

7/27/18

Announcements

- Final exam
 - Bonus due today
 - Wed, Aug 1 2:00 - 3:45 (be 10 min early)
 - Bolz 422 (here)
 - Midterm corrections due before

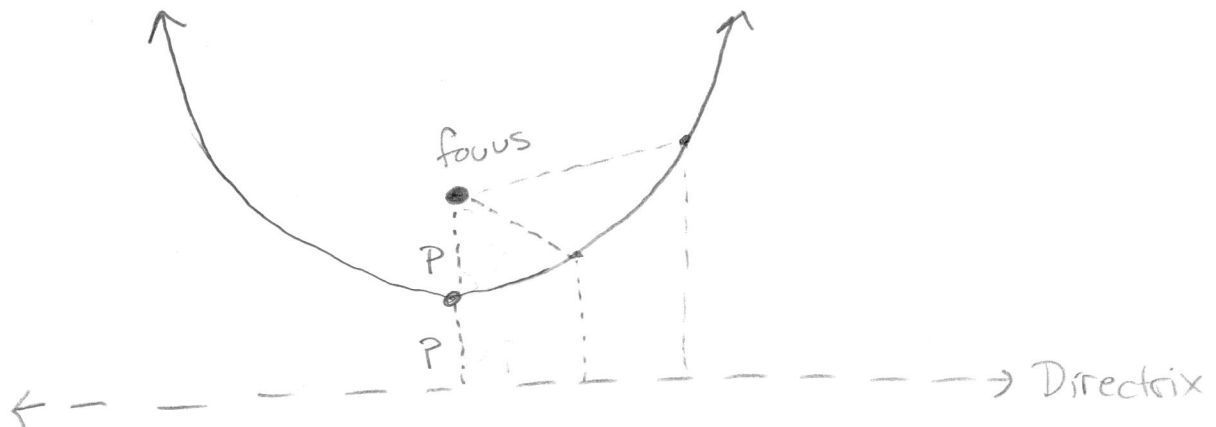
Monday

- Review session: MW 154, 12:30 - 3:00 - BRING QUESTIONS
- Office Hrs: MW 11:30 - 12:30 MW 549 (no DMir)

Today

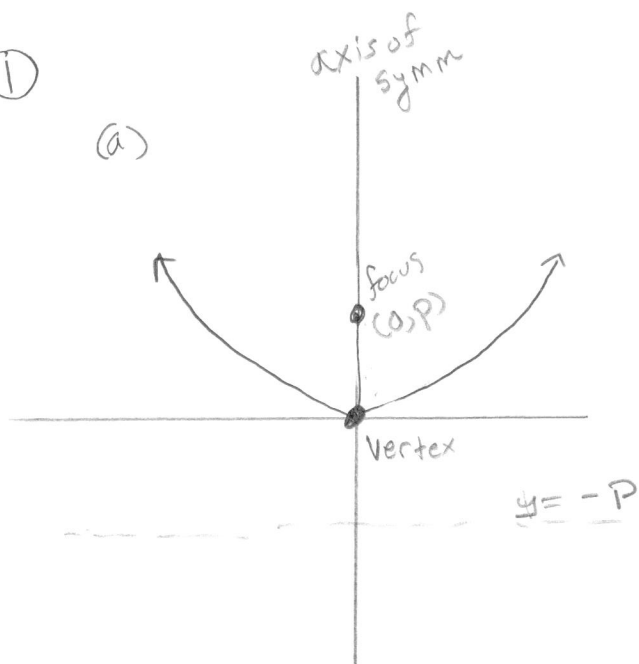
- The parabola (11.3)

Def: A parabola is the set of all points in the plane that are the same distance from a fixed line (the directrix) and a fixed point (the focus)
(we can actually make sense of this)

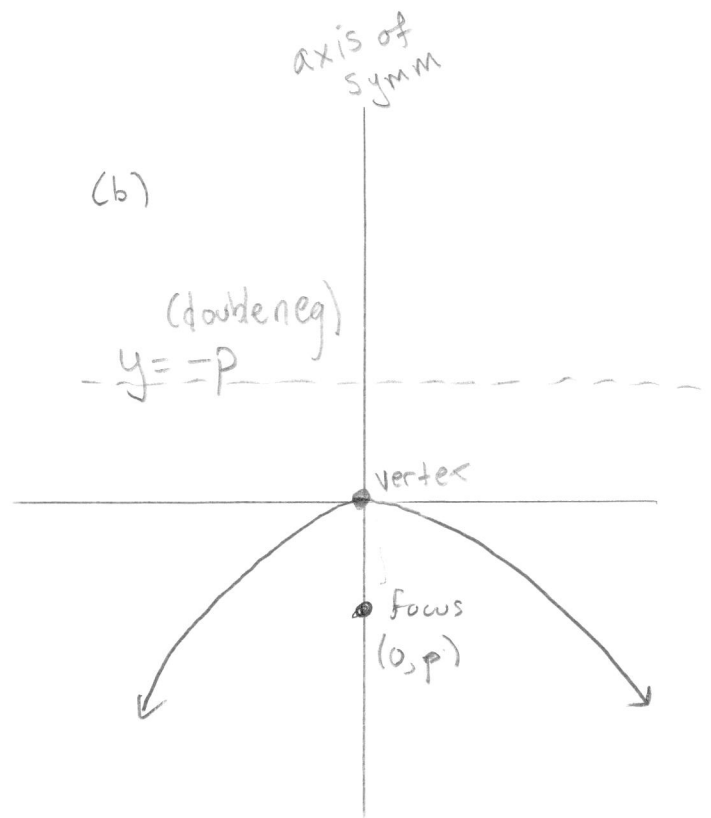


For us, parabolas will either be

①

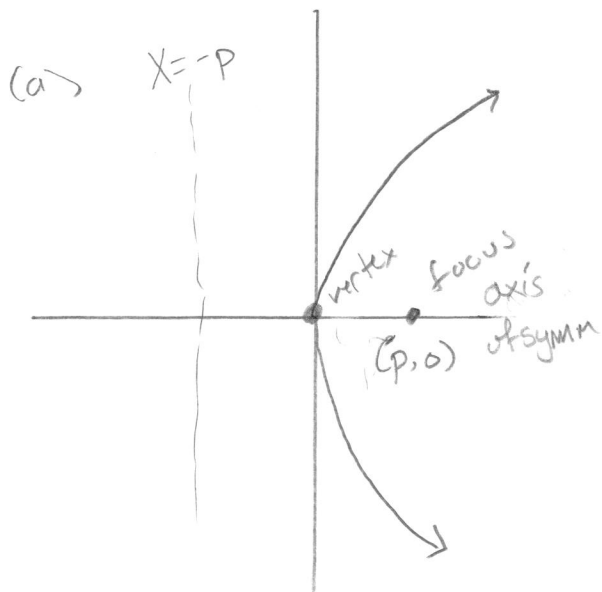


(b)

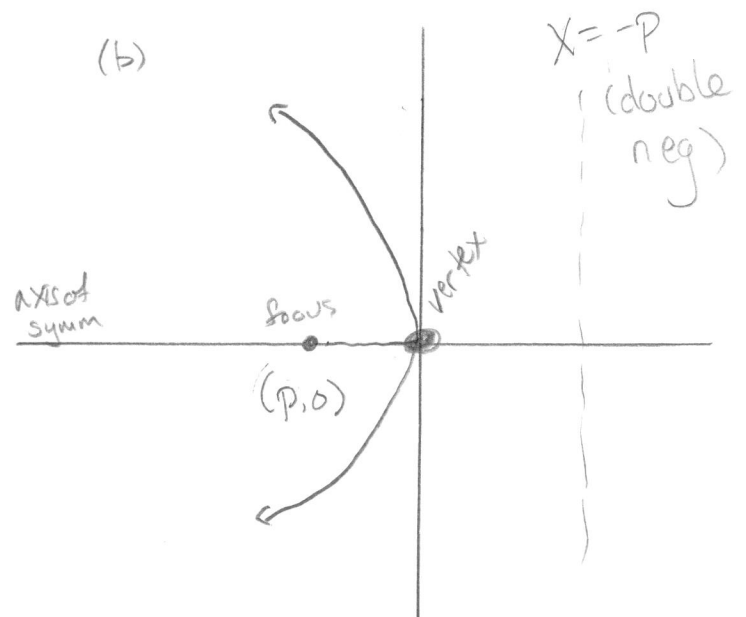


OR

②



(b)



②

$$\textcircled{1} \quad x^2 = 4py$$

• Axis of symmetry: y -axis

• Vertex: $(0, 0)$

• Focus: $(0, p)$ $p > 0 \Rightarrow (a)$
 $p < 0 \Rightarrow (b)$

• Directrix: $y = -p$ (in both cases)

$$\textcircled{2} \quad y^2 = 4px$$

x -axis

$(0, 0)$

$(p, 0)$ $p > 0 \Rightarrow (a)$
 $p < 0 \Rightarrow (b)$

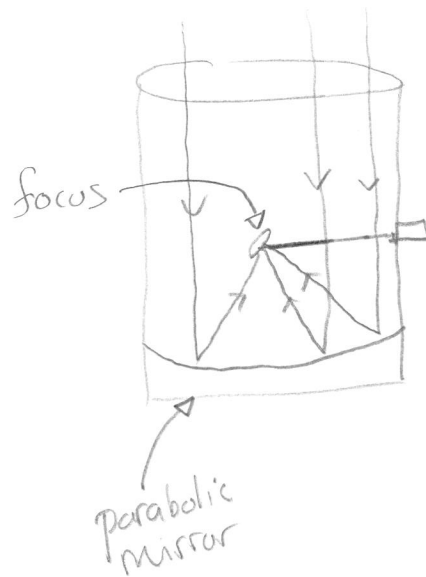
$x = -p$ (in both cases)

Note We're used to seeing parabolas as
 $(x = ay^2 + by + c)$
 $y = ax^2 + bx + c$ but w/ "change of coordinates"
 $(y^2 = 4px)$
 can always get $x^2 = 4py$ (convenient, b/c

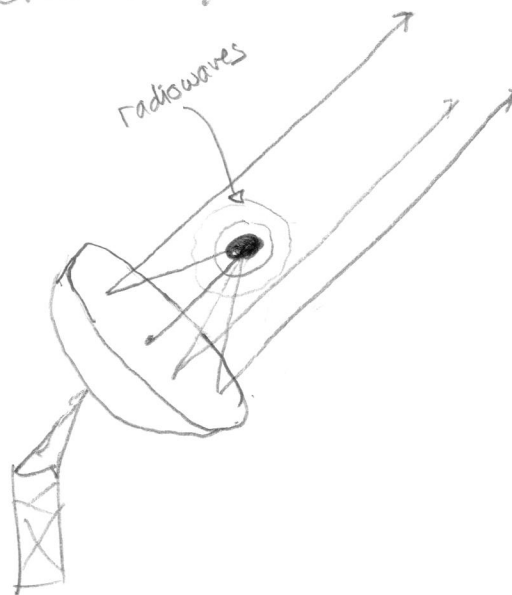
then we have "p")

Here, focus has some real importance:

Telescope



Or, similarly, satellite dish/radar (astronomy, weather, ...)



Radio astronomy \Rightarrow

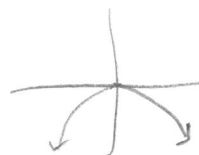
- CMB radiation (early evidence of Big Bang)
(thanks to pigeon poop)
- pulsars & quasars
- Liquid water on Mars (?)

Example

Given $x^2 = -8y$, identify axis of symm, vertex, focus, directrix, and graph it.

Axis of symm: Which way does it open?

$$x^2 = -8y \Rightarrow y = -\frac{1}{8}x^2 \Rightarrow \text{"normal"}$$



so y-axis

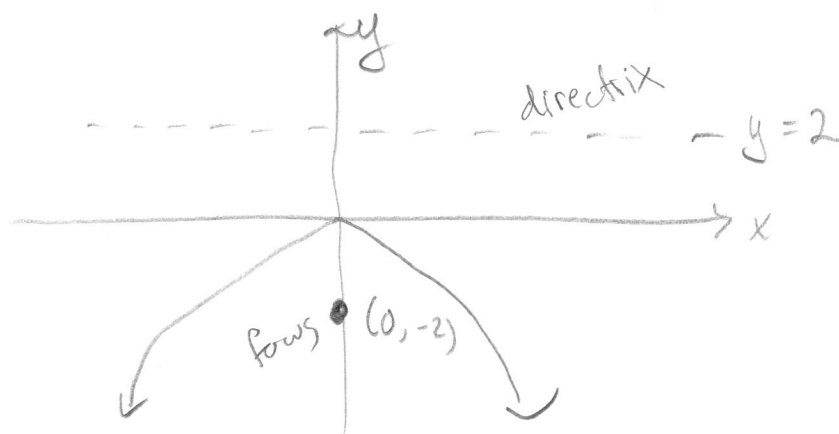
Vertex: (0, 0)

Focus? Standard form: $x^2 = 4py = -8y \Rightarrow p = -2$

so focus: (0, p) = (0, -2)

Directrix: $y = -p = -(-2) = 2$

Graph



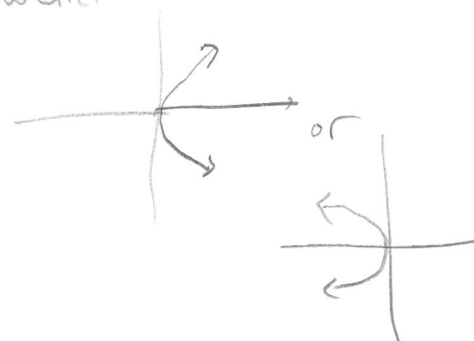
Example

Given $2y^2 = 32x$, identify axis of symm, vertex, focus, directrix, and graph it.

Axis of symm: $2y^2 = 32x \Rightarrow y^2 = 16x \Rightarrow$ 'weird'

so X-axis

vertex: (0,0)

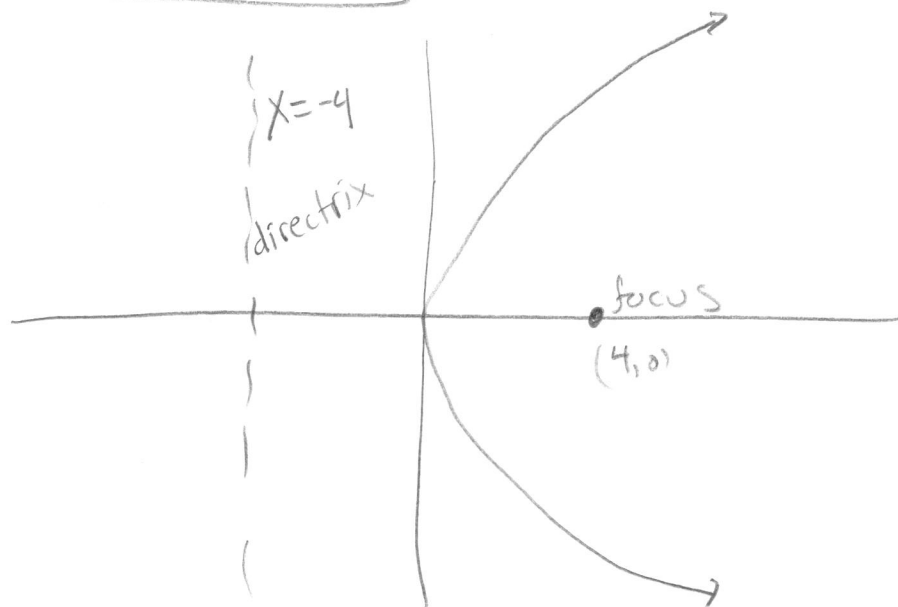


Focus? Standard form: $2y^2 = 32x \Rightarrow y^2 = 16x = 4px \Rightarrow p=4$

so Focus: (p,0) = (4,0)

Directrix: $x = -p = -4$

Graph



Example Determine the standard form of an equation of the parabola subject to given conditions.

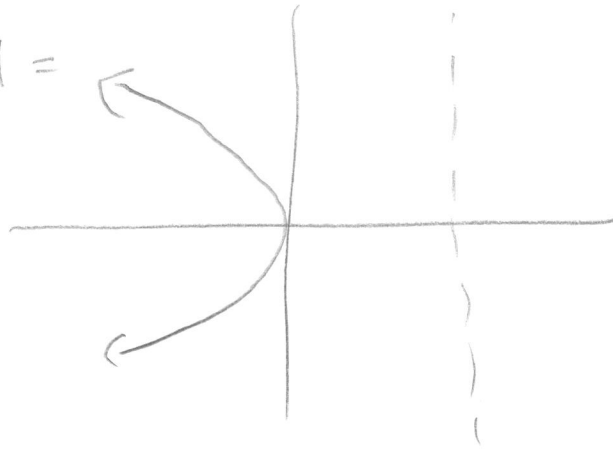
(a) vertex: $(0, 0)$

directrix: $x = 4$

(b) vertex: $(0, 0)$

directrix: $y = -2$.

(a) Directrix is $x = 4 =$

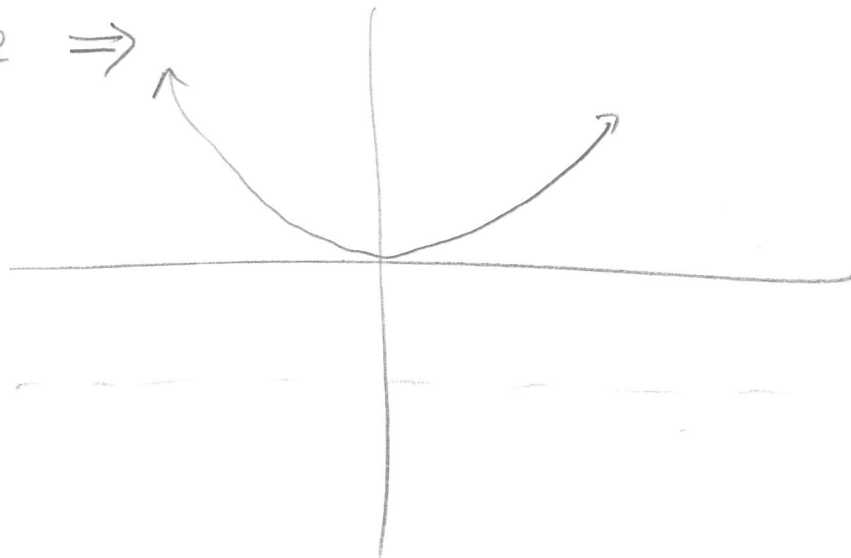


So we'll have form $y^2 = 4px$.

If directrix is $x = 4 = -p$, then $p = -4$.

So $\boxed{y^2 = 4(-4)x = -16x}$

(b) Directrix $y = -2 \Rightarrow$



So we'll have form $x^2 = 4py$.

If directrix is $y = -2 = -p$, then $p = 2$.

$$\text{So } \boxed{x^2 = 4(2)y = 8y}$$