7/25/18

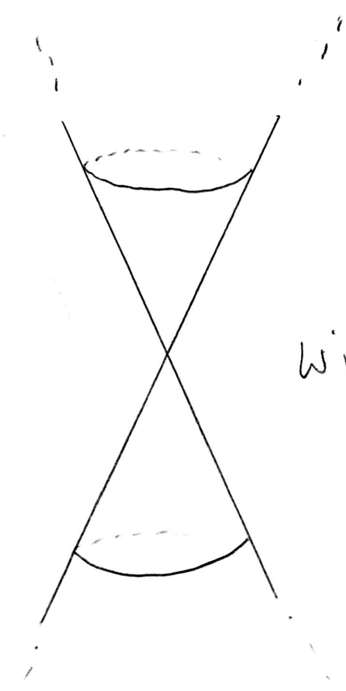
<u>Announcements</u>

- HW 6 due today
- Recommended ex.
- Final in <u>one</u> week
  'Bonus' Friday
- Review session (?)

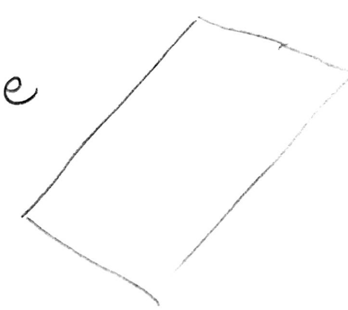
<u>Today</u>

- The ellipse (11.1)
- The hyperbola (11.2)

The ellipse, hyperbola, and parabola are conic sections.
obtained by slicing a (double) cone



with a plane

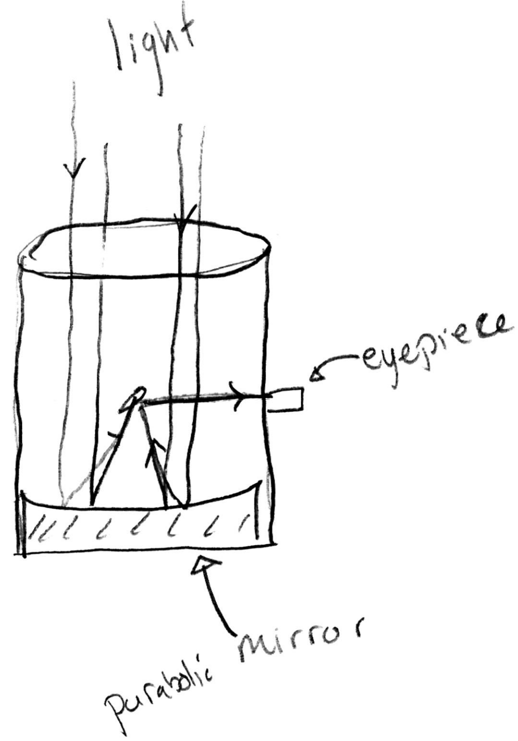


(visualization)

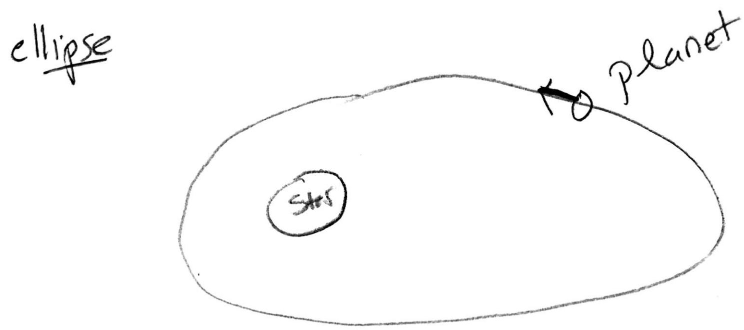
Why do we care?

(lots of reasons)

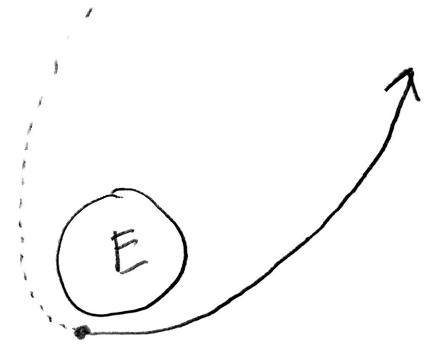
① Telescopes:



② orbits are conic sections



parabola

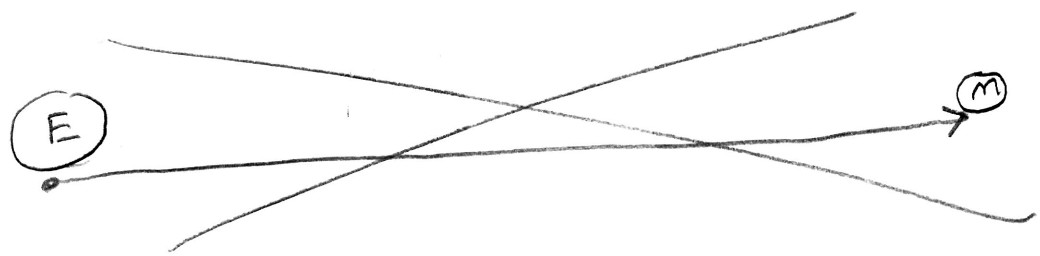


ship's vel. = escape vel.

hyperbola

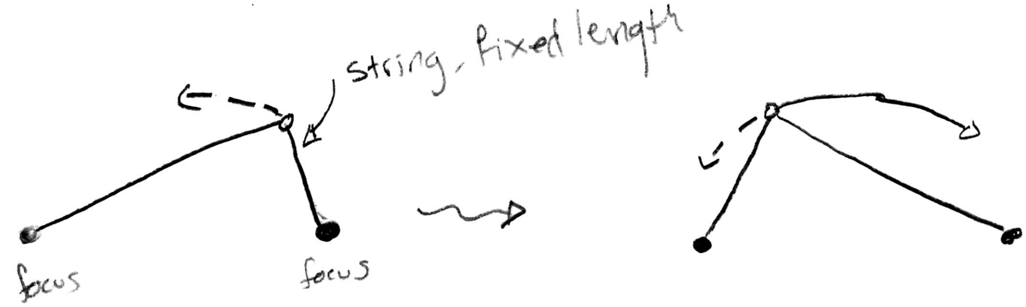
ship's vel > escape vel.

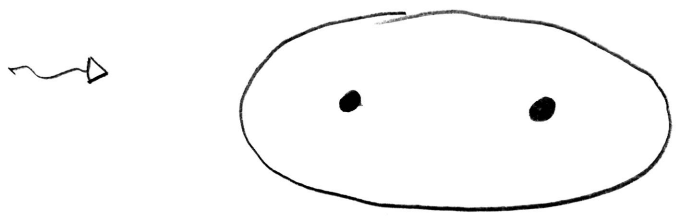
Apollo missions:



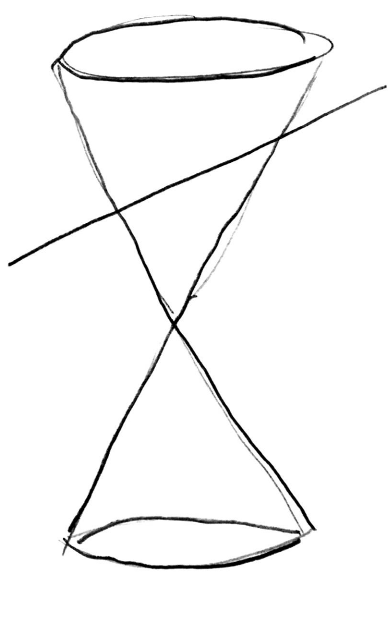
Ellipse

<u>Def</u>: An ellipse is the set of (x,y) such that the sum of the distances between (x,y) and two fixed points (foci, plural of focus) is a constant

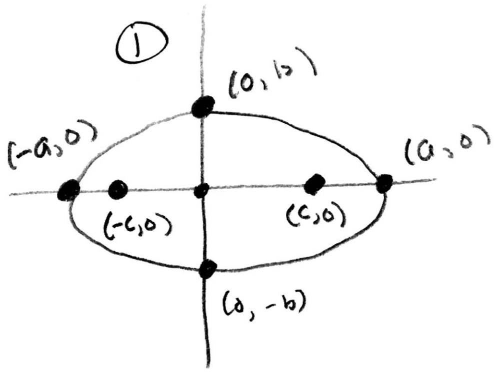




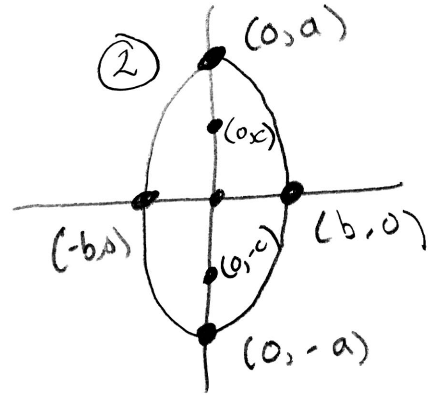
Conic section:



For us, ellipse will either be

① 

or

② 

*Let $a > b > 0$ and $c^2 = a^2 - b^2$ *

① • Equation: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

larger

• Major axis (longer): x-axis

• minor axis (shorter): y-axis

• Center: (0, 0)

• Foci: $(c, 0)$ and $(-c, 0)$

• vertices = endpts of major axis: $(a, 0)$ and $(-a, 0)$

• endpts of minor axis: $(0, b)$ and $(0, -b)$

② • Equation: $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$

larger

• Major axis: y-axis

• minor axis: x-axis

• center: (0, 0)

• Foci: $(0, c)$ and $(0, -c)$

• vertices: $(0, a)$ and $(0, -a)$

• endpts of minor axis: $(b, 0)$ and $(-b, 0)$

Full table p. 954

~~For both ① and ②, the eccentricity = $e = \frac{c}{a}$.~~

~~(e always between 0 and 1)~~

Example: Given $\frac{x^2}{16} + \frac{y^2}{9} = 1$, identify center, vertices, foci, ~~eccentricity~~, and graph it.

- $\boxed{\text{center: } (0,0)}$ (for us, always)

Let's get it to look like standard form:

$$\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1 \qquad a = 4, \quad b = 3$$

larger, so = a.

$\boxed{\text{Vertices: } (4,0) \text{ and } (-4,0)}$

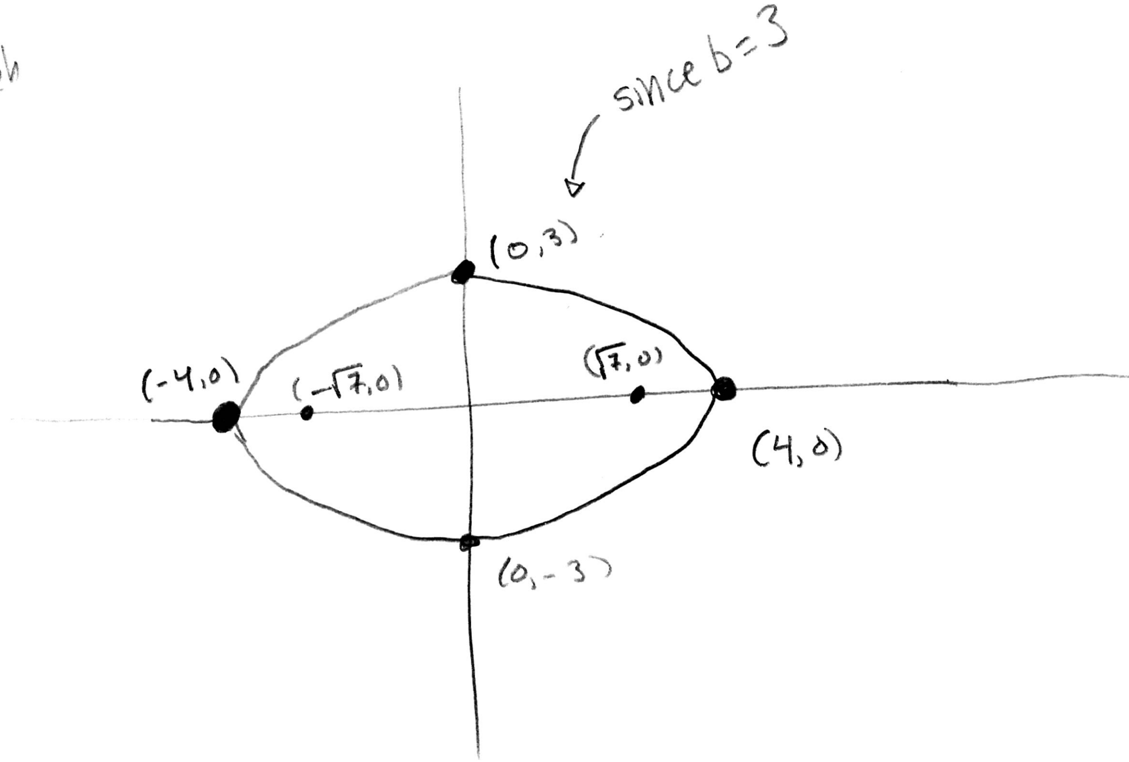
If $c^2 = a^2 - b^2 = 16 - 9 = 7 \Rightarrow c = \sqrt{7}$

(take pos)

$\boxed{\text{foci}: (c, 0) \text{ and } (-c, 0)}$

~~eccentricity: $e = \frac{c}{a} = \frac{\sqrt{7}}{4}$~~

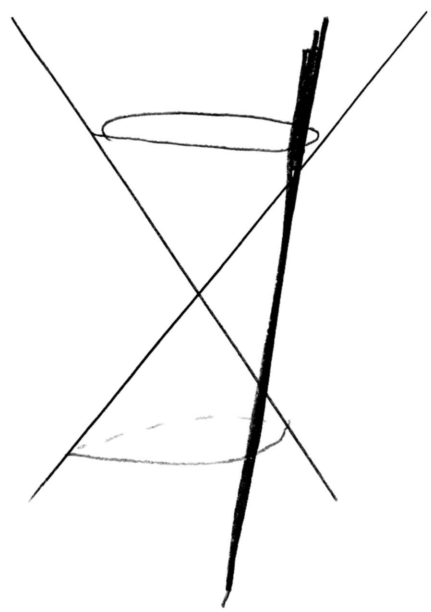
Graph



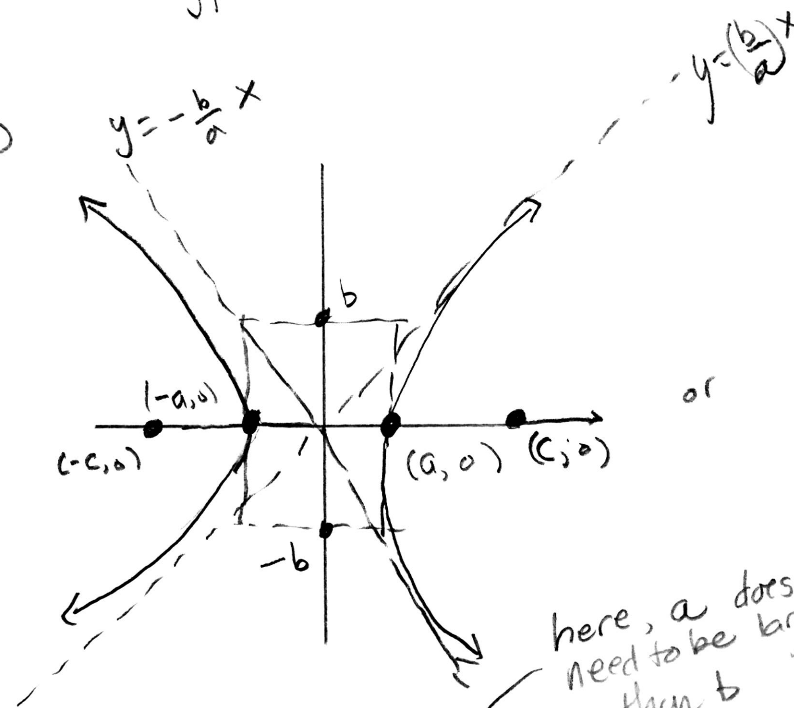
Hyperbola

Def: A *hyperbola* is the set of all (x,y) such that the difference in distance between (x,y) and two fixed pts. (foci) is a pos. constant

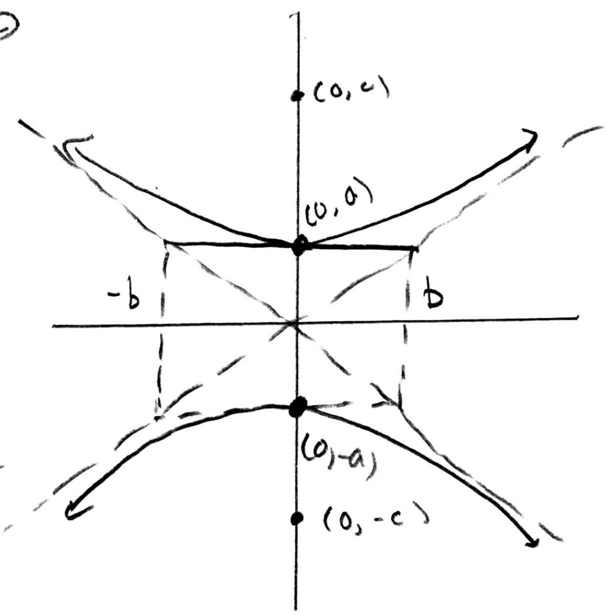
conic section:



For us, hyperbola will either be

① 

or

② 

here, a doesn't need to be larger than b

* Let $a, b > 0$, $c^2 = a^2 + b^2$ *

① • Eq'n $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

• Transverse axis: x-axis

• center: $(0,0)$

• Foci: $(c,0)$ and $(-c,0)$

• vertices: $(a,0)$ and $(-a,0)$

• Asymptotes: $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$

② • Eq'n: $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$

• Transverse axis: y-axis

• Center: $(0,0)$

• Foci: $(0,c)$ and $(0,-c)$

• vertices: $(0,a)$ and $(0,-a)$

• Asymptotes: $y = \frac{a}{b}x$ and $y = \frac{-a}{b}x$

Full table p. 970

Example Given $\frac{x^2}{4} - \frac{y^2}{9} = 1$, graph and identify center, vertices, foci, and asymptotes.

① Find center and vertices

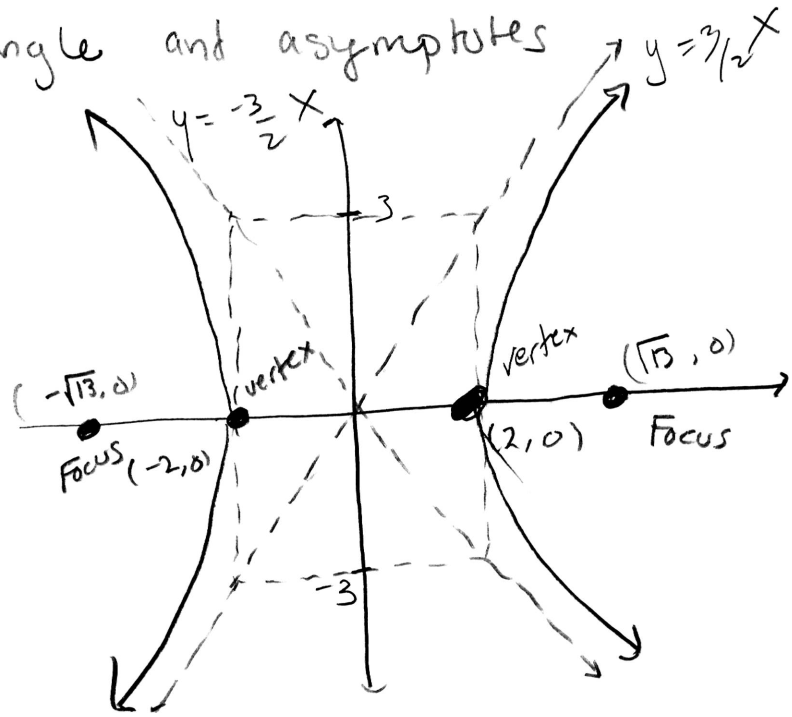
$$\frac{x^2}{2^2} - \frac{y^2}{3^2} = 1$$

$a = 2$

$b = 3$

$\boxed{\text{Center: } (0,0)}$ (always) $\boxed{\text{Vertices: } (2,0) \text{ and } (-2,0)}$

② Draw rectangle and asymptotes



③ Fill in and label: $c^2 = 2^2 + 3^2 \Rightarrow c^2 = 13 \Rightarrow c = \sqrt{13}$

$\boxed{\text{Foci: } (\sqrt{13}, 0) \text{ and } (-\sqrt{13}, 0)}$

$\boxed{\text{asymptotes } y = \frac{3}{2}x \text{ and } y = -\frac{3}{2}x}$