6/20/18

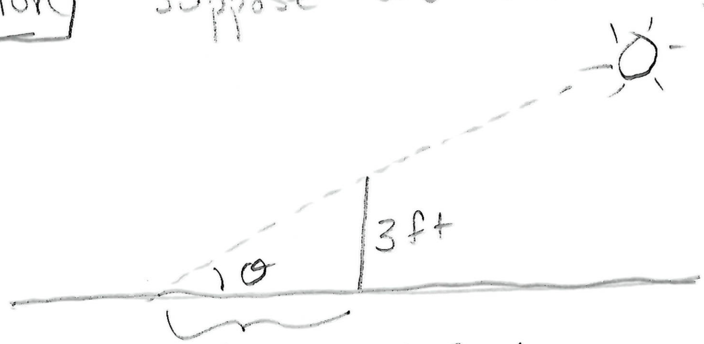
Announcements

- HW 2 due today
- Midterm next Tues 6:00 - 7:00 (please see me if conflict)
- Practice test on Carmen
- HW 3 due next Thurs

Today

- Inverse trig fns 5.7
  - Motivation
  - Idea of inverses
  - Definitions
  - Compositions

Motivation: Suppose we have a yard stick



and we measure this to be 4 ft

It would be nice if we could figure out what θ is, i.e. we already know

$\tan\theta = \frac{3}{4}$, and want to know what θ is.

Hope: build an inverse, $\tan^{-1}(x)$ (NOT $\frac{1}{\tan(x)}$)

so that $\tan^{-1}(3/4) = \theta$, then we'd know θ

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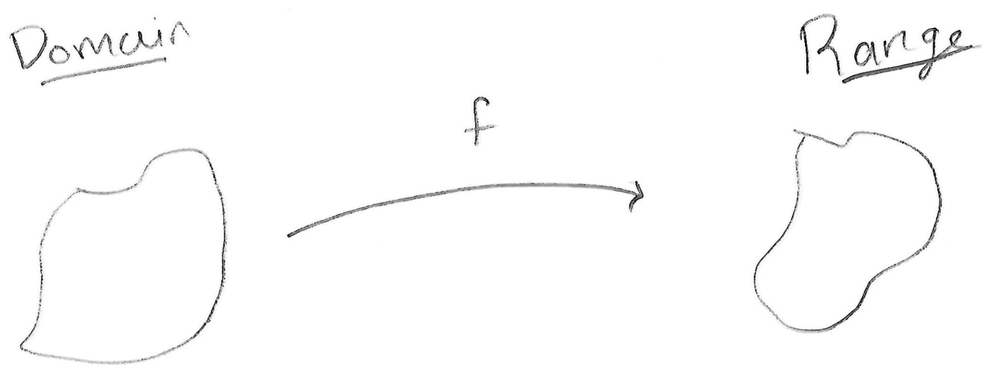
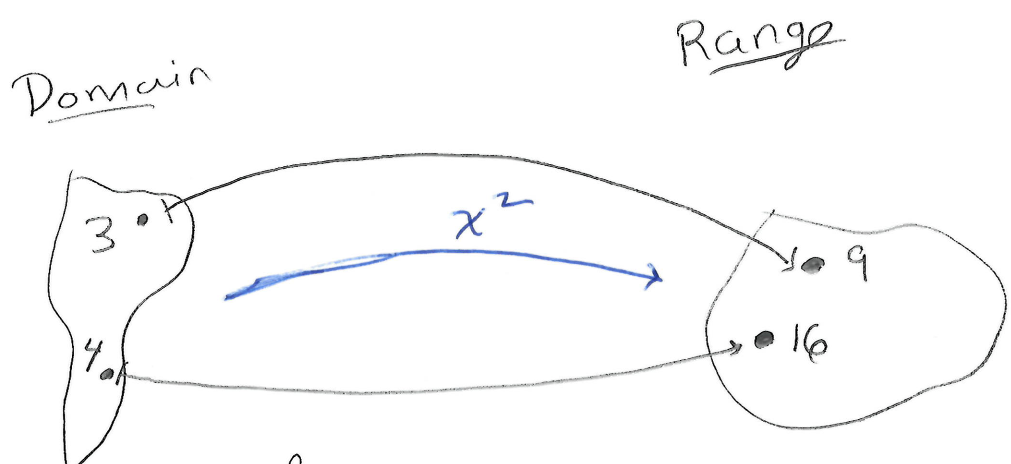
However, we will run into some problems, and have to do some work to get around it.

Idea of inverses

Think of functions as things that send numbers to some other numbers, i.e.



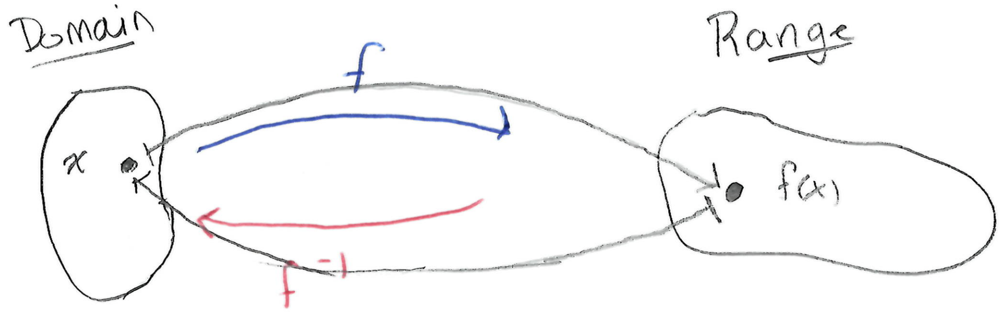
So if $f(x) = x^2$,



$$3 \xmapsto{f} 9$$

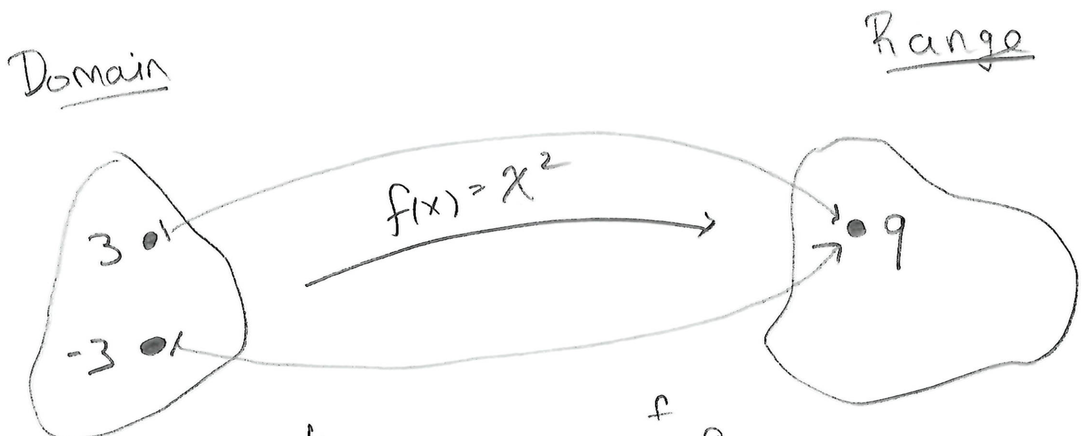
$$4 \xmapsto{f} 16$$

For inverses, we hope we can "undo" f, i.e.



Want $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

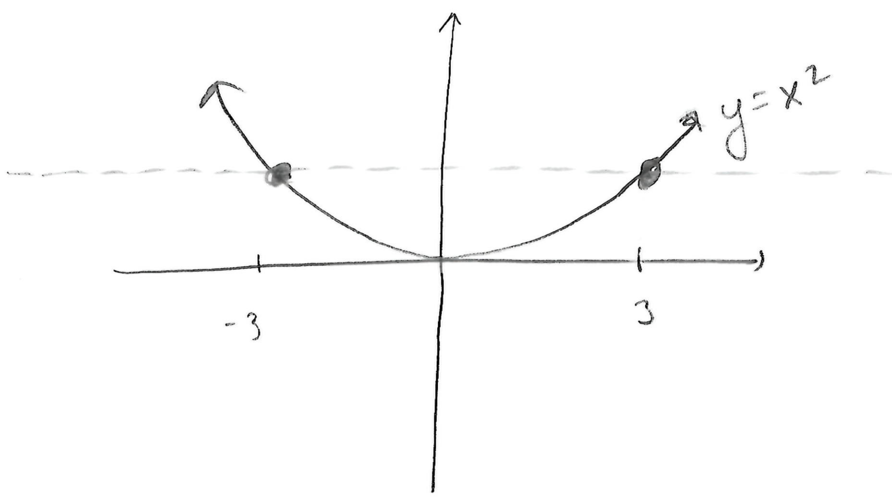
But this is not always possible, e.g.



$3 \xmapsto{f} 9$ and $-3 \xmapsto{f} 9$

So where should f^{-1} send 9?

This is a reflection of the fact that x^2 does not pass horizontal line test

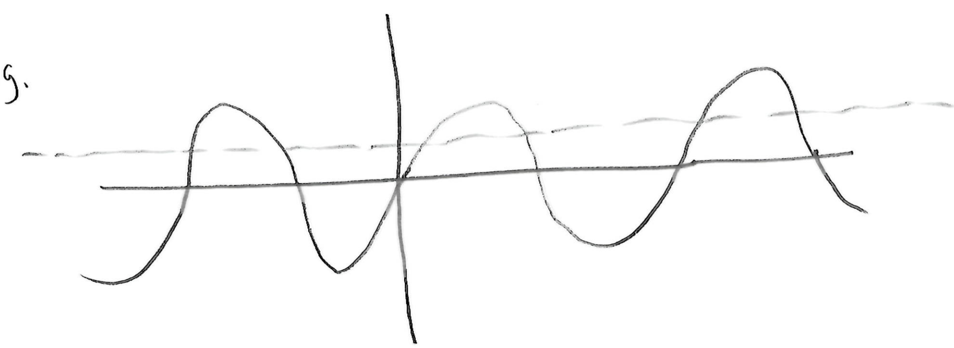


So functions only have inverses if this does not happen

$\Leftrightarrow$ pass hor. line test

Problem None of $\sin(x)$, $\cos(x)$, $\tan(x)$ pass hor. line test on their whole domain

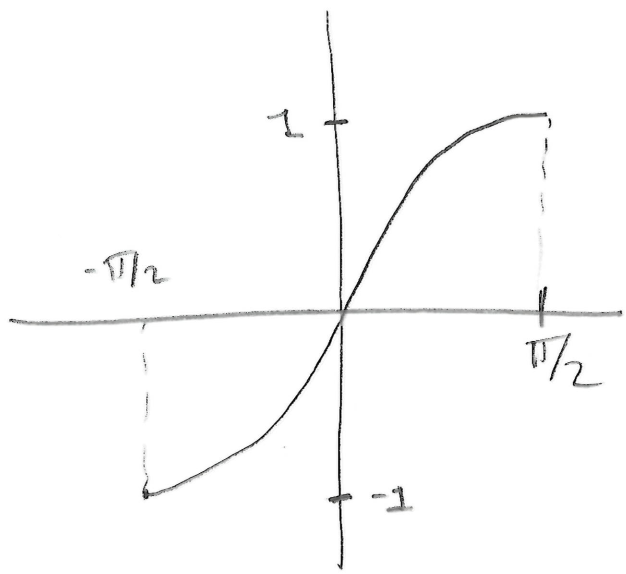
e.g.



Solution: Restrict the domains

Definitions

Sin(x) If we only look at $[-\pi/2, \pi/2]$,



$y = \sin(x)$ does pass hor. line test.

Def We define $\sin^{-1}(x)$ ← NOT $\frac{1}{\sin x}$ as the inverse function of $\sin(x)$ on this restricted domain. (Also called $\arcsin(x)$)

What this means:

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \underset{\sin^{-1}(x)}{\overset{\sin(x)}{\rightleftarrows}} [-1, 1]$$

Domain of $\sin^{-1}(x)$ is $[-1, 1]$

Range of $\sin^{-1}(x)$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\Rightarrow$ $\sin^{-1}(x)$ can only spit out numbers in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

So to say $\sin^{-1}(x) = t$ means that

$\sin(t) = x$ and t is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

e.g.

What is $\sin^{-1}(1)$?

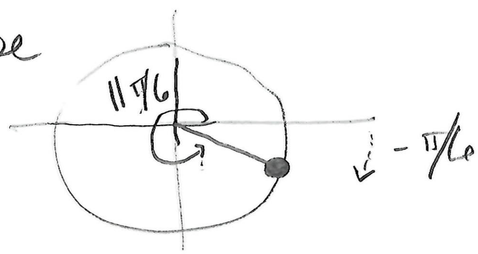
Sol/ If $\sin^{-1}(1) = t$, ← what we're looking for then $\sin(t) = 1$ and t in $[-\pi/2, \pi/2]$

$$\Rightarrow t = \boxed{\pi/2 = \sin^{-1}(1)}$$

What is $\sin^{-1}\left(-\frac{1}{2}\right)$?

Looking for angle whose sine is $-\frac{1}{2}$

One would be $11\pi/6$



But cannot be $11\pi/6$ because $11\pi/6$ not in $[-\pi/2, \pi/2]$.

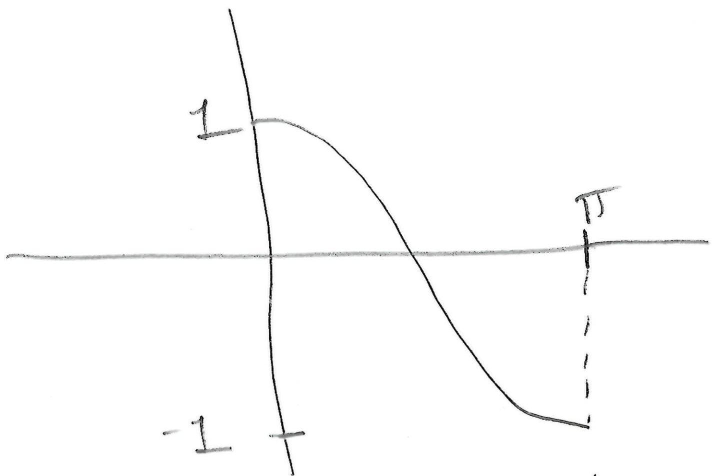
To say $\sin^{-1}\left(-\frac{1}{2}\right) = t$ ← what we're looking for means

$\sin t = -\frac{1}{2}$ and t is in $[-\pi/2, \pi/2]$.

So $\boxed{\sin^{-1}\left(\frac{-1}{2}\right) = -\pi/6}$

$\boxed{\cos(x)}$

We restrict $\cos(x)$ to $[0, \pi]$



NOT $\frac{1}{\cos x}$

and define $\cos^{-1}(x)$ as the inverse to $\cos(x)$ on this restricted domain. (Also called $\arccos(x)$).

<u>So</u>

$$[0, \pi] \underset{\cos^{-1}(x)}{\overset{\cos(x)}{\rightleftarrows}} [-1, 1].$$

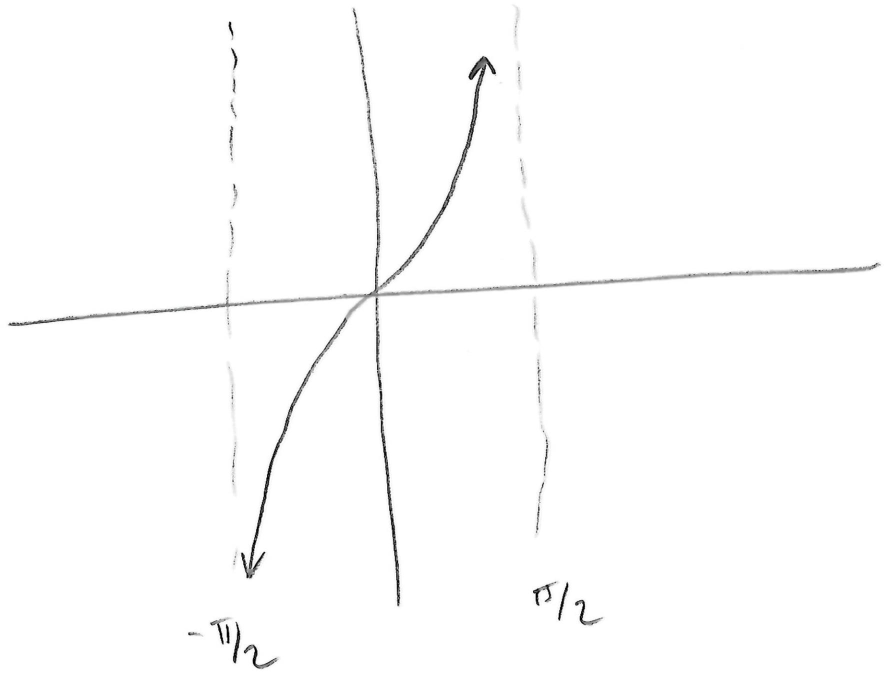
<u>Domain</u> of $\cos^{-1}(x)$ is $[-1, 1]$

<u>Range</u> of $\cos^{-1}(x)$ is $[0, \pi]$.

$\Rightarrow$ To say $\cos^{-1}(x) = t$ means that $\cos(t) = x$ <u>and</u> t is in $[0, \pi]$.

$\boxed{\tan(x)}$

Same story, but restrict $\tan(x)$ to $(-\pi/2, \pi/2)$



Notice that $\tan(x)$ is somewhat special in that its range is $(-\infty, \infty)$. So

$$(-\pi/2, \pi/2) \underset{\tan^{-1}(x)}{\overset{\tan(x)}{\rightleftarrows}} (-\infty, \infty)$$

Domain of $\tan^{-1}(x)$ is $(-\infty, \infty)$ (also called $\arctan(x)$)

Range of $\tan^{-1}(x)$ is $(-\pi/2, \pi/2)$

So to say $\tan^{-1}(x) = t$ means that $\tan(t) = x$ and t is in $(-\pi/2, \pi/2)$.

Compositions

Facts

① $\sin(\sin^{-1}(x)) = x$, provided $\sin^{-1}(x)$ was defined in the first place (i.e. x in domain of $\sin^{-1}$)

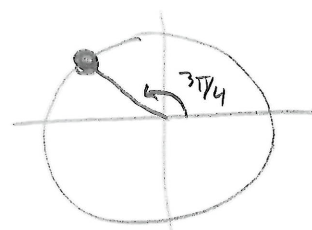
and same for $\cos x$ and $\tan x$

② $\sin^{-1}(\sin(x))$ NOT necessarily x.

and same for $\cos x$ and $\tan x$

e.g. What is $\sin^{-1}(\sin(3\pi/4))$?

Well, $\sin(3\pi/4) = \frac{\sqrt{2}}{2}$.



but $\sin^{-1}(\sin(3\pi/4))$

cannot be $3\pi/4$ because

$3\pi/4$ not in $[-\pi/2, \pi/2]$

Remember, to say $\sin^{-1}(\underset{\uparrow}{x}) = t$ means

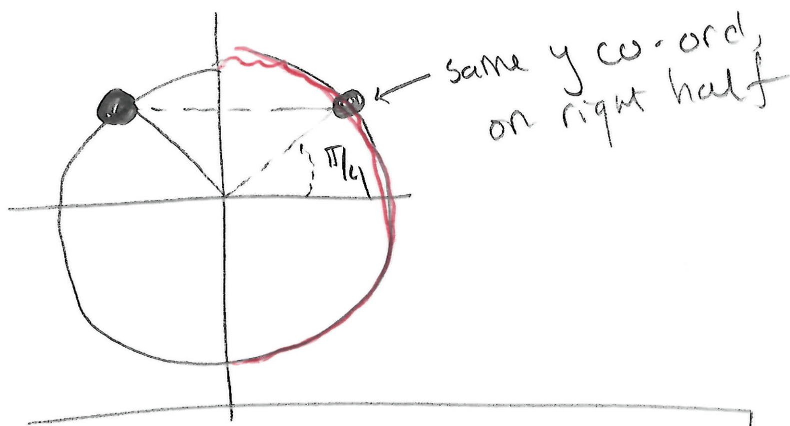
Some #

$\sin(t) = x$ and t is in $[-\pi/2, \pi/2]$

So to say $\sin^{-1}\left(\underbrace{\sin(3\pi/4)}_{\text{Some \#}}\right) = t$ means

$\leftarrow$ what we're looking for

$\underbrace{\sin(t) = \sin(3\pi/4)}_{\text{same y co-ord on unit circle}}$ and t in $\underbrace{[-\pi/2, \pi/2]}_{\text{on right half of unit circle.}}$



So $t = \boxed{\pi/4 = \sin^{-1}(\sin(3\pi/4))}$

not the same

(We won't worry about $\sec^{-1}(x)$, etc.)