Prob 1 a

$$\dot{\omega} = \frac{1}{J} u$$

$$\Rightarrow \ddot{\theta} = \frac{1}{J} u$$

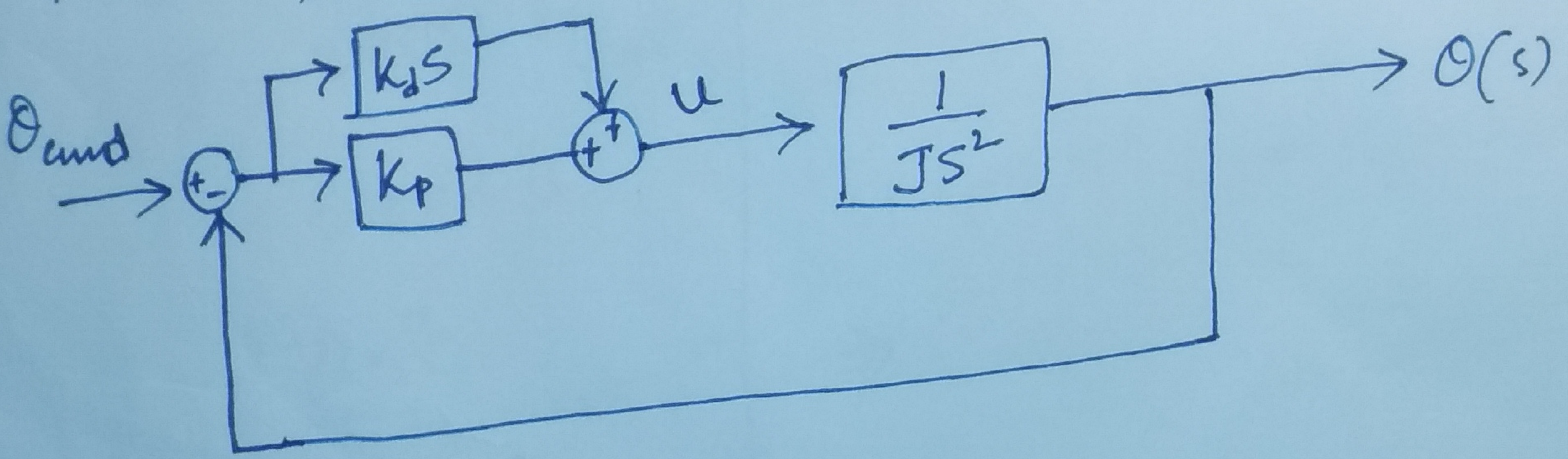
Laplace transform assuming $\theta(0) = 0$, $\dot{\theta}(0) = 0$

$$s^2 \theta(s) = \frac{1}{J} u(s)$$

$$\therefore \boxed{\theta(s) = \frac{1}{Js^2} u(s)} \quad — ①$$

Given, $\boxed{u = K_p(\theta_{cmd} - \theta) + K_D \frac{d}{dx}(\theta_{cmd} - \theta)}$ — ②

from ①, ②



$$\Rightarrow \left[(\theta_c - \theta) k_p + (\theta_c - \theta) k_d s\right] \times \frac{1}{Js^2} = \theta$$

$$\Rightarrow (k_p + k_d s)\theta_c - (k_p + k_d s)\theta = Js^2$$

$$\therefore \frac{\theta}{\theta_c}(s) = \frac{k_d s + k_p}{Js^2 + k_d s + k_p} = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

comparing Numerators & Denominators,

$$\Rightarrow \boxed{\omega_n = \sqrt{\frac{k_p}{J}}} \quad \& \quad \boxed{\zeta = \frac{k_d}{2\sqrt{k_p J}}}$$

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Substituting

$J = 2000 \text{ kg-m}^2$,

$\omega = 3 \text{ rad/s}$

$\zeta = 0.7$,

we get

$k_p = J\omega_n^2$

$\boxed{k_p = 18000}$

$k_d = 2\zeta\sqrt{k_p J}$

$= 2 \times 0.7 \times \sqrt{18000 \times 2000}$

$\therefore \boxed{k_d = 8400}$

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