

Verovatnoća i raspodele verovatnoća

Nedelja 4 - Vežbe

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Dogadaji

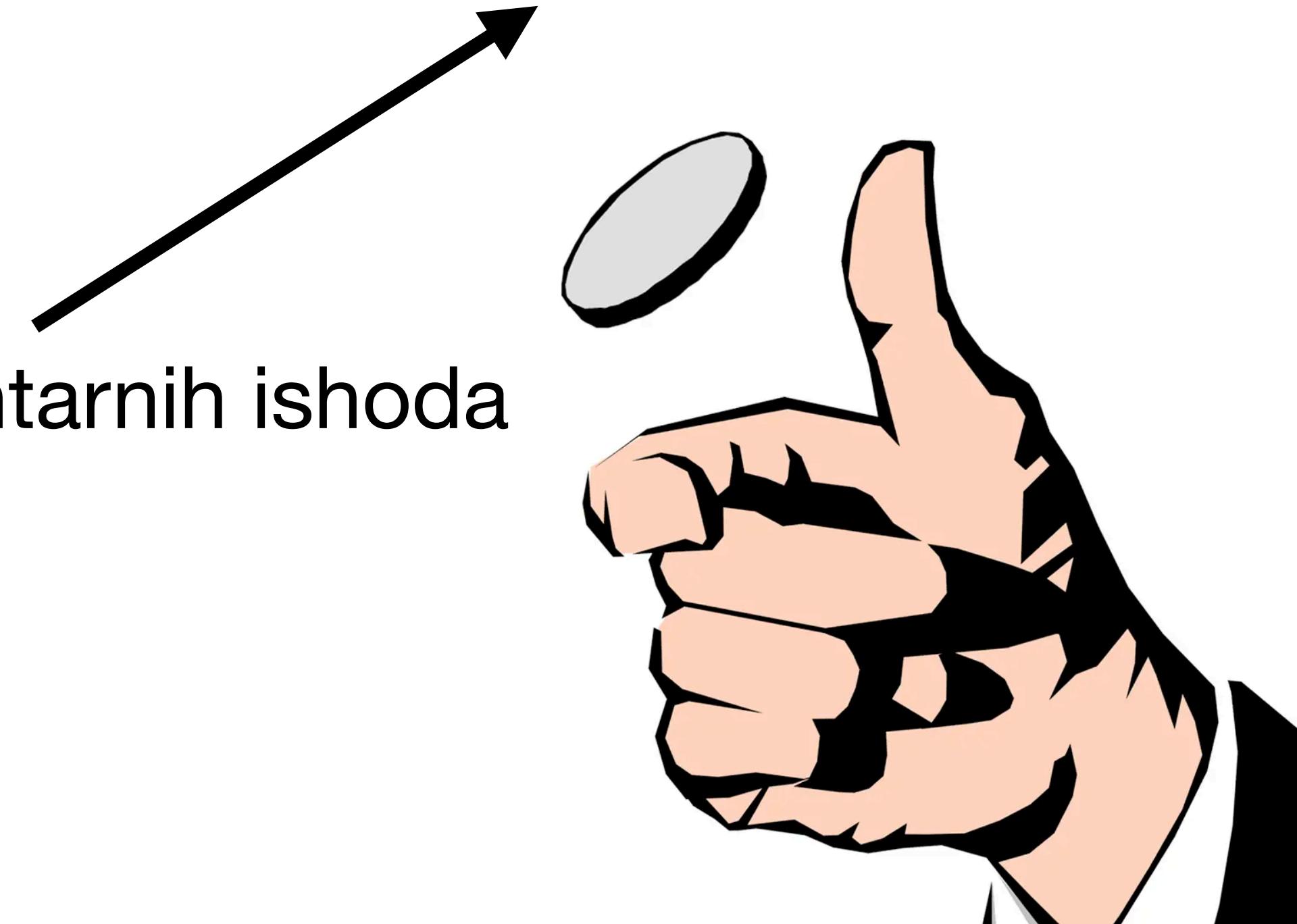
Deterministički

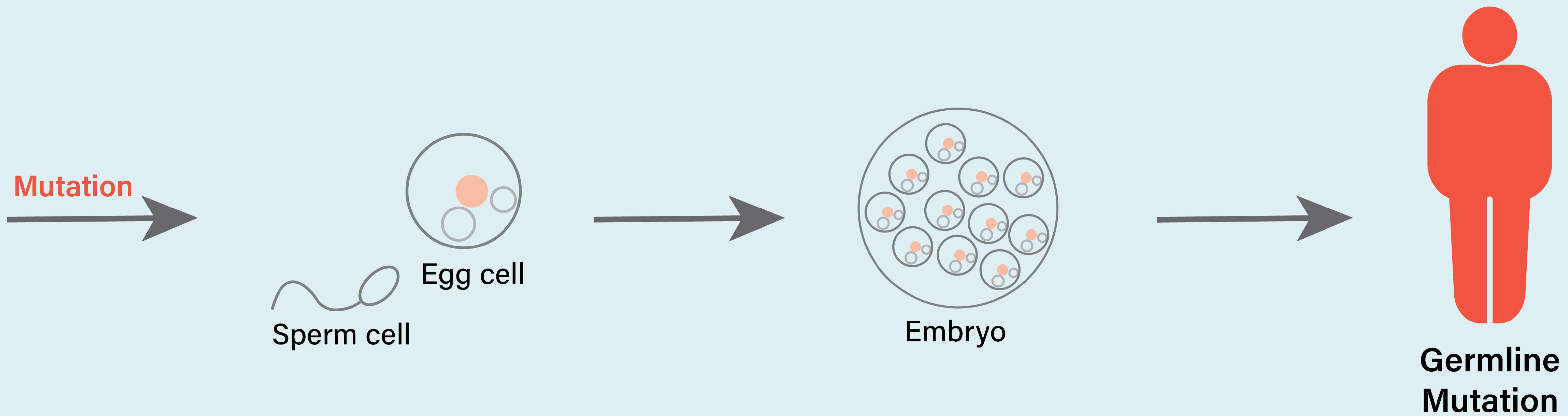
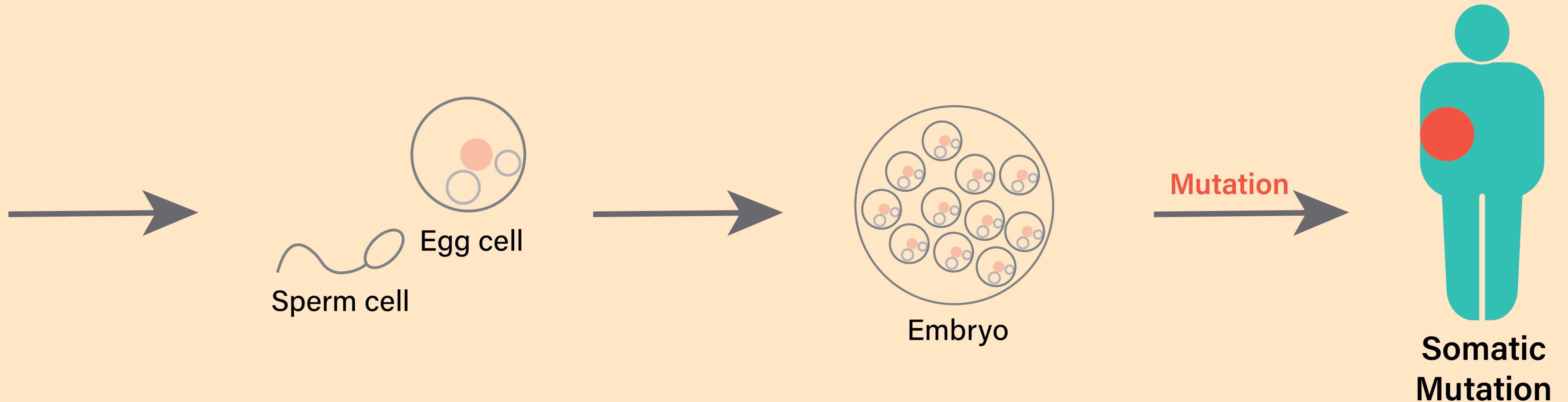
Bez Vitamina C → Skorbut

Slučajni (stohastični)

$$\omega = \{A, B, O, AB\}$$

Skup elementarnih ishoda

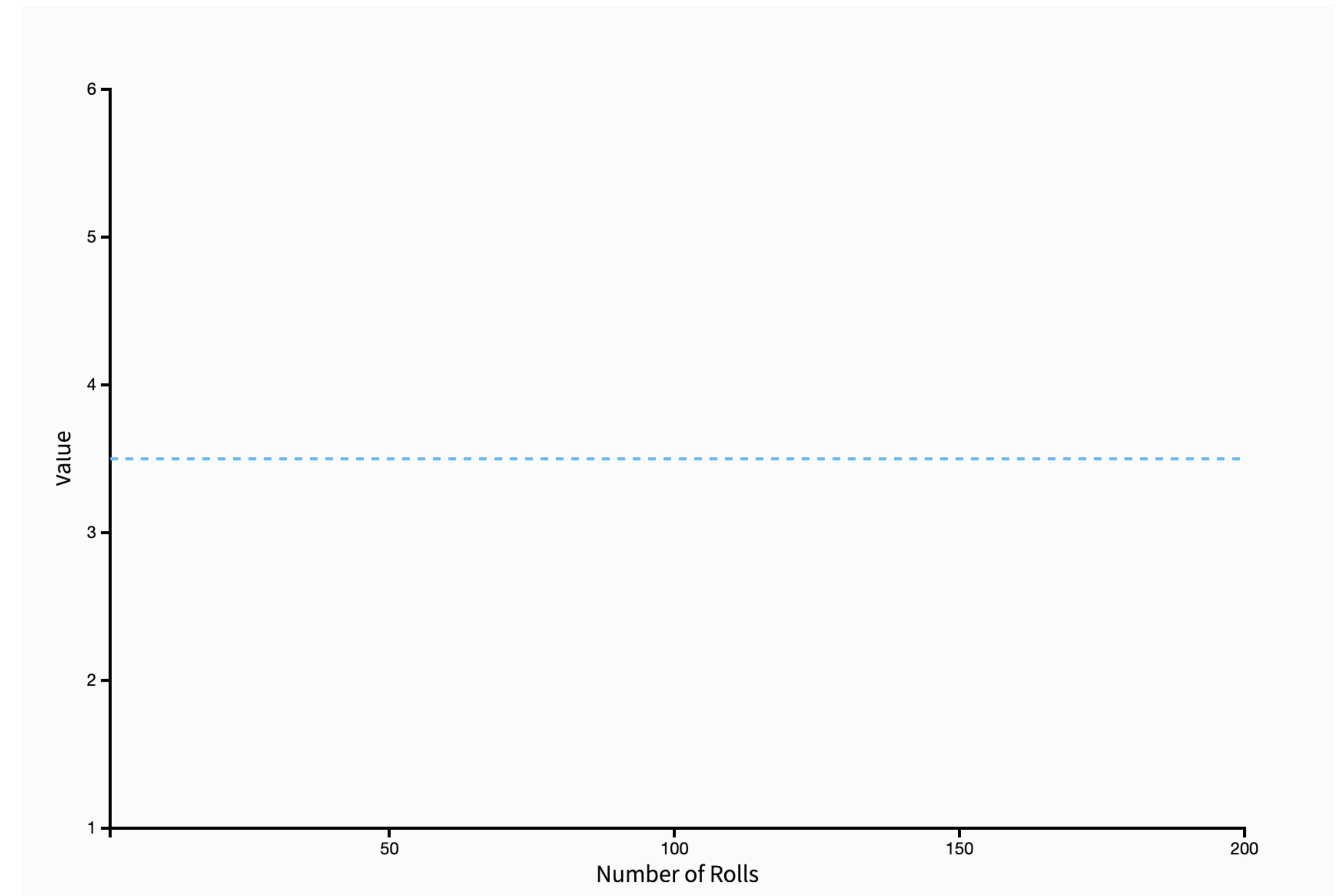
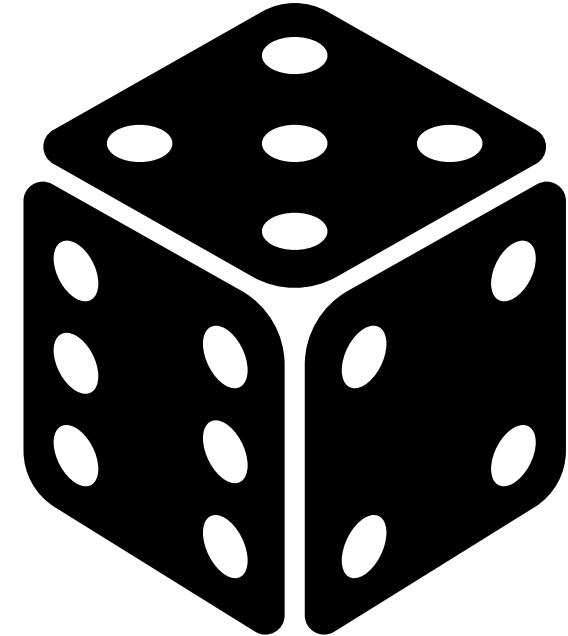




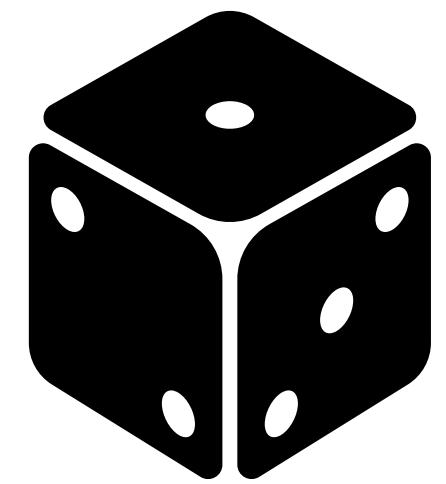
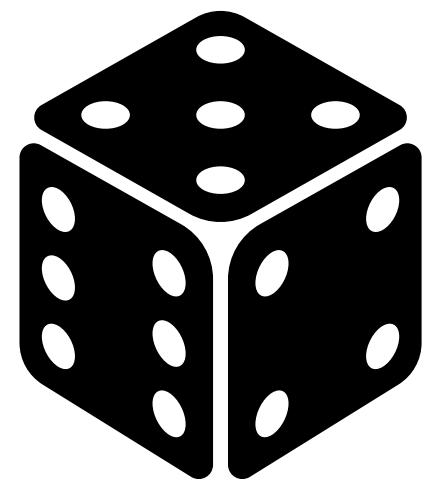
Verovatnoća

Verovatnoća je mera očekivanja nekog slučajnog događaja.

Od 0 (0%) do 1 (100%)



Verovatnoća



Verovatnoća

OUTCOME	ROLLING TWO DICE	ODDS	PERCENTAGE
2		35 to 1	2.78%
3		34 to 2	5.56%
4		33 to 3	8.33%
5		32 to 4	11.11%
6		31 to 5	13.89%
7		30 to 6	16.67%
8		31 to 5	13.89%
9		32 to 4	11.11%
10		33 to 3	8.33%
11		34 to 2	5.56%
12		35 to 1	2.78%



Verovatnoća

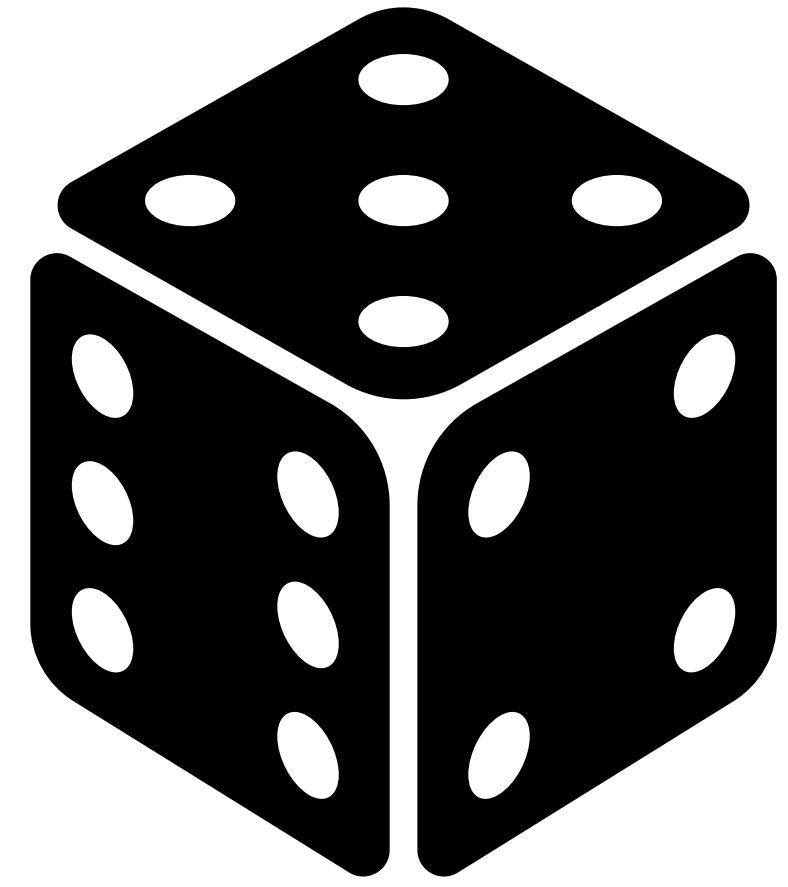
Podela

- Objektivna
 - Teorijska (matematička)
 - Empirijska (statistička)
- Subjektivna

Verovatnoća

Teorijska

- Pre merenja
- Svi mogući ishodi su jednakо verovatni



Verovatnoća

Empirijska

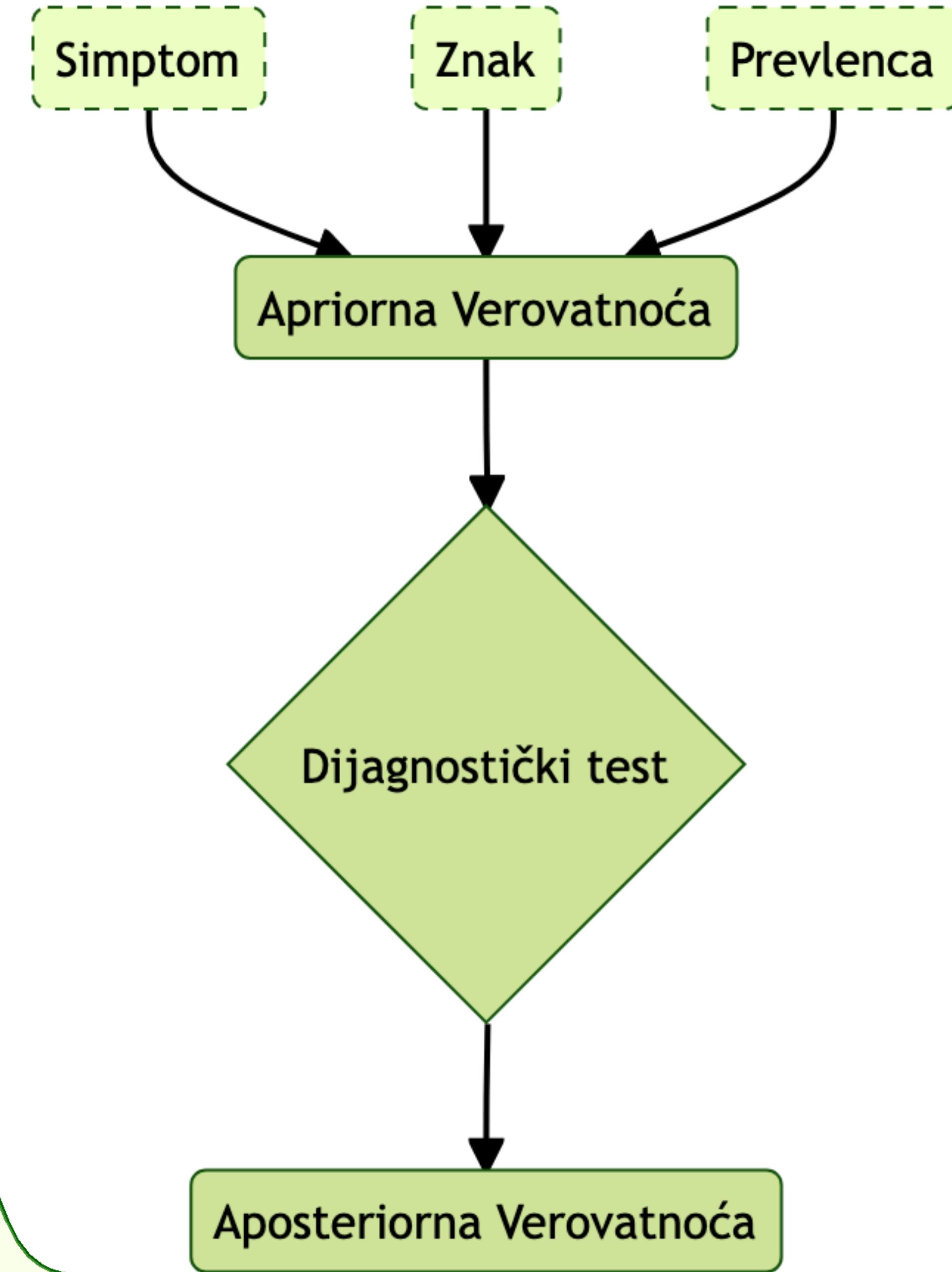
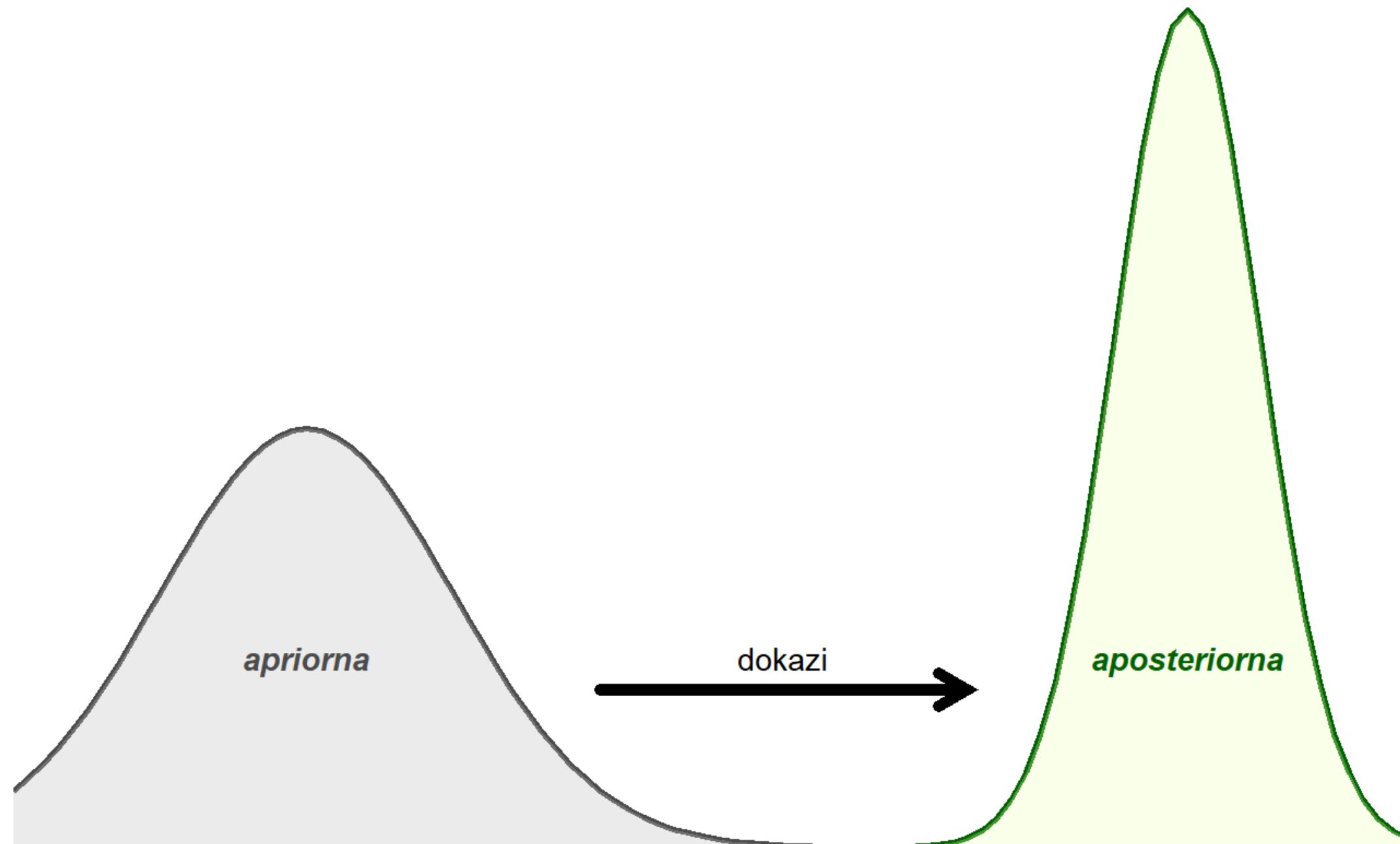
- **Empirijska verovatnoća** se određuje posle posmatranog događaja.

$$p = \frac{\text{Očekivano}}{\text{Ukupno}}$$

Krvna grupa	Relativna učestalost
O	45%
A	39%
B	12%
AB	4%

Verovatnoća Subjektivna

- Uverenje
- Ekspertsко mišljenje



Osobine Verovatnoće

- Nenegativnost $[0, 1]$
- Normiranost ($\text{zbir} = 1$)
- Aditivnost
- Verovatnoća događaja (p)
- Verovatnoća suprotnog događaja (q)
- Komplementarnost: $p + q = 1$

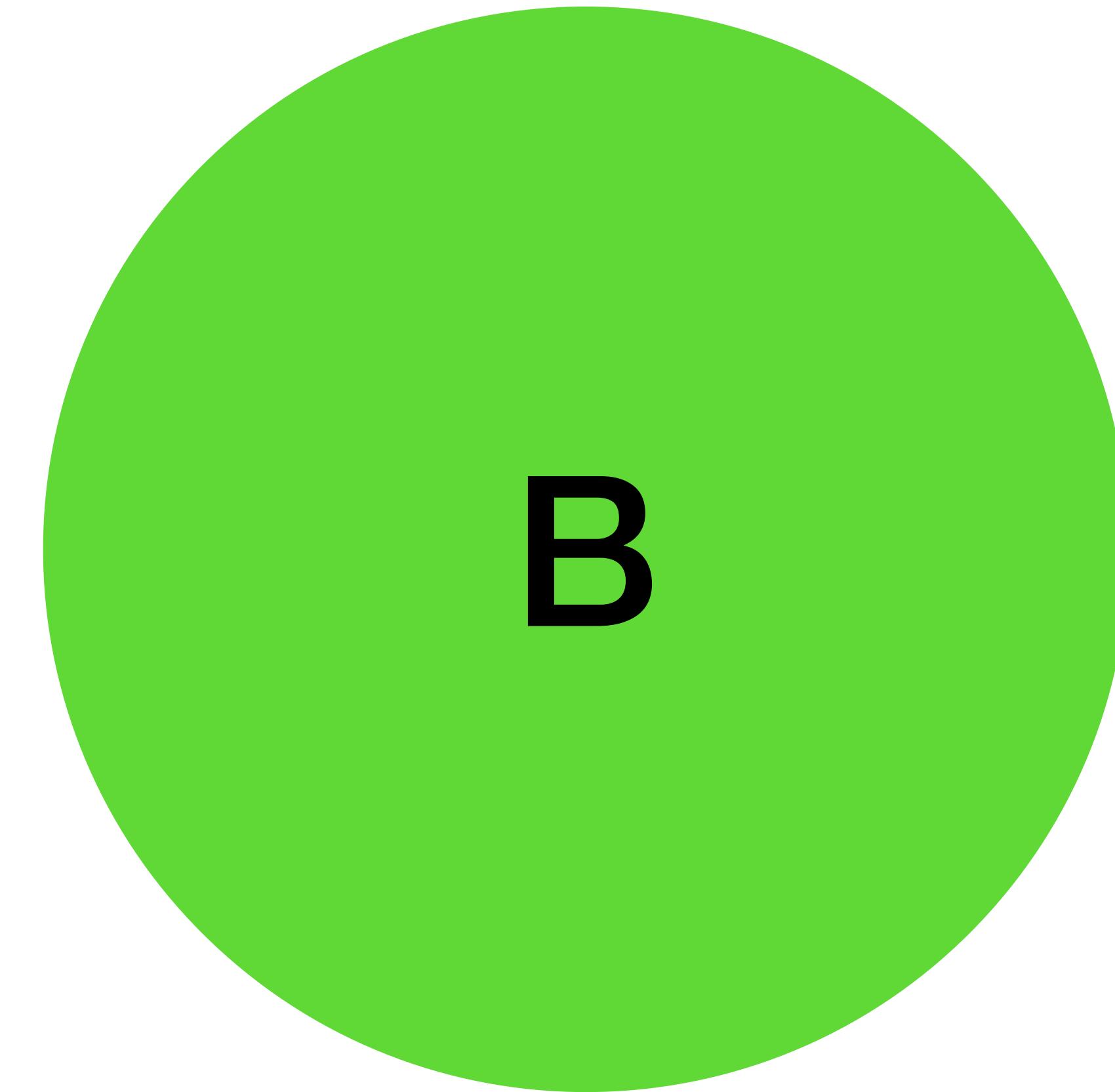
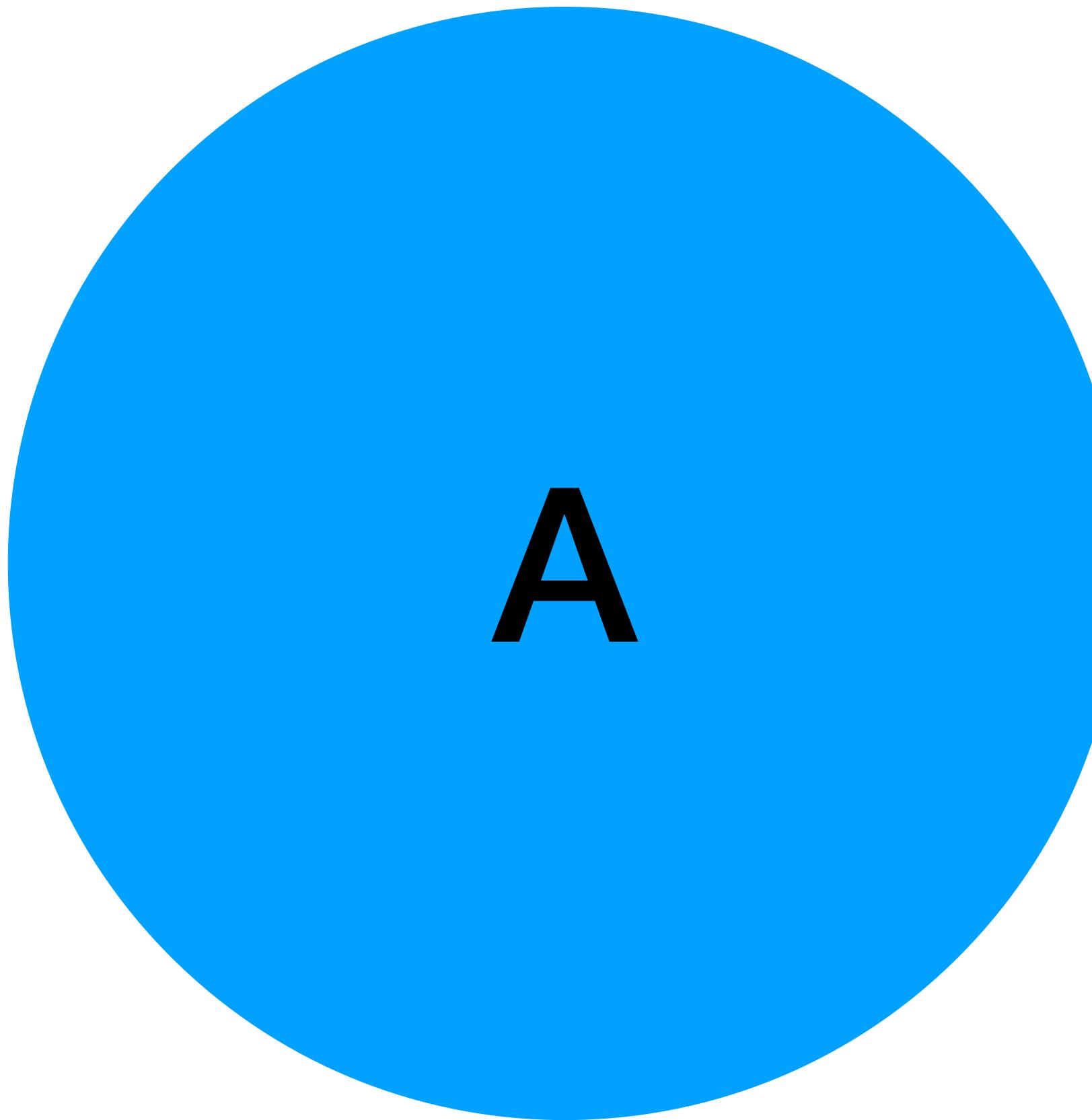
Isključivost

Događaji su isključivi ako ne mogu da se dogode istovremeno

- Krvna grupa
- Simptomi gripa
- Medicinski znakovi
- Dijagnoze

Zakoni

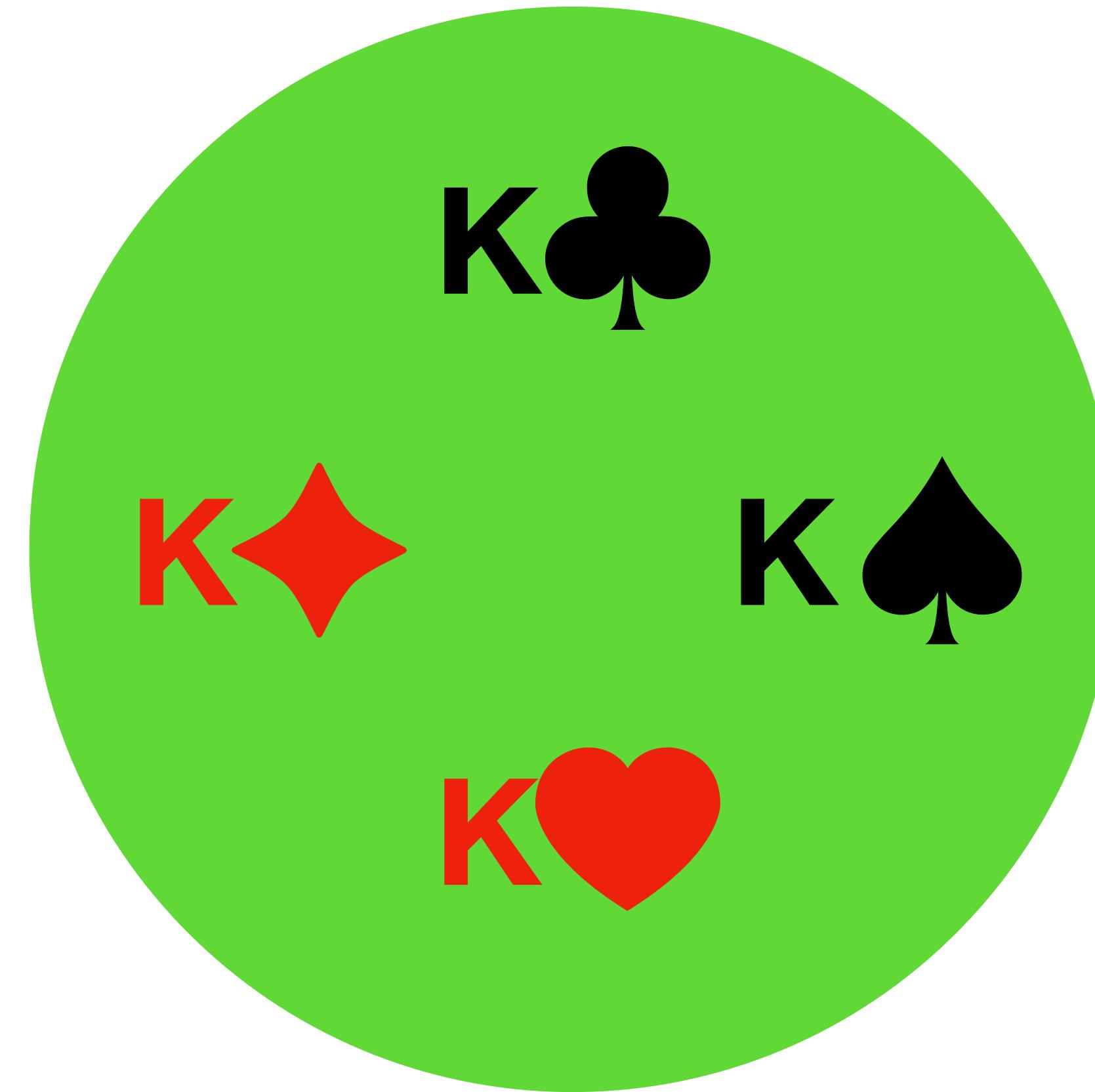
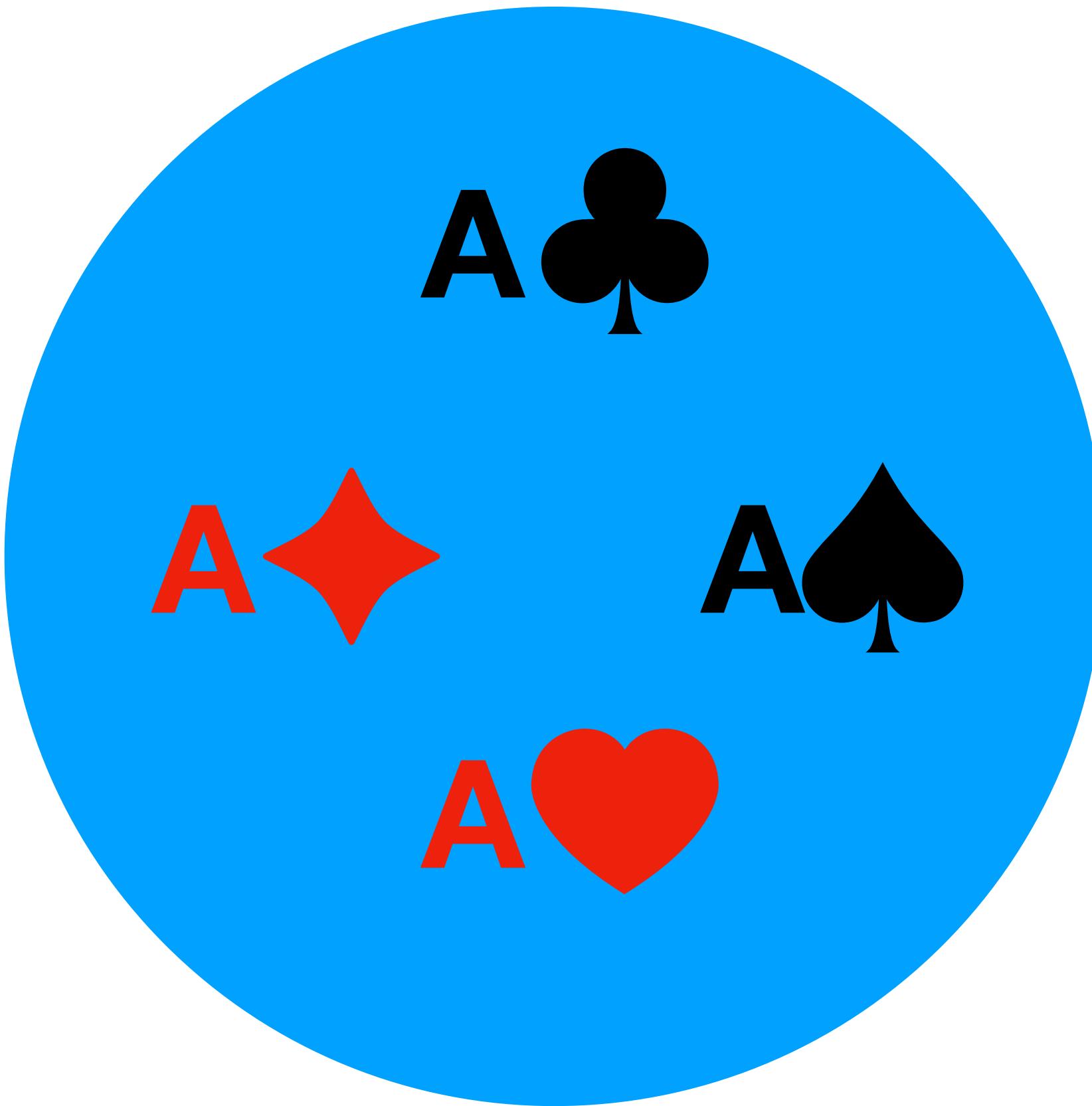
Adicija (sabiranje verovatnoća) isključivih događaja



$$P(A \cup B) = P(A) + P(B)$$

Zakoni

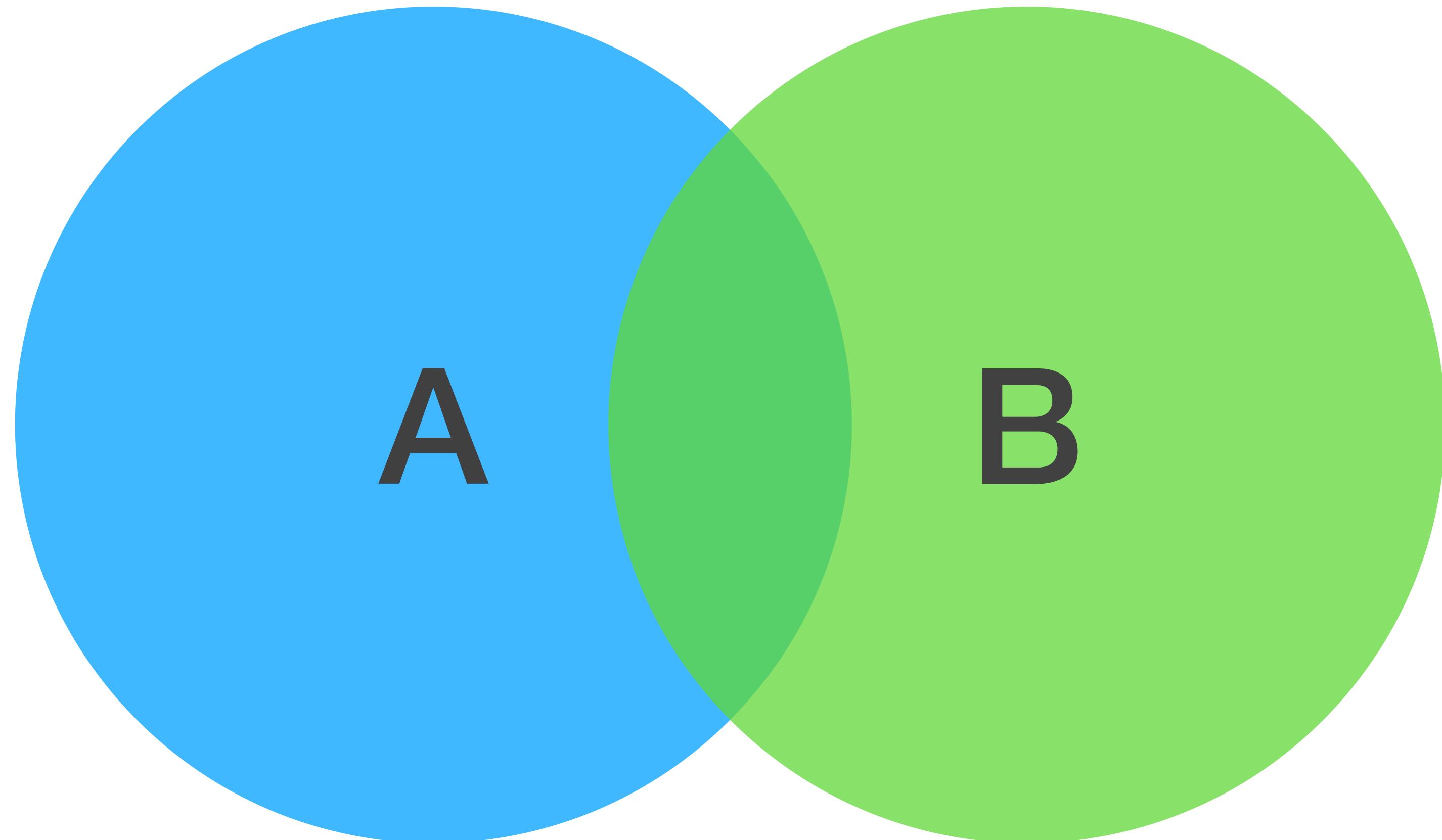
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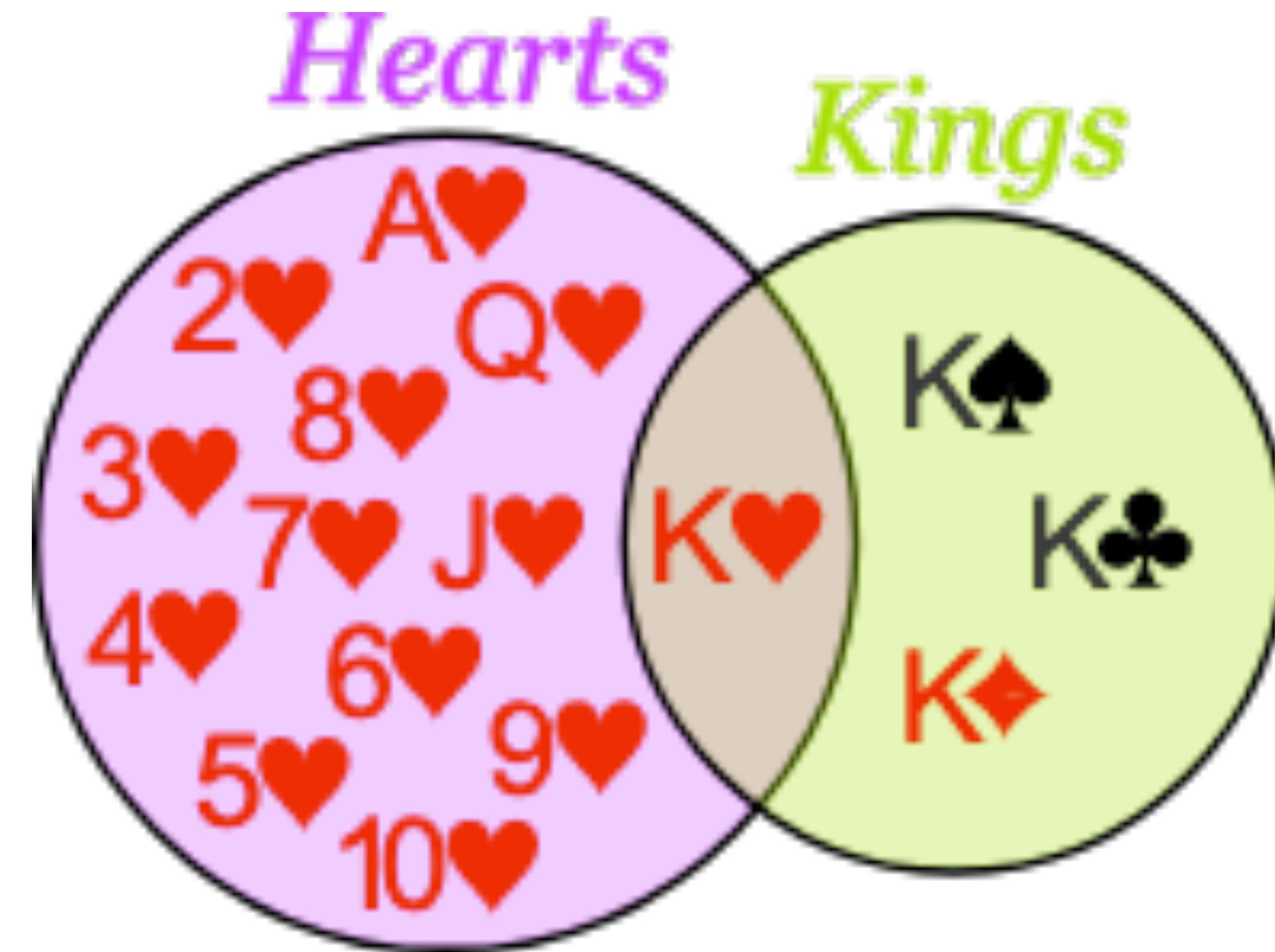
Adicija (sabiranje verovatnoća) neisključivih događaja



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Zakoni

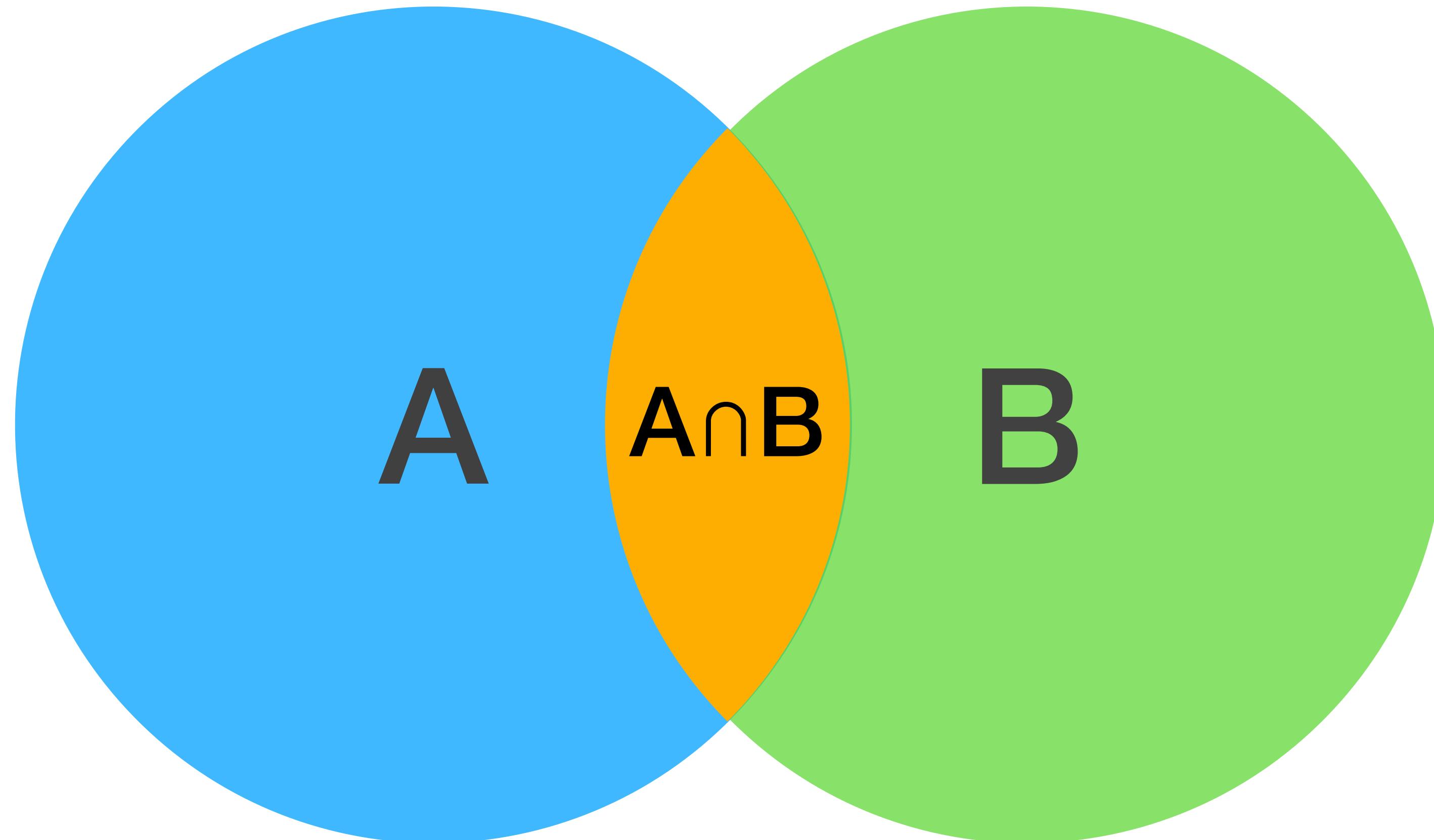
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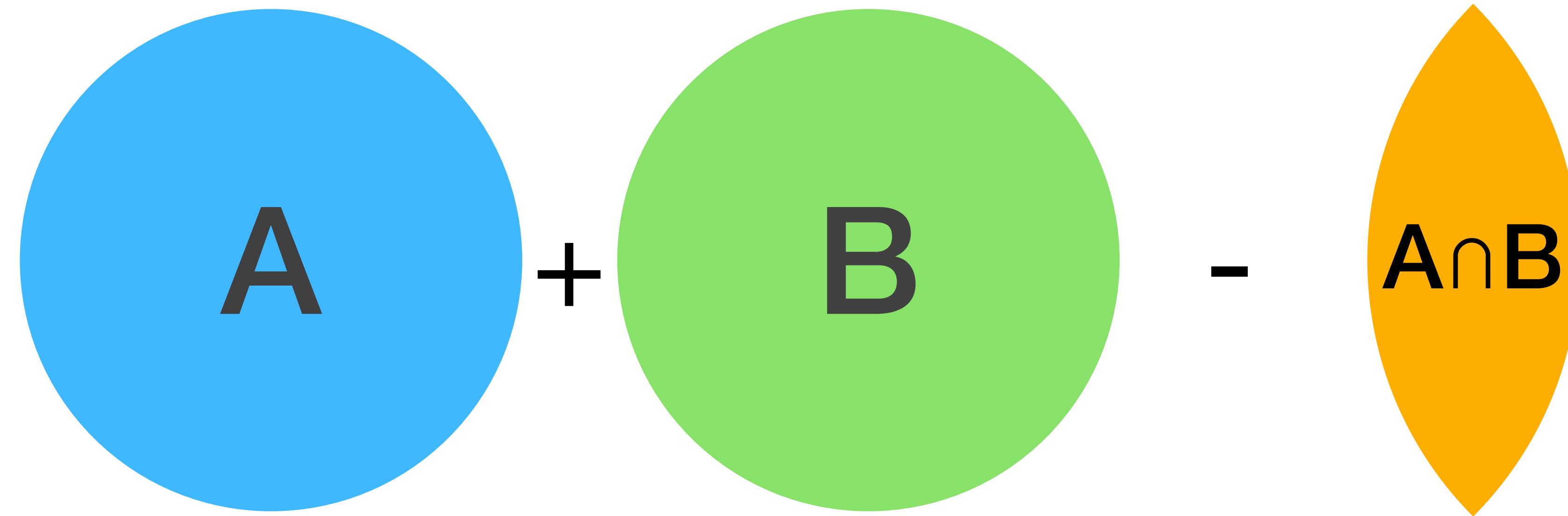
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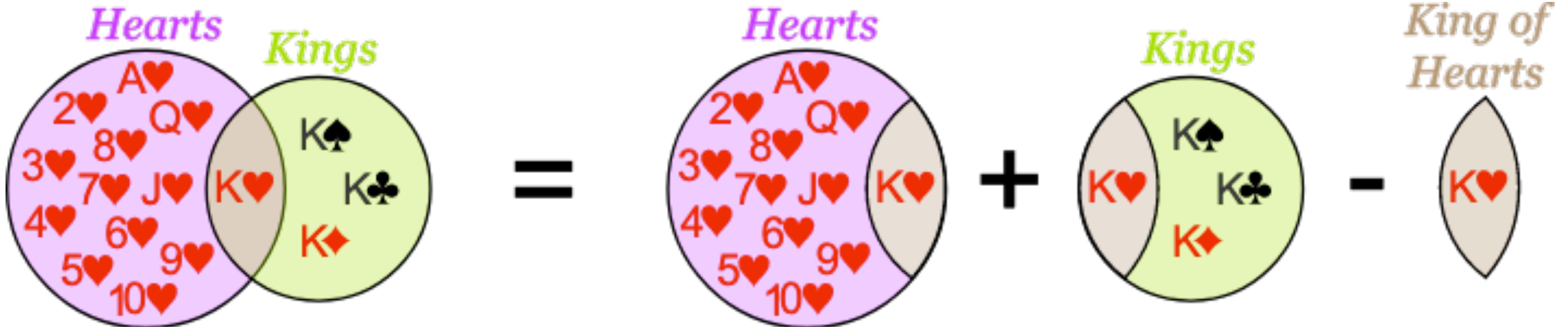
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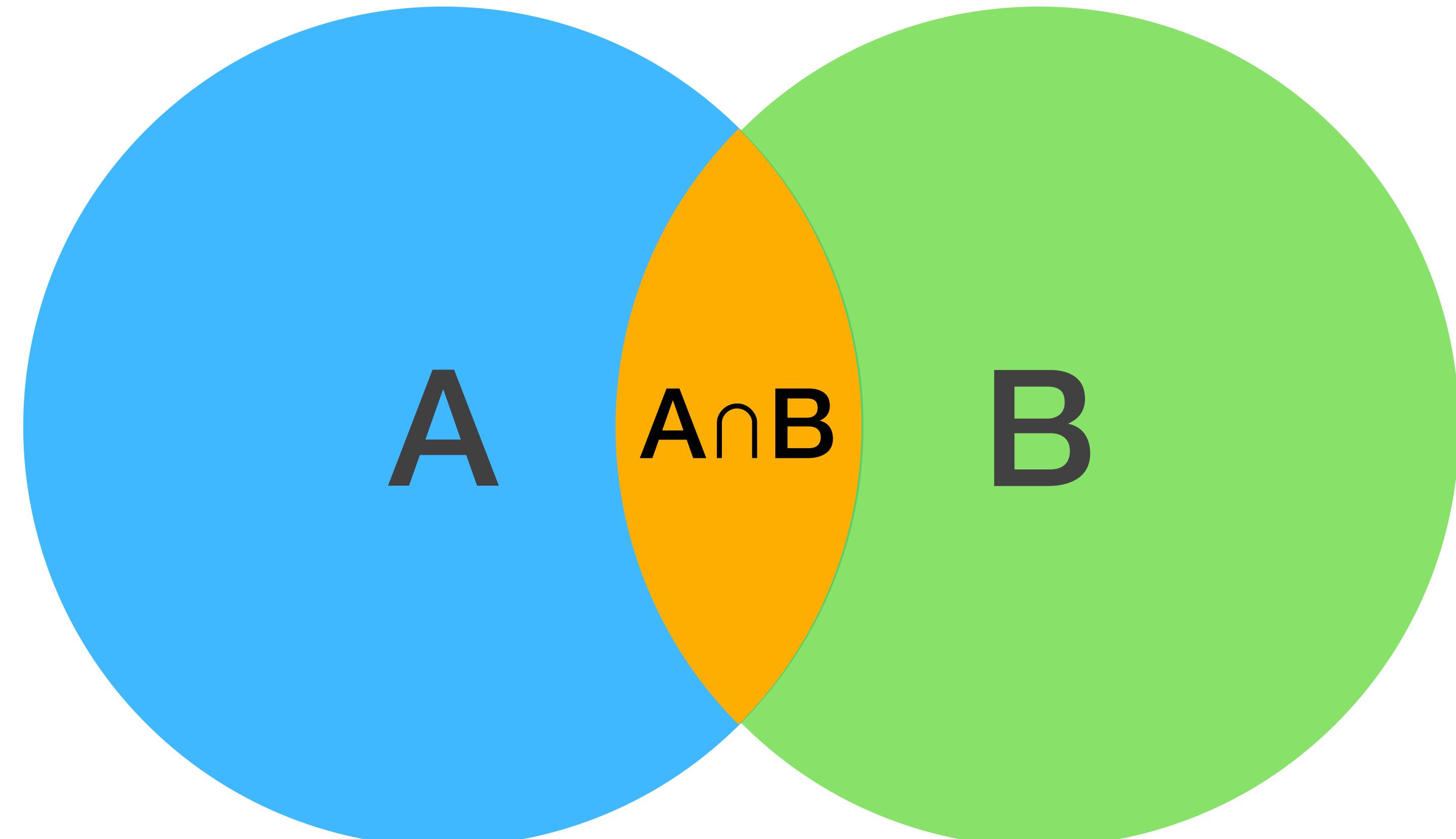
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$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

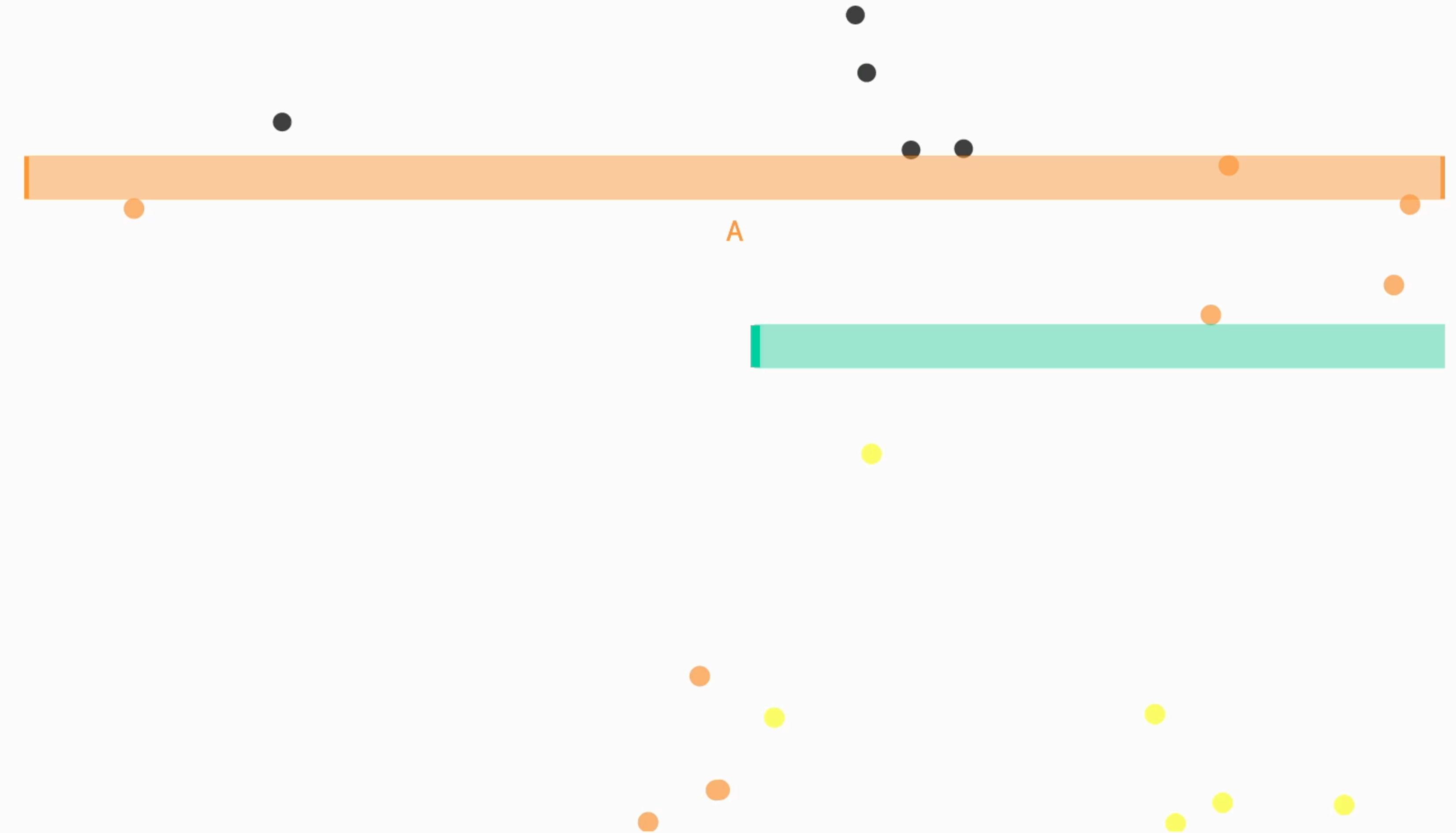
Zakoni

Zakon množenja za nezavisne



$$P(A \cap B) = P(A) \times P(B)$$

Zakoni Zavisne događaje



$$P(A \cap B) = P(A) \times P(B | A) = P(B) \times P(A | B)$$

CLINICAL SCENARIO

A 19-year-old college student has a severe sore throat and a mild fever (temperature 38.3°C), and he feels bad. The symptoms have been present for 4 days and initially started with a dry cough. There is a pharyngeal exudate, but only on the left side of the posterior pharynx. His neck reveals tender adenopathy. Should you assume he has streptococcal pharyngitis and start treatment?

Age

Group A streptococcus (GAS) rare under 3

3-14 years	+1
15-44 years	0
≥45 years	-1

Exudate or swelling on tonsils

No 0 Yes +1

Tender/swollen anterior cervical lymph nodes

No 0 Yes +1

Temp >38°C (100.4°F)

No 0 Yes +1

Cough

Cough present	0
Cough absent	+1

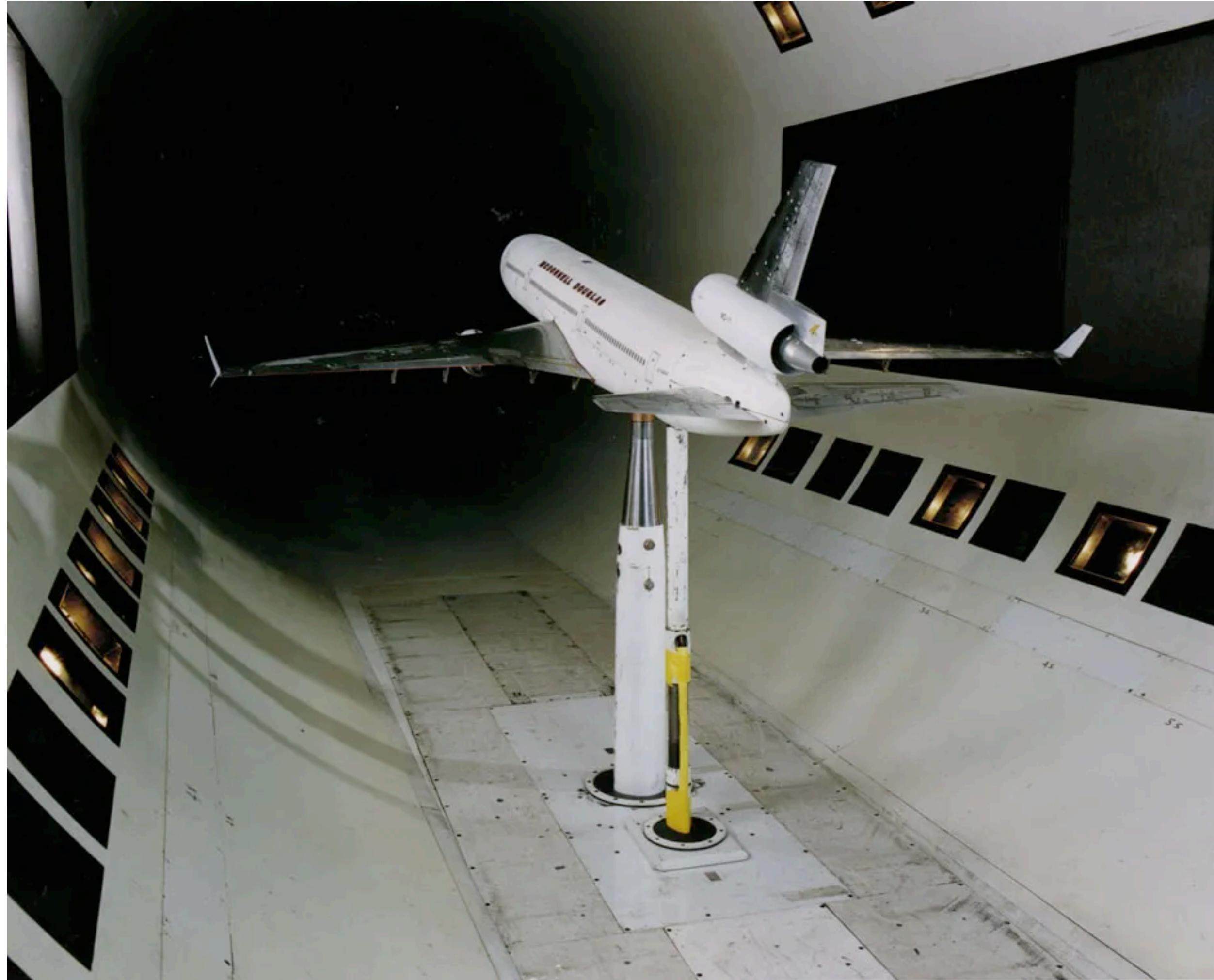
4 points

51% - 53% likelihood of strep

Consider rapid strep testing and/or culture. Empiric antibiotics may be appropriate depending on the specific scenario.

Teorijske Raspodele Verovatnoća

- **Teorijske raspodele verovatnoća** su specifični matematički modeli raspodela (distribucija) verovatnoća.
 - Binomna
 - Normalna



Matematički model



Stvarnost

Binomna Raspodela

Teorijske Raspodele Verovatnoća

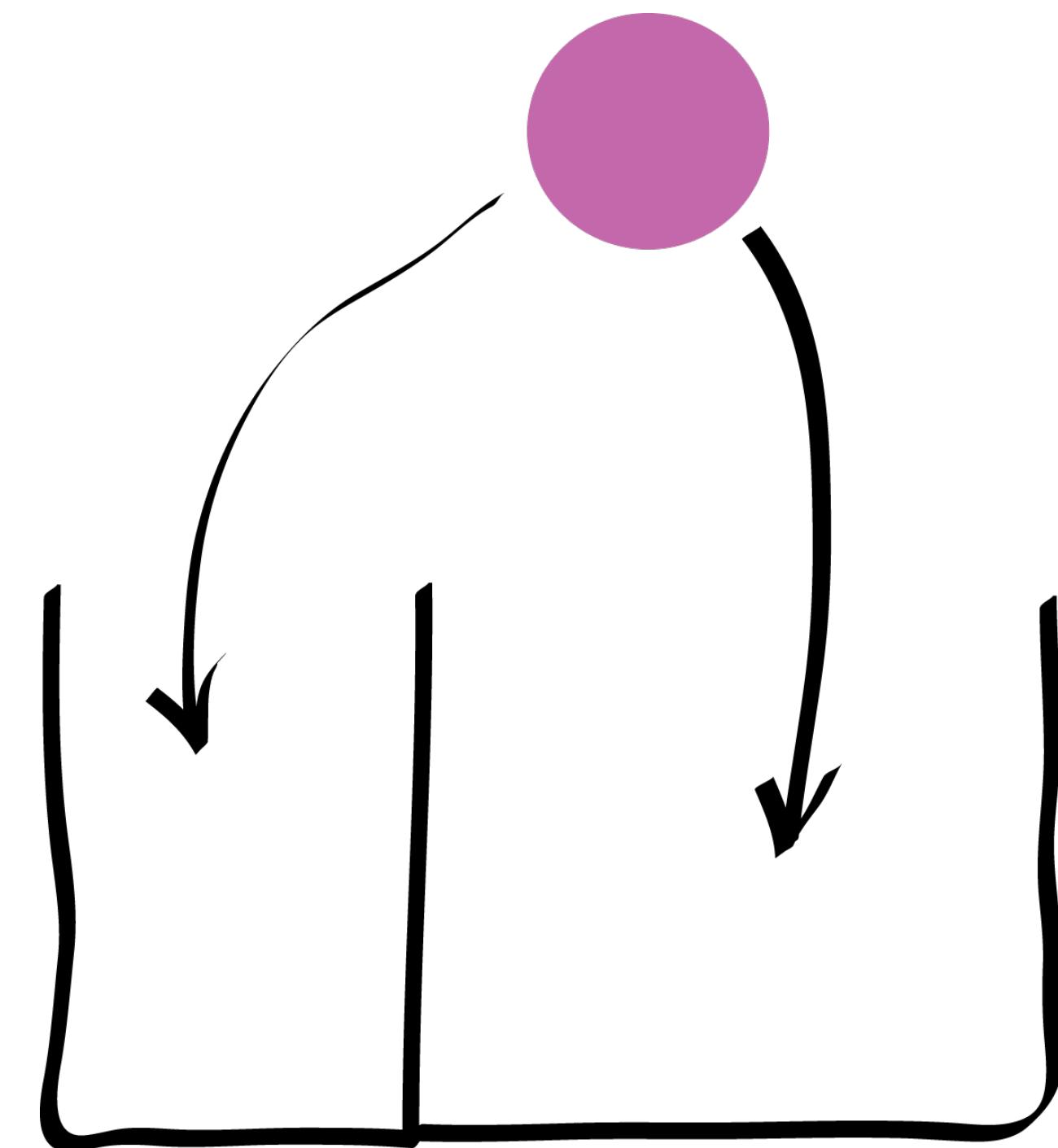
$$\bar{x} = np$$
$$sd = \sqrt{np(1 - p)}$$

- Uslovi:
 - Isključivi
 - Konstantne verovatnoće
 - Nezavisni

$$A \cap B = \emptyset$$

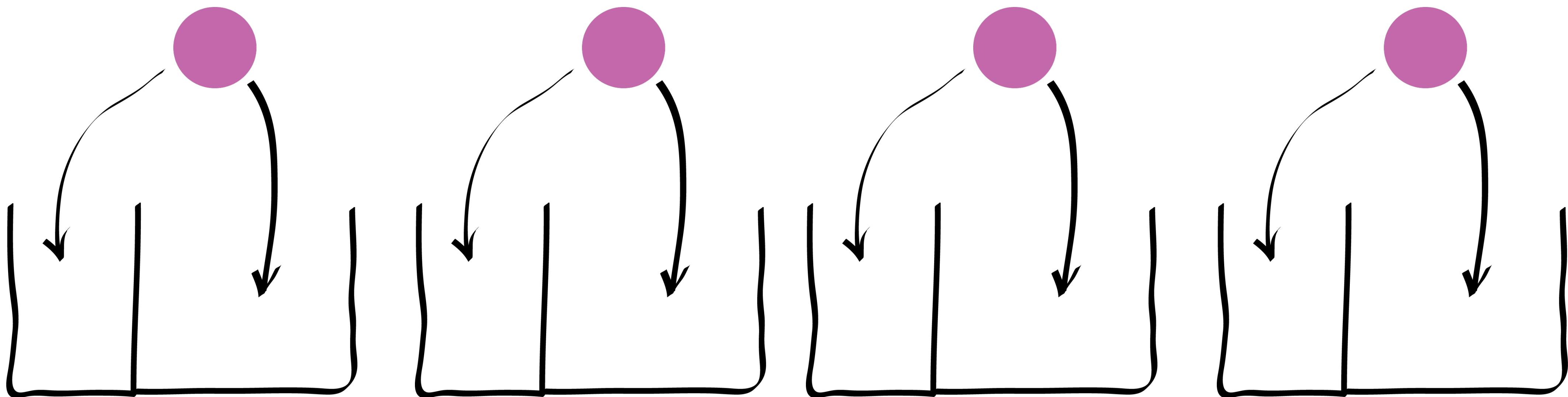
$p = \text{const.}$, za svaki događaj.

$$P(A \cap B) = P(A) \times P(B)$$



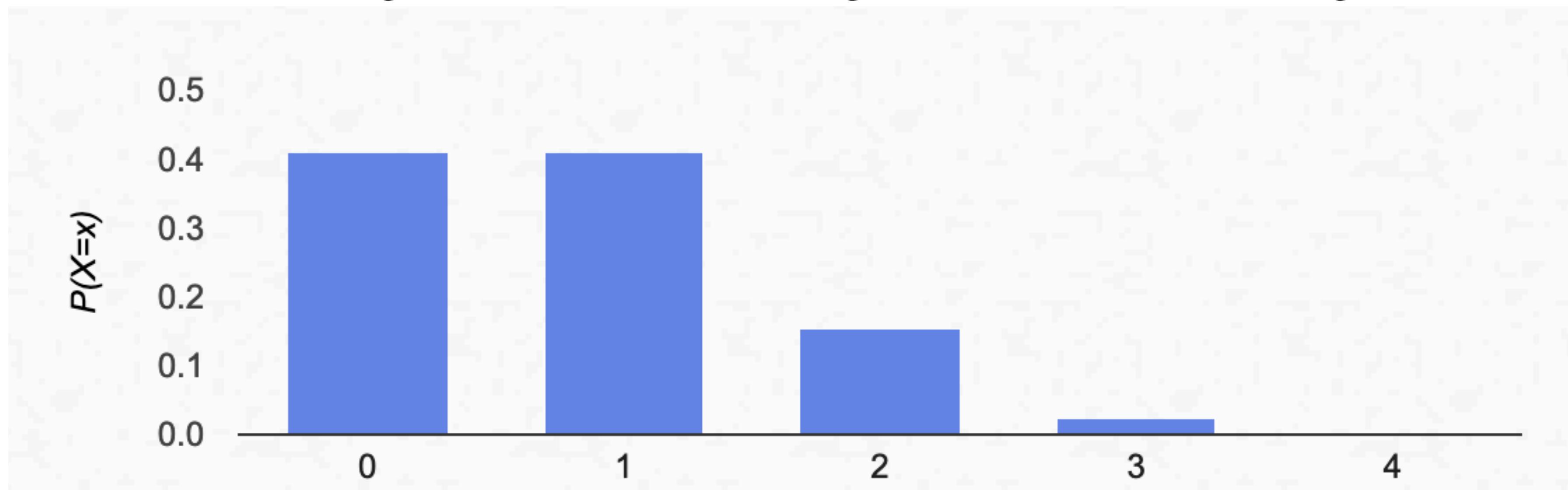
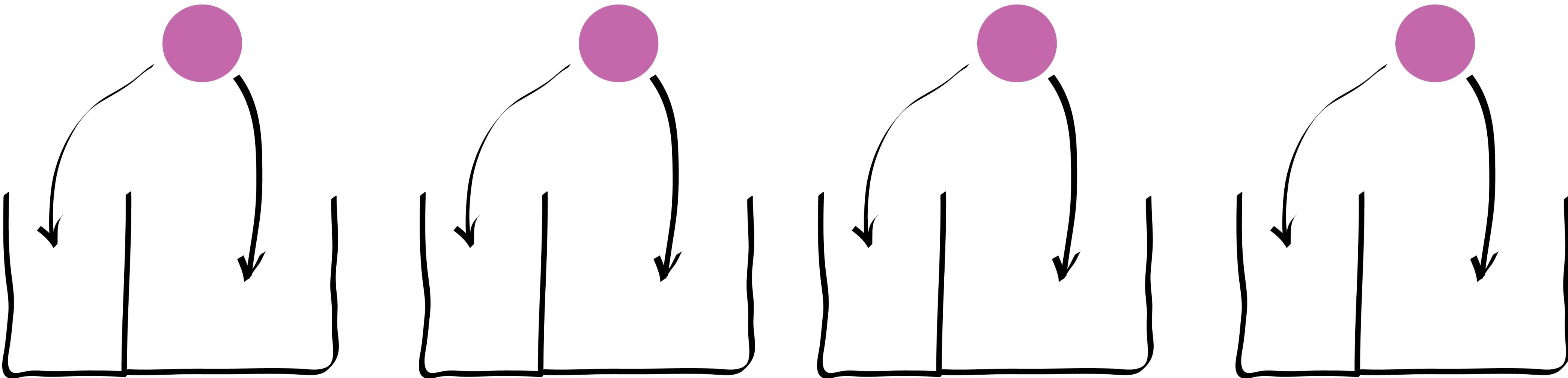
Binomna Raspodela

Teorijske Raspodele Verovatnoća



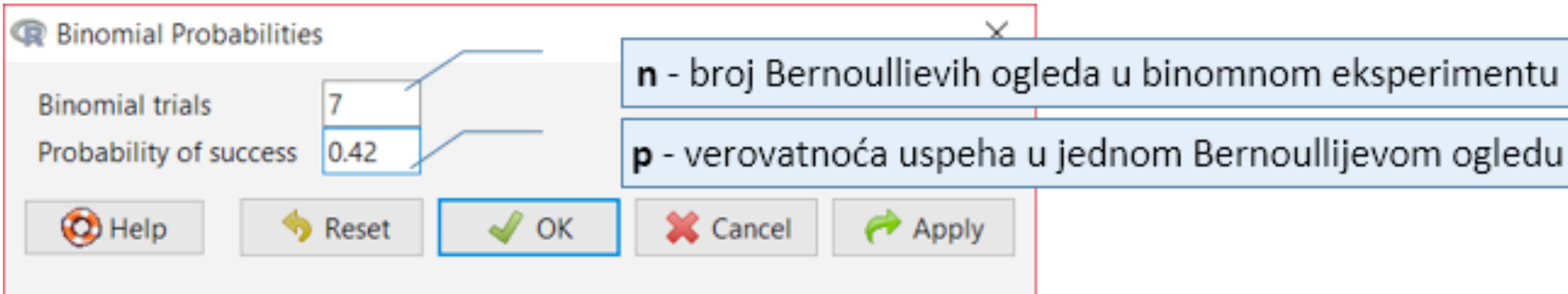
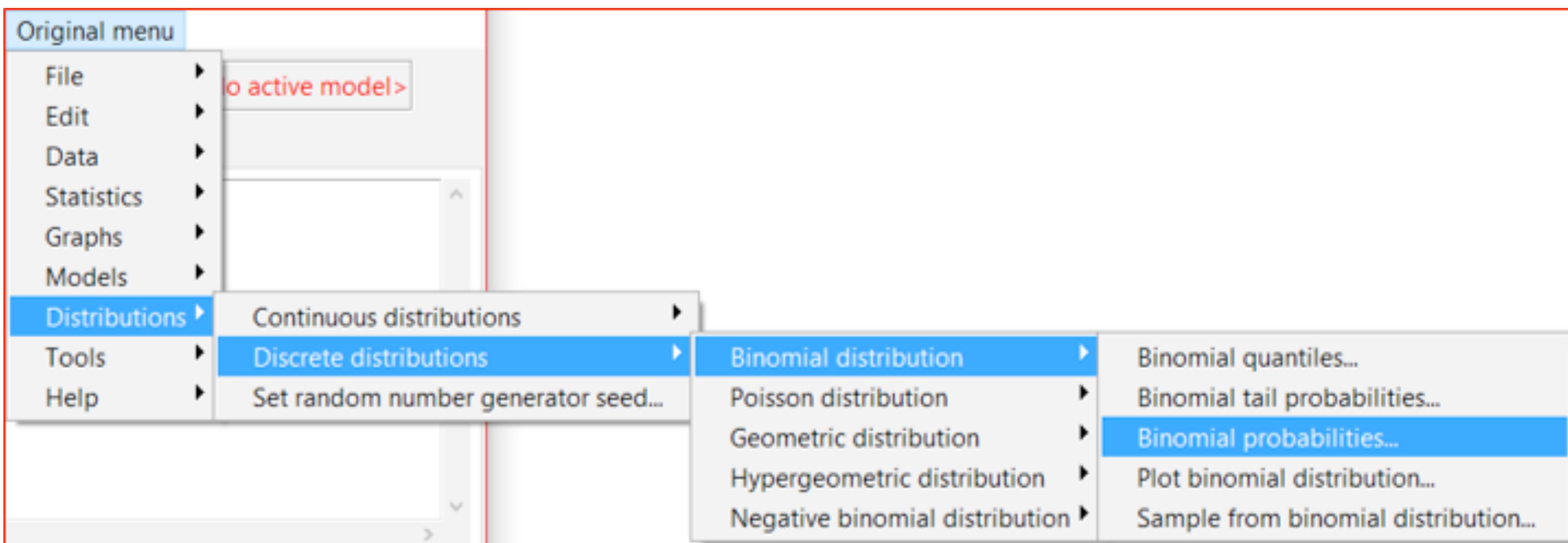
Binomna Raspodela

Teorijske Raspodele Verovatnoća



Učestalost hipertenzije u populaciji starijih od 65 godina iznosi 42%.

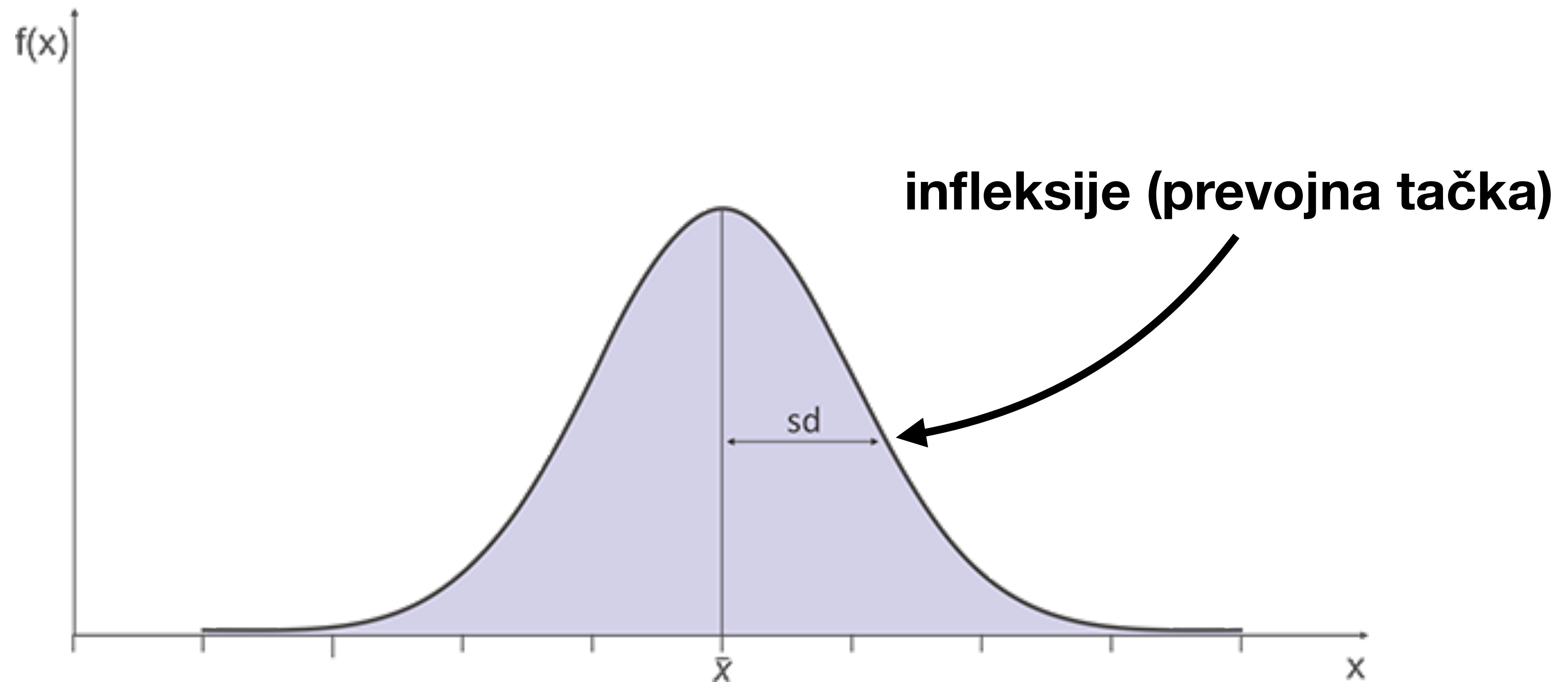
a) Kolika je verovatnoća da u slučajnom uzorku veličine 7 osoba, izabranom iz te iste populacije, dve osobe imaju hipertenziju?





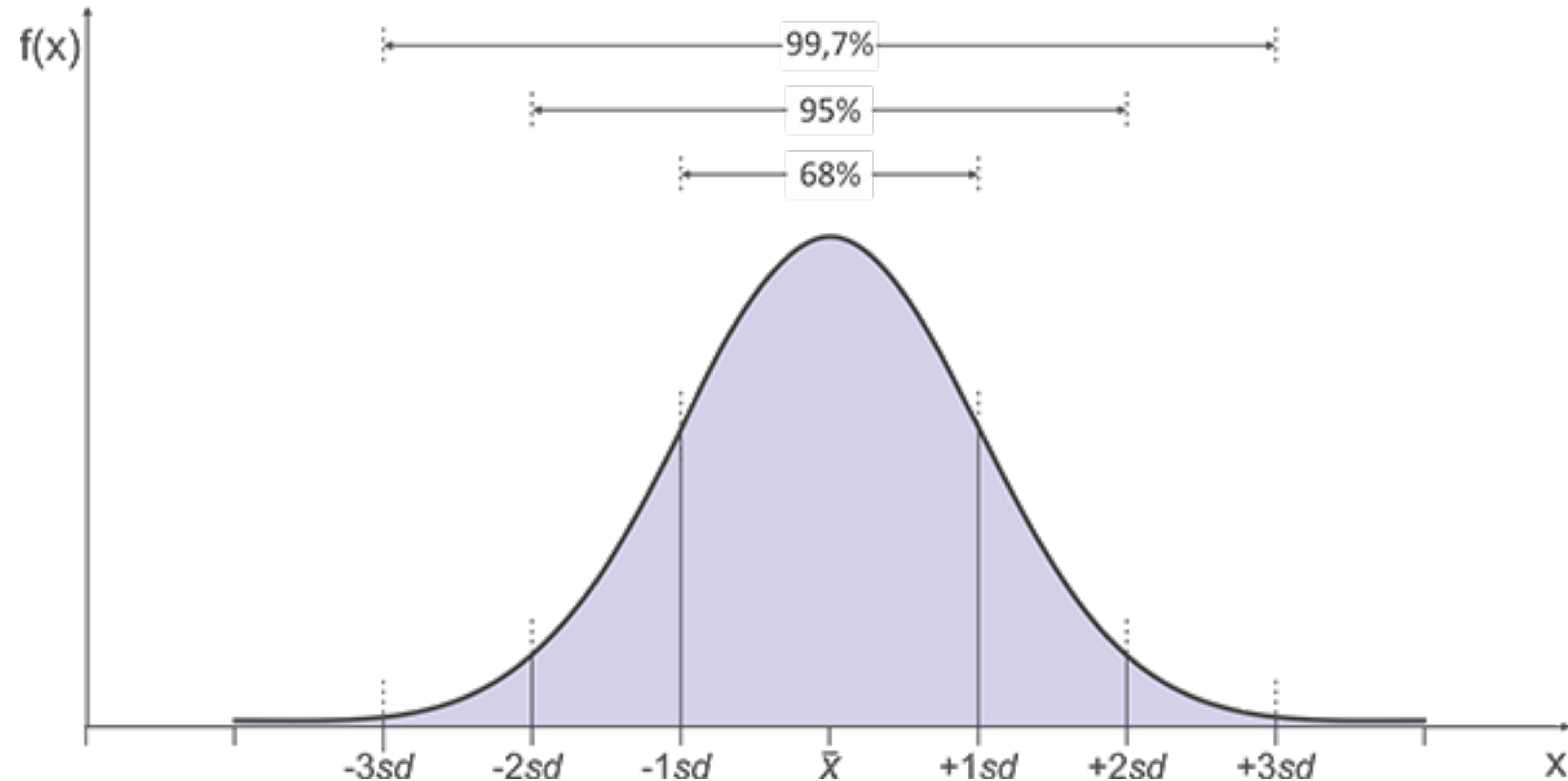
Normalna Raspodela

Teorijske Raspodele Verovatnoća



Normalna Raspodela

Teorijske Raspodele Verovatnoća



Normalna Raspodela

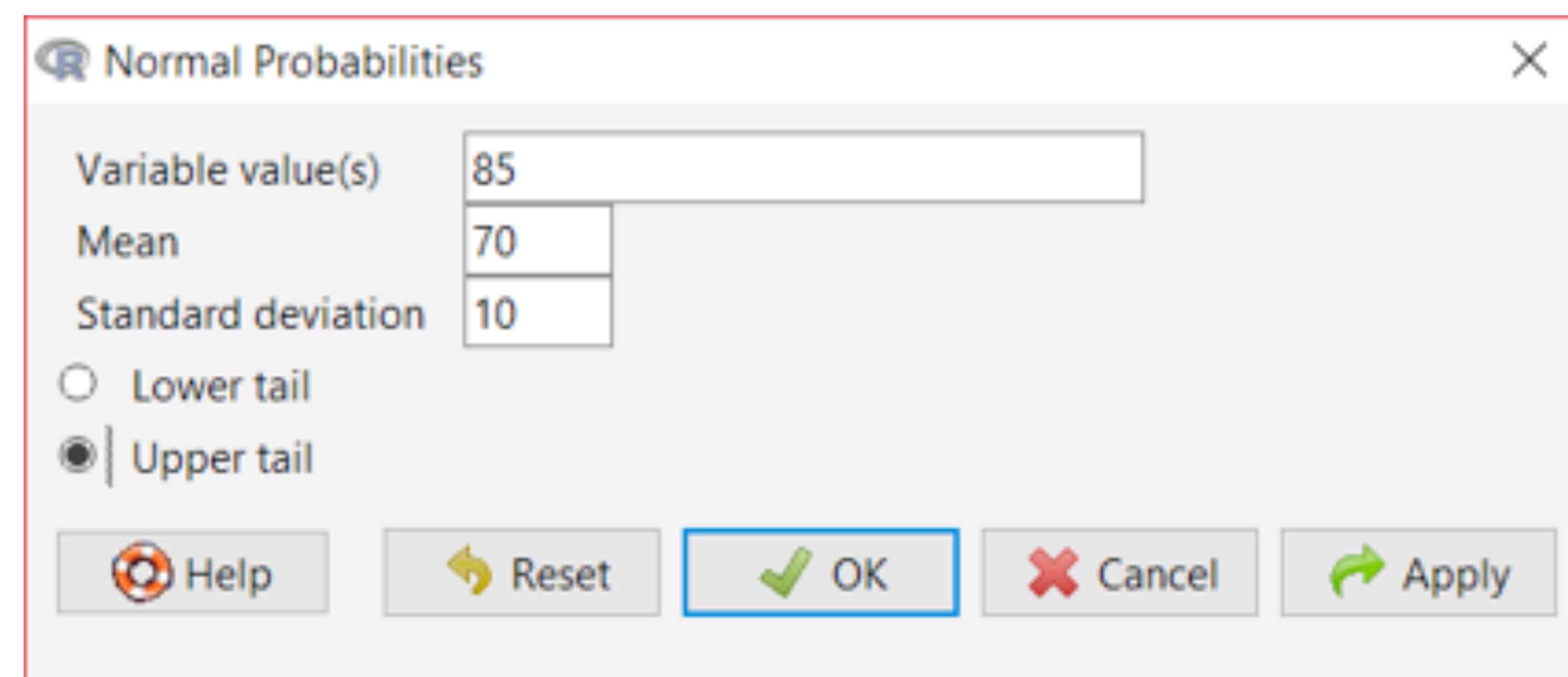
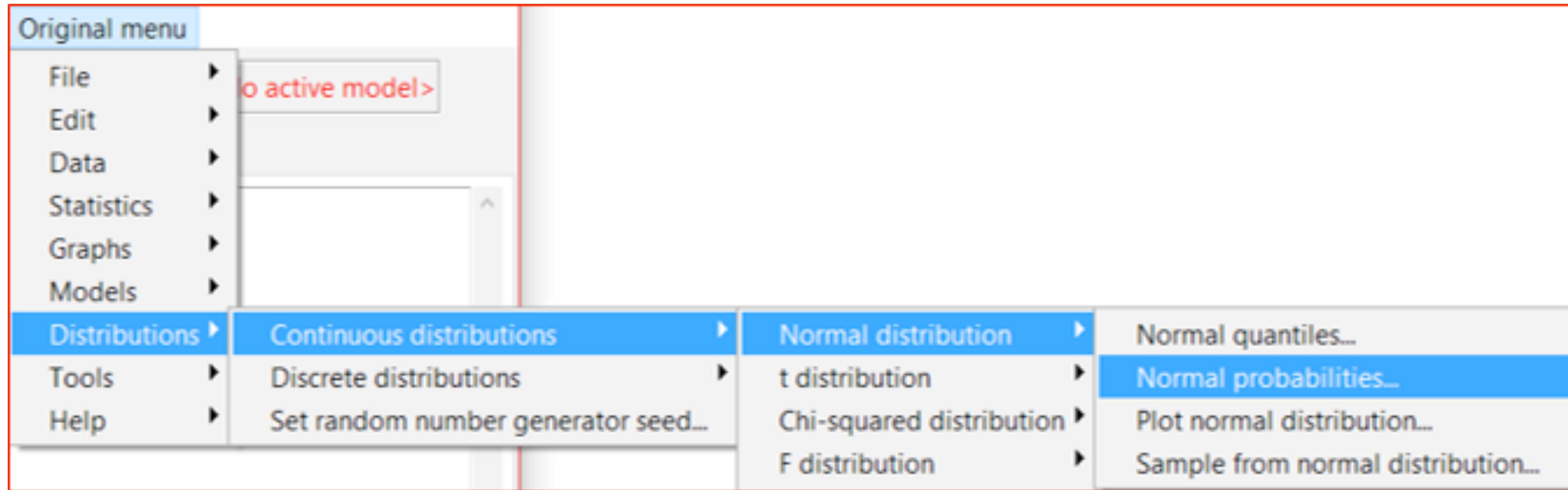
Teorijske Raspodele Verovatnoća

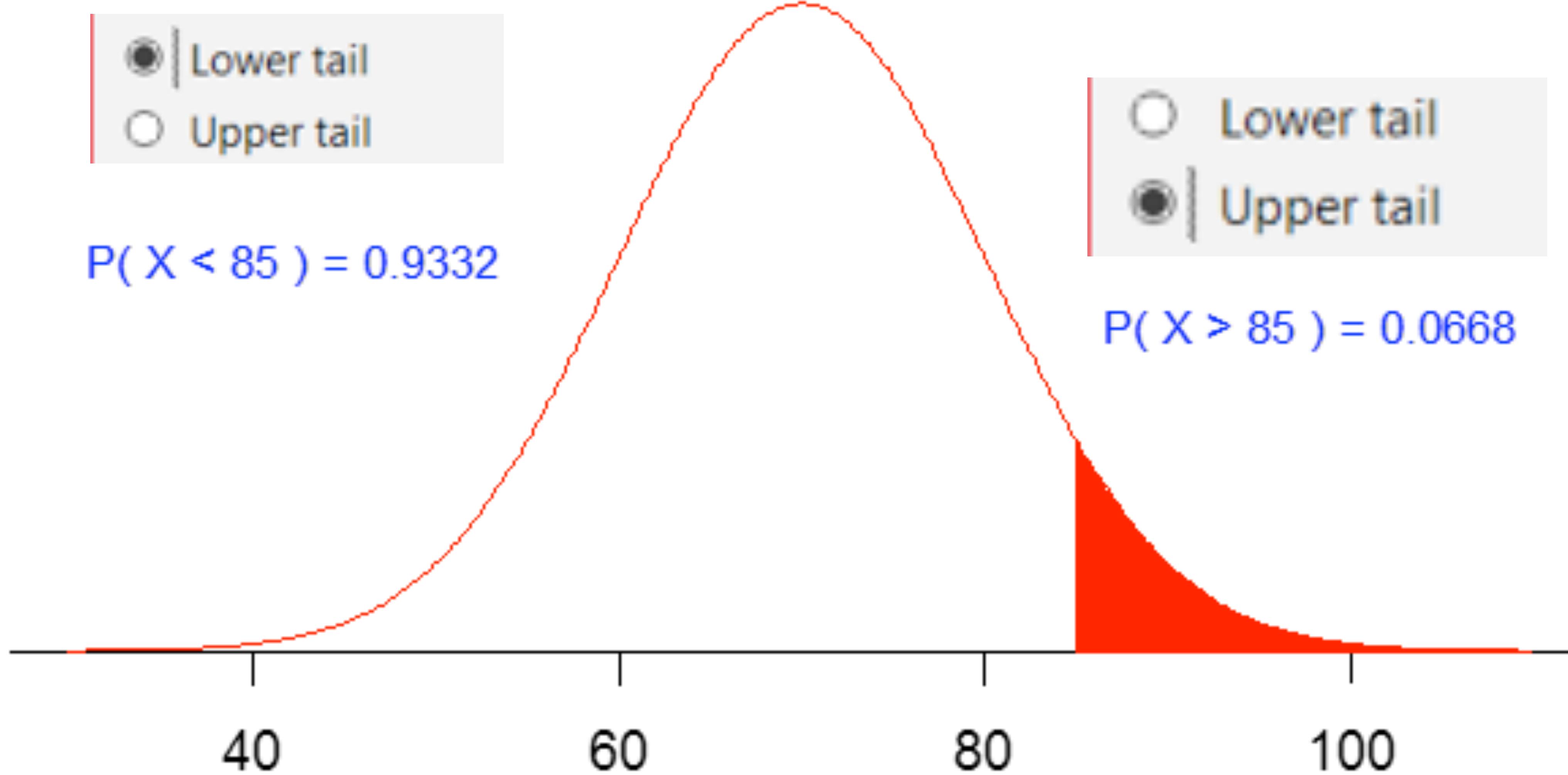
Površine ispod krive standardizovane normalne raspodele

z	0	1	2	3	4	5	6	7	8	9
0,0	0000	0040	0080	0120	0160	0199	0239	0279	0319	0359
0,1	0398	0438	0478	0517	0557	0596	0636	0675	0714	0753
0,2	0793	0832	0871	0910	0948	0987	1026	1064	1103	1141
0,3	1179	1217	1255	1293	1331	1368	1406	1443	1480	1517
0,4	1554	1591	1628	1664	1700	1736	1772	1808	1844	1879
0,5	1915	1950	1985	2019	2054	2088	2123	2157	2190	2224
0,6	2257	2291	2324	2357	2389	2422	2454	2486	2517	2549
0,7	2580	2611	2642	2673	2704	2734	2764	2794	2823	2852
0,8	2881	2910	2939	2967	2995	3023	3051	3078	3106	3133
0,9	3159	3186	3212	3238	3264	3289	3315	3340	3365	3389
1,0	3413	3438	3461	3485	3508	3531	3554	3577	3599	3621
1,1	3643	3665	3686	3708	3729	3749	3770	3790	3810	3830
1,2	3849	3869	3888	3907	3925	3944	3962	3980	3997	4015
1,3	4032	4049	4066	4082	4099	4115	4131	4147	4162	4177
1,4	4192	4207	4222	4236	4251	4265	4279	4292	4306	4319
1,5	4332	4345	4357	4370	4382	4394	4406	4418	4429	4441
1,6	4452	4463	4474	4484	4495	4505	4515	4525	4535	4545
1,7	4554	4564	4573	4582	4591	4590	4608	4616	4625	4633
1,8	4641	4649	4656	4664	4671	4678	4686	4693	4699	4706
1,9	4713	4719	4726	4732	4738	4744	4750	4756	4761	4767
2,0	4772	4778	4783	4788	4793	4798	4803	4808	4812	4817
2,1	4821	4826	4830	4834	4838	4842	4846	4850	4854	4857

U populaciji žena starosti između 25 i 50 godina vrednosti mokraćne kiseline u serumu su normalno raspoređene sa aritmetičkom sredinom 333 mmol/L i standardnom devijacijom 30 mmol/L .

a) Kolika je verovatnoća da slučajno izabrana osoba iz ove populacije ima vrednost mokraćne kiseline u serumu veću od 410 mmol/l ?





$$P(67 < X < 85) = 0.5511$$

$$P(X < 67) = 0.3821$$

$$P(X > 85) = 0.0668$$

● Lower tail

○ Upper tail

