

SHEET - An Introduction to Statistical Learning
Chapter 5 - Classification

23 september 2024

1 Resampling Methods

1.1 Courses' Demonstrations

Suppose that we wish to invest a fixed sum of money in two financial assets that yield returns of X and Y , respectively, where X and Y are random quantities. We will invest a fraction α of our money in X , and will invest the remaining $1 - \alpha$ in Y . Since there is variability associated with the returns on these two assets, we wish to choose α to minimize the total risk, or variance, of our investment. In other words, we want to minimize $Var(\alpha X + (1 - \alpha)Y)$. One can show that the value that minimizes the risk is given by

$$\alpha = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}} \quad (5.6)$$

Demonstration : $\alpha = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}$

$$Var(\alpha X + (1 - \alpha)Y) = \alpha^2 Var(X) + (1 - \alpha)^2 Var(Y) + 2\alpha(1 - \alpha)Cov(XY)$$

We search α that minimizes $Var(\alpha X + (1 - \alpha)Y)$

$$\frac{d}{d\alpha} Var(\alpha X + (1 - \alpha)Y) = 0$$

$$2\alpha Var(X) - 2(1 - \alpha)Var(Y) + 2(1 - \alpha)Cov(XY) - 2\alpha Cov(XY) = 0$$

$$2\alpha Var(X) - 2(1 - \alpha)Var(Y) + 2Cov(XY) - 2\alpha Cov(XY) - 2\alpha Cov(XY) = 0$$

$$\alpha Var(X) + (\alpha - 1)Var(Y) + Cov(XY) - 2\alpha Cov(XY) = 0$$

$$\alpha(Var(X) + Var(Y) - 2Cov(XY)) = Var(Y) - Cov(XY)$$

Finally

$$\alpha = \frac{Var(Y) - Cov(XY)}{Var(X) + Var(Y) - 2Cov(XY)}$$

$$\alpha = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}$$