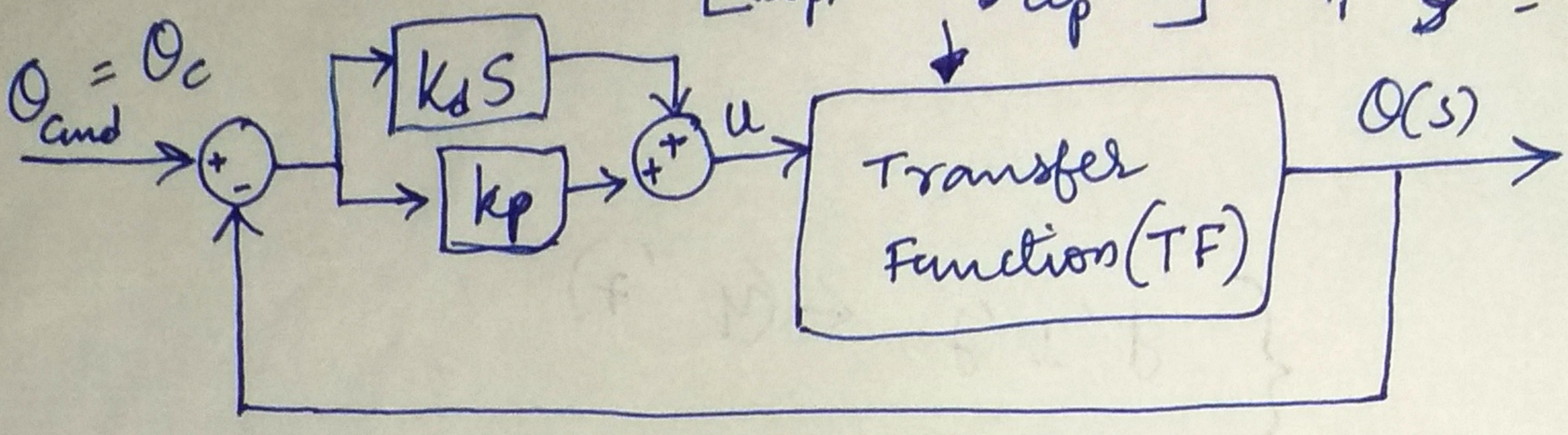
Prob 1B

$$\frac{\theta}{u}(s) = \frac{\left[\left(\frac{s}{\omega_z}\right)^2 + 2\zeta\frac{s}{\omega_z} + 1\right]}{Js^2\left[\left(\frac{s}{\omega_p}\right)^2 + 2\zeta\frac{s}{\omega_p} + 1\right]}$$

$\omega_z = 1.0$ rad/s
$\omega_p = 1.3$ rad/s
$\zeta = 0.002$



$$(\theta_c - \theta)(k_d s + k_p) \times TF = \theta(s)$$

$\Rightarrow$ closed-loop TF:

$$\frac{\theta}{\theta_c}(s) = \frac{(k_d s + k_p) \times TF}{1 + (k_p s + k_p) \times TF}$$

$\rightarrow$ Substituting TF,

$$\text{Numerator} = \frac{k_d}{\omega_z^2}\cdot s^3 + \left[\frac{2\zeta k_d}{\omega_z}\cdot + \frac{k_p}{\omega_z^2}\right]s^2 + \left[k_d + \frac{2\zeta k_p}{\omega_z}\right]s + k_p.$$

$$\text{Denominator} = \frac{J}{\omega_p^2}\cdot s^4 + \left[\frac{2\zeta J}{\omega_p} + \frac{k_d}{\omega_z^2}\right]s^3 + \left[J + \frac{2\zeta k_d}{\omega_z}\right]\cdot s^2 + \frac{k_p}{\omega_z^2}\cdot s^2 + \left[k_d + \frac{2\zeta k_p}{\omega_z}\right]s + k_p$$

Using the above numerator and denominator, Simulink Transfer function Block is created to ~~analyze~~ optimize the pole location while trying to achieve a damping factor ≥ 0.3.