SHEET - An Introduction to Statistical Learning
Chapter 5 - Classification

23 september 2024

1 Classification

1.1 Courses' Demonstrations

Suppose that we wish to invest a fixed sum of money in two financial assets that yield returns of X and Y , respectively, where X and Y are random quantities. We will invest a fraction α of our money in X, and will invest the remaining $1-\alpha$ in Y . Since there is variability associated with the returns on these two assets, we wish to choose α to minimize the total risk, or variance, of our investment. In other words, we want to minimize $Var(\alpha X + (1-\alpha)Y)$. One can show that the value that minimizes the risk is given by

$$\alpha = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}} \tag{5.6}$$

Demonstration : $\alpha = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}$

$Var(\alpha X + (1-\alpha)Y) = \alpha^2 Var(X) + (1-\alpha)^2 Var(Y) + 2\alpha(1-\alpha)Cov(XY)$

We search α **that minimizes** $Var(\alpha X + (1-\alpha)Y)$

$\frac{d}{d\alpha} Var(\alpha X + (1-\alpha)Y) = 0$

$2\alpha Var(X) - 2(1-\alpha)Var(Y) + 2(1-\alpha)Cov(XY) - 2\alpha Cov(XY) = 0$

$2\alpha Var(X) - 2(1-\alpha)Var(Y) + 2Cov(XY) - 2\alpha Cov(XY) - 2\alpha Cov(XY) = 0$

$\alpha Var(X) + (\alpha - 1)Var(Y) + Cov(XY) - 2\alpha Cov(XY) = 0$

$\alpha(Var(X) + Var(Y) - 2Cov(XY)) = Var(Y) - Cov(XY)$

Finally

$alpha = \frac{Var(Y)^2 - Cov(XY)}{Var(X)^2 + Var(Y)^2 - 2Cov(XY)}$

$\alpha = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}$