## **Problem 1:**

A simple representation of the attitude dynamics of a spacecraft (with no friction or gravity gradient) about a single axis can be represented by the following state equations:

$$\dot{\omega} = \frac{1}{I}u$$

where the control u is a control torque, J is the moment of inertia, and the states  $\omega$  and  $\theta$  represent the angular rate and attitude of the spacecraft respectively. The spacecraft attitude can be controlled using a feedback system where the attitude of the spacecraft is measured and compared to operators command input (the command attitude  $\theta_{cmd}$ ). A PD compensator on the tracking error  $(\theta_{cmd} - \theta)$  is used to determine the control,  $u = K(\theta cmd - \theta) + K_D \frac{d}{dx}(\theta cmd - \theta)$ .

a) Show that the closed loop transfer function is a second order system of the form:

$$\frac{\theta}{\theta_{cmd}}(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

and that we can specify any natural frequency and damping ratio by selecting appropriate values of the proportional and derivative gain – you should derive expressions that give the frequency and damping ratio in terms of the two gains and the inertia. Suppose the moment of inertia J is  $2000 \text{ kg-m}^2$ . Find gains K and  $K_D$  such that the natural frequency is 3 rad/sec and the damping ratio is 0.7.

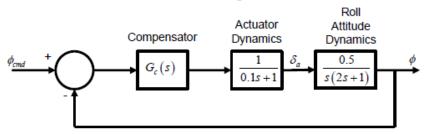
b) Suppose the spacecraft has a flexible structure. A simplified model of the attitude dynamics can be represented by the transfer function

$$\frac{\theta}{u}(s) = \frac{\left(\left(\frac{s}{\omega_z}\right)^2 + 2\zeta\frac{s}{\omega_z} + 1\right)}{Js^2\left(\left(\frac{s}{\omega_p}\right)^2 + 2\zeta\frac{s}{\omega_p} + 1\right)}$$

where  $\omega_z = 1.0$  rad/sec,  $\omega_p = = 1.3$  rad/sec, and  $\zeta = 0.002$ . The closed loop transfer function will have four poles. Using Matlab tools, find gains K and KD such that all of the poles are of the closed loop system are stable and have a damping ratio of at least 0.3.

## **Problem 2:**

Consider a roll attitude controller as shown in the diagram below:



We desire a controller that results in a stable response, has a steady-state error of  $\leq 10\%$  to a ramp input, and complex poles with frequency and damping ratio in the range 2.5 rad/sec  $\leq \omega_n \leq 3.5$  rad/sec and  $0.6 \leq \zeta \leq 0.8$ .

- a) Show that a proportional compensator,  $G_c(s) = K$ , will not provide the desired performance.
- b) Find a PD compensator,  $G_c(s) = K(T_c s + 1)$ , that meets the performance requirements.
- c) Find a PID compensator,  $G_c(s) = K \frac{(T_{z1}s + 1)(T_{z2}s + 1)}{s}$ , that meets the performance requirements and also results in zero steady-state error to a ramp input.
- d) Plot the response of the closed loop systems in parts b and c to a ramp input, and check that the steady-state error is as expected.

## **Problem 3:**

The non-linear equations of motion for the attitude dynamics of a spacecraft are shown below. The equations including the effects of the gravity gradient assuming the spacecraft is in a

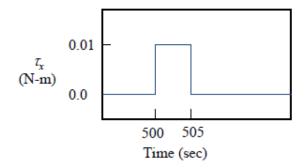
circular orbit with constant orbital rate  $n = \sqrt{\frac{GM}{r^3}}$ . The Euler angles  $\phi$ ,  $\theta$ , and  $\psi$  define the

orientation of the body axes relative to the Local Vertical Local Horizontal (LVLH) coordinate system. The LVLH system is rotating in the inertial system about its y-axis by the angular rate -n. The spacecraft includes three momentum wheels rotating about each of the three principal axes with angular momentums  $h_x$ ,  $h_y$ , and  $h_z$ . The torques supplied to each of the momentum wheels ( $\tau_x$ ,  $\tau_y$ , and  $\tau_z$ ) are the inputs to this dynamic system.

$$\begin{split} \dot{h}_x &= \tau_x \\ \dot{h}_y &= \tau_y \\ \dot{m}_z &= \tau_z \\ \dot{\omega}_x &= \frac{1}{J_x} \Big[ (J_y - J_z) \omega_y \omega_z - \dot{h}_x + h_y \omega_z - h_z \omega_y - 3n^2 (J_y - J_z) \sin\phi \cos\phi \cos^2\theta \Big] \\ \dot{\omega}_y &= \frac{1}{J_y} \Big[ (J_z - J_x) \omega_x \omega_z - \dot{h}_y + h_z \omega_x - h_x \omega_z + 3n^2 (J_z - J_x) \cos\phi \sin\theta \cos\theta \Big] \\ \dot{\omega}_z &= \frac{1}{J_z} \Big[ (J_x - J_y) \omega_x \omega_y - \dot{h}_z + h_x \omega_y - h_y \omega_x + 3n^2 (J_x - J_y) \sin\phi \sin\theta \cos\theta \Big] \\ \dot{\phi} &= \omega_x + \frac{\omega_y \sin\phi \sin\theta + \omega_z \cos\phi \sin\theta}{\cos\theta} + n \frac{\sin\psi}{\cos\theta} \\ \dot{\theta} &= \omega_y \cos\phi - \omega_z \sin\phi + n \cos\psi \\ \dot{\psi} &= \frac{\omega_y \sin\phi + \omega_z \cos\phi}{\cos\theta} + n \frac{\sin\psi \sin\theta}{\cos\theta} \end{split}$$

Consider a circular orbit with orbital rate  $n = 1.107 \times 10^{-3}$  rad/sec, such that the spacecraft has constant orientation in the LVLH system. The moments of inertia are  $J_x = 300 \text{ kg-m}^2$ ,  $J_y = 400 \text{ kg-m}^2$ , and  $J_z = 150 \text{ kg-m}^2$ . Develop a non-linear simulation of the spacecraft attitude dynamics in MATLAB.

Using the initial equilibrium conditions listed below simulate the response to a 0.01 N-m pulse in the roll axis (x-axis) momentum wheel starting at 500 seconds and ending at 505 seconds as shown below:



Simulate a long enough period of time to get an idea of the dynamic modes of the spacecraft (this will be a long time, 1000's of seconds). You can use a 1 second time step in your simulation. Plot the response of the three Euler angles versus time and comment on the differences in response when the y-axis momentum wheel is not turning (case a) compared to when the y-axis momentum wheel is spun up (case b).

The two initial conditions are:

- a. Initial state values:  $\phi = \theta = \psi = \omega_x = \omega_z = h_x = h_y = h_z = 0$ ,  $\omega_y = -n$ .
- b. Initial state values:  $\phi = \theta = \psi = \omega_x = \omega_z = h_x = h_z = 0$ ,  $\omega_y = -n$ , and  $h_y = -5$  N-m-s.