Problem 1:

A simple representation of the attitude dynamics of a spacecraft (with no friction or gravity gradient) about a single axis can be represented by the following state equations:

$$\dot{\omega} = \frac{1}{J} u$$

where the control u is a control torque, J is the moment of inertia, and the states ω and θ represent the angular rate and attitude of the spacecraft respectively. The spacecraft attitude can be controlled using a feedback system where the attitude of the spacecraft is measured and compared to operators command input (the command attitude θ_{cmd}). A PD compensator on the tracking error ($\theta_{cmd} - \theta$) is used to determine the control, $u = K(\theta cmd - \theta) + K_D \frac{d}{dx}(\theta cmd - \theta)$.

a) Show that the closed loop transfer function is a second order system of the form:

$$\frac{\theta}{\theta_{cmd}}(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

and that we can specify any natural frequency and damping ratio by selecting appropriate values of the proportional and derivative gain – you should derive expressions that give the frequency and damping ratio in terms of the two gains and the inertia. Suppose the moment of inertia J is 2000 kg-m^2. Find gains K and K_D such that the natural frequency is 3 rad/sec and the damping ratio is 0.7.

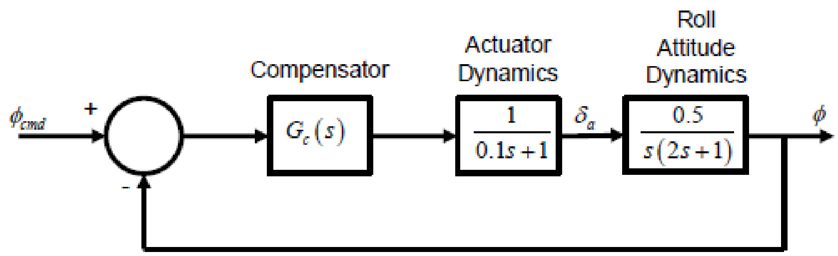
b) Suppose the spacecraft has a flexible structure. A simplified model of the attitude dynamics can be represented by the transfer function

$$\frac{\theta}{u}(s) = \frac{\left(\left(\frac{s}{\omega_z}\right)^2 + 2\zeta\frac{s}{\omega_z} + 1\right)}{Js^2\left(\left(\frac{s}{\omega_p}\right)^2 + 2\zeta\frac{s}{\omega_p} + 1\right)}$$

where ω_z = 1.0 rad/sec, ω_p = = 1.3 rad/sec, and ζ = 0.002. The closed loop transfer function will have four poles. Using Matlab tools, find gains K and KD such that all of the poles are of the closed loop system are stable and have a damping ratio of at least 0.3.

Problem 2:

Consider a roll attitude controller as shown in the diagram below:



We desire a controller that results in a stable response, has a steady-state error of $\leq 10\%$ to a ramp input, and complex poles with frequency and damping ratio in the range 2.5 rad/sec $\leq \omega_n \leq 3.5$ rad/sec and $0.6 \leq \zeta \leq 0.8$.

a) Show that a proportional compensator, $G_c(s) = K$, will not provide the desired performance.

b) Find a PD compensator, $G_c(s) = K(T_z s + 1)$, that meets the performance requirements.

c) Find a PID compensator, $G_c(s) = K\dfrac{(T_{z1}s+1)(T_{z2}s+1)}{s}$, that meets the performance requirements and also results in zero steady-state error to a ramp input.

d) Plot the response of the closed loop systems in parts b and c to a ramp input, and check that the steady-state error is as expected.

Problem 3:

The non-linear equations of motion for the attitude dynamics of a spacecraft are shown below. The equations including the effects of the gravity gradient assuming the spacecraft is in a circular orbit with constant orbital rate $n = \sqrt{\frac{GM}{r^3}}$. The Euler angles ϕ, θ, and ψ define the orientation of the body axes relative to the Local Vertical Local Horizontal (LVLH) coordinate system. The LVLH system is rotating in the inertial system about its y-axis by the angular rate -n. The spacecraft includes three momentum wheels rotating about each of the three principal axes with angular momentums h_x, h_y, and h_z. The torques supplied to each of the momentum wheels (τ_x, τ_y, and τ_z) are the inputs to this dynamic system.

$$\dot{h}_x = \tau_x$$

$$\dot{h}_y = \tau_y$$

$$\dot{h}_z = \tau_z$$

$$\dot{\omega}_x = \frac{1}{J_x}\left[\left(J_y - J_z\right)\omega_y\omega_z - \dot{h}_x + h_y\omega_z - h_z\omega_y - 3n^2\left(J_y - J_z\right)\sin\phi\cos\phi\cos^2\theta\right]$$

$$\dot{\omega}_y = \frac{1}{J_y}\left[\left(J_z - J_x\right)\omega_x\omega_z - \dot{h}_y + h_z\omega_x - h_x\omega_z + 3n^2\left(J_z - J_x\right)\cos\phi\sin\theta\cos\theta\right]$$

$$\dot{\omega}_z = \frac{1}{J_z}\left[\left(J_x - J_y\right)\omega_x\omega_y - \dot{h}_z + h_x\omega_y - h_y\omega_x + 3n^2\left(J_x - J_y\right)\sin\phi\sin\theta\cos\theta\right]$$

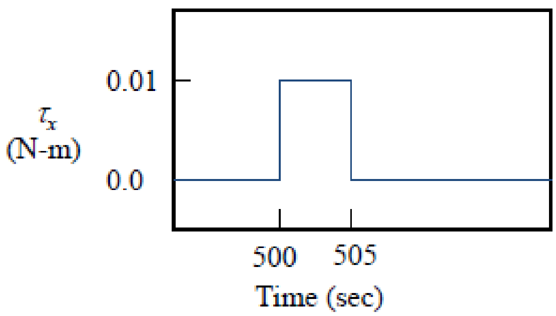
$$\dot{\phi} = \omega_x + \frac{\omega_y\sin\phi\sin\theta + \omega_z\cos\phi\sin\theta}{\cos\theta} + n\frac{\sin\psi}{\cos\theta}$$

$$\dot{\theta} = \omega_y\cos\phi - \omega_z\sin\phi + n\cos\psi$$

$$\dot{\psi} = \frac{\omega_y\sin\phi + \omega_z\cos\phi}{\cos\theta} + n\frac{\sin\psi\sin\theta}{\cos\theta}$$

Consider a circular orbit with orbital rate $n = 1.107 \times 10^{-3}$ rad/sec, such that the spacecraft has constant orientation in the LVLH system. The moments of inertia are $J_x = 300$ kg-m^2, $J_y = 400$ kg-m^2, and $J_z = 150$ kg-m^2. Develop a non-linear simulation of the spacecraft attitude dynamics in MATLAB.

Using the initial equilibrium conditions listed below simulate the response to a 0.01 N-m pulse in the roll axis (x-axis) momentum wheel starting at 500 seconds and ending at 505 seconds as shown below:



Simulate a long enough period of time to get an idea of the dynamic modes of the spacecraft (this will be a long time, 1000's of seconds). You can use a 1 second time step in your simulation. Plot the response of the three Euler angles versus time and comment on the differences in response when the y-axis momentum wheel is not turning (case a) compared to when the y-axis momentum wheel is spun up (case b).

The two initial conditions are:

a. Initial state values: $\phi = \theta = \psi = \omega_x = \omega_z = h_x = h_y = h_z = 0$, $\omega_y = -n$.
b. Initial state values: $\phi = \theta = \psi = \omega_x = \omega_z = h_x = h_z = 0$, $\omega_y = -n$, and $h_y = -5$ N-m-s.