6/8/18

Announcements
HW 1 due Wed

Today

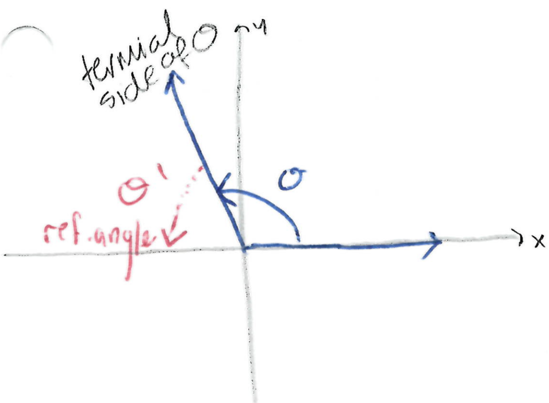
- Reference angles (5.3)
- Unit circle (5.4)
- Defining trig fns w/ unit circle (5.4)
- Domain/period/even and odd (5.4)

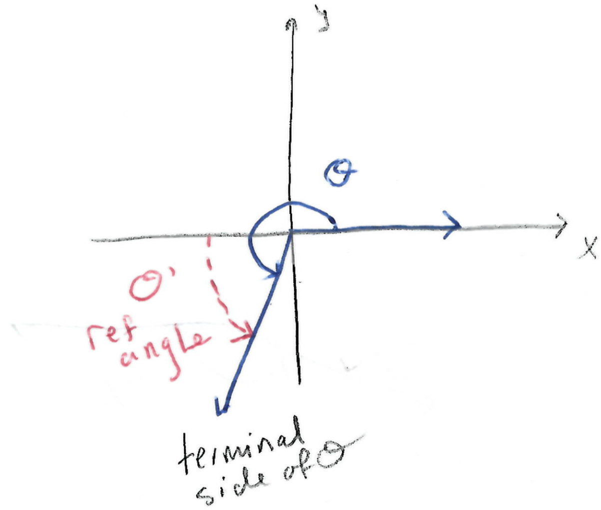
Reference Angles

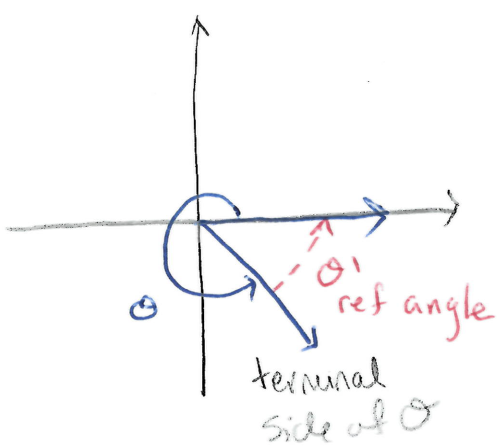
25 MIN

Def: Let θ be an angle in standard position. The reference angle θ' is the acute angle (between 0 and $\pi/2$) formed by terminal side of θ and horizontal axis.

Visually





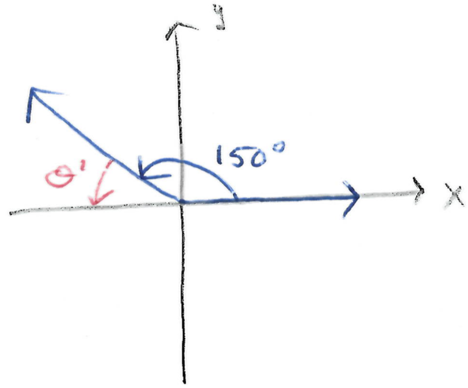


Note: θ' acute means reference angles are always positive.

e.g.

① Find ref angle for $\theta = 150°$ ~~and $\theta = -\frac{7\pi}{8}$~~

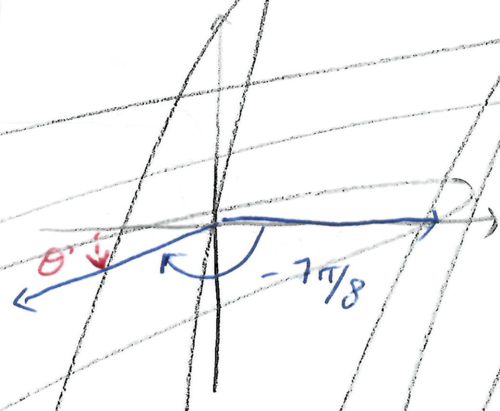
$\theta = 150°$ ⇝



Ref angle is $\theta' = 180° - 150° = \boxed{30°}$

~~$\theta = -7\pi/8$~~

~~$-7\pi/8$ is almost a half-rotation (in neg direction)~~

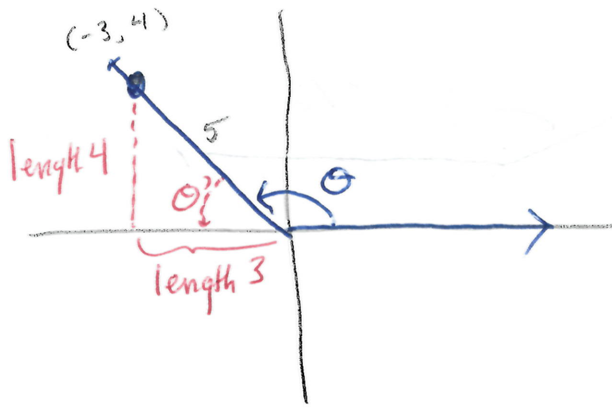


~~Ref angle is $\theta' = \pi - 7\pi/8 = \boxed{\pi/8}$~~

Why do we care?

Ref angles help us calculate trig fns. of non-acute angles

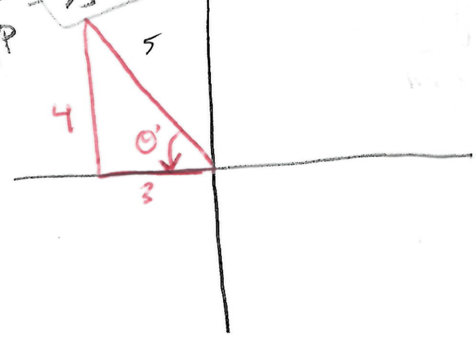
Suppose we know



On one hand, $\cos\theta = \frac{x}{r} = \frac{-3}{5}$. only differ by $\pm$

also, $\cos\theta' = \frac{\text{adj}}{\text{hyp}} = 3/5$

acute, just not in standard pos



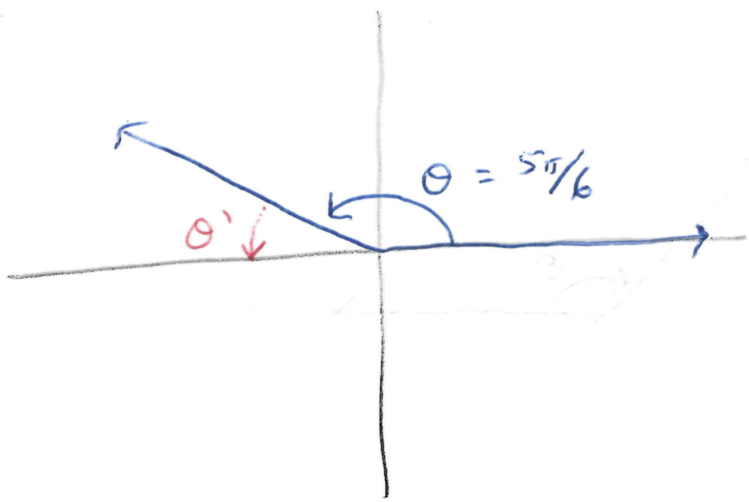
To find value of trig fns using ref. angles

① Determine fn. value on ref angle θ'

② Affix appropriate sign ($\pm$) based on quadrant e.g. Suppose we know $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.

What is $\cos(5\pi/6)$?

sketch



Ref angle $= \theta' = \pi - 5\pi/6 = \pi/6$.

$\Rightarrow \underset{\theta}{\cos(5\pi/6)} = \pm \underset{\theta'}{\cos(\pi/6)} = \pm \frac{\sqrt{3}}{2}$. x co-ord neg in QII

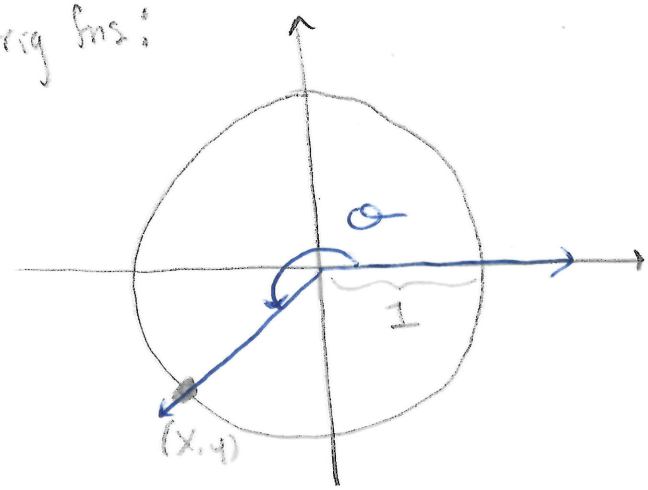
$\Rightarrow \cos(5\pi/6)$ is neg $\Rightarrow$ $\boxed{\cos(5\pi/6) = -\frac{\sqrt{3}}{2}}$

45 min left
10 min

Unit Circle

<u>Def</u>: The <u>unit circle</u> is the circle of radius 1 centered at origin

Relationship to trig fns:



pick any point (x,y) on unit circle to form an angle

By definition, $\sin\theta = \frac{y}{r} = y$

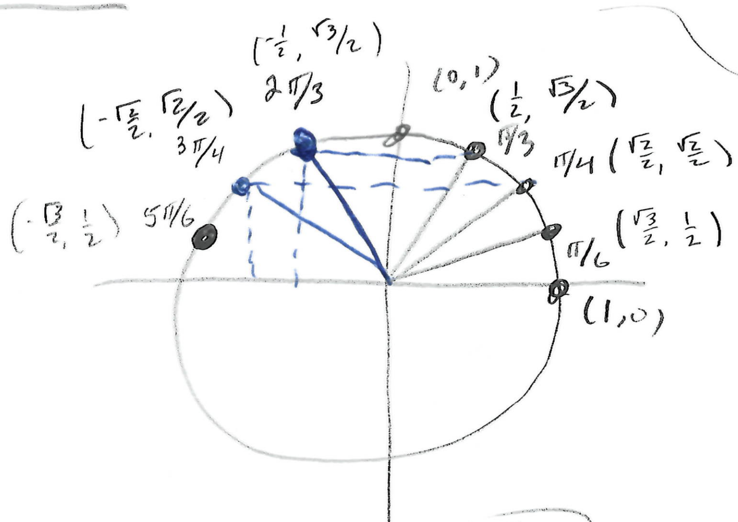
This is 1! (on unit circle)

$\cos\theta = \frac{x}{r} = x$

<u>Conclusion</u>: sin θ gives y co-ord of a pt on unit circle, and cos θ gives x co-ord

(can skip)
yeah, skip

Filling in unit circle



KNOW

$2\pi/3$ will have same y co-ord as $\pi/3$ and neg x co-ord $(\frac{-1}{2}, \frac{\sqrt{3}}{2})$

$3\pi/4$ will have same y co-ord as $\pi/4$ and neg x coord $(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

So e.g. ① $\sin(5\pi/6)$ = y co-ord on unit circle
$= \frac{1}{2}$

② $\cos(\frac{2\pi}{3})$ = x co-ord on unit circle $= -\frac{1}{2}$

③ $\cos(\frac{3\pi}{4})$ = x co-ord on unit circle $= \frac{-\sqrt{2}}{2}$.

* Full table p. 541 * AND PRINT OUT

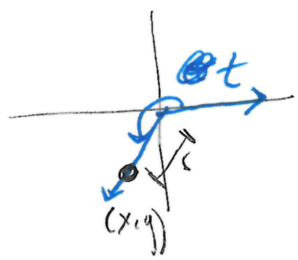
Defining trig fns w/ unit circle

35 ~~ac~~ left
Spend 15

We already know how to define sin(t) and cos t for any t.

How? Any "t" can be thought of as an angle (in radians) and we know how to calculate sine and cosine of any angle

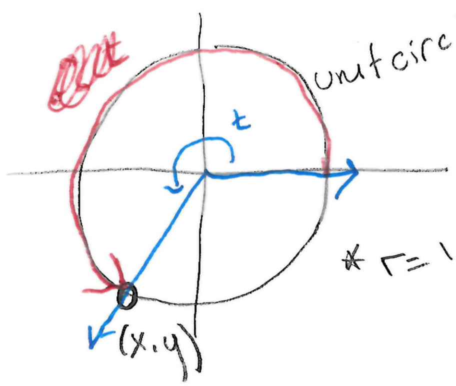
For any angle t



$\sin t = \frac{y}{r}$

$\cos t = \frac{x}{r}$

unit circle just makes life easier.



$\Rightarrow \sin t = y$

$\cos t = x$

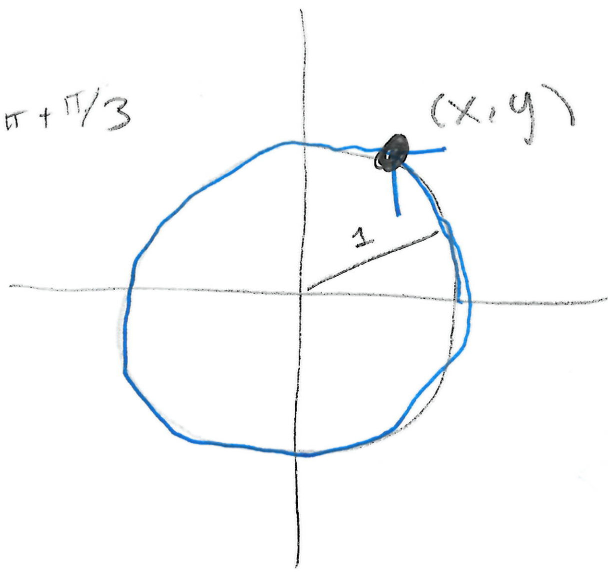
$* r = 1 *$

Also
distance of red = $\underbrace{\text{(measure of angle)}}_{t}\underbrace{\text{(radius)}}_{1}$ (from 5.1)

$\Rightarrow$ distance of red = t

Conclusion: Given any t

e.g. $t = 2\pi + \pi/3$



① Travel distance t on unit circle

② End at some point (x, y)

③ $\sin t = y$, $\cos t = x$

20 min

1 Domains, etc of trig fns

Why? any "t" can be thought of as an angle (in radians), and we know how to calculate sine and cosine of any angle.

Domains

- $\sin t$ and $\cos t$ defined for all t.

- $\tan t = \dfrac{\sin t}{\cos t}$ defined wherever $\cos t \neq 0$

x co-ord 0

⇝ all "t" except

$$\begin{cases} \pi/2,\ \pi/2 + 2\pi,\ \pi/2 - 2\pi, \text{ etc.} \rightsquigarrow \pi/2 + 2\pi n \\ -\frac{\pi}{2},\ -\frac{\pi}{2} + 2\pi,\ \ldots \rightsquigarrow -\frac{\pi}{2} + 2\pi n. \end{cases}$$

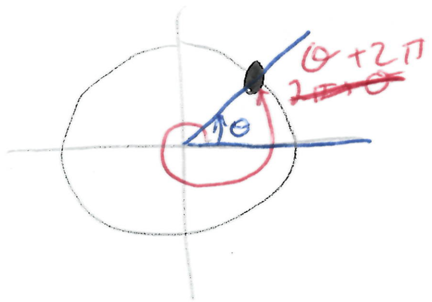
- $\csc(t) = \dfrac{1}{\sin t}$ defined wherever $\sin t \neq 0$

- $\sec(t) = \dfrac{1}{\cos t}$ defined wherever $\cos t \neq 0$

- $\cot t = \dfrac{1}{\tan t}$ defined wherever $\tan t \neq 0$

(*listed p. 543*)

Periodicity



know $\sin\theta = \sin(\theta + 2\pi)$ or 4π, etc

$\cos\theta = \cos(\theta + 2\pi)$

b/c θ and $\theta + 2\pi$ (or 4π, etc) have same

(x, y) co-ord on unit circle.

This is called periodic behavior

Fact (table p. 545)

- sine, cosine, cosecant, secant have period 2π,

  so e.g. $\sin(t + 2\pi) = \sin t$

- tangent and cotangent have period π, so e.g. $\tan(t + \pi) = \tan t$

Even/odd

Fact: $\sin(t)$ is an odd fn, meaning $\sin(-t) = -\sin(t)$ for all t

$\cos(t)$ is an even fn, meaning $\cos(-t) = \cos(t)$

All other fns are built from $\sin(t)$ and $\cos(t)$, so

e.g. $\tan(-t) = \dfrac{\sin(-t)}{\cos(-t)} = \dfrac{-\sin t}{\cos t} = -\tan t$

$\Rightarrow$ tan t is odd

Full table p. 546