

1. Aturan sinus : $a / \sin A = b / \sin B$
 $a+b/b = a/b + 1$
 $= \sin A / \sin B + 1$ (A)
2. $Y = \sin x - \cos x + 1$
Maksimum = $\sqrt{1^2 + 1^2 + 1}$
 $= 1 + \sqrt{2}$
Minimum = $1 - \sqrt{1^2 + 1^2}$
 $= 1 - \sqrt{2}$ (C)
3. $\sin^2 x - 2 \cos x = 1 \rightarrow 1 - \cos^2 x - 2 \cos x - 1 = 0$
 $\cos x (\cos x + 2) = 0$
 $\cos x = 0 \rightarrow x = \pi/2$
 $\sin x = 1$ (A)
4. $y = 1 \cdot \cos 2x + 1 \cdot \sin 2x + 1$
 $y_{\text{maks}} = \sqrt{1^2 + 1^2 + 1}$
 $= \sqrt{2} + 1$ (B)
5. $\tan 60 = a/\sqrt{2}/AD$
 $AD = a/\sqrt{2}/\tan 60$
 $= a/\sqrt{2}/\sqrt{3}$
 $= a/\sqrt{6} = a/6\sqrt{6}$ (B)
6. $\tan x = -2/3$, maka $5 \sin x + 6 \cos x / 2 \cos x - 3 \sin x$ dibagi $\cos x$
 $5 \tan x + 6 / 2 - 3 \tan x = 10/3 + 6 / 2 + 2 = 2/3$ (D)
7. $\sin \alpha = 1/\sqrt{3}$
 $\tan (1/2\pi - \alpha) + 3 \cos \alpha$
 $= 1 / \tan \alpha + 3 \cos \alpha$
 $= \sqrt{2} + 3 \cdot \sqrt{2}/\sqrt{3}$
 $= \sqrt{2} + \sqrt{6}$ (C)
8. $2 \tan^2 x + 3 \tan x - 2 = 0$
 $(2 \tan x - 1) (\tan x + 2) = 0$
 $\tan x = 1/2, \tan x = -2$
(tidak memenuhi)
 $\sin x = 2/5\sqrt{5}, \cos x = -1/5\sqrt{5}$
 $\sin x + \cos x = 1/5\sqrt{5}$
9. (i) $\cos A = b^2 + c^2 - a^2 / 2bc$
(ii) segitiga ADB $\rightarrow d^2 = c^2 + \frac{1}{4} b^2 - bc \cdot \cos A$
 $d^2 = c^2 + \frac{1}{4} b^2 - bc (b^2 + c^2 - a^2 / 2bc)$
 $d^2 = c^2 + \frac{1}{4} b^2 - \frac{1}{2} c^2 - \frac{1}{2} b^2 + \frac{1}{2} a^2$
 $d^2 = \frac{1}{2} a^2 - \frac{1}{4} b^2 + \frac{1}{2} c^2$ (B)
10. $(\sin x - \cos x)^2 = p^2$
 $\sin^2 x + \cos^2 x - 2 \sin x \cos x = p^2$
 $1 - p^2 = 2 \sin x \cos x$
 $\sin x \cos x = \frac{1}{2} (1 - p^2)$ (D)
11. $PQ^2 = PR^2 + RQ^2 - 2PR \cdot RQ \cos \text{sudut PQR}$
 $7^2 = 4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cos \text{sudut PQR}$
 $\cos \text{sudut PQR} = -8/10 = -4/5 \rightarrow \tan \text{sudut PQR}$
 $\cos \text{sudut PQR} = -8/10 = -4/5 \rightarrow \tan \text{sudut PQR} = -\sqrt{24}$ (D)
12. Gunakan rumus jari-jari lingkaran luar segitiga
 $R = BC / 2 \sin A = 30 / 2(1/3\sqrt{5}) = 9\sqrt{5}$ (D)
13. $\sin^2 A = 9/10$ untuk $\pi/2 < 2A < \pi \rightarrow \pi/4 < A < \pi/2$

$$\sin^2 A = 9/10 \rightarrow \sin A = 3/\sqrt{10}$$

$$\cos^2 A = 1 - \sin^2 A = 1 - 9/10 = 1/10$$

$$\cos A = 1/\sqrt{10}$$

$$\tan 2A = \sin 2A / \cos 2A = 2 \sin A \cos A / (1 - 2 \sin^2 A)$$

$$= 2(3/\sqrt{10})(1/\sqrt{10}) / (1 - 2(3/\sqrt{10})^2) = 6/10(-4/5) = 6/8 = -3/4 \text{ (B)}$$

$$14. \tan x - 2 \cot x - 1 = 0 \rightarrow \tan x - 2/\tan x - 1 = 0$$

$$\rightarrow \tan^2 x - \tan x - 2 = 0$$

$$\rightarrow (\tan x - 2)(\tan x + 1) = 0$$

$$\rightarrow \tan x = 2 \text{ atau } \tan x = -1$$

Karena terletak pada kuadran II, maka $\tan x = -1 \rightarrow x = 135 \rightarrow \sin 135 = \frac{1}{2}\sqrt{2}$ (B)

$$15. \sin 2x > \frac{1}{2} \rightarrow 30^\circ < 2x < 150^\circ \rightarrow 15^\circ < x < 75^\circ$$

Dapat diperoleh himpunan penyelesaian

$$HP = \{x | 15^\circ < x < 75^\circ\} \text{ (A)}$$

$$16. \text{Luas segitiga ABC}$$

$$= (3+2\sqrt{3})^2$$

$$AB = (6+4\sqrt{3})$$

$$BC = 7 \text{ cm}$$

AB = alas segitiga ABC

$$CD = \text{Tinggi segitiga ABC} = BC \cdot \sin \theta$$

Sehingga :

$$\text{Luas segitiga ABC} = \frac{1}{2} AB \cdot \sin \theta$$

$$\rightarrow (3+2\sqrt{3}) = \frac{1}{2} (6+4\sqrt{3}) \cdot 7 \cdot \sin \theta$$

$$\rightarrow (6+4\sqrt{3}) = (6+4\sqrt{3}) \cdot 7 \cdot \sin \theta$$

$$\sin \theta = (6+4\sqrt{3}) / (6+4\sqrt{3}) \cdot 7 = 1/7$$

Pada segitiga ABC:

$$A+B+C = 180^\circ$$

$$\rightarrow (A+C) = 180^\circ - A = 180^\circ - \theta$$

$$\sin(A+C) = \sin(180^\circ - \theta) = \sin \theta = 1/7 \text{ (A)}$$

$$17. \sin x = 8/10 = 4/5 \rightarrow \cos x = 3/5$$

Ingat rumus penjumlahan trigonometri :

$$\cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

Jadi,

$$\cos 3x + \cos x = 2 \cos 2x \cos x$$

$$= 2(1-2\sin^2 x)(\cos x)$$

$$= 2 \cdot -7/25 \cdot 3/5 = -42/125 \text{ (C)}$$

$$18. 2 \tan x / (1 + \tan^2 x) = 2 \sin x / \cos x / (1 + (\sin x / \cos x)^2)$$

$$= 2 \sin x \cos x / (\cos^2 x + \sin^2 x)$$

$$= 2 \sin x \cos x / 1 = 2 \sin 2x \text{ (B)}$$

$$19. 3 \cos(360-x) > 2 \sin^2 x$$

$$\rightarrow 3 \cos x > 2(1-\cos^2 x) \rightarrow 2 \cos^2 x + 3 \cos x - 2 > 0$$

$$\rightarrow (2 \cos x - 1)(\cos x + 2) > 0$$

Karena $(\cos x + 2)$ selalu positif, berarti tidak mempengaruhi pertidaksamaan, jadi tinggal menentukan nilai x yang memenuhi pertidaksamaan

$$(2 \cos x - 1) > 0 \rightarrow \cos x > \frac{1}{2} \text{ untuk } 0^\circ < x < 180^\circ$$

$$HP = \{0 \leq x < 60 \text{ atau } 300 < x \leq 360\} \text{ (D)}$$

20. Luas segitiga PQR = luas segitiga PQS + luas segitiga QSR

$$\frac{1}{2} PQ \cdot QR = \frac{1}{2} PQ \cdot ST + \frac{1}{2} QR \cdot SU$$

$$\rightarrow PQ \cdot QR = PQ \cdot ST + QR \cdot SU$$

$$\rightarrow 6 \cdot 4 = 6 \cdot OS \sin 45 + 4 \cdot QS \sin 45$$

$$\rightarrow 24 = 6 \cdot \frac{1}{2} \sqrt{2} QS + 4 \cdot \frac{1}{2} \sqrt{2} QS$$

$$\rightarrow QS = \frac{24}{5\sqrt{2}} = \frac{12}{5} \sqrt{2} \text{ (B)}$$

21. Ingat rumus:

$$A \sin x + B \cos x = k \cos (x - \alpha)$$

$$K = \sqrt{p^2 + (p+1)^2} = \sqrt{2p^2 + 2p + 1}$$

$$\tan \theta = \frac{p}{p+1}$$

Soal dapat ditulis

$$P \sin x + (p+1) \cos x = p + 2$$

$$\rightarrow K \cos(x - \theta) = p + 2$$

$$\rightarrow K = \frac{p+2}{\cos(x - \theta)}$$

Agar dapat penyelesaian maka haruslah

$$-1 \leq k \leq 1$$

$$\rightarrow -1 \leq \frac{p+2}{\sqrt{2p^2 + 2p + 1}} \leq 1 \rightarrow 0 \leq \frac{p+2}{\sqrt{2p^2 + 2p + 1}} \leq 1$$

$$\rightarrow p+2 \leq \sqrt{2p^2 + 2p + 1} \text{ (kedua ruas di kuadratkan)}$$

$$\rightarrow p^2 + 4p + 4 \leq 2p^2 + 2p + 1$$

$$\rightarrow p^2 - 2p - 3 \geq 0 \rightarrow (p-3)(p+1) \geq 0$$

$$HP = \{p \leq -1 \text{ atau } p \geq 3\} \text{ (A)}$$

22. $\frac{1}{\sin a} - \frac{1}{\cos a} = \cos a - \sin a / \sin a \cdot \cos a$

$$(\cos a - \sin a)^2 = \cos^2 a + \sin^2 a - 2 \sin a \cdot \cos a$$

$$= 1 - 2 \sin a \cdot \cos a = 1 - 2 \cdot \frac{8}{25} = \frac{9}{25}$$

Maka diperoleh: $\sin a - \cos a = \frac{3}{5}$ jadi

$$\frac{1}{\sin a} - \frac{1}{\cos a} = \cos a - \sin a / \sin a \cdot \cos a = \frac{3}{5} \cdot \frac{8}{25} = \frac{15}{8} \text{ (A)}$$

23. $\sin P + \sin Q = 2 \cdot \sin \frac{(P+Q)}{2} \cdot \cos \frac{(P+Q)}{2}$

Selanjutnya diperoleh:

$$2 \cdot \sin(x-25) \cdot \cos(45) - 1 \leq 0$$

$$2 \sin(x-25) \cdot \frac{1}{2} \sqrt{2} \leq 1 \rightarrow \sin(x-25) \leq \frac{1}{\sqrt{2}}$$

$$\text{Dapat diperoleh } \sin(x-25) \leq \frac{1}{2} \sqrt{2}$$

Perhatikan grafik $y = \sin(x-25)$

Karena $\sin(x-25) \leq \frac{1}{2} \sqrt{2}$ dan $0 \leq x \leq 360$, maka himpunanannya:

$$HP = \{x | 0 \leq x \leq 70 \text{ atau } 160 \leq x \leq 360\} \text{ (A)}$$

24. $2\sqrt{3} \cos 2x - 4 \sin x \cos x = 2$

$$\rightarrow 2\sqrt{3} \cos 2x - 4 \sin 2x = \sqrt{3} \cos 2x - \sin 2x = 1$$

$$K = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$$

$$\tan 2x = -\frac{1}{\sqrt{3}} \rightarrow \tan 2x = \tan 330 \rightarrow 2x = 330$$

$$\sqrt{3} \cos 2x - \sin 2x = 1 \rightarrow 2 \cos(2x - 330) = 1$$

$$\rightarrow \cos(2x - 330) = \frac{1}{2} \rightarrow \cos 2(x - 165) = \cos 60$$

$$\rightarrow 2(x - 165) = \pm 60 + k \cdot 360$$

$$\rightarrow x - 165 = \pm 30 + k \cdot 180$$

$$\rightarrow x = 165 \pm 30 + k \cdot 180$$

$$x = 165 + 30 + k \cdot 180 \rightarrow \text{HP} : \{195\} \text{ atau}$$

$$x = 165 - 30 + k \cdot 180 \rightarrow \text{HP} : \{135, 315\}$$

Jadi himpunan penyelesaiannya adalah

$$\{3/4 \pi, 13/12 \pi, 7/4 \pi\} (C)$$

$$25. \cos 2x + \sin x - 1 = 0$$

$$\rightarrow (\cos^2 x - \sin^2 x) + \sin x - (\cos^2 x + \sin^2 x) = 0$$

$$\rightarrow 2 \sin^2 x - \sin x = 0 \rightarrow \sin x (2 \sin x - 1) = 0$$

Berlaku:

$$\sin x = 0 \rightarrow x = 0, \pi, 2\pi \text{ atau } 2 \sin x - 1 = 0 \rightarrow \sin x = 1/2 \rightarrow x = \pi/6, 5\pi/6$$

$$\text{Jadi HP} = \{0, \pi/6, 5\pi/6, \pi, 2\pi\} (C)$$

$$26. \text{Gunakan rumus perbandingan sudut segitiga}$$

$$a/\sin A = b/\sin B = c/\sin C$$

$$6/\sin A = 5/\sin B = 4/\sin C$$

$$\text{Jadi } \sin \alpha : \sin \beta : \sin \gamma = 6 : 5 : 4 (C)$$

$$27. \cos(x-y) = \cos x \cdot \cos y + \sin x \cdot \sin y = 4/5$$

$$\cos x \cdot \cos y + 3/10 = 4/5 \rightarrow \cos x \cdot \cos y = 1/2$$

$$\tan x \cdot \tan y = \sin x \cdot \sin y / \cos x \cdot \cos y = 3/10 / 1/2 = 3/5 (D)$$

$$28. a \sin x + b \cos x = \sin(30+x)$$

$$a \sin x + b \cos x = \sin 30 \cdot \cos x + \cos 30 \cdot \sin x$$

$$= 1/2 \cos x + 1/2 \sqrt{3} \sin x = 1/2 \sqrt{3} \sin x + 1/2 \cos x$$

$$\text{Jadi } a = 1/2 \sqrt{3}; b = 1/2$$

$$\rightarrow a\sqrt{3} + b = 1/2 \sqrt{3} (\sqrt{3}) + 1/2 = 2 (D)$$

$$29. \text{Misalkan panjang BC} = x$$

Gunakan rumus cosinus

$$AB^2 = AC^2 + CB^2 - 2AC \cdot CB \cdot \cos \text{ sudut ACB}$$

$$7^2 = 5^2 + x^2 - 2 \cdot 5 \cdot x \cos 120 \rightarrow 49 = 25 + x^2 - 10x$$

$$x + 5x - 24 = 0 \rightarrow (x+8)(x-3) = 0$$

$$\rightarrow x = -8 \text{ atau } x = 3$$

Karena jarak harus positif maka $x = 3$ cm. Kelilingnya adalah $= 5 + 7 + 3$

$$= 15 \text{ cm} (B)$$

$$30. \sin 1/2 A = \sqrt{1 - \cos^2 1/2 A} = \sqrt{1 - (x+1/2x)} = \sqrt{x-1/2x}$$

$$\sin A = 2 \sin 1/2 A \cdot \cos 1/2 A = 2 \sqrt{x-1/2x} \cdot \sqrt{x+1/2x}$$

$$= \sqrt{x^2 - 1/x} (A)$$