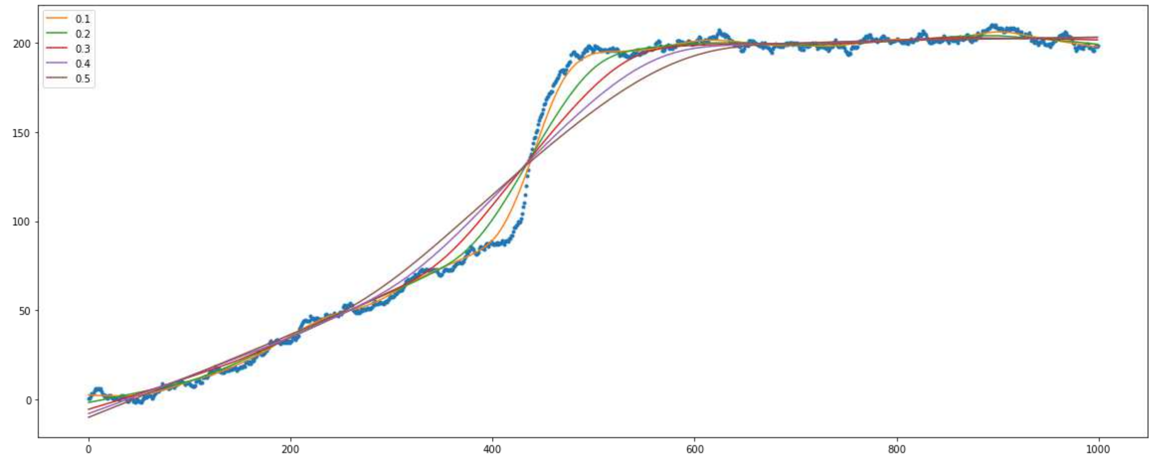
We also try to apply the LOESS method directly on the time series data:

In [18]:

```
lowess_ts = [sm.nonparametric.lowess(y_setar, ts, frac = bw, it = 0) for bw in bws]
```

In [19]:

```
fig, ax = plt.subplots(figsize = (20, 8))
ax.plot(ts, y_setar, marker = '.', linestyle = '')
for i in range(len(bws)):
    ax.plot(ts, lowess_ts[i][:, 1], marker = '', linestyle = '-', label = str(bws[i]))
ax.legend()
None
```



We see that the 0.1 bandwith most closely captures the regime shift, and does not seem to be too affected by local noise.

Part 3

Parameters for the histogram regression

In [20]:

```
## Number of intervals
n_bins = 10
## The breaks between the intervals
breaks = np.linspace(min(y_setar), max(y_setar), n_bins + 1)
```

In [21]:

```
## Initialize
h = np.diff(breaks)[0]
l = np.zeros(n_bins)
g = np.zeros(n_bins)
f_hat = np.zeros(n_bins)
h_hat = np.zeros(n_bins)
```

In [22]:

```
## Calc the hist regressogram, i.e. for each interval
for i in range(n_bins):
    x_bin = y_setar[(breaks[i] <= y_setar) & (y_setar < breaks[i+1])]
    assert(len(x_bin) > 5)
    l[i] = x_bin.mean()
    g[i] = np.sum((x_bin - l[i])**2) / len(x_bin)
    f_hat[i] = (n_bins*h)**(-1) * len(x_bin)
    h_hat[i] = g[i]/f_hat[i];
```

In [23]:

```
L = (l*h).cumsum()
H_hat = (h_hat*h).cumsum();
```

In [24]:

```
## Make confidence bands for the cumulated function. Def. (3.10).
## 95% confidence band, c is found in table 3.1
c_alpha = 1.273
```

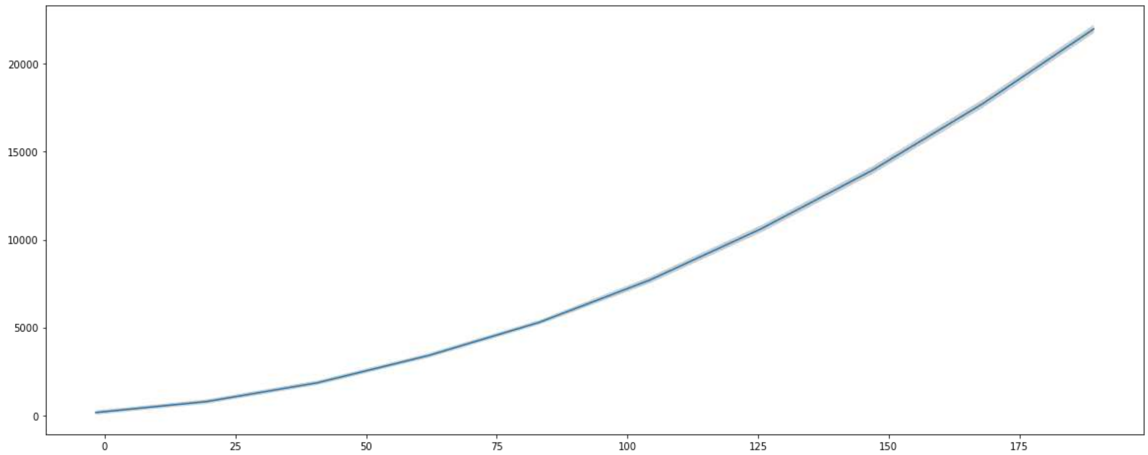
In [25]:

```
H_hat_b = H_hat[-1];
L_low = L - c_alpha * n_bins**(-0.5) * H_hat_b**(0.5) * (1 + H_hat/H_hat_b);
L_upr = L + c_alpha * n_bins**(-0.5) * H_hat_b**(0.5) * (1 + H_hat/H_hat_b);
```

We plot the computed cumulative means with confidence intervals:

In [26]:

```
fig, ax = plt.subplots(figsize = (20, 8))
ax.fill_between(breaks[0:-1], L_low, L_upr,  facecolor='#cccccc')
ax.plot(breaks[0:-1], L, marker = '', linestyle = '-')
None
```



We compare with the theoretical cumulative conditional mean and explore the assymptotic behaviour:

$$M(x) = \begin{cases} 0.2 + x & \text{if } x < 100 \\ 10 + 0.95x & \text{if } x \geq 100 \end{cases}$$

We compute the theoritical cumulative conditional mean piecewise:

For the first piece we have:

$$\int 0.2 + x \, \mathrm{d}x = 0.2x + \frac{1}{2}x^2 + C$$

And for the second piece we have:

$$\int 10 + 0.95x \, \mathrm{d}x = 10x + \frac{1}{2}0.95x^2 + C$$

In [27]:

```
piece_1 = lambda x: .2*x + .5 * x**2
piece_2 = lambda x: 10*x + .5 * 0.95 * x**2
```

We sum over the pieces:

In [28]:

```
np.sum([
    piece_1(100) - piece_1(min(y_setar)),
    piece_2(max(y_setar)) - piece_2(100)
])
```

Out[28]:

```
22410.585718956514
```

We see, that the theoritical commulative mean is very similar compared to the computed using the histogram approach:

In [29]:

```
L[-1]
```

Out[29]:

```
21990.377257477867
```

The small difference is considered to be a result of the histogram approach, altough it is not even within the confidence interval:

In [30]:

```
print(L_low[-1], '-', L_upr[-1])
```

```
21755.7060825 - 22225.0484325
```

For now we are not invesigrating this further, since the theoretical and computed com.sum. means are still very close.

Part 4

We load the data:

In [31]:

```
data_heat = pd.read_csv('DataPart4.csv')
data_heat.head(5)
```

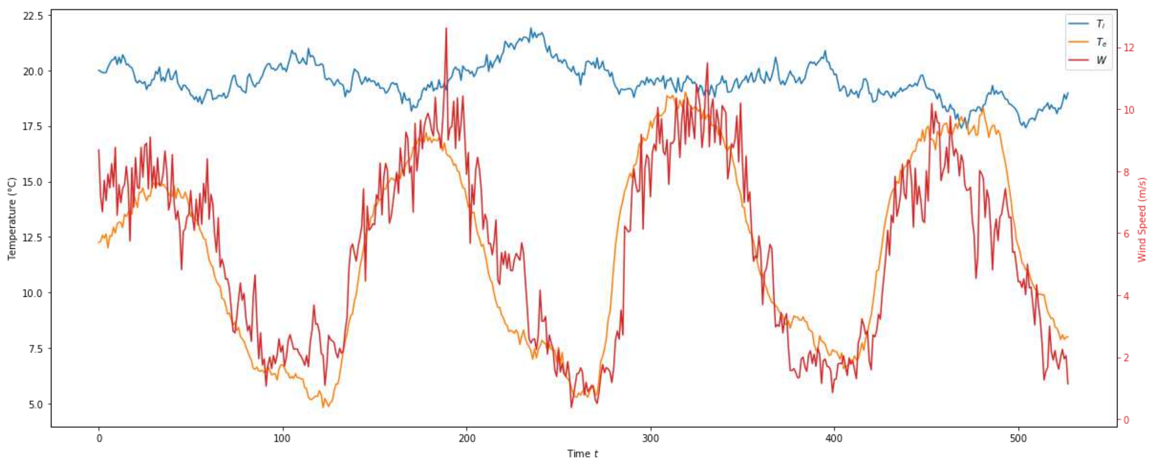
Out[31]:

	Ph	Ti	Te	W
0	1496.970334	20.002530	12.255610	8.678768
1	1411.792178	19.957051	12.298405	7.185271
2	1368.595696	19.904727	12.576470	6.684292
3	1404.547674	19.886161	12.455047	7.702868
4	1376.436161	19.908530	12.638600	7.047383

Lets take a quick look at data. Visible strong correlation between W and T_e, But also pattern in T_i which seem to raise at night (?). We see a clear cyclic pattern.

In [32]:

```
fig, ax = plt.subplots(figsize = (20, 8))
lns1 = ax.plot(data_heat['Ti'], label = r'$T_i$')
lns2 = ax.plot(data_heat['Te'], label = r'$T_e$')
ax.set_xlabel(r'Time $t$')
ax.set_ylabel('Temperature (°C)')
ax_sec = ax.twinx()
lns3 = ax_sec.plot(data_heat['W'], label = r'$W$', color = 'C3')
ax_sec.set_ylabel('Wind Speed (m/s)', color='C3')
ax_sec.tick_params('y', colors='C3')
lns = lns1+lns2+lns3
labs = [l.get_label() for l in lns]
ax.legend(lns, labs, loc=0)
None
```



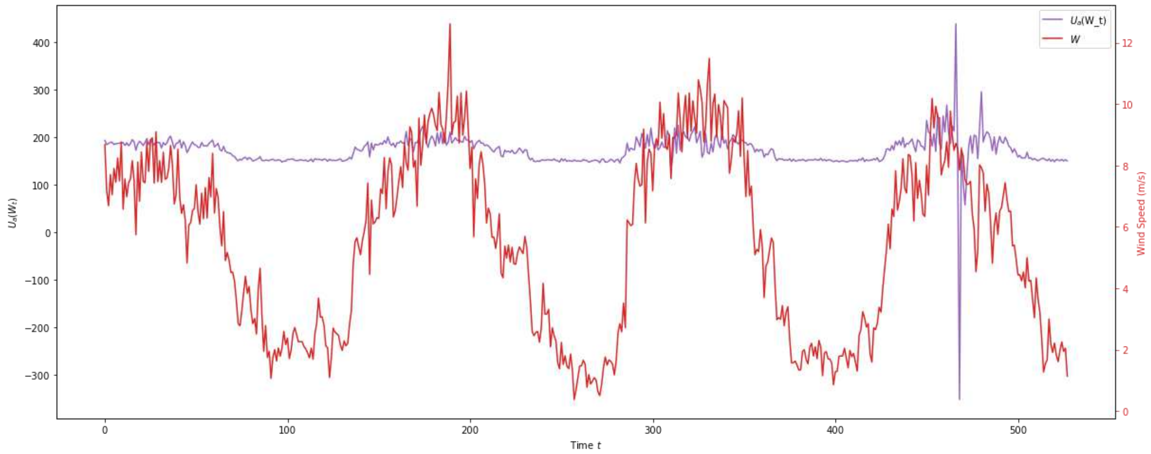
We calculate and plot $U_a(W_t)$ and plot is both side by side with W_t as a function of t and as a function of W_t.

In [33]:

```
W = data_heat['W']
U_W = data_heat['Ph'] / (data_heat['Ti'] - data_heat['Te'])
```

In [34]:

```
fig, ax = plt.subplots(figsize = (20, 8))
lns1 = ax.plot(U_W, label = r'$U_a($W_t$)$', color = 'C4')
ax.set_xlabel(r'Time $t$')
ax.set_ylabel(r'$U_a(W_t)$')
ax_sec = ax.twinx()
lns2 = ax_sec.plot(data_heat['W'], label = r'$W$', color = 'C3')
ax_sec.set_ylabel('Wind Speed (m/s)', color='C3')
ax_sec.tick_params('y', colors='C3')
lns = lns1+lns2
labs = [l.get_label() for l in lns]
ax.legend(lns, labs, loc=0)
None
```



Wee still see the cyclic pattern, but now we see that $U_a(W_t)$ seems to have a lower limit around ~150 (unit?). We also see that the function has extreem values when $T_i < T_e$.

We fit LOEES model for $U_a(W_t) \sim W_t$ - this time implemented directly using linear regressen steps (using numpy's polyfit).

In [35]:

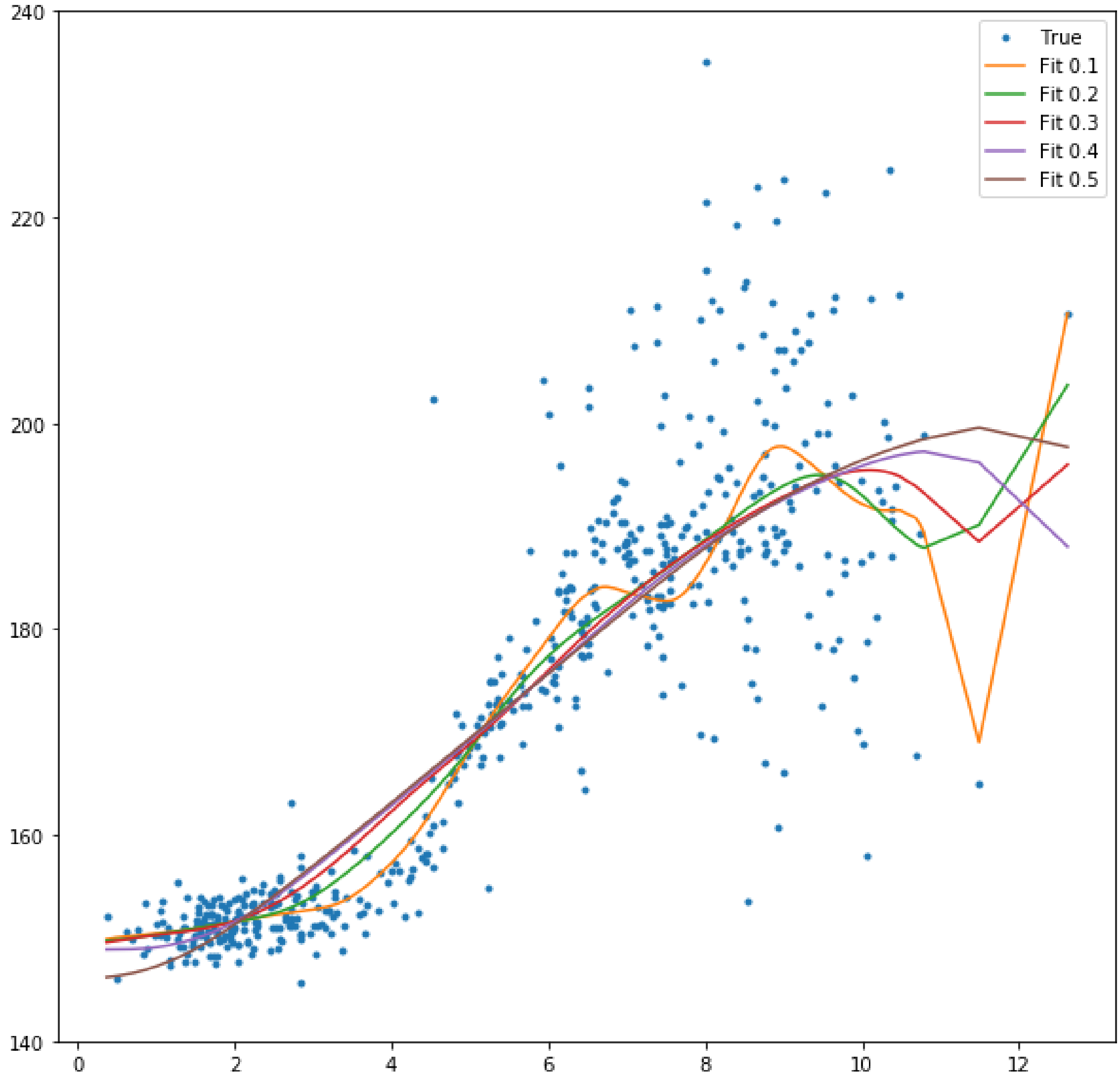
```
# Epanechnikov kernel
def ep_kernel(xall, x, h):
  ## Make the weights with an Epanechnikov kernel
  ## h has the same unit as x (i.e. it is on the same absolute scale, so if x is Watt,
 h is also given in Watt)
  u = np.abs(xall - x)
  u = u / h
  w = 3/4 * (1 - u**2)
  ## Set values with |u|>1 to 0
  w[np.abs(u) > 1] = 0
  return w
```

In [36]:

```
hs = bws*(W.max() - W.min())
fit = np.empty([len(hs), len(W)])
fit[:,:] = np.nan
for i in range(len(W)):
    for j in range(len(hs)):
        w = ep_kernel(W, W[i], hs[j])
        coefs = poly.polyfit(x = W, y = U_W, deg = 1, w = w)
        fit[j, i] = poly.polyval(W[i], coefs)
```

In [37]:

```
df = pd.DataFrame({'W': W, 'U_W': U_W})
for i in range(len(bws)):
    df['Fit %.1f' % bws[i]] = fit[i, :]

df = df.sort_values('W').reset_index(drop = True)

fig, ax = plt.subplots(figsize = (10, 10))
ax.plot(df['W'], df['U_W'], linestyle = '', marker = '.', label = 'True')
for i in range(len(bws)):
    ax.plot(df['W'], df['Fit %.1f' % bws[i]], linestyle = '-', marker = '', label = 'Fi
t %.1f' % bws[i])
ax.set_ylim(140, 240)
ax.legend()
None
```

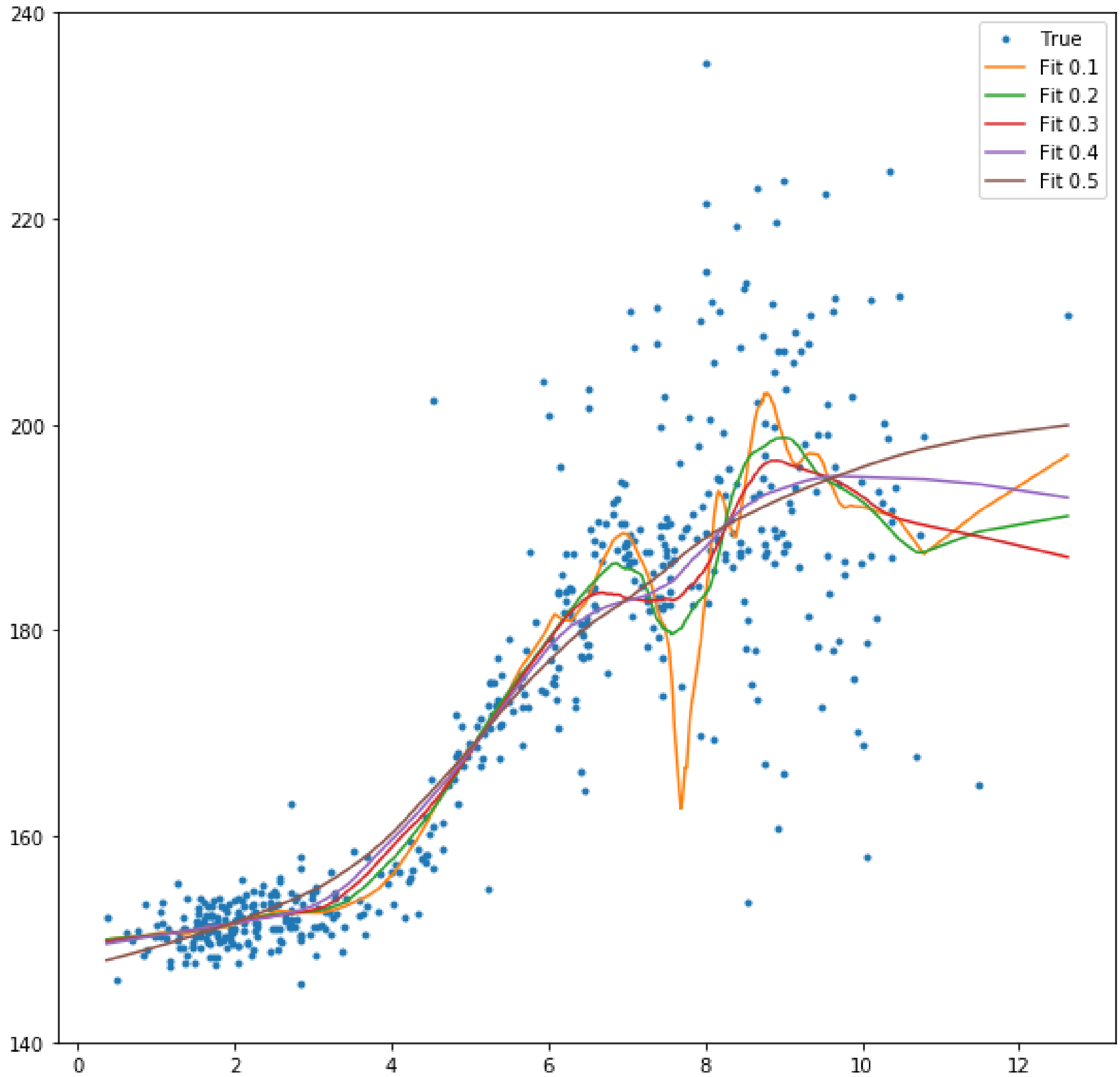


240
220
200
180
160
140
0
2
4
6
8
10
12
True
Fit 0.1
Fit 0.2
Fit 0.3
Fit 0.4
Fit 0.5

Just for comparison we estimate model using stat-models *lowess*-function. We get similar results, and difference it assumably because this uses a different kernel, and also some implementation/algorithm differences.

In [38]:

```
U_W_lowess = [sm.nonparametric.lowess(U_W, W, frac = bw, it = 0) for bw in bws]

df2 = pd.DataFrame({'W': W, 'U_W': U_W}).sort_values('W').reset_index(drop = True)

for i in range(len(bws)):
    df2['Fit %.1f' % bws[i]] = U_W_lowess[i][:,1]

fig, ax = plt.subplots(figsize = (10, 10))
ax.plot(df2['W'], df2['U_W'], linestyle = '', marker = '.', label = 'True')
for i in range(len(bws)):
    ax.plot(df2['W'], df2['Fit %.1f' % bws[i]], linestyle = '-', marker = '', label = 
'Fit %.1f' % bws[i])
ax.set_ylim(140, 240)
ax.legend()
None
```



Part 5

We load the data:

In [39]:

```
data = pd.read_csv('DataPart5.csv')
data.head(5)
```

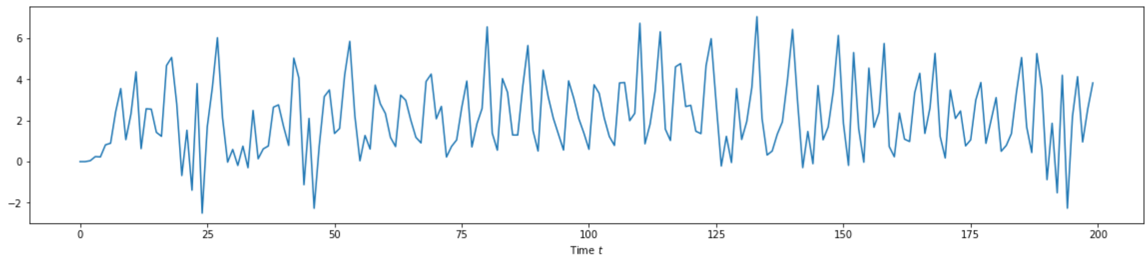
Out[39]:

	x
0	0.000000
1	0.000000
2	0.043702
3	0.244185
4	0.228373

We take a quick look at the data (first 200 obs.)

In [40]:

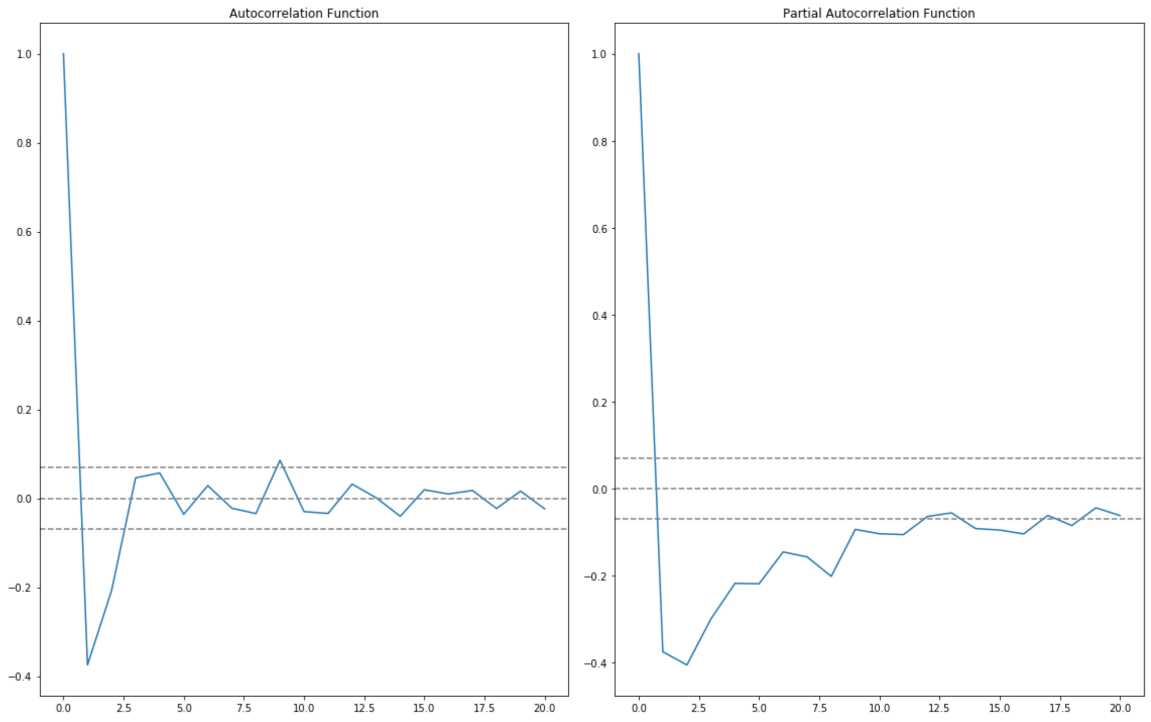
```
fig, ax = plt.subplots(figsize = (20, 4))
ax.set_xlabel(r'Time $t$')
ax.plot(data[:200])
None
```



We plot Autocorrelation and Partial Autocorrelation:

In [41]:

```
ts_diff = data['x'].values - data.shift(1)['x'].values
ts_diff = ts_diff[1:]
lag_acf = acf(ts_diff, nlags=20)
lag_pacf = pacf(ts_diff, nlags=20, method='ols')
fig, ax = plt.subplots(1, 2, figsize = (16, 10))
#Plot ACF:
ax[0].plot(lag_acf)
ax[0].axhline(y=0,linestyle='--',color='gray')
ax[0].axhline(y=-1.96/np.sqrt(len(ts_diff)),linestyle='--',color='gray')
ax[0].axhline(y=1.96/np.sqrt(len(ts_diff)),linestyle='--',color='gray')
ax[0].set_title('Autocorrelation Function')
#Plot PACF:
ax[1].plot(lag_pacf)
ax[1].axhline(y=0,linestyle='--',color='gray')
ax[1].axhline(y=-1.96/np.sqrt(len(ts_diff)),linestyle='--',color='gray')
ax[1].axhline(y=1.96/np.sqrt(len(ts_diff)),linestyle='--',color='gray')
ax[1].set_title('Partial Autocorrelation Function')
fig.tight_layout()
```



Bassed on the above we choose $(p, q) = (4, 4)$ for our $\mathrm{ARMA}(p, q)$-model. Also a higher values the inversion of the hassian-matrix fails.

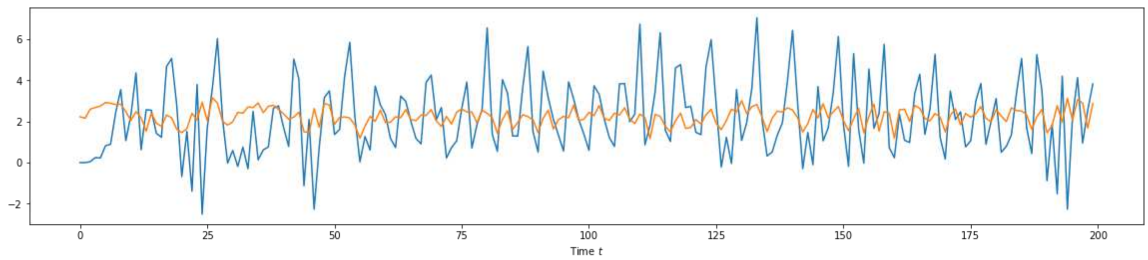
In [42]:

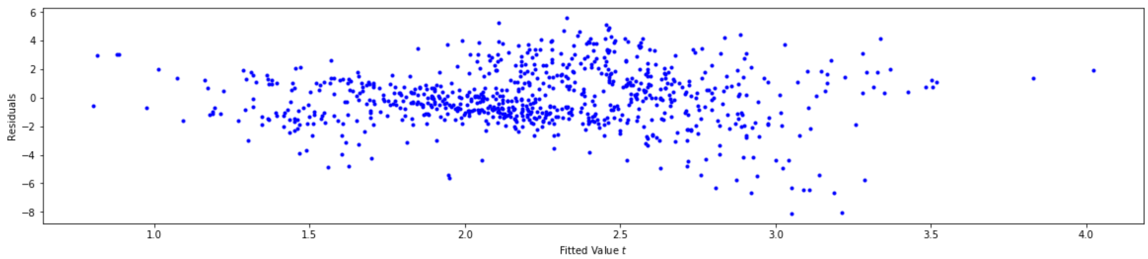
```
arma = statsmodels.tsa.arima_model.ARMA(data['x'].values, order = (4, 4)).fit()
```

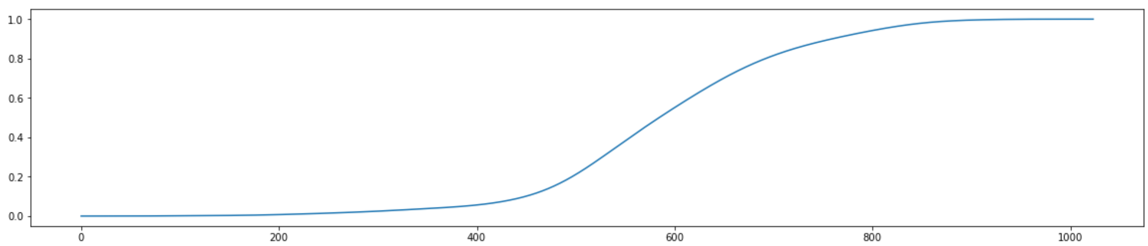
We plot the estimated values and residuals:

In [43]:

```
fig, ax = plt.subplots(figsize = (20, 4))
ax.set_xlabel(r'Time $t$')
ax.plot(data[:200])
ax.plot(arma.fittedvalues[:200])

resid = data['x'].values - arma.fittedvalues
fig, ax = plt.subplots(figsize = (20, 4))
ax.set_xlabel(r'Fitted Value $t$')
ax.set_ylabel(r'Residuals')
ax.plot(arma.fittedvalues, resid, color = 'b', marker = '.', linestyle='')

fig, ax = plt.subplots(figsize = (20, 4))
dens = sm.nonparametric.KDEUnivariate(resid)
dens.fit()
ax.plot(dens.cdf)

None
```







We are now "reasonably satisfied" and compute the LDF of the residuals:

I was not able to find any implementations of LDF in Python, so we will save the residuals an continue in R:

In [44]:

```
pd.DataFrame(resid, columns = ['resid']).to_csv('DataPart5_resid.csv', index = False)
```

See next page for R version.

Load required libs and sources:

In [1]:

```
library(readr)
library(ggplot2)

source('leaveOneOut.R')
```

Load residuals back in R:

In [2]:

```
resid <- read_csv('DataPart5_resid.csv', col_types = cols(
  resid = col_double()
))
```

In [3]:

```
x <- resid$resid
lags <-  c(1,2,3,4)
nBoot <- 30
```

The LDF-algorithm from ldf.R:

In [4]:

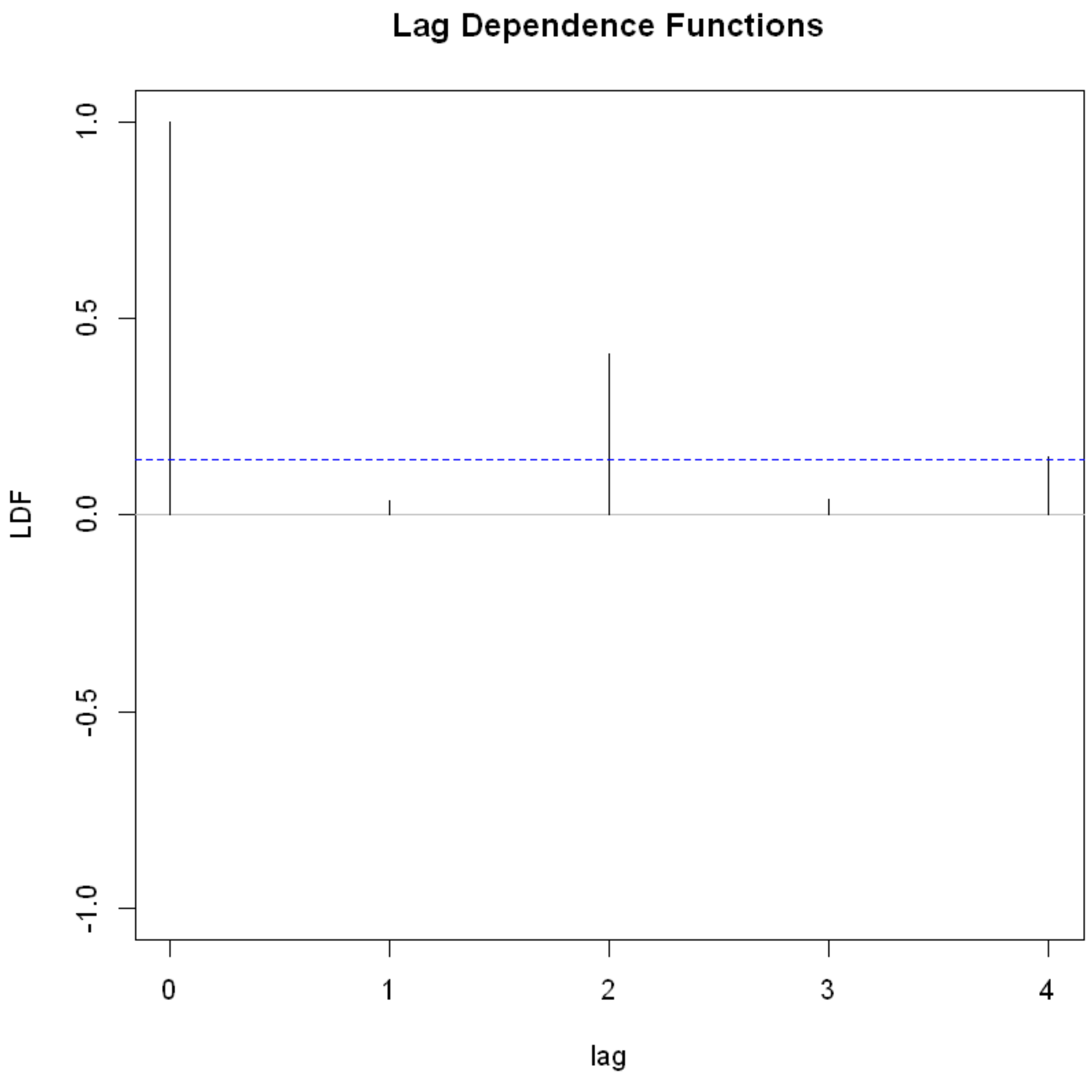
```r
## The result is kept in val
val <- vector()
##
for(i in 1:length(lags))
{
  ## Take the k
  k <- lags[i]
  ## Dataframe for modelling: xk is lagged k steps
  D <- data.frame(x=x[-(1:k)],xk=x[-((length(x)-k+1):length(x))])
  ## Leave one out optimization of the bandwidth with loess
  RSSk <- leaveOneOut(D, FALSE)
  ## Calculate the ldf
  RSS <- sum((D$x - mean(D$x))^2)
  val[i] <- (RSS - RSSk) / RSS
}

## Very simple bootstrapping
iidVal <- vector()
for(i in 1:nBoot)
{
  ## Bootstrapping to make a confidence band
  xr <- sample(x, min(length(x),100) ,replace=TRUE)
  ## Dataframe for modelling
  DR <- data.frame(x=xr[-1],xk=xr[-length(xr)])
  RSSk <- leaveOneOut(DR)
  ## The ldf is then calculated
  RSS <- sum((DR$x - mean(DR$x))^2)
  (iidVal[i] <- (RSS - RSSk) / RSS)
}
```

```
[1] "  Fitting for bandwidth 1 of 12"
[1] "  Fitting for bandwidth 2 of 12"
[1] "  Fitting for bandwidth 3 of 12"
[1] "  Fitting for bandwidth 4 of 12"
...
[1] "  Fitting for bandwidth 12 of 12"
```

Do you find any significant non-linearities?

In [5]:

```
lags <-  c(1,2,3,4)
plot(c(0,lags), c(1,val), type="n", ylim=c(-1,1), ylab="LDF", main="Lag Dependence Func
tions", xaxt="n", xlab="lag")
axis(1,c(0,lags))
abline(0,0,col="gray")
lines(c(0,lags), c(1,val), type="h")
## Draw the approximate 95% confidence interval
abline(h=quantile(iidVal,0.95), col="blue", lty=2)
```



Yes, as seen from above plot, yes we do indeed see significant non-linear lag dependencies for lag 2 and 4.

For a better model structure we propose a structure capable of campuring the non linearities, e.g. SETAR. Unfortunately it has not been possible to estimate number of regimes, lags, etc. for a SETAR-model on the data yet.