6/13/18

Today

- Brief review
- Graphing sin(x) and cos(x) } (5.5)
- Transformations

Announcements

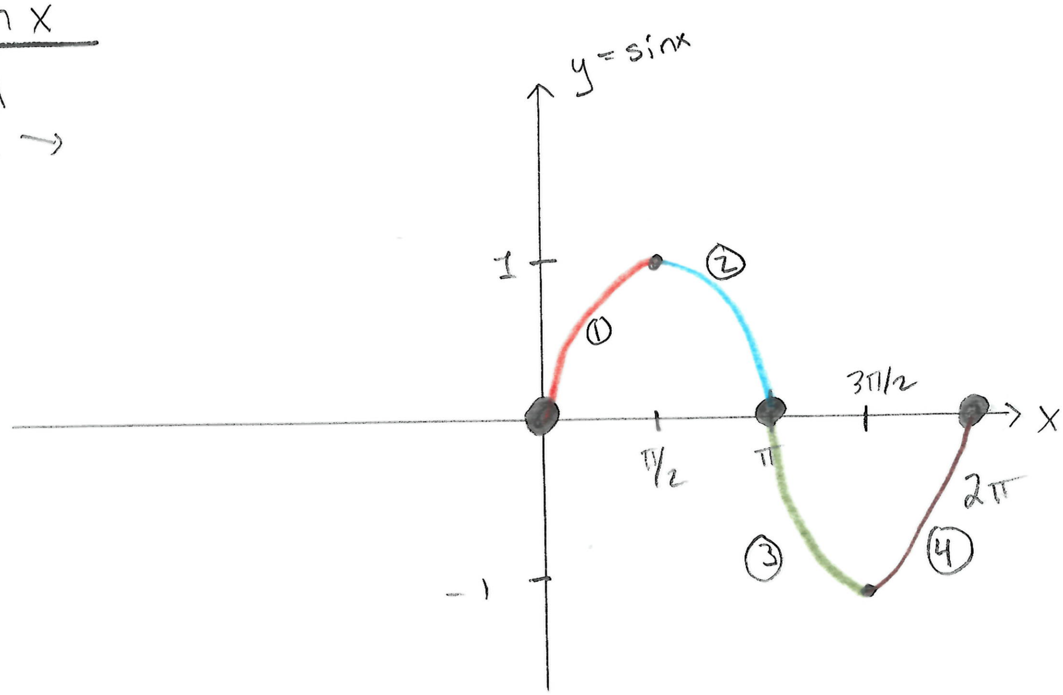
- HW 1 due today
- HW 2 due next Wed
- (unit circle) handout
- $\sin(t) = \sin t$
- office hours

- Defining/evaluating trig functions handout

Graphing $\sin x$ and $\cos x$

$y = \sin x$

we'll fill this in →



- First, recall that $\sin x$ has period 2π

$\Rightarrow$ we can graph $\sin x$ on $[0, 2\pi]$, then it repeats

- Find <u>key points</u> as a guide

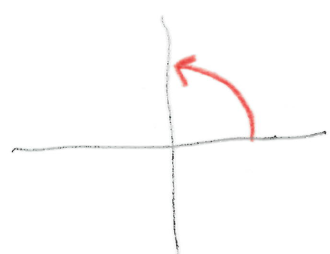
> $\sin(0) = 0$ > $\sin(2\pi) = 0$

> $\sin(\pi) = 0$ > $\sin(\pi/2) = 1$

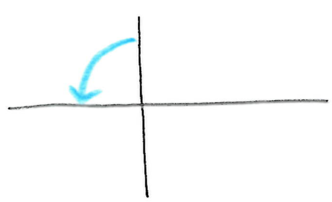
> $\sin(3\pi/2) = -1$

- In between?

① As we go from 0 to $\pi/2$ on unit circle y co-ord (sine) increases from 0 to 1



② From $\pi/2$ to π

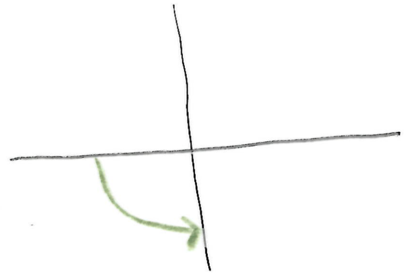


y co-ord decreases from 1 to 0

③ From π to $3\pi/2$

y co-ord decreases

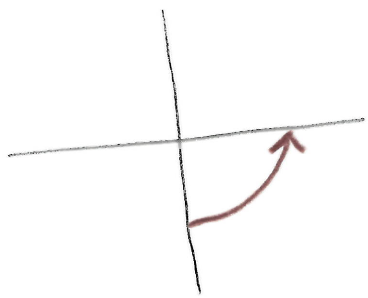
from 0 to -1



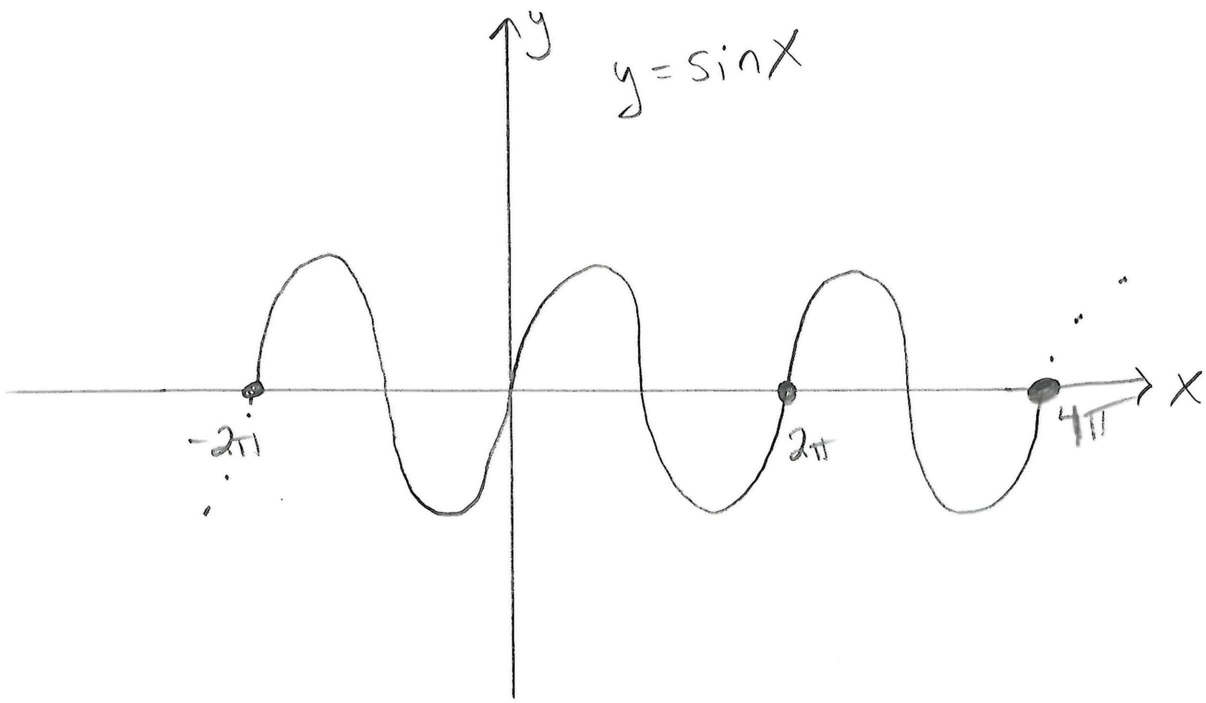
④ From $3\pi/2$ to 2π

y co-ord increases

from -1 to 0



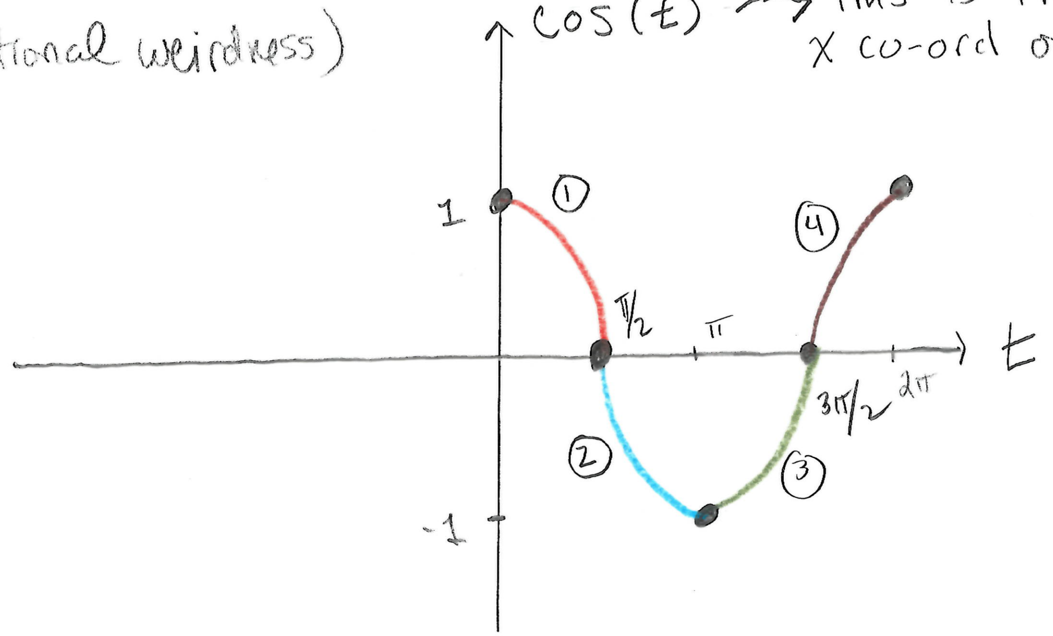
Then repeat, so:



$y = \cos x$

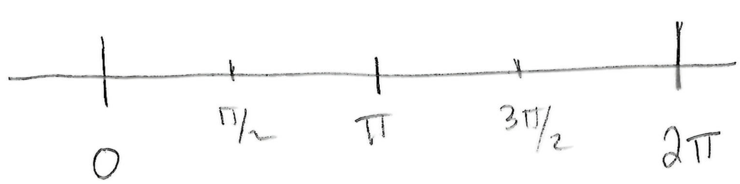
- Very similar, but now care about x co-ord on unit circle (rather than y)

We'll fill in →
(notational weirdness)



→ This is the (horizontal) x co-ord on unit circle

- Find key points as a guide.



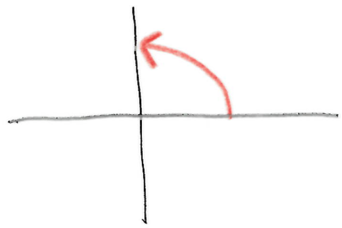
› $\cos 0 = 1$ › $\cos 2\pi = 1$

› $\cos \pi = -1$ › $\cos \pi/2 = 0$

› $\cos 3\pi/2 = 0$

• In between?

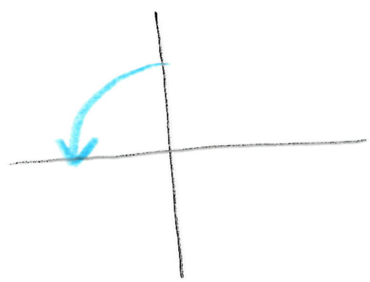
① As we go from 0 to $\pi/2$ on unit circle



x co-ord (cosine) decreases from 1 to 0

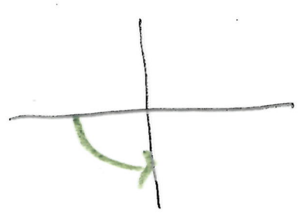
② From $\pi/2$ to π

x co-ord decreases from 0 to -1



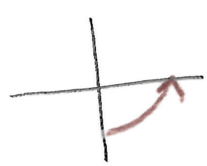
③ From π to $3\pi/2$

x co-ord increases from -1 to 0

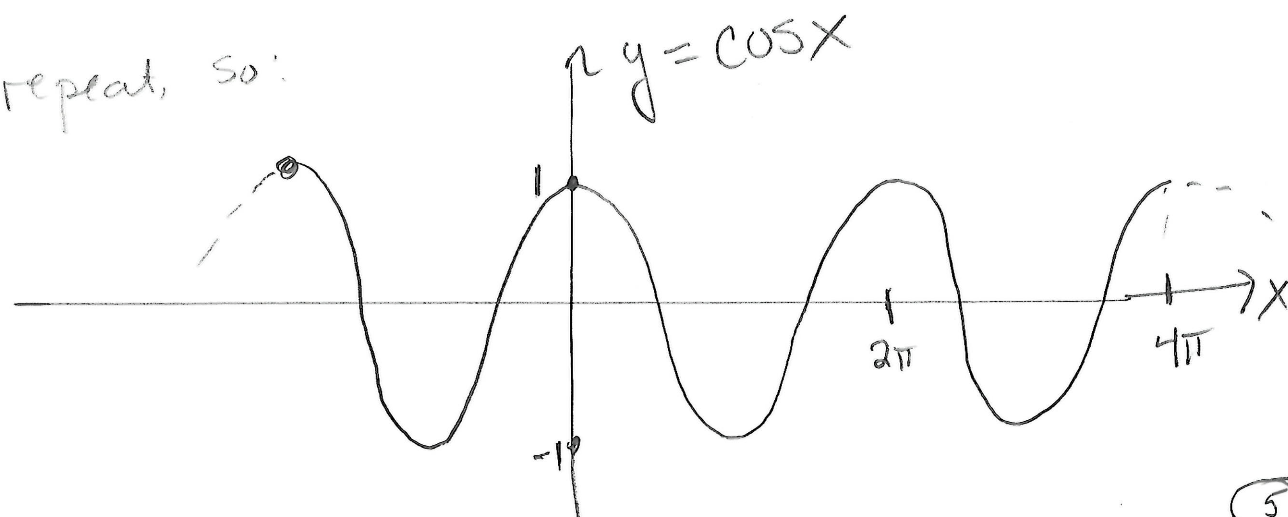


④ From $3\pi/2$ to 2π,

x co-ord increases from 0 to 1



Then repeat, so:



Transformations

$(B > 0)$

$y = A\sin(Bx - C) + D$
$y = A\cos(Bx - C) + D$

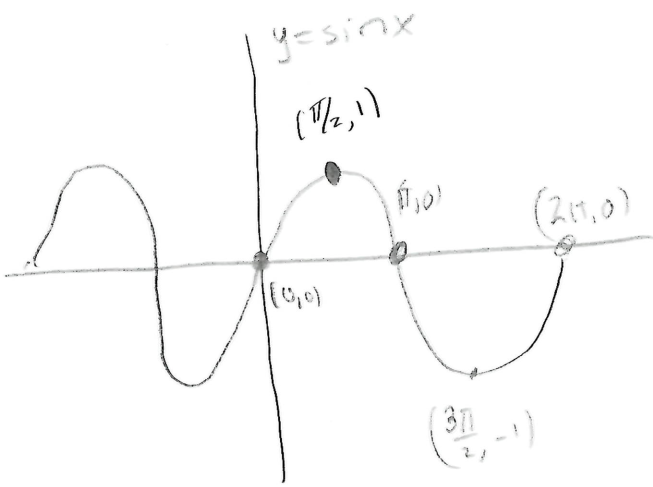
"variations" of what we just did

e.g. $y = 3\sin x + 2$

3: "vertical stretch"

2: move everything up 2

Build from $y = \sin x$



$y = 3\sin x$

every y co-ord multiplied by 3

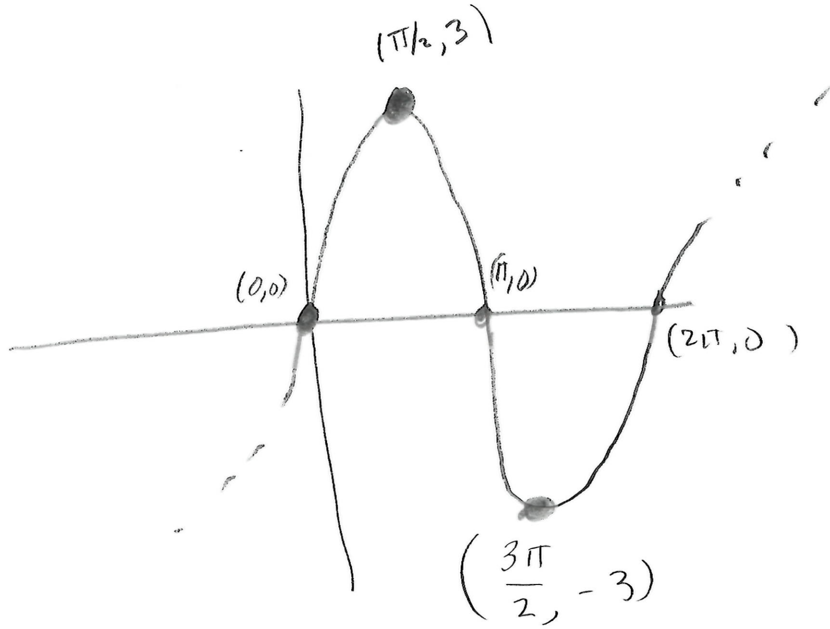
$(0,0) \rightsquigarrow (0,0)$

$(\frac{\pi}{2},1) \rightsquigarrow (\frac{\pi}{2},3)$

$(\pi,0) \rightsquigarrow (\pi,0)$

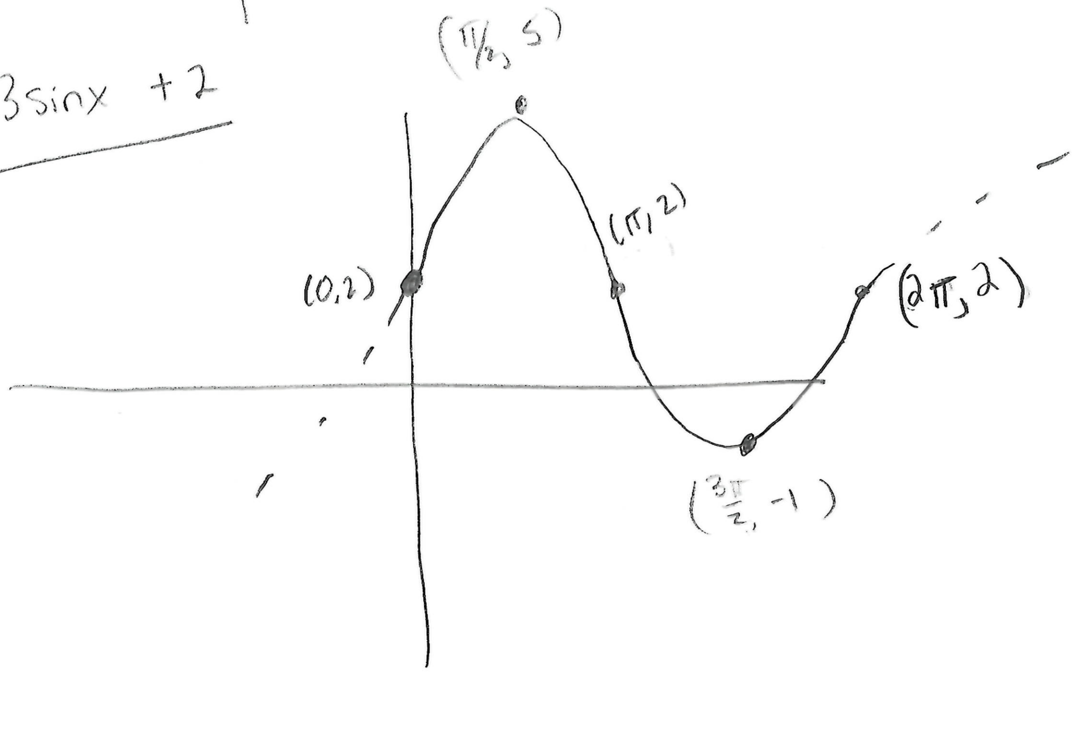
$(\frac{3\pi}{2},-1) \rightsquigarrow (\frac{3\pi}{2},-3)$

$(2\pi,0) \rightsquigarrow (2\pi,0)$



Then shift up 2

$y = 3\sin x + 2$



e.g. $y = \cos\left(\underbrace{\tfrac{1}{2}x - \tfrac{\pi}{4}}_{\text{call this } \theta}\right)$

$y = \cos\theta$ completes full period on $0 \leq \theta \leq 2\pi$

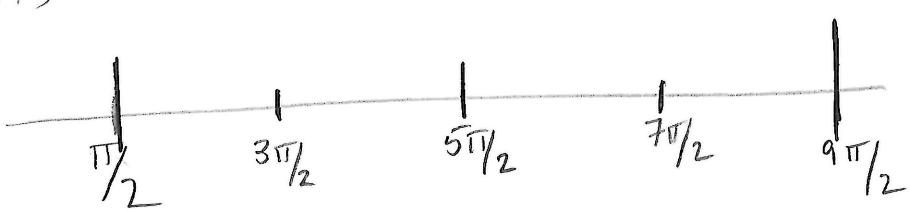
So $y = \cos\left(\tfrac{1}{2}x - \pi/4\right)$ completes full period when

$$0 \leq \tfrac{1}{2}x - \tfrac{\pi}{4} \leq 2\pi$$

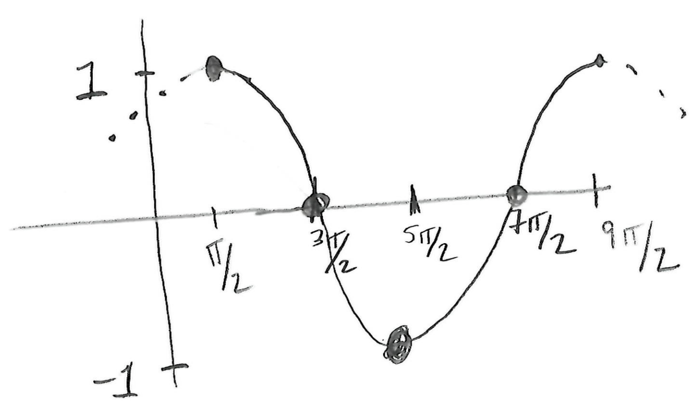
$$\Leftrightarrow \tfrac{\pi}{4} \leq \tfrac{1}{2}x \leq \tfrac{9\pi}{4}$$

$$\Leftrightarrow \underbrace{\tfrac{\pi}{2} \leq x \leq \tfrac{9\pi}{2}}_{\text{period is } \frac{9\pi}{2} - \frac{\pi}{2} = \frac{8\pi}{2} = 4\pi.}$$

Key points



- $\cos\left(\tfrac{1}{2}(\pi/2) - \pi/4\right) = \cos(0) = 1$
- $\cos\left(\tfrac{1}{2}\left(\tfrac{3\pi}{2}\right) - \pi/4\right) = \cos(\pi/2) = 0$
- $\cos\left(\tfrac{1}{2}\left(\tfrac{5\pi}{2}\right) - \tfrac{\pi}{4}\right) = \cos(\pi) = -1$
- $\cos\left(\tfrac{1}{2}\left(\tfrac{7\pi}{2}\right) - \tfrac{\pi}{4}\right) = \cos(6\pi/4) = 0$
- $\cos\left(\tfrac{1}{2}\left(\tfrac{9\pi}{2}\right) - \tfrac{\pi}{4}\right) = \cos(2\pi) = 1$



In general

$y = A\sin(Bx - C) + D$ $\quad (B > 0)$ otherwise, use even/odd e.g. $\sin(-x) = -\sin(x)$

$y = A\cos(Bx - C) + D$

- Amplitude is $|A|$ (vertical stretch)
- Period is $\frac{2\pi}{B}$
- Phase shift is $\frac{C}{B}$ (horizontal shift)
- Vertical shift is D
- One full period on $0 \leq Bx - C \leq 2\pi$

To graph $A\sin(Bx - C) + D$ (and cosine)

① Graph $\sin(Bx - C)$

- Solve $0 \leq Bx - C \leq 2\pi$
- Identify key points

② Multiply all y-vals by $A \rightsquigarrow$ graph $A\sin(Bx - C)$

③ Shift up/down by D