FMN100
Numerical Methods for Computer Graphics
Numerisk Analys, Matematikcentrum

Homework 3

To be uploaded no later than September 25.

TASKS

(1) Given the knot sequences $U_1 = \{0, 0, 1, 1\}$ and $U_2 = \{0, 0, 0, 1, 1, 1\}$, calculate the B-spline basis functions on each sequence.
(2) Implement an algorithm that calculates the value of the B-spline basis functions corresponding to a given knot vector and a given degree. Use the algorithm to plot the quadratic B-spline basis functions for the knot sequence $\{0, 0, 0, 0.3, 0.5, 0.5, 0.6, 1, 1, 1\}$. Experiment with different knot multiplicities.
(3) Implement an algorithm that evaluates if a given parameter value is in an interval with full support. The input variables should be: a knot sequence, a degree, a real number (the value of the parameter). Use your program to find out if $t = 0.12, 0.24, 0.4, 0.53, 0.78, 0.8$ are inside subintervals with full support for quadratic B-spline curves with the knot sequences given above.
(4) Use your previous algorithms to implement an algorithm that plots a B-spline curve given a knot sequence and a set of control points. Try it out for the knot sequence in Exercise 2, and the control points $(0, 0), (3, 4), (7, 5), (9, 2), (13, 1), (10, -1), (7, -1)$. Include the control polygon and the knot points in the plot.
(5) Suppose we have a clamped B-spline curve of degree p and $n + 1$ control points, and a knot vector of simple knots except for the first and last knots which have multiplicity $p + 1$. We could also construct the curve as a composite Bézier curve. Derive a relation between $n+1$ and the total number of control points of the composite curve's Bézier segments.

Carmen Arévalo, Claus Führer, 17 september 2017