Paper 2: The History and Mathematics of Timekeeping
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I. Discussion of Mathematical Idea

The measurement of time is the glue that allows everyday human life to function in a universal system, spanning conventional daily elements such as psychological organization, perspective of history, and age. Without a timekeeping system, the natural and formal sciences would be unpredictable and ambiguous due to their dependence on time. One specific science that is dependent on timekeeping is mathematics, where time can be used as a variable for speed, acceleration, and rates of growth. Per the laws of physics, the fabric of the universe follows when applying mathematical concepts to space, providing us with guaranteed synchronization in both daily life and mathematics ("History of Timekeeping Devices").

Methods of timekeeping have an ancient history, where devices of measurement are found to have taken different forms depending on the civilization and their respective available technology. Using the sun and moon as references, sundials are likely the first usage of timekeeping by the ancient Egyptians around 1500 BCE. Water clocks also surfaced around the same time, as the obvious advantage over a sundial was the consistent availability of natural lighting. Other civilizations, such as the Greeks, Chinese, and Babylonians, followed by adopting sundials and water clocks as their primary methods of timekeeping. Candle clocks were, as well in these civilizations, utilized when methods of timekeeping during the night when water clocks were unavailable ("From the Clockwork World View to Irreversibility" 61-65). It wouldn't be until around the eleventh century that the next recorded development appeared: the hourglass, which served as a vital device for navigating waters. The reliability of the innovative hourglass was its stable functionality, since the unstable waters did not affect the time measurement quality. The design of the hourglass was another aspect that increased the simplicity of the device. Similar to the function of the water clock and candle clock, the hourglass's sand emptying from top to bottom marks a certain interval of past time ("History of Timekeeping Devices"). Intervals of time for the hourglass typically signified the passing off of one hour, hence the term 'hourglass'.

In the fourteenth century, Henry de Vick developed one of the first mechanical clocks, and though it took on a similar look to the sundial, its function relied on weight rather than natural lighting. An issue with the initial design of the mechanical clock was its practicality, since the first mechanical clocks were massive in size and typically mounted or embedded on churches and clock towers. One century later, the mainspring was invented, allowing the production of smaller mechanical clocks for commercial use. Later innovations surfaced between the sixteenth and seventeenth centuries, entailing the development of harmonic oscillators and the pendulum, which shaped the foundation of today's timekeeping devices (Georgi).

Leonardo da Vinci and Galileo Galilei significantly contributed to the advancement of timekeeping through their work with pendulums. In the late 15th century, Da Vinci created the earliest known pendulum drawings, laying the foundation for using pendulums in clocks. Galileo's research in 1582 contributed to this advancement by showing that a pendulum's frequency is dependent on its length rather than its weight. These contributions set the stage for Dutch scientist Christiaan Huygens' invention of the pendulum clock in 1656, significantly enhancing the accuracy of time measurement by improving the technology sixfold in comparison to the previous mechanisms. The earlier versions of Huygens' pendulum clocks were remarkably precise for the seventeenth century, where the margin of error was less than a minute per day. Later improvements further reduced this margin to ten seconds. The widespread adoption of minute and second hands in clocks then followed, a feature that became common due to the enhanced accuracy. Other flaws in the pendulum clocks were corrected in the eighteenth and nineteenth centuries, where, respectively, chronometers were introduced to avoid temperature variations, which affected a clock's reliability. Electric-powered clocks improved the general control over pendulum clocks ("History of Timekeeping Devices").

Quartz timers, in the 1940s, replaced electric-powered clocks as the primary engine of the mechanical clock due to their enhanced time measurement. The significance of the quartz timer opened the door for smaller mechanical clocks, such as the wristwatch and alarm clock. The body of the clock consisted of an integrated circuit, which allowed for the compact arrangement of numerous transistors and resistors. As a result, wristwatches obtained the ability to perform complex functions from the electrical vibrations from the quartz-crystal resonator. The first quartz wristwatches retained the traditional minute-hour hands, though the moving parts were made entirely redundant in succeeding models ("History of Timekeeping Devices").

II. Historical Impact

The measurement of work and activities through timekeeping has been a fundamental aspect of human civilization, enabling individuals and societies to organize their lives efficiently. Historically, timekeeping devices have been utilized as tools for managing various tasks, from agricultural labor and industrial production to daily routines. By assigning specific durations to activities, people could allocate their time wisely, improve productivity, and coordinate collective efforts. In agricultural communities, the timing of planting and harvesting relied on precise time measurements, ensuring a maximized harvest. Similarly, the Industrial Revolution is known as an era of heightened time consciousness, where factories relied on clocks to create schedules in order to enhance productivity ("History of Timekeeping Devices"). The measurement of work and activities through timekeeping has structured daily life and has also played an integral role in the development and advancement of societies throughout history.

The religious impact of timekeeping served as a rhythm for guided rituals, prayers, and festivals. In western religions, such as Christianity, Judaism, and Islam, specific times of day are dedicated for daily prayers, where timekeeping devices have been used to accurately determine prayer sessions. Moreover, eastern religions that entailed spiritual traditions, such as Buddhism and Hinduism, integrated timekeeping into meditation and devotion, aligning with cosmic cycles. In the sense that ancient civilizations, such as the Mayans and Egyptians, developed complex calendars based on celestial observations to mark religious events and ceremonies ("History of Timekeeping Devices"). Timekeeping thus plays a significant role in the structure of religious life and practices, emphasizing the significance of the role of timekeeping.

The role precise timekeeping devices play in global synchronization is through time standardization, where the universal understanding of time-enabled coordination had been made possible through internationally recognized time zones. Prior to standardization, each region typically functioned in an independent time zone, which consequently created confusion in transportation and communication. The adoption of standard time zones, based on meridians of longitude, allowed for synchronized schedules and common transportation timetables ("History of Timekeeping Devices"). In a perfect world, there should be twenty-four time zones in total; however, politics, policy, and sovereign borders append to the time zone boundaries, reducing the total number of world time zones to twenty-one. For instance, modern-day China, where the mainland is comparable to the size of the mainland United States, functions under one single time zone. As the constant development of technology progresses, timekeeping will play an integral role in better measuring world time zones and synchronization amongst current disconnected regions.

Timekeeping devices have a significant influence on mathematical modeling, where their development transformed the accuracy and efficiency of creating models to simulate the physical behavior of the devices, optimizing their design and function. As timekeeping technology progressed from sundials to atomic clocks, mathematical models grew increasingly complex. Mathematic model development contributed to broader mathematical fields such as numerical analysis, in which algorithms are developed to obtain approximate solutions to complex mathematical problems (Jespersen and Fitz-Randolph). This evolution illustrates the deep interplay between technological advancements and mathematical theory.

III. Future Implications

The impact on mathematics through the development of time measurement reached a key milestone when the pendulum clock was invented in the 17th century. In the same time period, harmonic oscillation was discovered and became a core building block in mathematical applications, such as physics. The connection between harmonic oscillation and physics, or more specifically, the elements of waves, circulates through complex numbers. The research on wave phenomena is paved by combining the imaginary number i with real numbers in algebra, trigonometry, and calculus. Euler's Identity, $e^{i\pi} = -1$, where i is the imaginary number, is another heavily influenced topic by harmonic oscillation; it demonstrates a profound connection between complex exponentials and trigonometric functions. Algebraic definitions of trigonometric functions, such as sine and cosine in terms of complex exponentials, simplify many mathematical expressions, specifically combinations of complex exponentials (Georgi).

A major mathematical area in the development of clocks is harmonic oscillation, as previously mentioned. Applied as a fundamental idea in wave physics, this concept involves systems with only one degree of freedom, where the motion is linear and time-invariant, meaning it is unchanged over time. The motion of a harmonic oscillator in clocks is inspired by the applications of Hooke's law, $F = -kx$, where F is the spring force, k is the spring constant, and x is the stretch of the spring. Hooke's law describes the force exerted by a spring, and is in correlation with Newton's second law, $F = ma$, where m is the mass and a is the acceleration . Note that a differential equation condenses the oscillatory behavior of the system, and combined with a constant to the sine and cosine of angular frequency, multiplies time. A single, constant frequency follows Hooke's law, which is characterized by simple harmonic motion. This feature is crucial for clocks, as it ensures the regular, predictable motion necessary for accurate timekeeping, typically manifested in mechanisms like pendulums or balance wheels (Georgi). As the evolution of timekeeping progresses, harmonic oscillation will serve as the foundation of future developments.

The most advanced and accurate device of timekeeping to date is the atomic clock, where the most precise model clocks in at 30 billionths of a second per year. The first suggestion of atomic transitions came in 1879, and proved to be successful in the mid-20th century, when Louis Essen successfully created the first successful atomic clock using the caesium-133 atomic transition. Now, in the 21st century, atomic clocks utilize other elements such as hydrogen and rubidium, granting atomic clocks a more minimal size, increased stability, and less power ("History of Timekeeping Devices"). German physicist R.L. Mossbauer discovered the Mossbauer Effect in 1958, which demonstrated that under specific circumstances, atomic nuclei emit gamma rays with stable frequencies. These emissions, which have an incredibly high quality factor of over 10 billion, are much better than those from cesium oscillators, which have allowed scientists to test Einstein's ideas about how photons behave under the influence of gravity. Despite the challenges of producing strong and pure gamma-ray signals for resonant devices, nuclear emissions have longer natural lifetimes, making them potential candidates for frequency standards. The future of atomic clocks, and timekeeping as a whole, signifies an active objective to improve the precision of timekeeping using gamma-ray resonation (Jespersen and Fitz-Randolph).

Smartphones and modern wristwatches are the most common and convenient ways for people to keep track of time digitally at any given point. The practicality of smartphones combines other useful clock functions, such as alarms, timers, and stopwatches. All non-analog wristwatches are also, by convention, digital. Introduced in the 1970s, digital wristwatches use plastic parts and transistors, whereas modern 'classic' wristwatches were created over the past one hundred years of developments ("History of Timekeeping Devices"). Innovations in the 21st century brought the smartphone and wristwatch into one powerful device, which dominates the consumer markets for wearable technology. Wearable technologies, such as smart watches, smart glasses, and virtual reality, will eventually utilize artificial intelligence and thus, further cloud the future of timekeeping. The later possibilities timekeeping holds seem boundless, which defines the innovative direction of time visualization as ambiguous.

IV. Citations

1. "From the Clockwork World View to Irreversibility: The Newtonian Clockwork Universe." *Complexity Explained*, Springer, Berlin/Heidelberg, 2007, pp. 61–65, doi:10.1007/978-3-540-35778-0_3.
2. Georgi, Howard. *The Physics of Waves*. Prentice Hall, 1993, pp. 1-33.
3. "History of Timekeeping Devices." Wikipedia, 28 Nov. 2023, en.wikipedia.org/wiki/History_of_timekeeping_devices.
4. Jespersen, James, and Fitz-Randolph, Jane. *From Sundials to Atomic Clocks: Understanding Time and Frequency*. National Institute of Standards and Technology, U.S. Department of Commerce, 1999.