

Pembahasan paket test 5

$$1) \sin(\pi/3 - x) = \sin \pi/3 \cos x - \cos \pi/3 \sin x$$

$$= \frac{1}{2} \sqrt{3} \times \frac{4}{5} - \frac{1}{2} \left(-\frac{3}{5} \right)$$

$$= \frac{1}{10} (4\sqrt{3} + 3)$$

$$2) > \sin x + \cos y = 1$$

$$\sin^2 x + 2 \sin x \cos y + \cos^2 y = 1 \dots (1)$$

$$> \cos x + \sin y = \frac{3}{2}$$

$$\cos^2 x + 2 \cos x \sin y + \sin^2 y = \frac{9}{4} \dots (2)$$

$$> (1) \& (2)$$

$$\sin^2 x + 2 \sin x \cos y + \cos^2 y = 1$$

$$\cos^2 x + 2 \cos x \sin y + \sin^2 y = \frac{9}{4}$$

$$1 + 2(\sin x \cos y + \cos x \sin y) + 1 = \frac{13}{4}$$

$$\sin x \cos y + \cos x \sin y = \frac{5}{8}$$

$$\sin(x + y) = \frac{5}{8}$$

$$> \sin^2(x + y) = 2 \sin(x + y) \cos(x + y)$$

$$= 2 \times \frac{5}{8} \times \frac{\sqrt{39}}{8}$$

$$= \frac{5}{32} \sqrt{39}$$

$$3) \cos(70^\circ + x) = \cos((40^\circ + x) + 30^\circ)$$

$$= \cos(40^\circ + x) \cos 30^\circ - \sin(40^\circ + x) \sin 30^\circ$$

$$= (\sqrt{1 - a^2}) \frac{1}{2} \sqrt{3} - a \cdot \frac{1}{2}$$

$$= \frac{1}{2} (\sqrt{3} (1 - a^2) - a)$$

$$4) \cot 105^\circ \tan 15^\circ = \cot(90^\circ + 15^\circ) \tan 15^\circ$$

$$= \tan 15^\circ \tan 15^\circ$$

$$= (\tan(45^\circ - 30^\circ))$$

$$= \left(\frac{1 - 1/\sqrt{3}}{1 + 1 \cdot 1/\sqrt{3}} \right)^2$$

$$= \left(\frac{\sqrt{3} - 1}{\sqrt{3} + 1} \right)^2$$

$$= \left(\frac{\sqrt{3} - 1}{\sqrt{3} + 1} \cdot \frac{\sqrt{3} - 1}{\sqrt{3} - 1} \right)^2$$

$$= (2 - \sqrt{3})^2$$

$$= 7 - 4\sqrt{3}$$

$$5) (\cos x + \sin x)^2 / (\cos x - \sin x)^2$$

$$= \cos^2 x + 2 \sin x \cos x + \sin^2 x / \cos^2 x - 2 \sin x \cos x + \sin^2 x$$

$$= 1 + \sin 2x / 1 - \sin 2x$$

$$6) > \cos^2 \frac{1}{2} \theta = x + 1 / 2x$$

$$2x \cos^2 \frac{1}{2} \theta - x = 1$$

$$x (2 \cos^2 \frac{1}{2} \theta - 1) = 1$$

$$x \cos \theta = 1$$

$$x + 1 / \cos \theta = \sec \theta$$

$$> x^2 - 1/x^2 = 1 / \cos^2 \theta - \cos^2 \theta$$

$$= \sec^2 \theta - \cos^2 \theta$$

$$= 1 + \tan^2 \theta - \cos^2 \theta$$

$$= \tan^2 \theta + \sin^2 \theta$$

$$7) 1 - \cot a = -1/3$$

$$\cot a = 1 + 1/3 = 4/3$$

$$\sin 2a + \cos 2a = 2 \sin a \cos a + 2 \cos^2 a - 1$$

$$= 2 \cdot 3/5 \cdot 4/5 + 2 (4/5)^2 - 1$$

$$= 24/25 + 32/25 - 25/25$$

$$= 31/25$$

$$8) 3 \sin A + 4 \cos B = 6$$

$$9 \sin^2 A + 24 \sin A \cos B + 16 \cos^2 B = 36 \dots (1)$$

$$3 \cos A + 4 \sin B = 1$$

$$9 \cos^2 A + 24 \cos A \sin B + 16 \sin^2 B = 1 \dots (2)$$

$$(1) \& (2)$$

$$9 \sin^2 A + 24 \sin A \cos B + 16 \cos^2 B = 36$$

$$9 \cos^2 A + 24 \cos A \sin B + 16 \sin^2 B = 1$$

$$9 + 24 \sin A \cos B + \cos A \sin B + 16 = 37$$

$$\sin A \cos B + \cos A \sin B = 12/24 = \frac{1}{2}$$

$$\text{Sudut } A + \text{sudut } B + \text{sudut } C = \text{sudut } 180^\circ$$

$$\sin C = \sin (180^\circ - (A + B)) = \sin (A + B)$$

$$= \sin A \cos B + \cos A \sin B$$

$$= \frac{1}{2}$$

$$9) \alpha + \beta + \gamma = 180^\circ$$

$$\beta + \gamma = 180^\circ - \alpha$$

$$A / \sin \alpha = b / \sin \beta = c / \sin \gamma$$

$$B \sin \gamma = c \cdot \sin \beta / \sin \gamma$$

$$B - c / b + c$$

$$= C \cdot \sin \beta / \sin \gamma - c / c \cdot \sin \beta / \sin \gamma + c$$

$$= c (\sin \beta / \sin \gamma - 1) / c (\sin \beta / \sin \gamma + 1)$$

$$= \sin \beta - \sin \gamma / \sin \beta + \sin \gamma$$

$$= 2 \cos \frac{1}{2} (\beta + \gamma) \sin \frac{1}{2} (\beta - \gamma) / 2 \sin \frac{1}{2} (\beta + \gamma) \cos \frac{1}{2} (\beta - \gamma)$$

$$= \cos \frac{1}{2} (180^\circ - \alpha) \sin \frac{1}{2} (\beta - \gamma) / \sin \frac{1}{2} (180^\circ - \alpha) \cos \frac{1}{2} (\beta - \gamma)$$

$$= \sin \frac{1}{2} \alpha \sin \frac{1}{2} (\beta - \gamma) / \cos \frac{1}{2} \alpha \cos \frac{1}{2} (\beta - \gamma)$$

$$= \tan \frac{1}{2} \alpha \cdot \tan \frac{1}{2} (\beta - \gamma)$$

$$= \tan \frac{1}{2} (\beta - \gamma) / \cot \frac{1}{2} \alpha$$

$$10) (\sin 2x + \sqrt{3} \cos 2x)^2 - 5 = \cos (\phi/6 - 2x)$$

$$(\sin 2x + \sqrt{3} \cos 2x)^2 - 5 = \cos \phi/6 \cos 2x + \sin \phi/6 \sin 2x$$

$$(\sin 2x + \sqrt{3} \cos 2x)^2 - 5 = \frac{1}{2} \sqrt{3} \cos 2x + \frac{1}{2} \sin 2x$$

$$(\sin 2x + \sqrt{3} \cos 2x)^2 - 5 = \frac{1}{2} (\sin 2x + \sqrt{3} \cos 2x)$$

$$\text{Misal: } \sin 2x + \sqrt{3} \cos 2x = p$$

$$p^2 - \frac{1}{2} p - 5 = 0$$

$$2p^2 - p - 10 = 0$$

$$(2p - 5)(p + 2) = 0$$

$$P = 5/2 \text{ (tidak mungkin) } \quad V \quad p = 2$$

$$\sin 2x + \sqrt{3} \cos 2x = -2$$

$$2 \cos (2x - 30^\circ) = -2$$

$$\cos (2x - 30^\circ) = -1$$

$$\cos (2x - 30^\circ) = \cos 180^\circ$$

$$2x - 30^\circ = + - 180^\circ + k \cdot 360^\circ$$

$$> 2x = 210 + k \cdot 360^\circ$$

$$X = 105^\circ + k \cdot 180^\circ$$

$$X = 105^\circ, 285^\circ$$

$$> 2x = -150 + k \cdot 360$$

$$X = -75^\circ + k \cdot 180^\circ$$

$$X = -255^\circ, -75^\circ, 105^\circ, 285^\circ$$

Jadi, banyaknya nilai x yang memenuhi adalah 4

$$11) k = \sqrt{1^2 + (-\sqrt{3})^2} = 2$$

$$\tan \alpha = -\sqrt{3} / 1 = -\sqrt{3}$$

$$\alpha = 2\pi / 3$$

$$\cos(x - 2\pi / 3) = \cos \pi / 2$$

$$x - 2\pi / 3 = + - \pi / 2 + k \cdot 2\pi$$

$$x = \pi / 2 + 2\pi / 3 + k \cdot 2\pi$$

$$x = 7\pi / 6 + k \cdot 2\pi$$

$$\text{Untuk } K = 0 \text{ ----} > \quad x = 7\pi / 6$$

$$x = -\pi / 2 + 2\pi / 3 + k \cdot 2\pi$$

$$x = \pi / 6 + k \cdot 2\pi$$

$$\text{Untuk } K = 0 \text{ ----} > \quad x = \pi / 6$$

$$0 < x < \pi / 6 \quad \text{atau} \quad 7\pi / 6 < x < 2\pi$$

Jadi, pilihannya yang memenuhi adalah $\pi / 12 < x < \pi / 6$

$$12) 2 - \sin \theta / \cos \theta \leq \cos \theta / \sin \theta$$

$$2 - \sin \theta / \cos \theta - \cos \theta / \sin \theta \leq 0$$

$$2 - \sin \theta - \sin^2 \theta - \cos^2 \theta / \sin \theta \cos \theta \leq 0$$

$$2 - \sin \theta - (\sin^2 \theta + \cos^2 \theta) / \sin \theta \cos \theta \leq 0$$

$$2 \sin \theta - 1 / \sin \theta \cos \theta \leq 0$$

$$> 2 \sin \theta - 1 = 0$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \pi / 6$$

$$> \sin \theta = 0$$

$$\theta = 0$$

$$\cos \theta = 0 \rightarrow \theta = \pi / 2$$

Himpunan penyelesaiannya adalah $0 < \theta < \pi / 6$

$$13) \sin (x + \pi / 3) + \sin (x - \pi / 3) \geq \frac{1}{2}$$

$$2 \sin \frac{1}{2} ((x + \pi / 3) + (x - \pi / 3))$$

$$\cos \frac{1}{2} ((x + \pi / 3) - (x - \pi / 3)) \geq \frac{1}{2}$$

$$2 \sin x \cos \pi / 3 \geq \frac{1}{2}$$

$$2 \sin x (\frac{1}{2}) \geq \frac{1}{2}$$

$$\sin x \geq \frac{1}{2}$$

$$\sin x = \frac{1}{2} \rightarrow x = \pi / 6, 5\pi / 6$$

Jadi, nilai x yang memenuhi adalah $\pi / 6 \leq x \leq 5\pi / 6$

$$14) \cos \alpha = k^2 + 2^2 - 1 @ 2 / 2.k.2 < 7/8$$

$$k^2 + 3 / 4k - 7/8 < 0$$

$$2(k^2 + 3) - 7k / 8k < 0$$

$$2k^2 - 7k + 6 / 8k < 0$$

$$(2k - 3)(k - 2) / 8k < 0$$

$$k = 3/2 \vee k = 0$$

Jadi, k yang memenuhi adalah $3/2 < k < 2$

$$15) \sin x + \sin 2x > \sin 3x$$

$$(\sin x - \sin 3x) + \sin 2x > 0$$

$$2 \cos 2x \sin (-x) + 2 \sin x \cos x > 0$$

$$-2 \sin x (\cos 2x - \cos x) > 0$$

$$-2 \sin x (2 \cos^2 x - \cos x - 1) > 0$$

$$-2 \sin x (2 \cos x + 1) (\cos x - 1) > 0$$

$$\sin x = 0 \quad \vee \quad \cos x = -\frac{1}{2} \quad \vee \quad \cos x = 1$$

$$\text{➤ } \sin x = 0 \quad \text{----} > x = 0^\circ, 180^\circ$$

$$\text{➤ } \cos x = -1/2 \quad \text{----} > x = 120^\circ, 240^\circ$$

$$\text{➤ } \cos x = 1 \quad \text{----} > x = 0^\circ, 360^\circ$$

Jadi, x yang memenuhi adalah $0 < x < 120^\circ, 180^\circ < x < 240^\circ$

$$16) \alpha = 360^\circ / 8 = 45^\circ$$

$$X^2 = 6^2 + 6^2 - 2 \cdot 6 \cdot 6 \cos 45^\circ$$

$$X^2 = 36 + 36 - 72(\frac{1}{2} \sqrt{2})$$

$$X = \sqrt{36(2 - \sqrt{2})}$$

$$X = 6\sqrt{2 - \sqrt{2}}$$

Jadi, keliling segi delapan tersebut adalah: $8(6\sqrt{2 - \sqrt{2}}) = 48\sqrt{2 - \sqrt{2}}$

$$17) \cos A = \frac{AB^2 + AC^2 - BC^2}{2 AB \cdot AC} = \frac{6^2 + 3^2 - (3\sqrt{7})^2}{2 \cdot 6 \cdot 3} = -1/2$$

$$\cos A = -1/2 \quad \text{----} > \sin A = \frac{1}{2} \sqrt{3}$$

$$= \frac{1}{2} AB \cdot AC \cdot \sin A$$

$$= \frac{1}{2} \cdot 6 \cdot 3 \cdot \frac{1}{2} \sqrt{3}$$

$$= \frac{9}{2} \sqrt{3}$$

Jadi volume prisma adalah:

$$\frac{9}{2} \sqrt{3} (20) = 90 \sqrt{3}$$

$$18) \cos(A + B) = \cos \phi / 3$$

$$\cos A \cos B - \sin A \sin B = \frac{1}{2}$$

$$\frac{5}{8} - \sin A \sin B = \frac{1}{2}$$

$$\sin A \sin B = \frac{1}{8}$$

$$\text{Jadi, } \cos (A - B) = \cos A \cos B + \sin A \sin B$$

$$= 5/8 + 1/8$$

$$= 6/8$$

$$= 3/4$$

$$19) \sin (\alpha - \beta) = 3/5$$

$$\sin \alpha \cos \beta - \cos \alpha \sin \beta = 3/5$$

$$1/5 - \cos \alpha \sin \beta = 3/5$$

$$\cos \alpha \sin \beta = 2/5$$

$$\sin (\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$= 1/5 - 2/5$$

$$= -1/5$$

$$20) (\cos 145^\circ + \cos 35^\circ) - \cos 45^\circ$$

$$= 2 \cos (145 + 35 / 2) \cos (145 - 35 / 2) - \cos 45^\circ$$

$$= 2 \cos 90 \cos 55 - \frac{1}{2} \sqrt{2}$$

$$= 2(0) \cos 55 - \frac{1}{2} \sqrt{2}$$

$$= -1/2 \sqrt{2}$$

$$21) \cos 140^\circ - \cos 100^\circ / \sin 140^\circ - \sin 100^\circ$$

$$= -2 \sin 120^\circ \sin 20^\circ / 2 \cos 120^\circ \sin 20^\circ$$

$$= -\tan (120^\circ)$$

$$= \sqrt{3}$$

$$22) 3 \sin (3x + x / 2) \cos (3x - x / 2)$$

$$2 \sin 2x \cos x$$

$$2(\sin x \cos x) \cos x$$

$$4 \cdot 4/5 \cdot 3/5 \cdot 3/5$$

$$= 144/125$$

$$23) \sin (x - 60^\circ) + \sin (x + 60^\circ) = p$$

$$2 \sin ((x - 60^\circ) + (x + 60^\circ) / 2)$$

$$\cos \left(\frac{(x - 60^\circ) + (x + 60^\circ)}{2} \right) = p$$

$$2 \sin x \cos (-60) = p$$

$$2 \sin x \cdot \frac{1}{2} = p$$

$$\sin x = p$$

$$\text{Jadi, } \sin 2x = 2 \sin x \cos x$$

$$= 2p \sqrt{1 - p^2}$$

$$24) \cos A = \frac{4}{5} \text{ maka } \sin A = \frac{3}{5}$$

$$\sin B = \frac{1}{\sqrt{5}} \text{ maka } \cos B = \frac{2}{\sqrt{5}}$$

$$\sin C = \sin (180^\circ - (A + B))$$

$$= \sin (A + B)$$

$$= \sin A \cos B + \cos A \sin B$$

$$= \frac{3}{5} \cdot \frac{2}{\sqrt{5}} + \frac{4}{5} \cdot \frac{1}{\sqrt{5}}$$

$$= \frac{10}{5\sqrt{5}}$$

$$= \frac{2}{\sqrt{5}}$$

$$25) \sin^2 2x - 2 \sin x \cos x - 2 = 0$$

$$\sin 2x - \sin 2x - 2 = 0$$

$$(\sin 2x - 2)(\sin 2x + 1) = 0$$

$$\sin 2x = 2 \text{ (tidak mungkin)} \vee \sin 2x = -1$$

$$\sin 2x = -1$$

$$\sin 2x = \sin 270$$

$$2x = 270 + k \cdot 360$$

$$x = 135 + k \cdot 180$$

$$\text{Untuk } k=0, \text{ maka } x = 135$$

$$\text{Untuk } k=1, \text{ maka } x = 315$$

$$2x = (180 - 270) + k \cdot 360$$

$$x = -45 + k \cdot 180$$

Untuk $k = 1$, maka $x = 135$

Untuk $k=2$, maka $x = 315$

Jadi, himpunan penyelesaiannya adalah $\{315, 315\}$

$$26) \cos 2x - 3 \cos x + 2 = 0$$

$$2 \cos^2 x - 1 - 3 \cos x + 2 = 0$$

$$(2 \cos x - 1)(\cos x - 1) = 0$$

$$\text{➤ } \cos x = \frac{1}{2}$$

$$\cos x = \cos 60$$

$$\text{➤ } x = 60 + k.360$$

$$\text{Untuk } k = 0, \text{ maka } x = 60 = \pi/3$$

$$\text{➤ } x = -60 + k.360$$

$$\text{Untuk } k = 1, \text{ maka } x = 300 = 5\pi/3$$

$$\text{➤ } \cos x = 1$$

$$\cos x = \cos 0$$

$$x = 0 + k.360$$

$$\text{Untuk } k = 0, \text{ maka } x = 0$$

$$\text{Untuk } k = 1, \text{ maka } x = 360 = 2\pi$$

Jadi himpunan penyelesaiannya adalah $\{0, \pi/3, 5\pi/3, 2\pi\}$

$$27) 2 \cos (2x - 60^\circ) = 1$$

$$\cos (2x - 60^\circ) = \cos 60^\circ$$

$$\text{➤ } 2x - 60^\circ = 60 + k.360$$

$$2x = 120 + k.360$$

$$x = 60 + k.180$$

$$\text{Untuk } k = 0, \text{ maka } x = 60$$

$$\text{➤ } 2x - 60^\circ = -60 + k.360$$

$$2x = 0 + k.360$$

$$x = 0 + k.180$$

$$\text{Untuk } k=1, \text{ maka } x = 180$$

Jadi himpunan penyelesaiannya adalah $\{60^\circ, 180^\circ\}$

$$28) 3 \cos (360 - x) > 2 \sin^2 x$$

$$3 \cos x > 2 \sin^2 x$$

$$3 \cos x > 2(1 - \cos^2 x)$$

$$2 \cos^2 x + 3 \cos x - 2 > 0$$

$$(2 \cos x - 1)(\cos x + 2) > 0$$

$$\cos x = \frac{1}{2} \quad \vee \quad \cos x = -2 \text{ (tidak mungkin)}$$

$$\cos x = \cos 60$$

$$X = + - 60 + k.360$$

$$X = 60, 300$$

$$29) \sin(x - 20) + \sin(x + 70) - 1 \geq 0$$

$$2 \sin\left(\frac{(x - 20) + (x + 70)}{2}\right)$$

$$\cos\left(\frac{(x - 20) + (x + 70)}{2}\right) \geq 1$$

$$2 \sin(x + 25) \cos(-45) \geq 1$$

$$2 \sin(x + 25) \frac{1}{2} \sqrt{2} \geq 1$$

$$\sin(x + 25) = \frac{1}{2} \sqrt{2}$$

$$\sin(x + 25) = \sin 45$$

$$\text{➤ } x + 25 = 45 + k.360$$

$$x = 20 + k.360$$

$$x = 20$$

$$\text{➤ } x + 25 = (180 - 45) + k.360$$

$$x = 110 + k.360$$

$$x = 110$$

$$30) QS^2 = PQ^2 + PS^2 - 2 PQ PS \cos 45^\circ$$

$$QS^2 = 8^2 + (8\sqrt{2})^2 - 2.8.8\sqrt{2}\left(\frac{1}{2}\sqrt{2}\right)$$

$$QS = \sqrt{64} = 8$$

Perhatikan segitiga QSR

$$QS / \sin R = QR / \sin R$$

$$8 / \sin 30^\circ = QR / \sin 60^\circ$$

$$QR = 16\left(\frac{1}{2}\sqrt{3}\right) = 8\sqrt{3}$$