Inventory and Production Management
Group 8

Inventory assignment

N. Baehr (2076515), S. Friendlaender (2070091), M. Pietruk (2075739), E. Rimkus (2070805)

1

The given dataset contained EasyCoffee's data on the historical demand for 104 consecutive weeks. Since Mr. Stark wants to revise the smoothing constants once every year, we split the dataset to obtain annual data, i.e. data for every 52 weeks. This way the predictions that are made based on the most suitable model, correspond to the data only from the previous year, so they should be the most relevant. Then, to avoid over-fitting and get less biased performance measures we further split the data into training (80%) and testing sets (20%).

We will perform the prediction of the future demands using the best performing model from the second year. It is for that reason, why we believe it is best to choose the model based on only the performance of the models in the second year, as it is going to be that given model (with those smoothing constants) used to make predictions and as we consider only the values of demand in the last year, we consider the most relevant data to make decision for today.

1.1 SES

Firstly, we utilized the SES method to predict the future demand values.The SES method is a forecasting method in which the results are based on the following equations :
$\hat{y}_{t+1} = \alpha y_t + (1-\alpha)\hat{y}_t)$
$\hat{y}_{t+h|t} = \hat{y}_{t+1|t} = \hat{y}_{t+1}$ h=1,2,....
$\hat{Y}_h = h \cdot \hat{y}_t$ h=1,2,....
To initialize the SES, we need to establish the initial value $\hat{y}_1$ (of the second year). As suggested in the reader, we will fix this value to be the mean of the first 10 observations from the second year. So in our case, $\hat{y}_1 = \frac{1}{10}\sum_{n=1}^{10} y_n$.

The next step is to find, such a value of α that will be optimal for constructing predictions. Using the data of the second year, we construct predictions for the demand for all possible values of α, ranging form 0 to 1 by step of 0.001.

Since we chose Mean Squared Error (MSE) as our prediction performance indicator. For each possible α, we calculate the corresponding MSE, which is given by $\frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2$, where $\hat{y}_t$ is the predicted values, and y_t the real values of demand.

Now, knowing the predicted values in the last week of the second year ($\hat{y}_{104}$) we are able to construct predictions for the upcoming 6 weeks. To achieve this, firstly we predict the value of the demand in week 105 using the previously mentioned formula : $\hat{y}_{t+1} = \alpha y_t + (1-\alpha)\hat{y}_t$. Then, using the $\hat{Y}_h = h \cdot \hat{y}_t$ we are able to obtain predictions of the demand for the first 6 weeks of the third year.

It is also important to keep track of the naive error of each prediction as it will be useful later on. We can compute the aforementioned naive error for the prediction of week k (So in our case k$\in <1,6>$, where k$\in \mathbb{Z}$) using the following formula : $naive\_error(k) = \sqrt{MSE\_Year\_2} * k$

1.2 Holt's Trend Model

The second forecasting method will be used, namely the Holt's Trend Method.
Similarly to the SES method, firstly we need to initialize the values of the parameters. In this case, however instead of using the mean of a couple of first observations we will utilize the regression on the first 10 observations. The constant coefficient of the regression is our level, and the slope is the trend of the demand.

After achieving the initial values for the parameters ℓ_t - level and b_t - trend, we can move on to the next stage of constructing the predictions, which is the selection of optimal β^* and α values. To do that, we will perform a process analogical to the α selection process in the SES method.
Now, we are equipped with necessary parameters to construct predictions for the first 6 weeks of the first year. Combining the values we obtained in the previous step, with the set of equations showed below we are able to freely predict the future values of the demand.
$\ell_t = \alpha y_t + (1-\alpha)(\ell_{t-1} + b_{t-1})$
$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1}$
$\hat{y}_{t+h} = \ell_t + hb_t$

Again, the optimal α and β values are obtained using MSE. In that case we obtain that the MSE equals 1054,963.

1.3 Conclusions and predictions

Since the RMSE of the Holt's trend model in year 2 was 1054.962, which is lower than the RMSE of the SES level model was 1083.924, we are going to use the Holt's Trend method with the calculated smoothing constants for the second year (the estimated level l= 5553.3 and the and the estimated trend b = 34.1758) to make predictions for the next 6 weeks.

$\hat{\mu}$	$\hat{Y}$	naive error
5587.5	5587.5	1055.0
5621.7	11209	2109.9
5655.9	16865	3164.9
5690.0	22555	4219.8
5724.2	28279	5274.8
5758.4	34038	6329.8

Table 1: forecasts and error standard deviations

2

In this part, we are asked to perform an analysis of the future performance of the company, considering that the current inventory policy is maintained. Firstly, we should point out all the known factors influencing the performance:
S=32000
v=120
p =120*1.1=132 (profit margin 1.1)
L=2 (weeks)

R=4 (weeks)
r_week=0.25/52
A=500
B2= 0.05

Knowing these values, we are able to analyze the future performance of the company. For this purpose, we will calculate the following performance measures: P1, ESPRC, P2, and costs of ordering, holding, shortage, as well as total costs. Firstly, it is imperative to obtain the mean and standard deviation of the demand during the replenishment cycle. After obtain the aforementioned values we are able to obtain the safety factor using the formula: $k = \frac{S-\mu_{RL}}{\sigma_{RL}}$.

Next, we are calculating the parameters for the gamma of the demand in the review period $\rho_R = (\frac{\mu_R}{\sigma_R})^2$

$\rho_L = \rho_R \frac{L}{R}$

$\rho_{RL} = \frac{(R+L)\rho_R}{R}$

$\lambda_R = \frac{\mu_R}{\sigma_R^2}$

Now, we can calculate the value of P1 knowing that $P1 = F_{R+L}(S)$, where R+L follows the gamma distribution, with the parameters calculated above.

As the next step we can easily calculate the value of ESPRC- which is the expected shortage per replenishment cycle. Using what we have obtained so far we can easily obtain this value using the following formula: $ESPRC = \mathcal{GL}(S, \rho_{RL}, \lambda_R) - \mathcal{GL}(S, \rho_R, \lambda_R)$. As seen, we use Gamma Loss Function instead of the Normal one. This is because of the nature of distribution, Gamma distribution fits it better than the normal distribution.

Having obtained the value of ESPRC we can easily calculate P2- which is the fraction of demand that is satisfied in the arbitrary replenishment cycle. We can do that by using the following formula: $P2 = 1 - \frac{ESPRC}{\mu_R}$

Now we can focus on analyzing the costs of such an inventory management style:

Firstly, the ordering cost is simply $\frac{A*52}{R}$

The holding cost can also be easily showcased as: $(k\sigma_{RL} + \frac{1}{2}\mu_R)vr$

Finally, the shortages costs are: $B2 * ESPRC * \frac{52}{R}$

Unsurprisingly summing up those 3 values amounts to calculating the total costs. So,

$TC = \frac{A*52}{R} + (k\sigma_{RL} + \frac{1}{2}\mu_R)vr + B2 * ESPRC * \frac{52}{R}$

Utilizing all the formulas showcased above we reach the following results:

P1 service rate	0.4101
P2 service rate	0.8436
ESPRC	3528.21
Order-up-to-level	32,000
Order value	6500
Holding costs	283,347.27
Shortage costs	275,200.10
Total costs	565,047.37

3

In this part, we are faced with finding the value of S, i.e. the up-to-reorder point amount, which will minimize the total cost (TC) that we have determined in the previous question. This can

be easily done as, the TC is very much dependent on the value of S, through the shortages cost, particularly the ESPRC value, and through the holding costs, particularly the value of safety stock k.
$TC = \frac{A*52}{R} + (k\sigma_{RL} + \frac{1}{2}\mu_R)vr + B2 * ESPRC * \frac{52}{R}$ where
$k = \frac{S-\mu_{RL}}{\sigma_{RL}}$ and $ESPRC = \mathcal{GL}(S, \rho_{RL}, \lambda_R) - \mathcal{GL}(S, \rho_R, \lambda_R)$
We can achieve the goal of minimizing TC by iteratively calculating it for different values of S and seeing which one yields the lowest value. For the purpose of this exercise, we decided to check values of S that ranged from 1 to 100000, using integer values only. After applying this method we obtained the result: S=35332. This value of S resulted in the following amounts of the performance measures we defined earlier:

P1 service rate	0.6153
P2 service rate	0.9153
ESPRC	1909.44
Order-up-to-level	35,332
Order value	6500
Holding costs	383,307.27
Shortage costs	148,936.13
Total Cost	538,743.40

KPIs assuming no goodwill costs, R,S

4

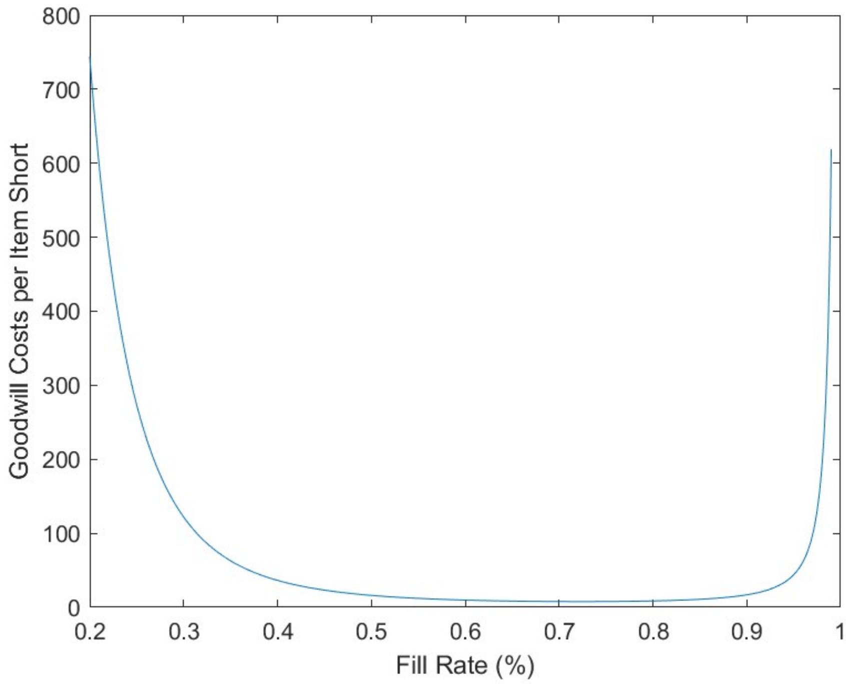
Mr. Stark demanded that the fill rate of 99% should be reached. In this part, we will calculate the order-up-to level S, which allows such a value of P2.
As mentioned in the previous part the value of S has an effect on P2 through the dependence of the ESPRC on S. Hence, to obtain the value of S, which satisfies the fill rate requirement of Mr. Stark, we need to solve the following equation: $0 = ESPRC(S) - (1-0.99)\mu_R = \mathcal{GL}(S, \rho_{RL}, \lambda_R) - \mathcal{GL}(S, \rho_R, \lambda_R) - (1-0.99)\mu_R$. Solving this equation for S allows us to achieve the desired fill rate level.
Due to the change of the value of S it is also important to note the changes in the remaining performance measures:

P1 service rate	0.9353 ↑
P2 service rate	0.9900 ↑
ESPRC	225.55 ↓
Order-up-to-level	44,044.70 ↑
Order value	6500
Holding costs	644,688.36 ↑
Shortage costs	17,592.97 ↓
Total costs	668,781.33 ↑

KPIs for service rate of 99%, R,S

5

For different values of the fill rate (P2) we firstly calculate the corresponding order-up-to level S and then what we took as the goodwill cost of B1. In this way we can create a graph that depicts the relationship between the fill rate and the goodwill cost, where fill rate is the independent and goodwill cost dependent variable.



We also obtained the following results:
The implied goodwill cost at the fill rate target 99% = 618.8614
Initial B1(with S=32000)= 36,744.135
Optimal B1(with S such that its optimal for the desired fill rate) = 139,584.7

6

Ignoring stockout costs means that the total costs is just the sum of holding cost and ordering cost. So since we want to re-set the length of the review period R to a maximum of 1 year, we just solve the minimization problem of total cost with respect to R. So for R between 1 week and 52 weeks we want to find such an R for which total cost is minimal.It is important to mention that the values of ρ_R and σ_R will also be readjusted accordingly. Moreover, once this new review period R is calculated we also derive the new order-up-to level S (given that P2 = 0.99). Finally, we include the estimated goodwill costs (from Ex. 5) and determine the overall total cost for the estimated parameters R,S. Following the steps mentioned above the following results are reached:

P1 service rate	0
P2 service rate	0.9900 ↑
ESPRC	2,852.51 ↓
Order-up-to-level	33,225.76 ↑
Order value	500
Holding costs	4,251,454.86 ↑
Shortage costs	17,115.05 ↓
Goodwill costs	1,765,307.94
Total costs	6,034,377.86 ↑

KPIs for service rate of 99% with goodwill costs, R,S

7

The R,S policy offers a regular opportunity to adjust the order-up-to-level, S, and the review period, R, if the demand is changing over time and if certain requirements are made regarding the service levels. EasyCoffee is located in a few different places and the demand for coffee machines is more likely to change over time as the more locations there are, the more changes in demand are likely. It was mentioned that the product life cycles have decreased and the number of stock keeping units have grown tremendously over the years. Assuming that the trend is likely to continue (due to for example, profit maximisation strategies of suppliers and using different materials or remaining price competitive by offering lower quality), the demand for the coffee machines is likely to increase. It's important to mention that Pokland, the supplier, has always been very reliable and delivered on time. The loyalty factor of a supplier plays a role in choosing the optimal strategy as the management may value more expensive, but certain services, instead of less expensive, higher variance services. Following an R,S policy gives the inventory manager a chance to periodically assess the optimal way for tackling the inventory issues like improving service levels and decreasing costs on regular basis. It simply offers more flexibility than other inventory policies.
However, one could argue that the coffee market has rather stationary demand (people who drink coffee, usually drink it on regular basis and stick to their drinking habits). As a result, s,Q policy could be a optimal strategy. However, if the quantity order, Q, is set too low or too high, high and frequent goodwill costs (as well as losing a market share) and ordering costs for the primary case could hurt the firm tremendously as so could the holding costs for the latter.
With an s,S policy, the risk of having frequent goodwill costs decreases as the undershoots from the s,Q policy are non existent, yet the holding costs may hurt the company financially.
For s,Q and s,S, an assumption of no undershoots is made.

8 Key performance indicators

P1 service rate	0.4101
P2 service rate	0.8436
ESPRC	3528.21
Order-up-to-level	32,000
Order value	6500
Holding costs	283,347.27
Shortage costs	275,200.10
Total costs	565,047.37

Table 2: KPIs currently, R,S

P1 service rate	0.6153 ↑
P2 service rate	0.9153 ↑
ESPRC	1909.44 ↓
Order-up-to-level	35,332 ↑
Order value	6500
Holding costs	383,307.27 ↑
Shortage costs	148,936.13 ↓
Total costs	538,743.40 ↓

Table 3: KPIs assuming no goodwill costs, R,S

P1 service rate	0.9353 ↑
P2 service rate	0.9900 ↑
ESPRC	225.55 ↓
Order-up-to-level	44,044.70 ↑
Order value	6500
Holding costs	644,688.36 ↑
Shortage costs	17,592.97 ↓
Total costs	668,781.33 ↑

Table 4: KPIs for service rate of 99%, R,S

P1 service rate	0 ↑
P2 service rate	0.9900 ↑
ESPRC	2,852.51 ↓
Order-up-to-level	33,225.76 ↑
Order value	500 ↓
Holding costs	4,251,454.86 ↑
Shortage costs	17,115.05 ↓
Goodwill costs	1,765,307.94
Total costs	6,034,377.86 ↑

Table 5: KPIs for service rate of 99% with goodwill costs, R,S

P1 service rate	0.3876 ↓
P2 service rate	0.9810 ↑
ESPRC	5,415.33 ↑
Order-up-to-level	40,690.90 ↑
Order value	4146.01 ↓
Holding costs	480,000.00 ↑
Shortage costs	422,395.90 ↑
Total costs	902,475.63 ↑

Table 6: KPIs, s,Q

P1 service rate	0.8574 ↑
P2 service rate	0.9900 ↑
ESPRC	5,415.33 ↑
Order-up-to-level	40,690.90 ↑
Order value	3,260.40 ↓
Holding costs	610,363.56 ↑
Shortage costs	422,395.90 ↑
Total costs	1,032,822.16 ↑

Table 7: KPIs, s,S

Arrows (↑ and ↓) are comparing KPIs from policies with the current policy (Table 2).

9

Given the results from the tables in section 8, the recommendation of making sure that the P2 service rate is at 99% level can be made. It comes at higher costs (around 20% more), but both P1 and P2 service rates have increased. The shortage costs have decreased more than 15 fold, but the holding costs have more than doubled (correlation inbetween the two costs) as more than 12,000 more units will have to be stored. However, the goodwill costs have not been accounted for and can be of great weight in the final costs (as seen in table 5). Other policies, s,Q and s,S, have almost doubled the costs for the company and as a result, it's hard to recommend them, but it's up to Mr Stark to make a decision regarding the policy.