$\boxed{6/15/18}$

Today

- Graphing sec(x) and csc(x)
- Graphing tan(x) and cot(x)

Announcements

- HW 2 due Wed
- Midterm 1 is Tuesday after next.

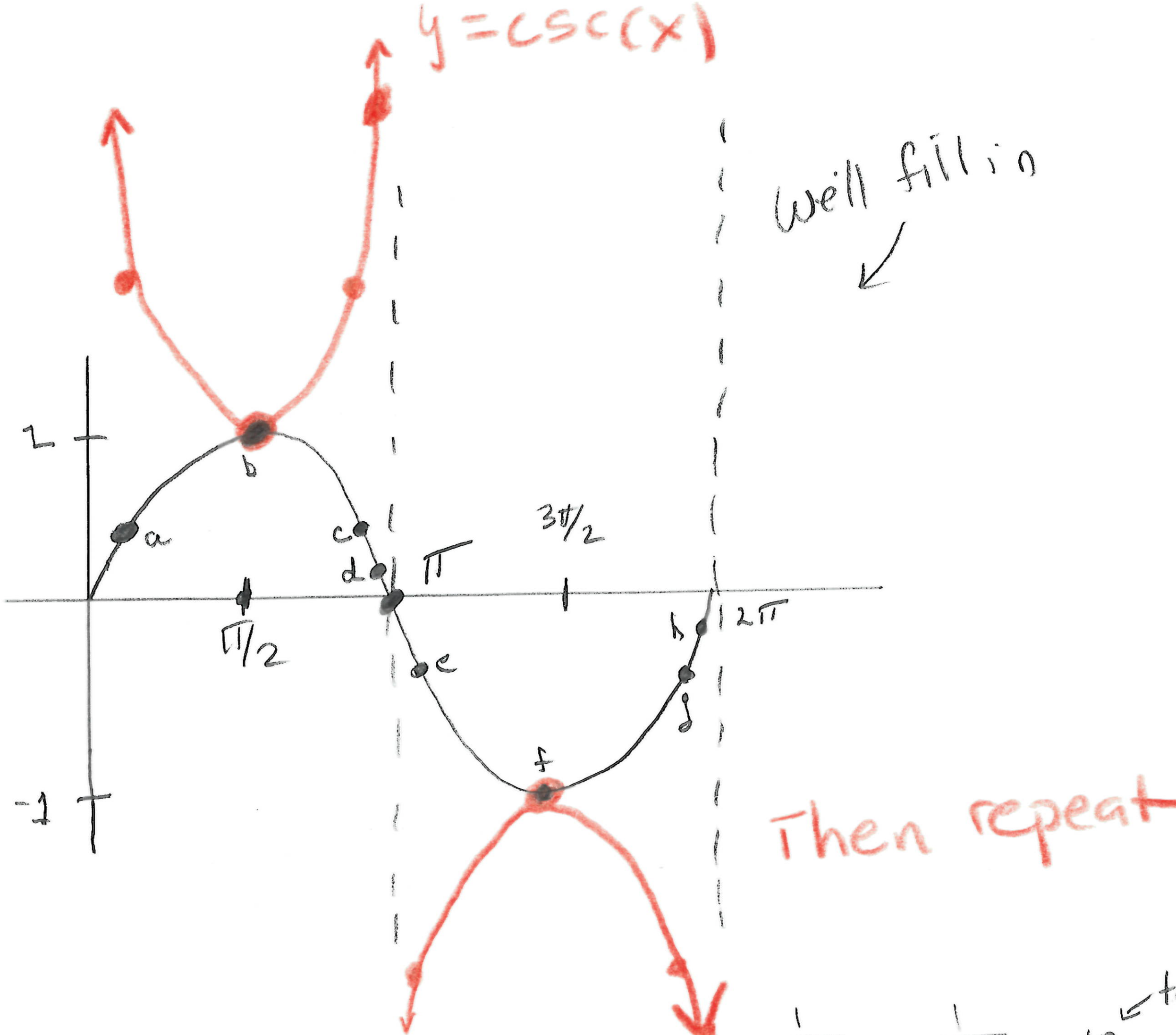
$\boxed{\sec(x) \text{ and } \csc(x)}$

- Remember that $\csc(x) = \frac{1}{\sin x}$ and $\sec(x) = \frac{1}{\cos x}$.

$\Rightarrow$ we'll build graph of csc(x) from sin x and sec(x) from cos(x) csc(x)

One full period of sin(x)

LEAVE SPACE IN NOTES

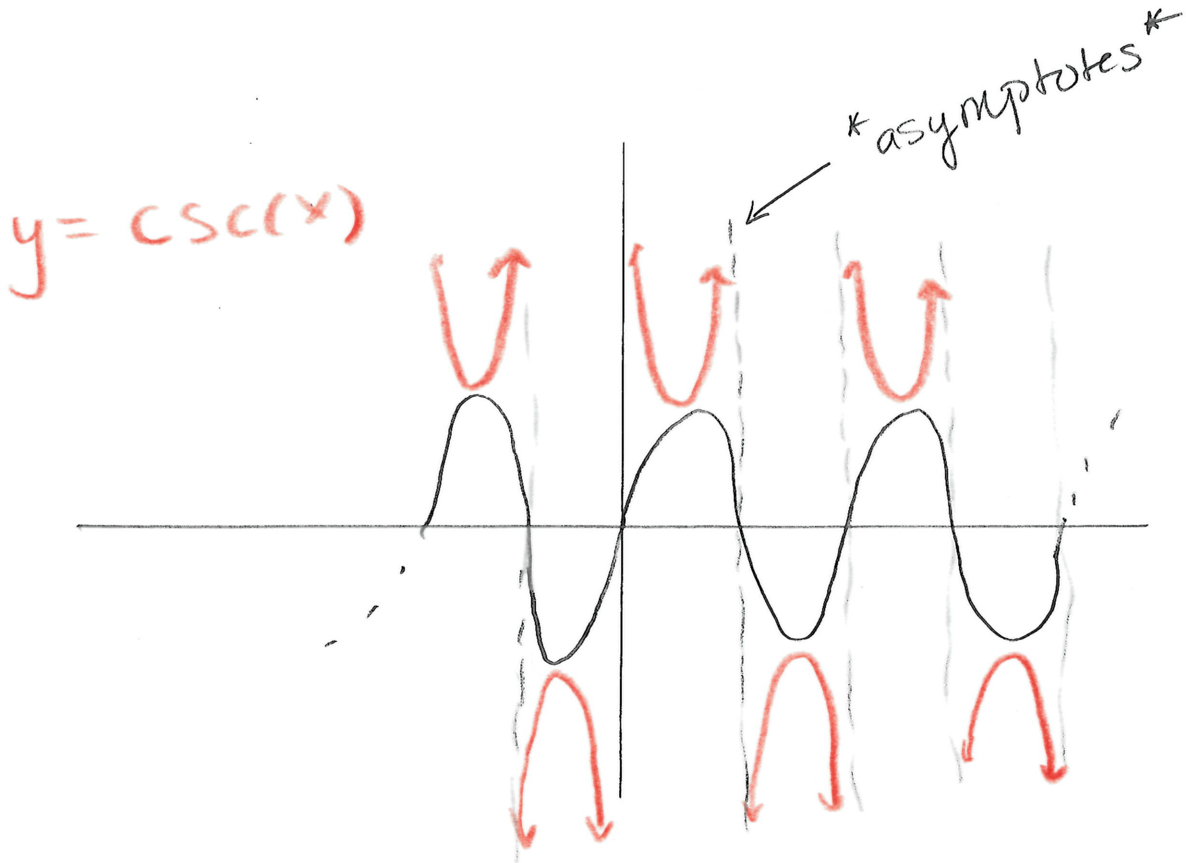


- at a, sin X is e.g. $\frac{1}{10}$. $\Rightarrow csc(x) = \frac{1}{\sin x} = \frac{1}{\frac{1}{10}} = 10$ ↙ tall
- at b, $\sin x = 1 \Rightarrow csc(x) = \frac{1}{\sin x} = \frac{1}{1} = 1$
- at c, $\sin(x) = \frac{1}{10} \Rightarrow csc(x) = 10$
- at d, sin(x) is e.g. $\frac{1}{100} \Rightarrow csc(x) = \frac{1}{\frac{1}{100}} = 100$ ↙ very tall

- at e, sin(x) is e.g. $-\frac{1}{10} \Rightarrow \csc(x) = -10$
- at f, $\sin x = -1 \Rightarrow \csc(x) = -1$
- at g, $\sin(x) = -\frac{1}{10} \Rightarrow \csc(x) = -10$
- at h, sin(x) is e.g. $-\frac{1}{100} \Rightarrow \csc(x) = -100$

Intuition: $\frac{1}{\text{small}}$ = big $\Rightarrow$ when sin(x) nears 0, csc(x) blows up (very pos. or very neg).

So



Domain of $y = \csc(x)$ is all numbers except $0, \pi, -\pi, 2\pi, -2\pi$, etc.

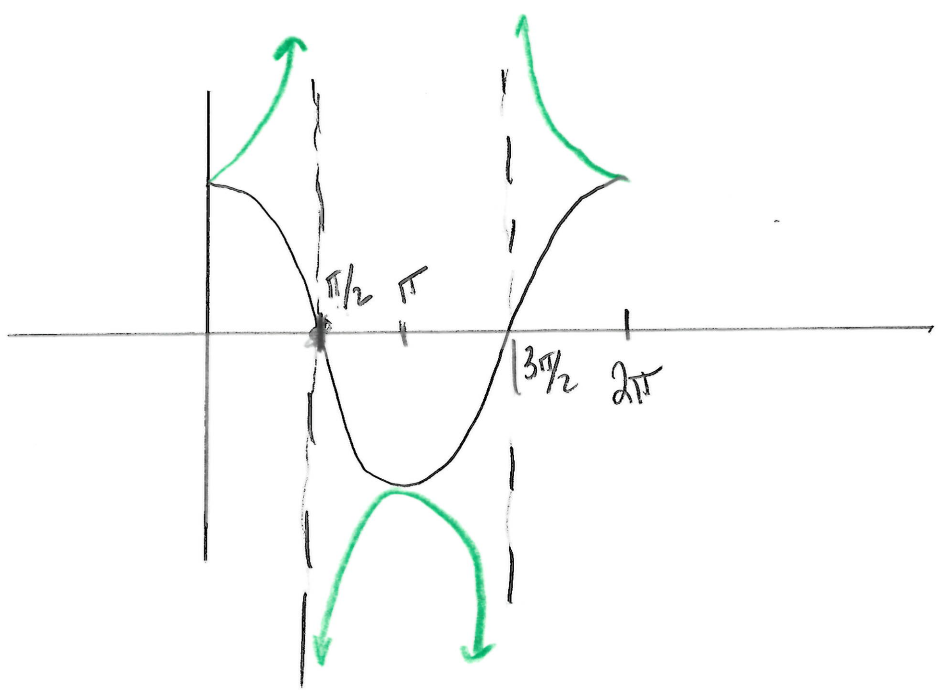
Range of $y = \csc(x)$ is all $y \geq 1$ or $y \leq -1$

Period of $y = \csc(x)$ is 2π

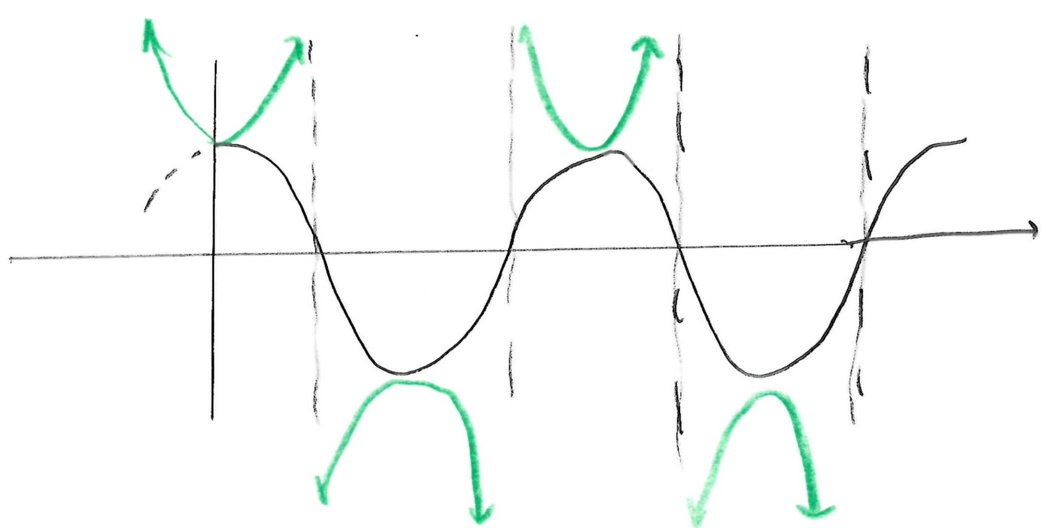
sec(x)

· Play same game but with cos(x) instead of sinx b/c $\sec(x) = \frac{1}{\cos(x)}$.

One full period of cos(x)



Then repeat



- **Domain** of $y = \sec(x)$ is all numbers except

  $\pi/2, -\pi/2, 3\pi/2, -3\pi/2, \ldots$

  $\Rightarrow$ all x except $n\pi + \pi/2$

- **Range** of $y = \sec(x)$ is all $y \geq 1$ or $y \leq -1$

- **Period** of $y = \sec(x)$ is 2π.

Dealing w/ transformations

e.g. Graph $y = 3\sec(2x) + 1$

Strategy

1. Graph $y = 3\cos(2x)$, then "flip"
2. Then shift up 1
   (usually easier this way)

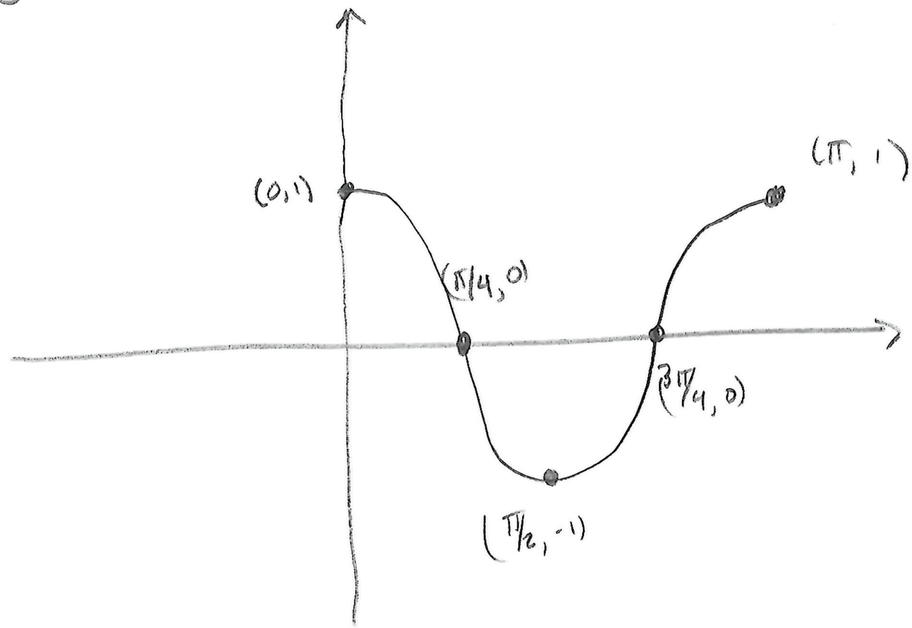
1. $y = 3\underbrace{\cos(2x)}$

   Solve $0 \le 2x \le 2\pi$

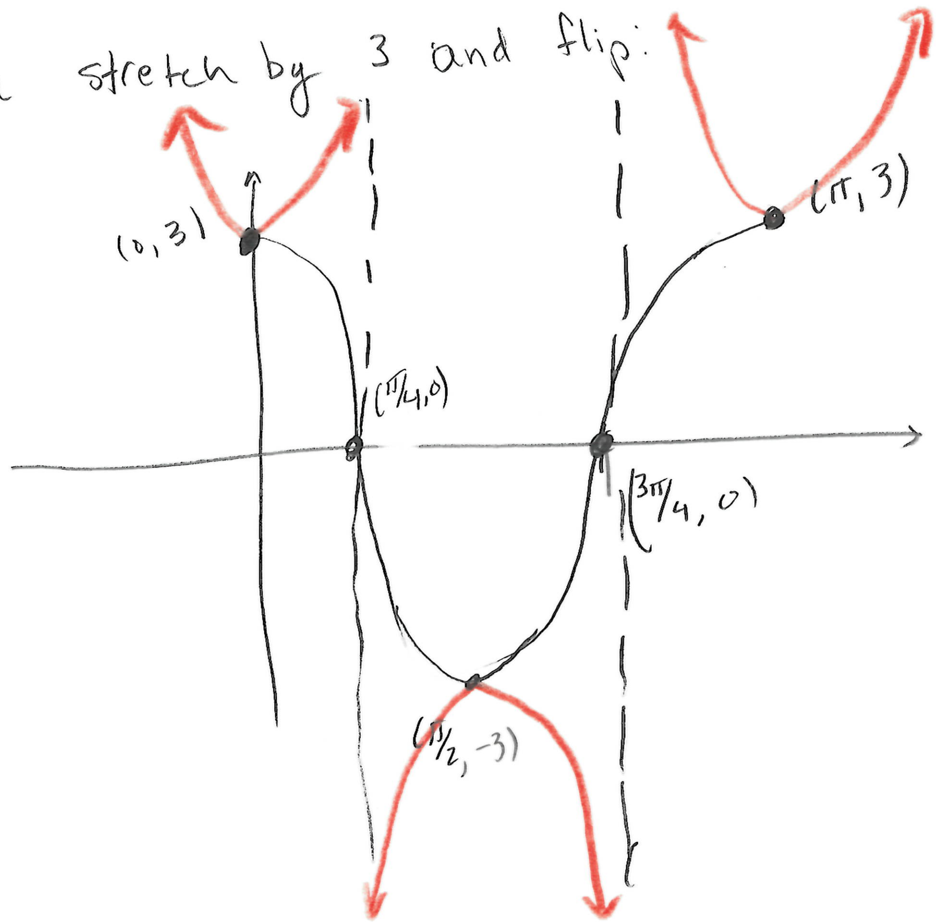
   $\Rightarrow 0 \le x \le \pi$ is interval of one full period.

Keypoints

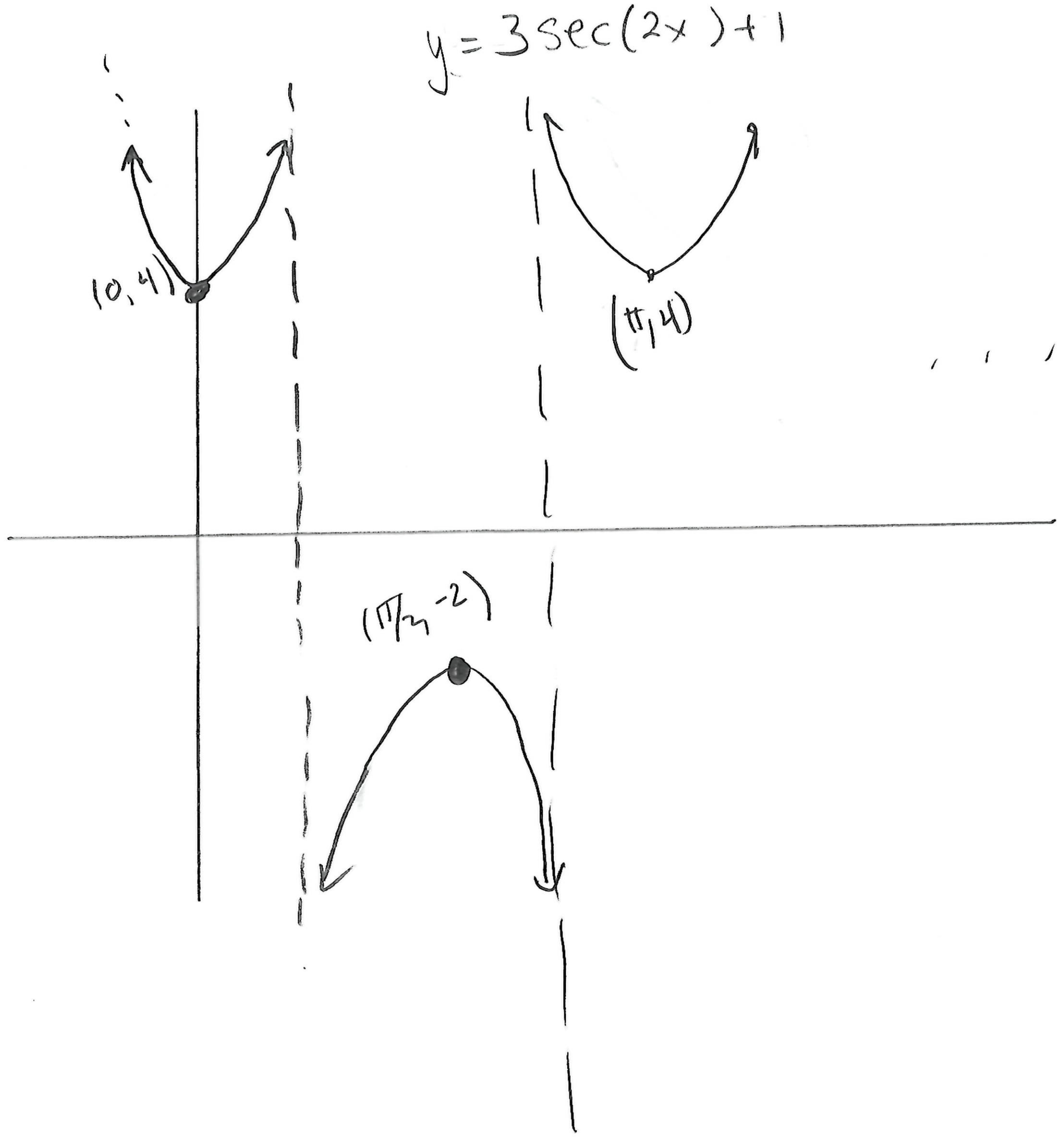
So $y = \cos(2x)$



Then stretch by 3 and flip:



Then shift up 1



$\boxed{\tan(x) \text{ and } \cot(x)}$

tan(x)

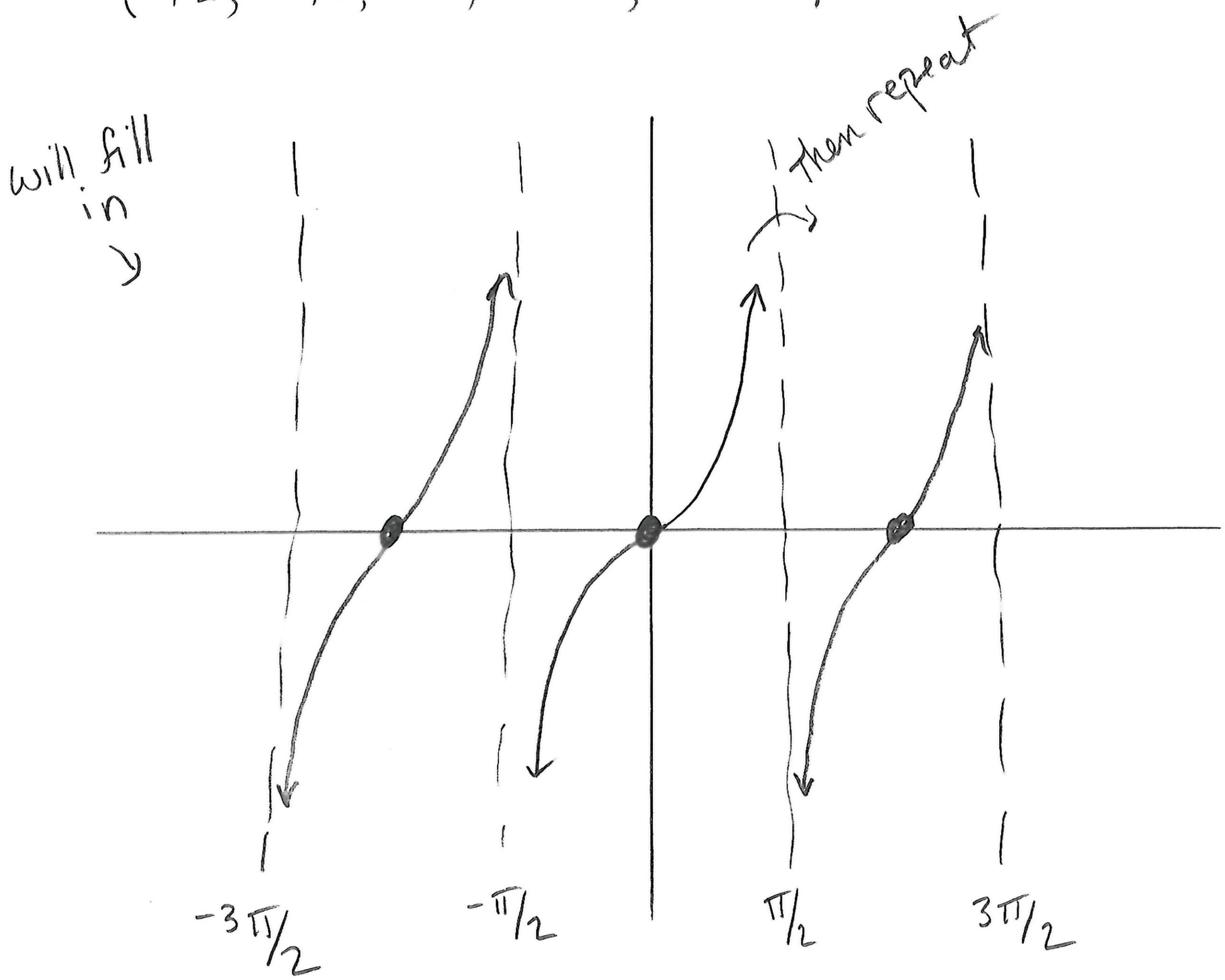
Recall: $\tan x = \dfrac{\sin x}{\cos x}$ has period π

will be 0 when $\sin(x) = 0$

$(0, \pi, -\pi, 2\pi, -2\pi, \dots)$

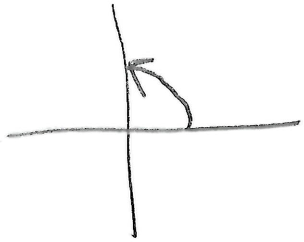
and have asymptotes when $\cos(x) = 0$

$(\pi/2, 3\pi/2, -\pi/2, -3\pi/2, \dots)$



Just need to decide pos vs. neg.

when $0 \leq x \leq \pi/2$

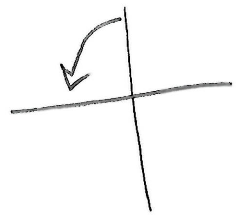


$\sin(x) \geq 0$ and

$\cos(x) \geq 0$

so $\tan(x) \geq 0$ between 0 and $\pi/2$

when $-\pi/2 \leq x \leq \pi/2$



$\sin(x) \geq 0$ and

$\cos(x) \leq 0$

$\Rightarrow \tan(x) \leq 0$ between $-\pi/2$ and 0

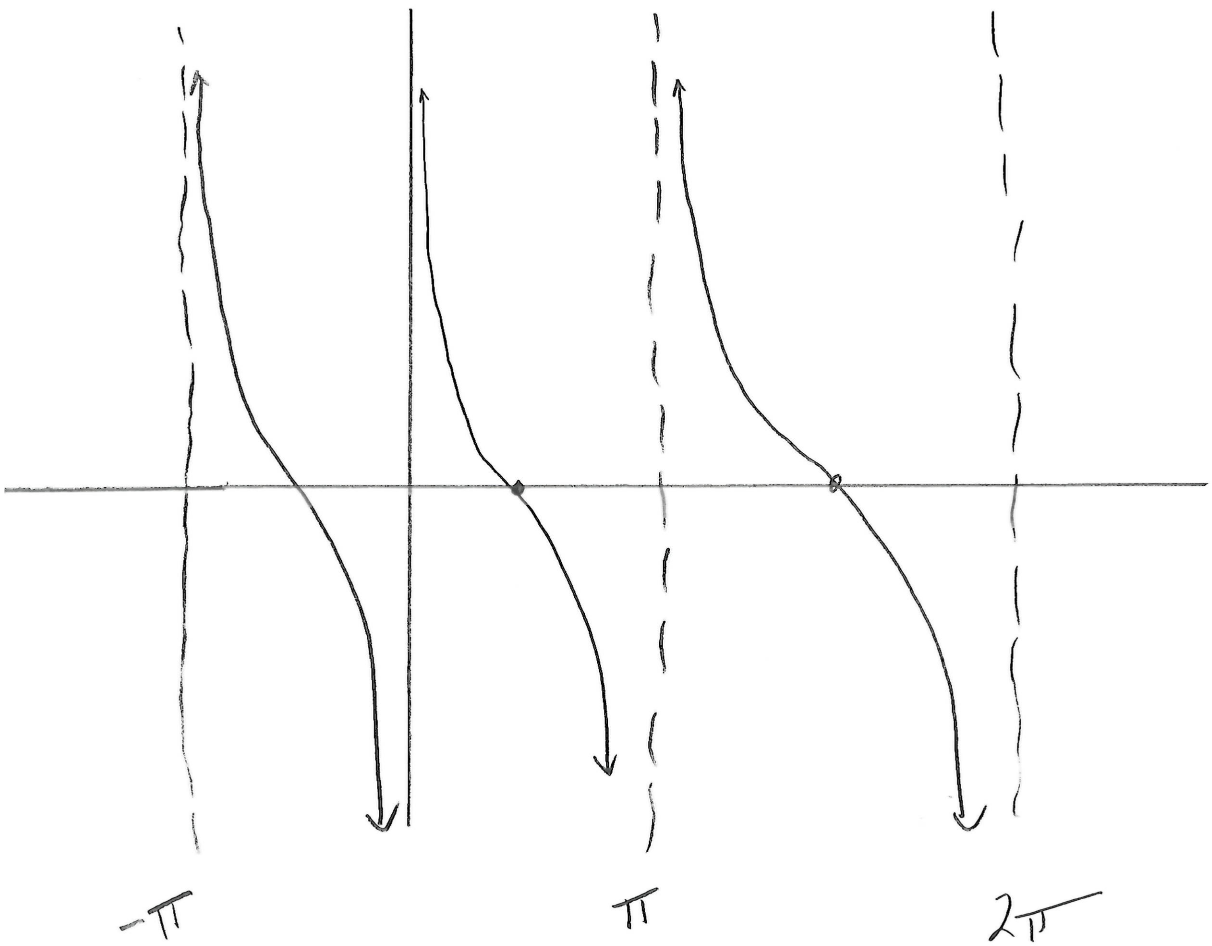
<u>Domain</u> of $y = \tan(x)$ is all x except $\pi/2, -\pi/2, 3\pi/2, -3\pi/2, \ldots$

<u>Range</u> is all numbers

<u>Period</u> is π

$y = \cot x$

Same story, but with $\cot x = \dfrac{\cos x}{\sin x}$, you get



Domain of $y = \cot x$ is all x except $0, \pi, -\pi, 2\pi, -2\pi, \ldots$

Range is all numbers

Period is π

Transformations

e.g. Graph $y = \tan(\underbrace{x - \pi/4}_{\theta})$

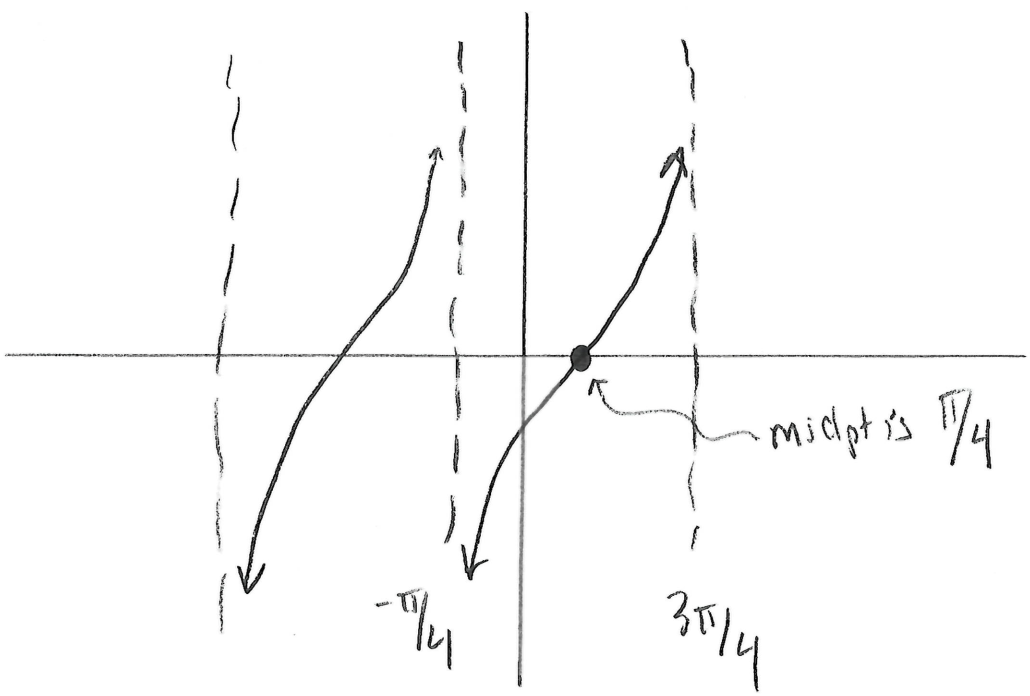
Sol $\tan\theta$ completes a full period for $-\pi/2 < \theta < \pi/2$

$\Rightarrow \tan(x - \pi/4)$ completes a full period

for $-\pi/2 < x - \pi/4 < \pi/2$

$\Rightarrow -\pi/4 < x < 3\pi/4$

asymptote



In general

To graph $y = A\tan(Bx - C) + D$

1. Solve $-\frac{\pi}{2} < Bx - C < \pi/2$
   → gives asymptotes
2. Plot x-intercept half way between asymptotes
3. Graph w/out vertical shift
   (* if A is neg, have to reflect about x-axis *)
4. Then shift up/down by D

For $A\cot(Bx - C) + D$, same thing, except

Solve $0 < Bx - C < \pi$ instead in (1).