7/18/18

Announcements

- Exam Thu 6:00 - 7:00
  EA 170
- New office: MW 549

Today

- Vectors §8.4
  - Definition(s)
  - Operations
  - component form
  - Unit vectors
  - Putting together

Def(s)

A vector is a quantity with both a magnitude ("length" or "size") and a direction.

Intuitively, vectors are arrows

Most important distinction: vector vs scalar

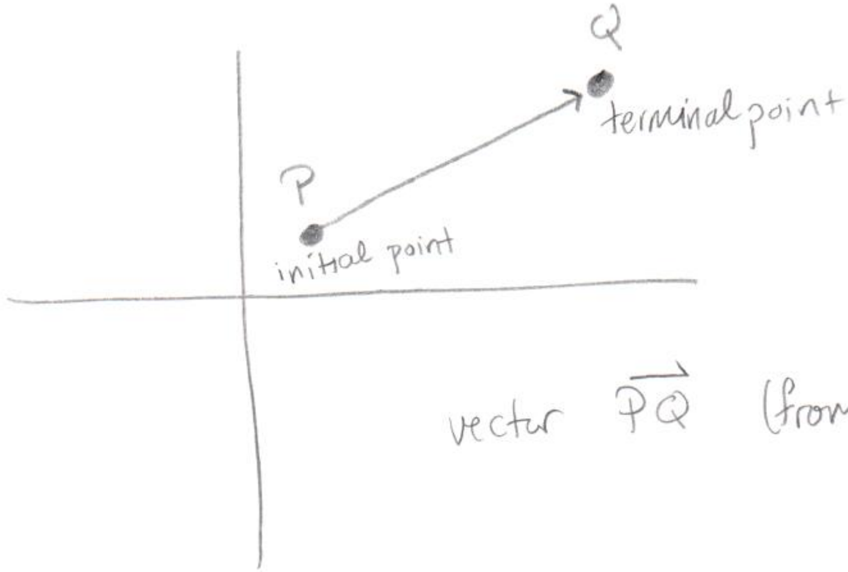
number

Scalar	Vector
One value – its size	Described by both a size and direction.
e.g. 6, π, 7896, ...	e.g. Motion of an object in a straight line
An object's speed 60 mph	An object's velocity $\underbrace{\text{60 mph}}_{\text{size}}$ $\underbrace{\text{eastwards}}_{\text{direction.}}$

(vectors are important)

Def: A vector in the plane is a line segment w/ a given direction

e.g.

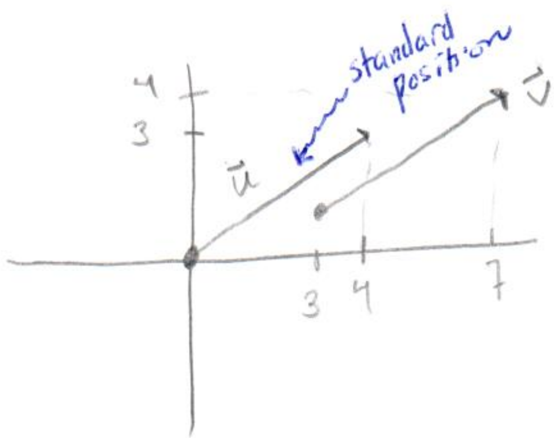


vector $\overrightarrow{PQ}$ (from P to Q)

Its magnitude, $\|\overrightarrow{PQ}\|$, is its length.

⚠ Two vectors are equal if and only if they have both same magnitude and direction.

e.g. Is the vector $\vec{u}$ from (0,0) to (4,3) the same as the vector $\vec{v}$ from (3,1) to (7,4)



(look same).

Magnitudes?

$$\|\vec{u}\| = \sqrt{(4-0)^2 + (3-0)^2} = \sqrt{16+9} = 5$$

same ✓

$$\|\vec{v}\| = \sqrt{(7-3)^2 + (4-1)^2} = \sqrt{16+9} = 5$$

Directions?

$$\text{slope of } \vec{u} = \frac{\Delta y}{\Delta x} = \frac{3-0}{4-0} = 3/4$$

same ✓

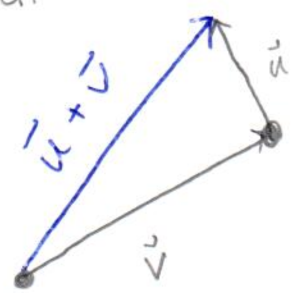
$$\text{slope of } \vec{v} = \frac{\Delta y}{\Delta x} = \frac{4-1}{7-3} = 3/4$$

so yes, $\boxed{\vec{u} = \vec{v}}$

Operations on vectors

Adding Given

add by placing "tip to tail"



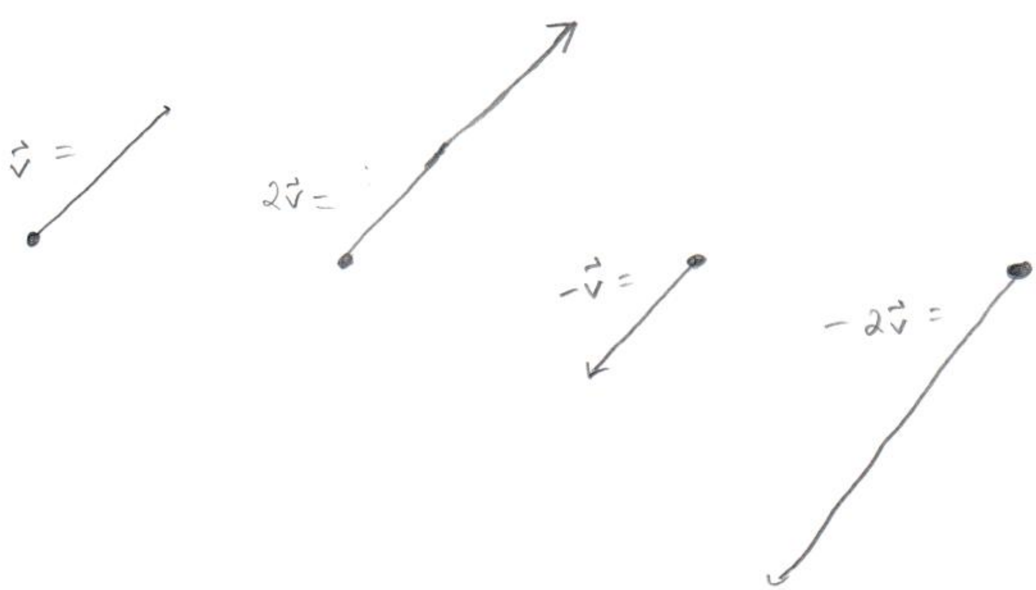
Scalar multiplication

Let k be a number, $\vec{v}$ a vector. Then

$k\vec{v}$ has
- magnitude $|k|\,\|\vec{v}\|$
- Direction: same as $\vec{v}$ if k pos
  opposite $\vec{v}$ is k neg.

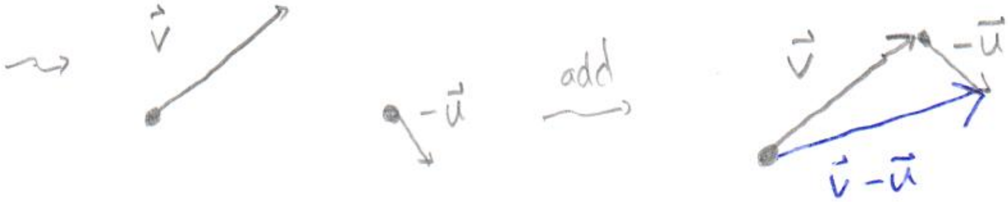
(if $k=0$, have zero vector $\vec{0}$)

e.g.



Subtraction

$\vec{v} - \vec{u} = \vec{v} + (-\vec{u})$ so given 



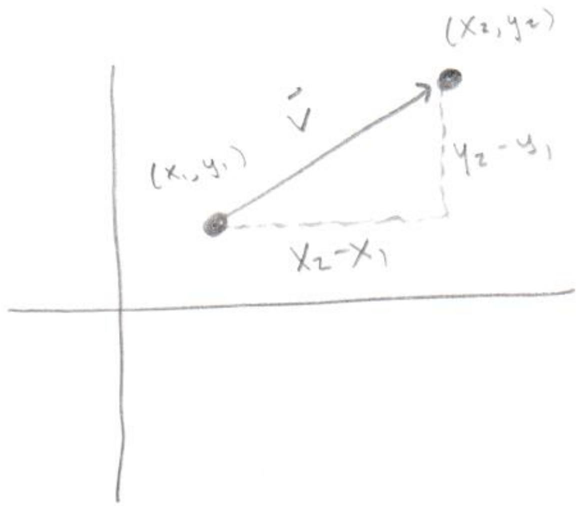
* No vector multiplication *

$\boxed{\text{Component form}}$

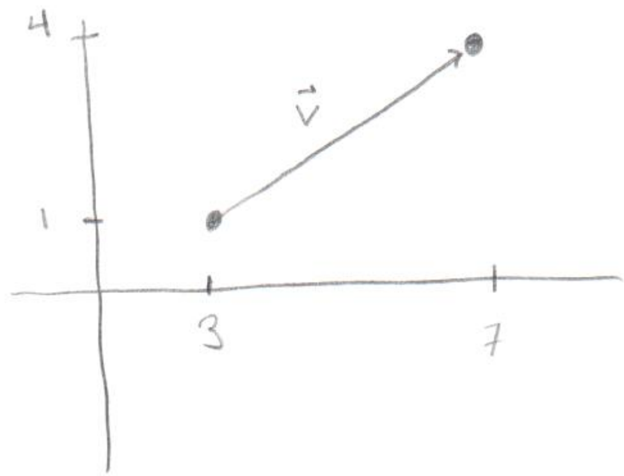
Lets us actually compute $\vec{u}+\vec{v}$, $\vec{u}-\vec{v}$, etc.

(especially if can't draw)

Given



$\vec{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$ in component form e.g write $\vec{v}$ from $(3,1)$ to $(7,4)$ in component form.



$$\vec{v} = \langle 7-3, 4-1 \rangle = \boxed{\langle 4, 3 \rangle}$$

Computations can now be done "component wise"

e.g. $\vec{u} = \langle a_1, b_1 \rangle$ and $\vec{v} = \langle a_2, b_2 \rangle$ are equal iff $a_1 = a_2$ and $b_1 = b_2$.

e.g. Given $\vec{u} = \langle -3, 8 \rangle$ and $\vec{v} = \langle 2, -5 \rangle$, find

a) $5\vec{u}$ b) $2\vec{u} - 3\vec{v}$

a) $5\vec{u} = 5\langle -3, 8 \rangle = \langle 5(-3), 5(8) \rangle = \boxed{\langle -15, 40 \rangle}$

b) $2\vec{u} - 3\vec{v} = 2\langle -3, 8 \rangle - 3\langle 2, -5 \rangle$

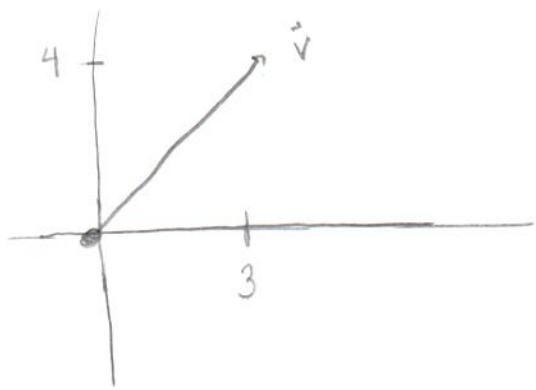
$= \langle -6, 16 \rangle - \langle 6, -15 \rangle$

$= \langle -6-6, 16-(-15) \rangle = \boxed{\langle -12, 31 \rangle}$

Unit Vectors

Def: A vector with length 1 is called a unit vector.

e.g. Given $\vec{v} = \langle 3, 4 \rangle$, find a unit vector in the same direction as $\vec{v}$.



Want to multiply $\vec{v}$ by a scalar so it has length 1.

what is "current" length of $\vec{v}$? $\|\vec{v}\| = \sqrt{3^2 + 4^2} = 5$.

Agree $\frac{1}{5}\vec{v}$ same direction as $\vec{v}$? (Yes, $\frac{1}{5} > 0$)

And $\left\| \frac{1}{5}\vec{v} \right\| = \left|\frac{1}{5}\right| \|\vec{v}\| = \frac{1}{5}(5) = 1$

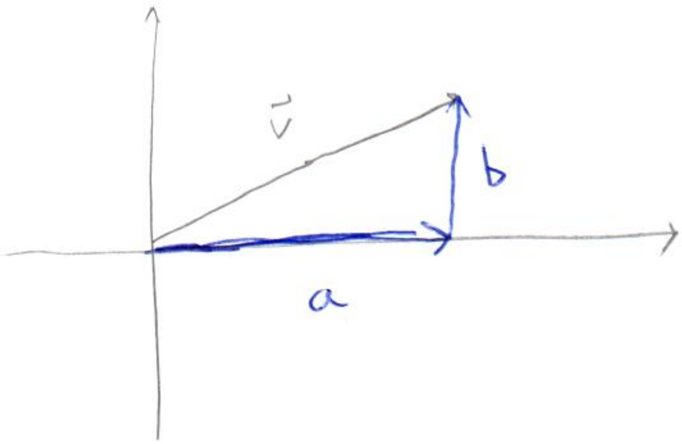
so $\boxed{\frac{1}{5}\vec{v}}$

In general, unit vector in same direction as $\vec{v}$ is $\frac{\vec{v}}{\|\vec{v}\|}$

Two special unit vectors

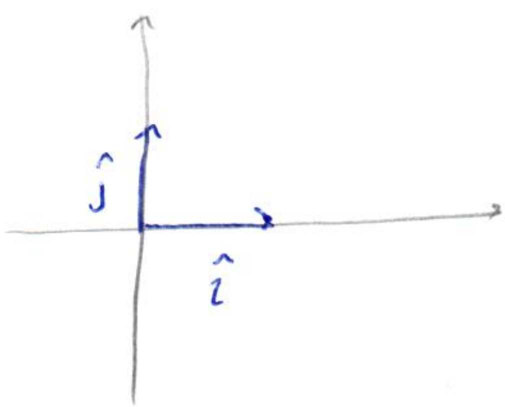
If $\vec{v} = \langle a, b \rangle$, $\vec{v}$ has "a" as its x co-ord

'b' as its y co-ord.



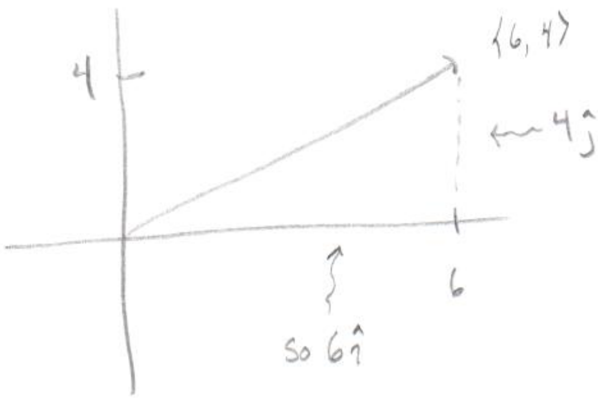
Let $\hat{\imath}$ (not "i" imag. #) be unit vector in pos x-dir.

$\hat{\jmath}$ be unit vector in pos y-dir.



Then $\vec{v} = \langle a, b \rangle = \underbrace{a\hat{\imath}}_{\text{a units right}} + \underbrace{b\hat{\jmath}}_{\text{b units up}}$

e.g. Write $\langle 6,4\rangle$ and $\langle 3,-2\rangle$ in terms of $\hat{\imath}$ and $\hat{\jmath}$ and add them.



$\Rightarrow \langle 6,4\rangle = 6\hat{\imath} + 4\hat{\jmath}$.

Similarly, $\langle 3,-2\rangle = 3\hat{\imath} - 2\hat{\jmath}$.

Then $\langle 6,4\rangle + \langle 3,-2\rangle = 6\hat{\imath} + 4\hat{\jmath} + 3\hat{\imath} - 2\hat{\jmath}$

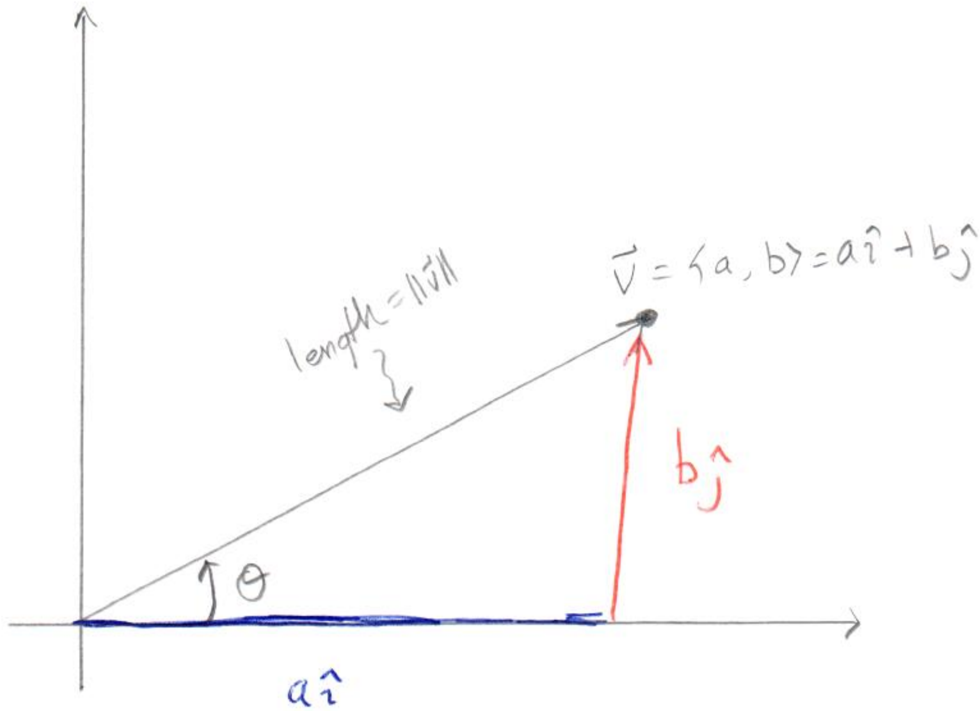
$= 9\hat{\imath} + 2\hat{\jmath} = \langle 9,2\rangle$.

⚠ Don't multiply!

$(6\hat{\imath} + 4\hat{\jmath})(3\hat{\imath} - 2\hat{\jmath}) = 18\hat{\imath}^2 + \dots$ NO

Putting it all together

We now have a bunch of ways to think about/write vectors



$\|\vec{v}\| = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$ $(a \neq 0)$.

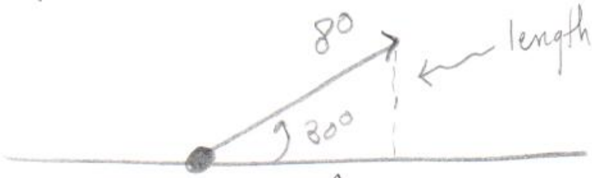
- $\cos\theta = \frac{a}{\|\vec{v}\|} \Rightarrow a = \|\vec{v}\|\cos\theta$
- $b = \|\vec{v}\| \sin\theta$.

So

$$\vec{v} = \langle a, b \rangle = \langle \|\vec{v}\|\cos\theta, \|\vec{v}\|\sin\theta \rangle = a\hat{\imath} + b\hat{\jmath} = \|\vec{v}\|\cos\theta\,\hat{\imath} + \|\vec{v}\|\sin\theta\,\hat{\jmath}$$

e.g. Sam pulls an object w/ force of 80 lbs at an angle of 30° from horizontal. Write force vector $\vec{F}$ in terms of $\hat{\imath}$ and $\hat{\jmath}$

Magnitude $= \|\vec{F}\| = 80$ lbs.



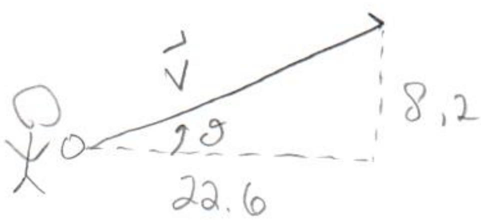
length $= 80\sin(30°) = 80\left(\frac{1}{2}\right) = 40$.

length $= 80\cos(30°) = 80\left(\frac{\sqrt{3}}{2}\right) = 40\sqrt{3}$

so $\boxed{\vec{F} = 40\sqrt{3}\,\hat{\imath} + 40\,\hat{\jmath}}$

e.g. A ball is thrown with initial velocity $\vec{v} = 22.6\hat{\imath} + 8.2\hat{\jmath}$.

Find speed at which the ball was thrown and angle above horizontal



Initial speed = magnitude of $\vec{v} = \|\vec{v}\| = \sqrt{22.6^2 + 8.2^2} \approx \boxed{24 \text{ m/s}}$

To find θ, since $\tan\theta = \dfrac{8.2}{22.6}$ and $\underbrace{\theta \text{ in QI}}_{\text{so } \tan^{-1}(x) \text{ gives right value.}}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{8.2}{22.6}\right) \approx \boxed{19.9°}$