$\boxed{6/6/18}$

Lecture

Today

- Trig functions of acute angles ($S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$)
- Pythagorean identities
- ~~Cofunction identities~~
- Trig functions of any angle

$\boxed{\text{Trig functions of acute angles}}$

Def An angle θ is acute if $0^\circ \leq \theta < 90^\circ$

$(0 \leq \theta < \pi/2)$

(less than 90°)

We define $\sin\theta$, $\cos\theta$, etc. for acute angles using right triangles

SOH CAH TOA

Def

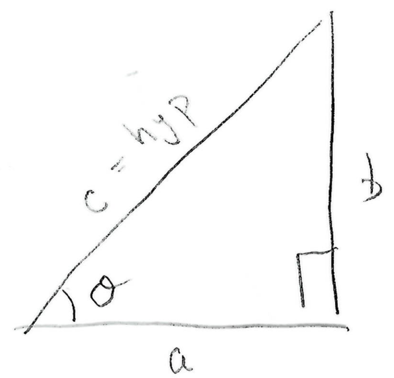
- $\sin\theta = \dfrac{opp}{hyp}$ • $\cos\theta = \dfrac{adj}{hyp}$ • $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{opp}{hyp}}{\frac{adj}{hyp}} = \dfrac{opp}{adj}$

define

- $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{hyp}{adj}$ • $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{hyp}{adj}$ • $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{adj}{opp}$

Note: Book deals w/ $\csc\theta$, $\sec\theta$, $\cot\theta$ in a slightly different order

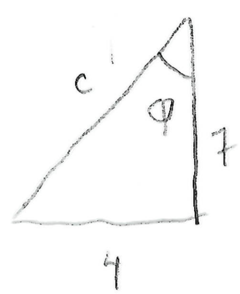
e.g.

① 

θ is acute

- $\sin\theta = \dfrac{b}{c}$ • $\cos\theta = \dfrac{a}{c}$ • $\tan\theta = \dfrac{b}{a}$.

② Suppose given



What is $\sin\varphi$?

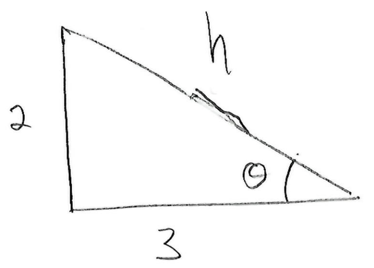
Sol: We know $\sin\varphi = \dfrac{opp}{hyp} = \dfrac{4}{c}$. what is c?

$4^2 + 7^2 = c^2$ (pyth. thm) $\Rightarrow 16 + 49 = c^2 \Rightarrow 65 = c^2$

$\Rightarrow c = \sqrt{65}$, so $\boxed{\sin\varphi = \dfrac{4}{\sqrt{65}}}$ for practice evaluate other trig fns.

(Can skip if needed)

③ Find the exact value of all six trig fns. for θ



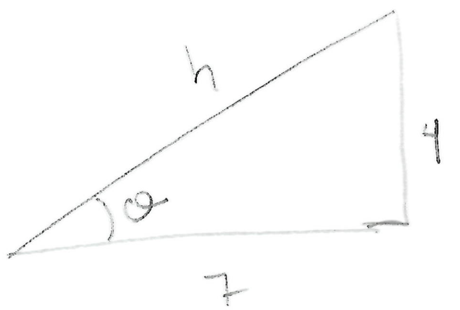
Sol: First, what is h?

Know $3^2 + 2^2 = h^2 \Rightarrow 9 + 4 = h^2 \Rightarrow 13 = h^2 \Rightarrow h = \sqrt{13}$

- $\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{2}{\sqrt{13}}$
- $\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{3}{\sqrt{13}}$
- $\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{2}{3}$
- $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{2}{\sqrt{13}}} = \dfrac{\sqrt{13}}{2}$
- $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{3}{\sqrt{13}}} = \dfrac{\sqrt{13}}{3}$
- $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{\frac{2}{3}} = \dfrac{3}{2}.$

(4) Suppose $\tan\theta = \frac{4}{7}$. What are other trig fns? (θ acute)

Sol: $\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{7}$



What is h? $7^2+4^2=h^2 \Rightarrow 49+16=h^2 \Rightarrow 65=h^2$

$\Rightarrow h=\sqrt{65}$ (take positive)

- $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{\sqrt{65}}$ • $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{7}{\sqrt{65}}$

- $\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{65}}{4}$ • $\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{65}}{7}$ • $\cot\theta = \frac{1}{\tan\theta} = \frac{7}{4}$

Special acute angles

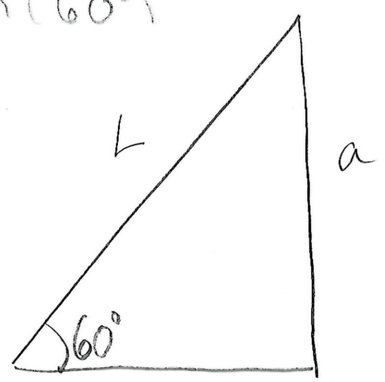
Trig fns of

30° ($\pi/6$ rad), 45° ($\pi/4$ rad), 60° ($\pi/3$ rad)

are known

Full table on p. 513

e.g. $\sin(60^\circ)$



↑ this side will always be $\frac{1}{2}L$

$$\sin(60^\circ) = \frac{\text{opp}}{\text{hyp}} = \frac{a}{L}$$

also, $(\frac{1}{2}L)^2 + a^2 = L^2 \Rightarrow \frac{1}{4}L^2 + a^2 = L^2 \Rightarrow a^2 = \frac{3}{4} \cdot L^2$

$$\Rightarrow a = \sqrt{\frac{3}{4}L^2} \Rightarrow a = \frac{\sqrt{3}}{2}L$$

Then $\boxed{\sin(60^\circ) = \frac{a}{L} = \frac{\frac{\sqrt{3}}{2}\cancel{L}}{\cancel{L}} = \frac{\sqrt{3}}{2}}$

Pythagorean identities

① $\underbrace{\sin^2\theta + \cos^2\theta = 1}$ $\quad \sin^2\theta = (\sin\theta)^2$ ② $\tan^2\theta + 1 = \sec^2\theta$ ③ $1 + \cot^2\theta = \csc^2\theta$

* Knowing this is enough *

If we know $\sin^2\theta + \cos^2\theta = 1$, then divide by $\cos^2\theta$

$$\Rightarrow \quad \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

So ① ⇒ ② and similarly ① ⇒ ③

e.g. Given $\csc\theta = \frac{5}{4}$ for an acute angle θ, what is $\cot\theta$?

Sol: We know $1 + \cot^2\theta = \csc^2\theta$

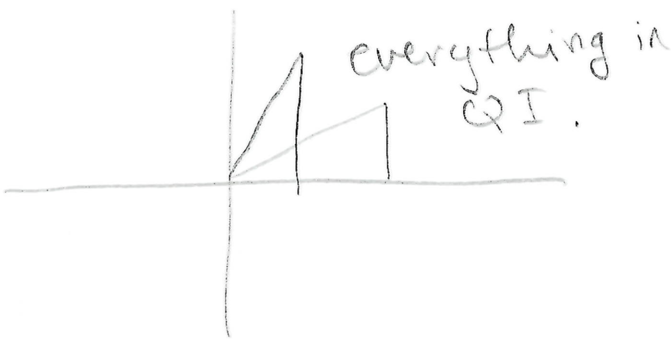
$$\Rightarrow 1 + \cot^2\theta = \left(\frac{5}{4}\right)^2 \Rightarrow 1 + \cot^2\theta = \frac{25}{16}$$

$$\Rightarrow \frac{16}{16} + \cot^2\theta = \frac{25}{16} \Rightarrow \cot^2\theta = \frac{9}{16}$$

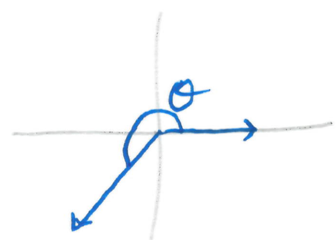
$$\Rightarrow \boxed{\cot\theta = 3/4}$$

Trig fns of any angle

We've defined trig fns of acute angles, i.e.

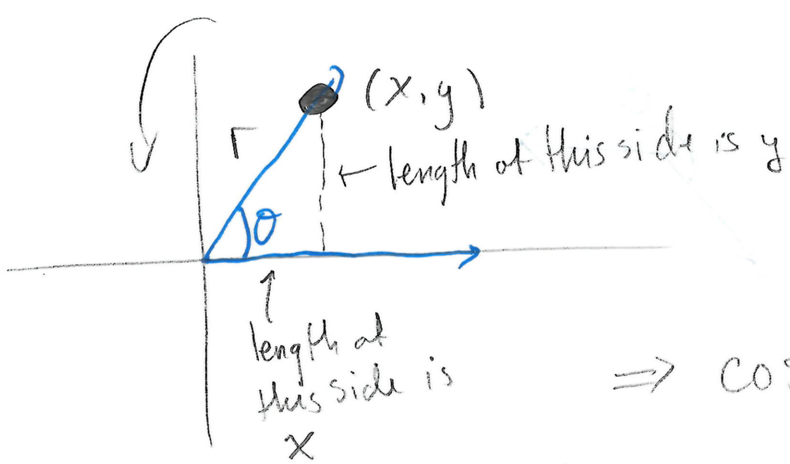


but what about



?

Well, for an acute angle, e.g. $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$



$\Rightarrow \cos\theta = \dfrac{x}{r}$

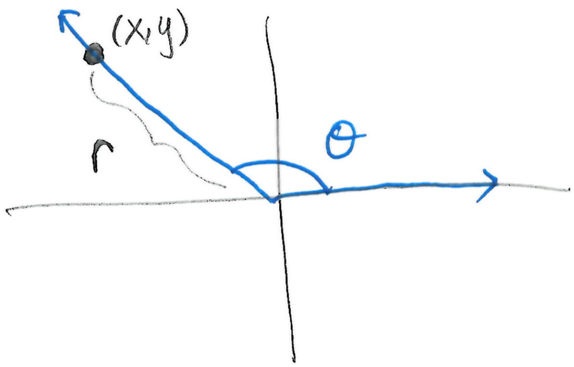
and $r^2 = x^2 + y^2$

$\Rightarrow r = \sqrt{x^2+y^2}$

and $\sin\theta = \dfrac{y}{r}$

Def For any angle θ in standard position, pick a point on terminal side

e.g

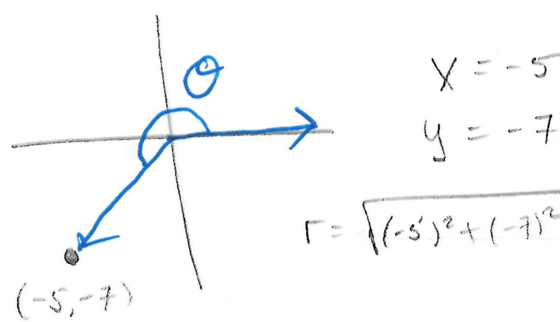


- $\sin\theta = \frac{y}{r}$
- $\cos\theta = \frac{x}{r}$
- $\tan\theta = \frac{y}{x}$ ← provided $\neq 0$

- $\csc\theta = \frac{r}{y}$ ← provided $y \neq 0$
- $\sec\theta = \frac{r}{x}$ ← " "
- $\cot\theta = \frac{x}{y}$

e.g. Suppose $(-5,-7)$ is on the terminal side of angle θ in standard pos What is value of trig fns?

Sol: Sketch:



$x = -5$

$y = -7$

$r = \sqrt{(-5)^2 + (-7)^2} = \sqrt{74}$

- $\sin\theta = \frac{y}{r} = \frac{-7}{\sqrt{74}}$
- $\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{74}}$
- $\tan\theta = \frac{y}{x} = \frac{7}{5}$

- $\csc\theta = \frac{r}{y} = \frac{-\sqrt{74}}{7}$
- $\sec\theta = \frac{r}{x} = \frac{-\sqrt{74}}{5}$
- $\cot\theta = \frac{x}{y} = \frac{5}{7}$.