Introduction Asset Pricing - Assignment 1

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Part 1: Descriptive statistics

In this assignment we will analyze the data for returns of Five Fama/French factors from emerging markets. The five Fama/French factors are Mkt-RF, SMB, HML, RMW, and CMA. Market excess return (Mkt-RF): This factor measures the excess return of the overall stock market above the risk-free rate of return. It is often used as a proxy for systematic risk or the overall level of market risk. SMB (Small Minus Big) is the average return on the nine small stock portfolios minus the average return on the nine big stock portfolios. HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios. RMW (Robust Minus Weak) is the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios. CMA (Conservative Minus Aggressive) is the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios.

The data includes returns for all factors between year 2000 to present with frequency of 1 month. The dataset of returns of emerging markets consists of these countries: Brazil, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Qatar, Saudi Arabia, South Africa, South Korea, Taiwan, Thailand, Turkey, United Arab Emirates. The raw data comes from Bloomberg for 1995.

The excess returns considered in this analysis comes from the set of same emerging markets and also from the same time frame with periodicity of data equal 1 month. The raw data comes from Bloomberg. The portfolios are divide based on size into two groups, portfolios Big stocks are those in the top 90% of June market cap for the country, and small stocks are those in the bottom 10%. Moreover, each of these two groups are then further divided into three subgroups based on volume of investment, low investment are those in lower than 30th percentile of investment, medium are those between 30th and 70th percentile of investment and high are those with 70th percentile or higher investment. The independent 2x3 sorts on size and Inv produce six value-weight portfolios, small aggressive (high Inv), small neutral, small conservative (low Inv), big aggressive, big neutral, and big conservative. For the sake of conciseness we will from now on use these names to describe the different portfolios: smalloinv, smallinv2, smallhiinv, bigloinv, biginv2, bighiinv.

Here is summary statistics for the excess returns of the six groups of portfolios:

```
##   SMALL LoINV          ME1 INV2         SMALL HiINV         BIG LoINV     
##  Min.   :-25.5100   Min.   :-26.2200   Min.   :-30.8300   Min.   :-24.3900  
##  1st Qu.: -2.3975   1st Qu.: -2.3575   1st Qu.: -2.9775   1st Qu.: -2.3475  
##  Median :  0.9600   Median :  1.1900   Median :  1.0400   Median :  0.7700  
##  Mean   :  0.9013   Mean   :  0.8832   Mean   :  0.4849   Mean   :  0.8303  
##  3rd Qu.:  4.7375   3rd Qu.:  4.8000   3rd Qu.:  4.2625   3rd Qu.:  4.4450  
##  Max.   : 16.7500   Max.   : 19.0200   Max.   : 22.0200   Max.   : 15.8600  
##     ME2 INV2          BIG HiINV      
##  Min.   :-27.1100   Min.   :-31.7100  
##  1st Qu.: -2.5175   1st Qu.: -2.8475  
##  Median :  0.8400   Median :  0.7000  
##  Mean   :  0.6827   Mean   :  0.6318  
##  3rd Qu.:  4.3625   3rd Qu.:  4.8175  
##  Max.   : 18.6500   Max.   : 21.4000  
```

Part 1: Fama-French Five Factor Model

```r
### First factor: return on market portfolio ###
rme <- (Emerging[,2])/100
rme <- as.numeric(rme$`Mkt-RF`)
### Second factor: small minus big ###
smb <- (Emerging[,3])/100
smb <- as.numeric(smb$`SMB`)
```

```
### Third factor: high minus low ###
hml <- (Emerging[,4])/100
hml <- as.numeric(hml$`HML`)
### Fourth factor: robust minus weak ###
rmw <- (Emerging[,5])/100
rmw <- as.numeric(rmw$`RMW`)
### Fifth factor: conservative minus aggressive ###
cma <-(Emerging[,6])/100
cma <- as.numeric(cma$`CMA`)
### Risk free return ###
rf <- (Emerging[,7])/100
rf <- as.numeric(rf$`RF`)
### Mean of rme ###
mrme <- mean(rme)

### Mean of rf ##
mrf <- mean(rf)

### Gross return on market portfolio ###
rm<-1+rme+rf


### Five industry portfolios ###

### Net returns in fractions ###
rall<-excessreturnsinvestment[,2:7]/100

### Excess returns (calculated as net return - net return risk free) ###
dim.invport<-dim(excessreturnsinvestment[,2:7])
ralle<-rall-rf*matrix(1,nrow=dim.invport[1],ncol=dim.invport[2])

### Net return and excess return of cnsmr, manuf, hitec, hlth and other ###
smalloinv<-rall[,1]
smalloinve<-smalloinv-rf
smallinv2<-rall[,2]
smallinv2e<-smallinv2-rf
smallhiinv<-rall[,3]
smallhiinve<-smallhiinv-rf
bigloinv<-rall[,4]
bigloinve<-bigloinv-rf
biginv2<-rall[,5]
biginv2e<-biginv2-rf
bighiinv<-rall[,6]
bighiinve<-bighiinv-rf

lm.smalloinv.ff5<-lm(smalloinve~rme+smb+hml+rmw+cma)
lm.smallinv2.ff5<-lm(smallinv2e~rme+smb+hml+rmw+cma)
lm.smallhiinv.ff5<-lm(smallhiinve~rme+smb+hml+rmw+cma)
lm.bigloinv.ff5<-lm(bigloinve~rme+smb+hml+rmw+cma)
lm.biginv2.ff5<-lm(biginv2e~rme+smb+hml+rmw+cma)
lm.bighiinv.ff5<-lm(bighiinve~rme+smb+hml+rmw+cma)
homoskedastic_smalloinv=coeftest(lm.smalloinv.ff5)[1,]
heteroskedastic_smalloinv=coeftest(lm.smalloinv.ff5,
```

```r
                                   vcov = vcovHC(lm.smalloinv.ff5, type = "HC0"))[1,]
homoskedastic_smallinv2=coeftest(lm.smallinv2.ff5)[1,]
heteroskedastic_smallinv2=coeftest(lm.smallinv2.ff5,
                                   vcov = vcovHC(lm.smallinv2.ff5, type = "HC0"))[1,]
homoskedastic_smallhiinv=coeftest(lm.smallhiinv.ff5)[1,]
heteroskedastic_smallhiinv=coeftest(lm.smallhiinv.ff5,
                                    vcov = vcovHC(lm.smallhiinv.ff5, type = "HC0"))[1,]

homoskedastic_bigloinv=coeftest(lm.bigloinv.ff5)[1,]
heteroskedastic_bigloinv=coeftest(lm.bigloinv.ff5,
                                  vcov = vcovHC(lm.bigloinv.ff5, type = "HC0"))[1,]
homoskedastic_biginv2=coeftest(lm.biginv2.ff5)[1,]
heteroskedastic_biginv2=coeftest(lm.biginv2.ff5,
                                 vcov = vcovHC(lm.biginv2.ff5, type = "HC0"))[1,]

homoskedastic_bighiinv=coeftest(lm.bighiinv.ff5)[1,]
heteroskedastic_bighiinv=coeftest(lm.bighiinv.ff5, vcov =
                                    vcovHC(lm.bighiinv.ff5, type = "HC0"))[1,]
results = rbind(homoskedastic_smalloinv, heteroskedastic_smalloinv,
                homoskedastic_smallinv2,
                heteroskedastic_smallinv2, homoskedastic_smallhiinv,
                heteroskedastic_smallhiinv,
                homoskedastic_bigloinv, heteroskedastic_bigloinv,
                homoskedastic_biginv2,
                heteroskedastic_biginv2, homoskedastic_bighiinv,
                heteroskedastic_bighiinv)


# Add a new column to results with stars indicating significance level
Significance <- ifelse(abs(results[, "Estimate"]/results[, "Std. Error"]) > 1.96, "***",
                ifelse(abs(results[, "Estimate"]/results[, "Std. Error"]) > 1.64, "**",
                ifelse(abs(results[, "Estimate"]/results[, "Std. Error"]) > 1.28,
                       "*", "")))
results <- cbind(results, Significance)

# Use kable() from kableExtra package to generate a table with stars

library(kableExtra)
table_results = knitr::kable(head(results[, c(1,2,5)],13), format = "markdown")
table_results = kable_styling(table_results, full_width = FALSE,
                              position = "left", font_size = 10)


# Print table
table_results
```

	Estimate	Std. Error	Significance
homoskedastic_smalloinv	0.00134103277932025	0.000367953929915994	***
heteroskedastic_smalloinv	0.00134103277932025	0.0003347089685312	***
homoskedastic_smallinv2	0.000630419278379523	0.000387395622066692	*
heteroskedastic_smallinv2	0.000630419278379523	0.000357781629319907	**
homoskedastic_smallhiinv	-0.00150501900003184	0.000434343062397255	***
heteroskedastic_smallhiinv	-0.00150501900003184	0.000426335227730637	***

	Estimate	Std. Error	Significance
homoskedastic_bigloinv	-0.000766554090583998	0.000457524637845748	**
heteroskedastic_bigloinv	-0.000766554090583998	0.000433322721344076	**
homoskedastic_biginv2	-0.00133207256996875	0.000465540322187779	***
heteroskedastic_biginv2	-0.00133207256996875	0.000408945365437507	***
homoskedastic_bighiinv	0.002086193068655	0.000511730121117035	***
heteroskedastic_bighiinv	0.002086193068655	0.000475690718350022	***

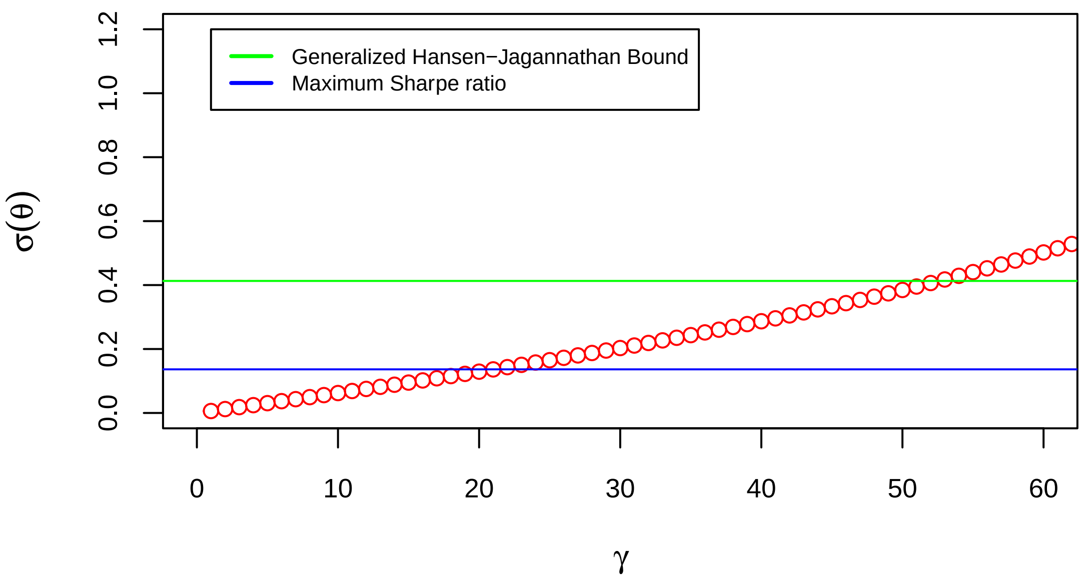
We test the FF5 by checking the p-values corresponding to the α=0 in the output. Therefore, the null hypothesis is H_0: α=0. The alternative hypothesis is H_1: $\alpha \neq 0$. We reject the null hypothesis if the p-value to a corresponding test is less than the significance level.
* indicates a coefficient that is significant at the 0.05 level (i.e., p-value < 0.05). ** indicate a coefficient that is significant at the 0.01 level (i.e., p-value < 0.01). *** indicate a coefficient that is significant at the 0.001 level (i.e., p-value < 0.001) As we can see from the table there's rarely difference in significance level while testing for homo- and heteroskedesticity. However, the p-value is always less than 0.05, leading to a rejection of the hypothesis (α=0) at 5% significance level. As a result, we conclude that the FF5 is not valid for any of the portfolios at a 5% significance level.

Part 2: Hansen-Jagannathan Bounds

```r
gammas = seq(0.1,60, by=0.1)
sigma.theta = rep(NaN, 600)
for (i in 1:600)
{
  theta.non<-rm^(-gammas[i])
  c<-1/mean(theta.non)
  theta<-c*theta.non
  sigma.theta[i] = sd(theta)

}
sharpe.smalloinv<-mean(smalloinve)/sd(smalloinve)
sharpe.smallinv2<-mean(smallinv2e)/sd(smallinv2e)
sharpe.smallhiinv<-mean(smallhiinve)/sd(smallhiinve)
sharpe.bigloinv<-mean(bigloinve)/sd(bigloinve)
sharpe.bighiinv<-mean(bighiinve)/sd(bighiinve)
sharpe.biginv2<-mean(biginv2e)/sd(biginv2e)
sharpe_max<-max(sharpe.smalloinv, sharpe.smallinv2,
                sharpe.smallhiinv, sharpe.bigloinv, sharpe.bighiinv, sharpe.biginv2)


hjb<-sqrt(t(colMeans(ralle))%*%solve(cov(ralle))%*%(colMeans(ralle)))
hjb=as.numeric(hjb)
```



The graph above showcases the values of $\sigma(\theta)$ for values of γ that we've created earlier. Furthermore, it compares it to the values of the Maximum Sharpe ratio (Hansen-Jagannathan Bound) and the Generalised Hansen-Jagannathan Bound. Generalised Hansen-Jagannathan Bound equals 0.412882202888747. From the graph we can tell that the Generalised Hansen-Jagannathan Bound realises the inequality for $\gamma \gtrapprox 52$ and the Maximum Sharpe ratio (Hansen-Jagannathan Bound) realises the inequality for $\gamma \gtrapprox 20$.

```
tab <- matrix(c(sharpe.smalloinv,sharpe.smallinv2,sharpe.smallhiinv,
                sharpe.bigloinv,sharpe.bighiinv,sharpe.biginv2,
                sharpe_max,hjb), ncol=8, byrow=TRUE)
colnames(tab) <- c('sharpe.smalloinv','sharpe.smallinv2',
                   'sharpe.smallhiinv','sharpe.bigloinv','sharpe.bighiinv',
                   'sharpe.biginv2','sharpe_max','hjb')
rownames(tab) = c("Values")
tab=as.table(tab)
tab
```

```
##        sharpe.smalloinv sharpe.smallinv2 sharpe.smallhiinv sharpe.bigloinv
## Values       0.13645372       0.13075977        0.05778317      0.12394846
##        sharpe.bighiinv sharpe.biginv2 sharpe_max        hjb
## Values      0.07572335     0.09297325 0.13645372 0.41288220
```

From the table above we see that the value of the Generalized Hansen-Jagannathan Bound is bigger than any of the Sharpe Ratios.