$\boxed{6/22/18}$

Announcements

- Midterm Tues 6:00 - 7:00
  EA 170
- HW 3 due Thurs

Today

- Algebra w/ trig } 6.1
- Verify identities }
- Sum and difference formulas } 6.2

$\boxed{\text{Algebra w/ trig}}$ ⇝ re-writing stuff

Idea: Simplify expressions using identities we already know

Recall (don't write these all down)

① Reciprocals / Quotients

- $\csc(x) = \frac{1}{\sin(x)}$ (also implies $\frac{1}{\csc(x)} = \frac{1}{\frac{1}{\sin(x)}} = \sin x$)
- $\sec(x) = \frac{1}{\cos x}$ (also implies $\frac{1}{\sec(x)} = \cos(x)$)
- $\tan(x) = \frac{\sin x}{\cos x}$ and $\cot(x) = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

② Pythagorean identities

- $\sin^2 x + \cos^2 x = 1$, which implies
- $\tan^2 x + 1 = \sec^2 x$
- $1 + \cot^2 x = \csc^2 x$

③ Even/odd

- $\sin(-x) = -\sin(x)$ (which implies that

$$\csc(-x) = \frac{1}{\sin(-x)} = \frac{1}{-\sin x} = -\csc(x))$$

- $\cos(-x) = \cos x$ (which implies $\sec(-x) = \sec(x)$)
- $\tan(-x) = -\tan x$ (which implies $\cot(-x) = -\cot(x)$)

e.g. Simplify $\dfrac{1 - \sec^2 t}{\tan t - \tan t \sec t}$

Sol The top factors: $1 - \sec^2 t = (1 - \sec t)(1 + \sec t)$

since $(1 - \sec t)(1 + \sec t) = 1 + \cancel{\sec t - \sec t} - \sec^2 t$

The bottom factors: $\tan t - \tan t \sec t$

$= \tan t (1 - \sec t)$

Therefore, $\dfrac{1-\sec^2 t}{\tan t - \tan t \sec t} = \dfrac{(1-\cancel{\sec t})(1+\sec t)}{\tan t\,(1-\cancel{\sec t})}$

$$= \frac{1+\sec t}{\tan t} = \frac{1}{\tan t} + \frac{\sec t}{\tan t}$$

$$= \cot t + \frac{\frac{1}{\cos t}}{\frac{\sin t}{\cos t}} = \cot t + \frac{1}{\cancel{\cos t}} \cdot \frac{\cancel{\cos t}}{\sin t}$$

$$= \cot t + \frac{1}{\sin t} = \boxed{\cot t + \csc t}$$

$\boxed{\text{Verify identities}}$

Idea: We'll be asked to show $\square = \bigcirc$

Strategy: Algebraically manipulate (re-write without changing value) RHS until we obtain LHS or vice vera. e.g.

$$\square = \square = \square = \bigcirc.$$

e.g.

① Verify that $\sin(-x) + \csc x = \cot x \cdot \cos x$ is an identity.

What this means: show LHS = RHS for all x.

Sol: $RHS = \cot x \cdot \cos x = \dfrac{\cos x}{\sin x} \cdot \cos x$

$$= \frac{\cos^2 x}{\sin x} \quad \text{(pause)}$$

Recall $\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$.

(resume) Therefore, $\dfrac{\cos^2 x}{\sin x} = \dfrac{1 - \sin^2 x}{\sin x}$

$$= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} = \csc(x) - \sin x \quad \text{(pause)}$$

Note that $-\sin x = \sin(-x)$

(resume) So $\csc(x) - \sin x = \csc(x) + \sin(-x)$.

Therefore, RHS = LHS, as desired.

Note This is an argument / proof. We are asked to show that RHS = LHS, so we cannot assume it is true. So, e.g. saying

"RHS = LHS

RHS + 2 = LHS + 2

⋮

1 = 1, so they are equal"

is not a solution, for two reasons:

1. We began by assuming the very thing we are supposed to prove (begging the question)

2. The conclusion we got was "1 = 1" which has nothing to do with what we were asked.

Sum and difference formulas

Sum formulas (p. 623) (don't erase)

① $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

② $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

③ $\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$

(Note: Since $\tan(a+b) = \dfrac{\sin(a+b)}{\cos(a+b)}$ by def,

if we know ① and ②, we could obtain ③)

Difference formulas are a consequence of the sum formulas.

e.g. What is $\sin(a-b)$?

Sol: Note that $a-b = a+(-b)$, so

$$\sin(a-b) = \sin(\underbrace{a+(-b)}_{\text{now use sum formula}})$$

$$= \sin(a)\cdot \underbrace{\cos(-b)}_{\substack{=\cos(b) \\ \text{since even}}} + \cos(a)\cdot \underbrace{\sin(-b)}_{\substack{=-\sin(b) \\ \text{since odd}}}$$

$$= \sin(a)\cos(b) - \cos(a)\sin(b)$$

and we could do the same thing with $\cos(a-b)$ and $\tan(a-b)$. (Try it!)

Difference formulas

① ' $\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$

② ' $\cos(a-b) = \cos(a)\cdot\cos(b) + \sin(a)\cdot\sin(b)$

③ ' $\tan(a-b) = \dfrac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}$.

Tip: To get from ① to ①', just changed $+ \rightsquigarrow -$

② to ②', changed $- \rightsquigarrow +$

③ to ③', changed $+ \rightsquigarrow -$ and $- \rightsquigarrow +$

(table w/ side-by-side p. 623)

What's the point?

We can use stuff we know to figure out stuff we didn't know;

e.g. ① What is $\cos(5\pi/12)$?

Sol/ $5\pi/12$ is not on the unit circle, so before, we would have been out of luck.

Can we write $5\pi/12$ as the sum/diff of two other angles, the cosine of which we know?

(Yep) $\frac{5\pi}{12} = \frac{3\pi}{12} + \frac{2\pi}{12} = \underset{\text{"a"}}{\frac{\pi}{4}} + \underset{\text{"b"}}{\frac{\pi}{6}}$

So $\cos(5\pi/12) = \underbrace{\cos(\pi/4 + \pi/6)}_{\text{and we can use sum formula to figure this out}}$

$$= \cos(\pi/4) \cdot \cos(\pi/6) - \sin(\pi/4)\sin(\pi/6)$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}}$$

② What is $\sin(5\pi/12)$?

Sol/ Like in prev. e.g. $\sin\left(\frac{5\pi}{12}\right) = \sin(\pi/4 + \pi/6)$

$= \sin(\pi/4)\cos(\pi/6) + \cos(\pi/4)\sin(\pi/6)$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \boxed{\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}}$

(3) Suppose we know that

- $\cos\alpha = 3/7$ and α in Q IV
- $\sin\beta = \frac{7}{25}$ and β in Q II

What is $\sin(\alpha+\beta)$?

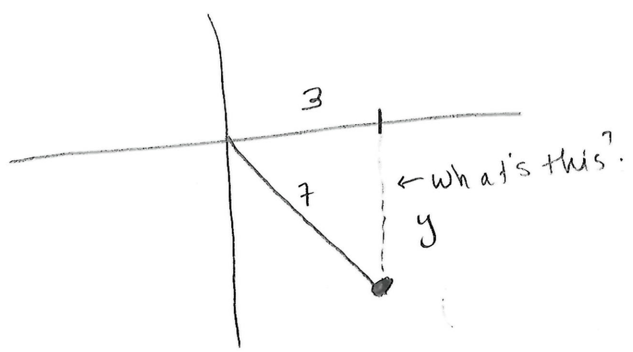
Know

Sol/ Whatever else is true, $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

Need

$\underline{\sin\alpha}$: Know $\cos\alpha = \frac{3}{7} = \frac{x}{r}$ in Q IV

$\rightsquigarrow$

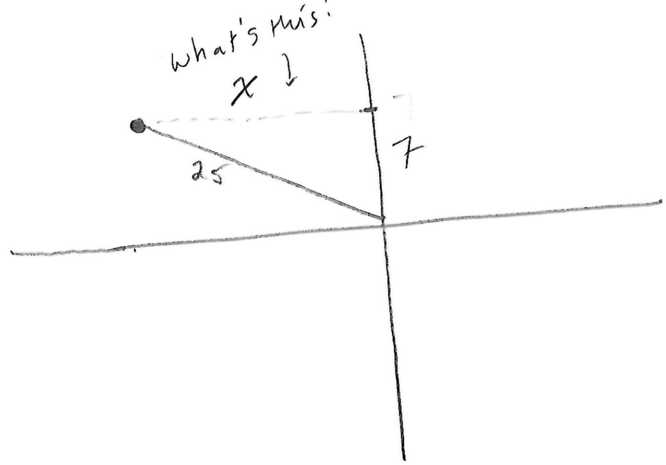


$3^2 + y^2 = 7^2 \Rightarrow 9 + y^2 = 49 \Rightarrow y^2 = 40 \Rightarrow y = \pm\sqrt{40}$

From picture, $y = -\sqrt{40}$

Then $\sin\alpha = \frac{y}{r} = \frac{-\sqrt{40}}{7}$

$\underline{\cos\beta}$: Know $\sin\beta = \frac{7}{25} = \frac{y}{r}$ in Q II



$$x^2 + 7^2 = 25^2 \Rightarrow x^2 + 49 = 625$$

$$\Rightarrow x^2 = 576 \Rightarrow x = \pm 24$$

From picture, $x = -24$

Then $\cos\beta = \frac{x}{r} = \frac{-24}{25}$.

Thus, $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

$$= \frac{-\sqrt{40}}{7} \cdot \frac{-24}{25} + \frac{3}{7} \cdot \frac{7}{25}$$

$$= \boxed{\frac{\sqrt{40} \cdot 24 + 21}{7 \cdot 25}}$$