

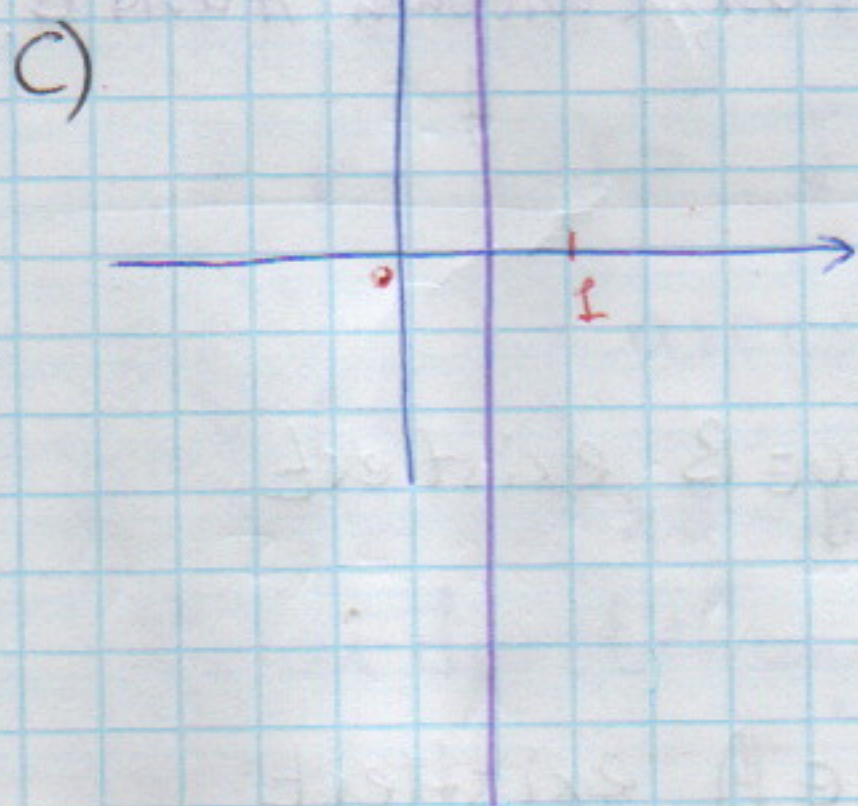
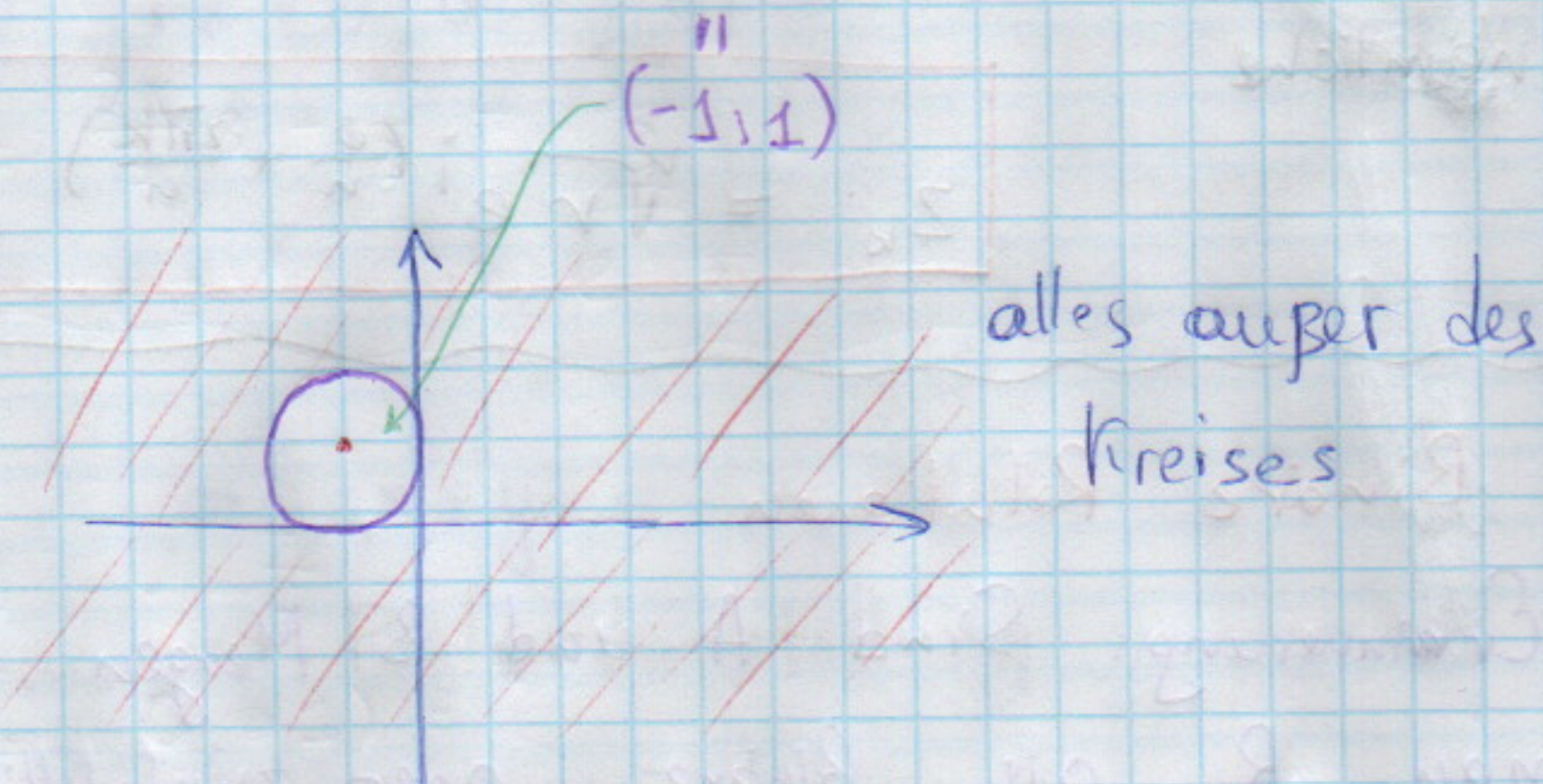
# Übungsblatt 2

## Aufgabe 1



$$b) B := \{z \in \mathbb{C} : |z + 1 - i| > 1\} =$$

$$= \{z \in \mathbb{C} : |z - \underbrace{(i - 1)}_{(-1, 1)}| > 1\}$$



## Aufgabe 2

$$a) \left| \left( \frac{2+2i}{1-i} \right)^6 \right| = \frac{|(2+2i)^6|}{|(1-i)^6|} = \frac{\sqrt{4+4}^6}{\sqrt{2}^6} = \frac{\sqrt{8}^2 \cdot \sqrt{8}^2 \cdot \sqrt{8}^2}{\sqrt{2}^2 \cdot \sqrt{2}^2 \cdot \sqrt{2}^2}$$

$$= \frac{8^3}{2^3} = 8^2 = \underline{\underline{64}}$$

$$b) |(6+2i)(2+i)| = |6+2i| \cdot |2+i| = \sqrt{36+4} \cdot \sqrt{4+1} =$$

$$= \sqrt{40} \cdot \sqrt{5} = \sqrt{200} = 10\sqrt{2}$$

$$c) \left| \left( \frac{\frac{\sqrt{3}}{2} - \frac{1}{2}i}{2^{15}} \right)^{15} \right| = \frac{\left| \left( \frac{\sqrt{3}}{2} - \frac{1}{2}i \right)^{15} \right|}{2^{15}} = \frac{\sqrt{3+1}^{15}}{2^{15}}$$

$$= \frac{2^{15}}{2^{15}} = \underline{\underline{1}}$$



# Aufgabe 3

a)  $z_1 = -2 - 2i$

"  
(-2, -2)

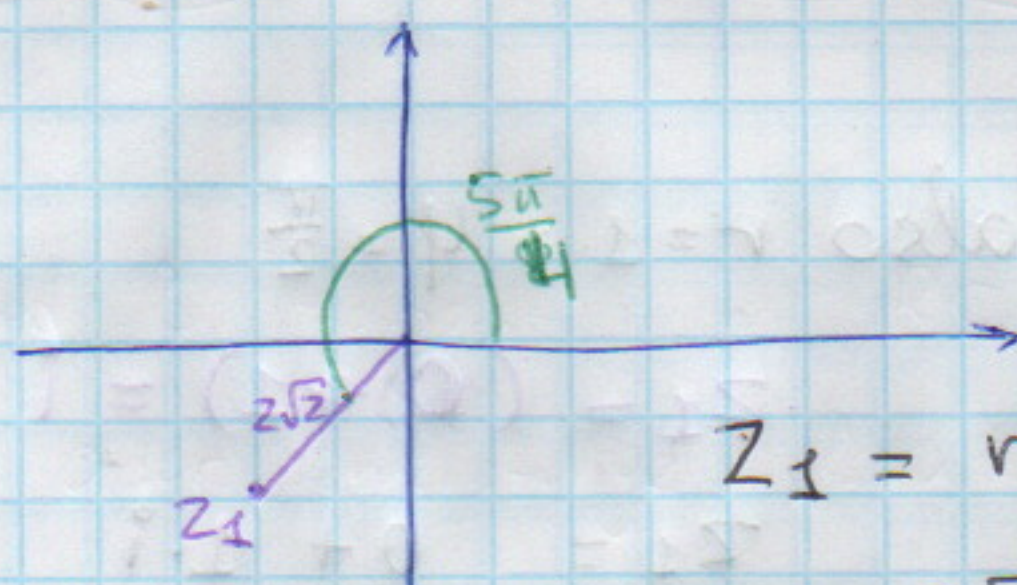
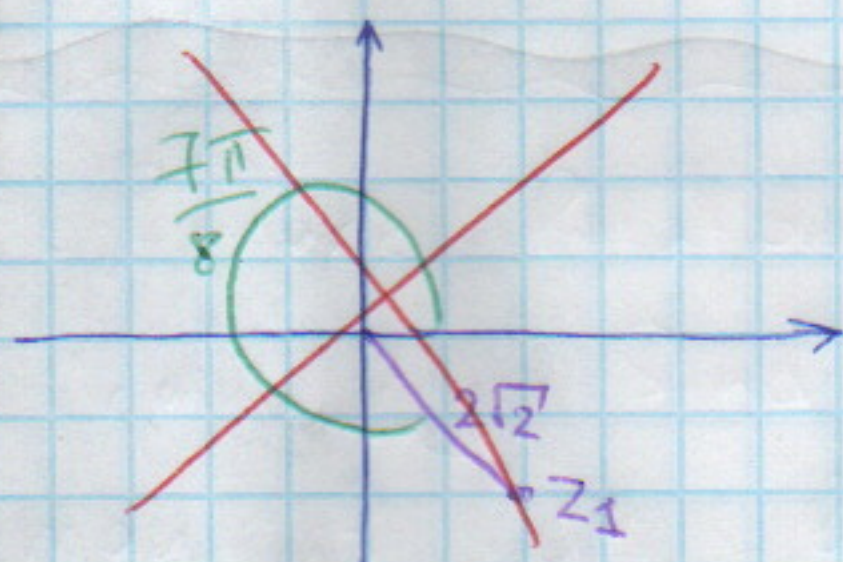
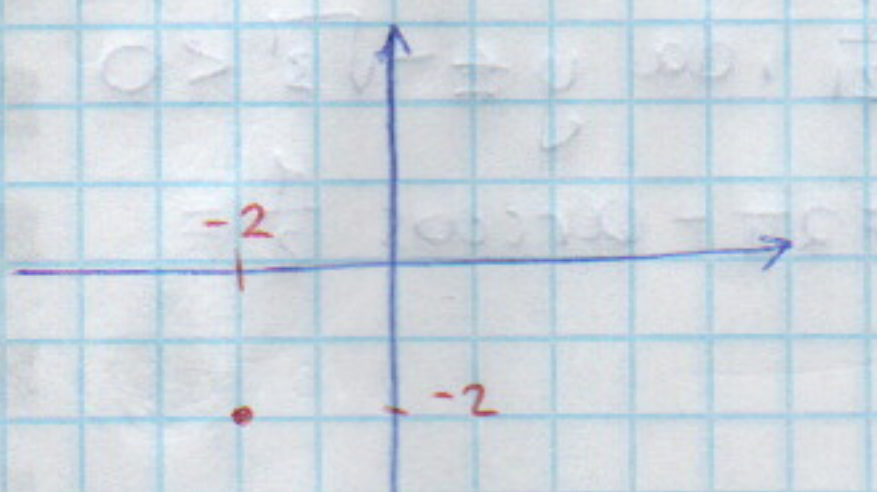
in Polardarstellung:

1)  $r = \sqrt{x^2 + y^2} = \sqrt{4+4} = \sqrt{8} = \underline{2\sqrt{2}}$

2)  $\varphi = 2\pi - \arccos \frac{x}{|z|}$ , da  $y = -2 < 0$

$\varphi = 2\pi - \arccos \frac{-2}{2\sqrt{2}} = 2\pi - \arccos -\frac{1}{\sqrt{2}} =$

$= 2\pi - \frac{\pi}{4} = \frac{8\pi}{4} - \frac{\pi}{4} = \underline{\underline{\frac{7\pi}{4}}}$



$z_1 = r e^{i\varphi} =$   
 $= \underline{2\sqrt{2}} \cdot e^{i \frac{5\pi}{4}}$

b)  $z_2 = -3i$

"  
(0, -3)

in Polardarstellung:

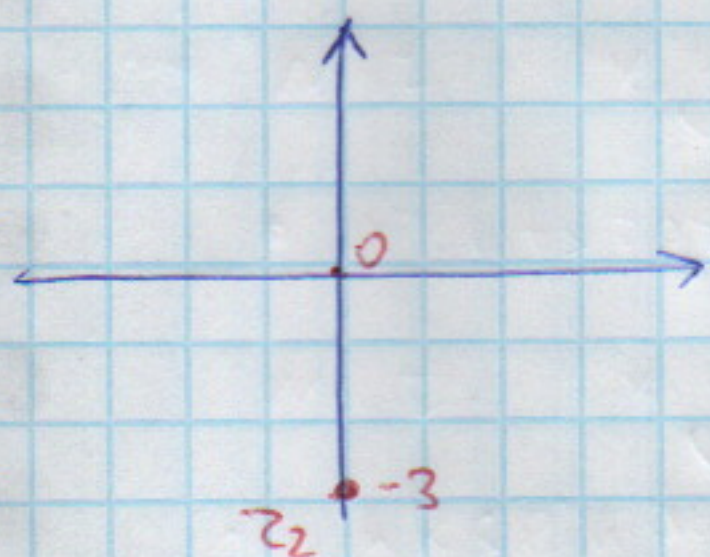
1)  $r = \sqrt{9} = \underline{3}$

2)  $\varphi = 2\pi - \arccos \frac{x}{|z|}$ , da  $y = -3 < 0$

$\varphi = 2\pi - \arccos \frac{0}{3} = 2\pi - \arccos 0 = 2\pi - \frac{\pi}{2} =$

$= \frac{4\pi}{2} - \frac{\pi}{2} = \underline{\underline{\frac{3\pi}{2}}}$

$z_2 = r e^{i\varphi} = \underline{3 e^{i \frac{3\pi}{2}}}$



c)  $z_3 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

"  
(\frac{\sqrt{3}}{2}, \frac{1}{2})

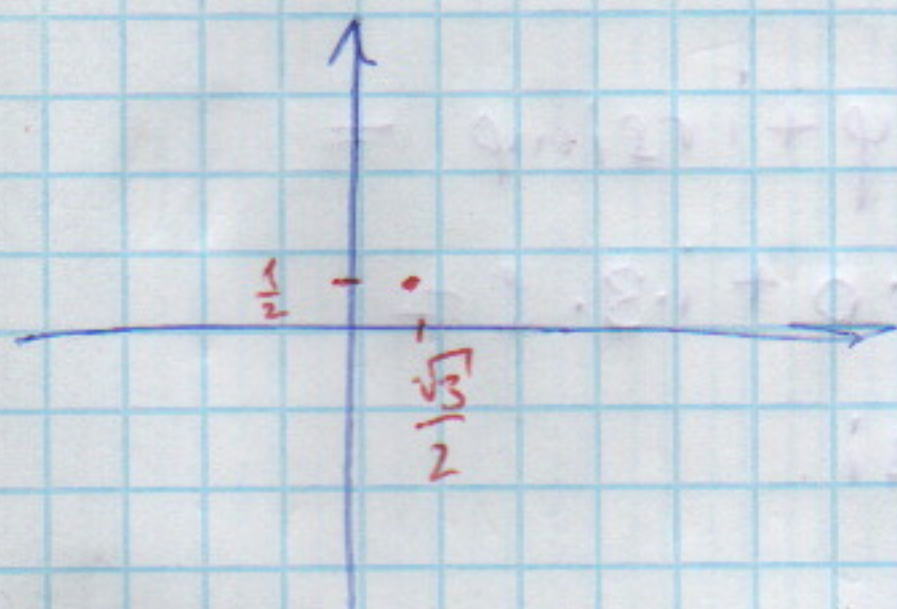
in Polardarstellung:

1)  $r = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = \underline{1}$

2)  $\varphi = \arccos \frac{x}{|z|}$ , da  $y = \frac{1}{2} > 0$

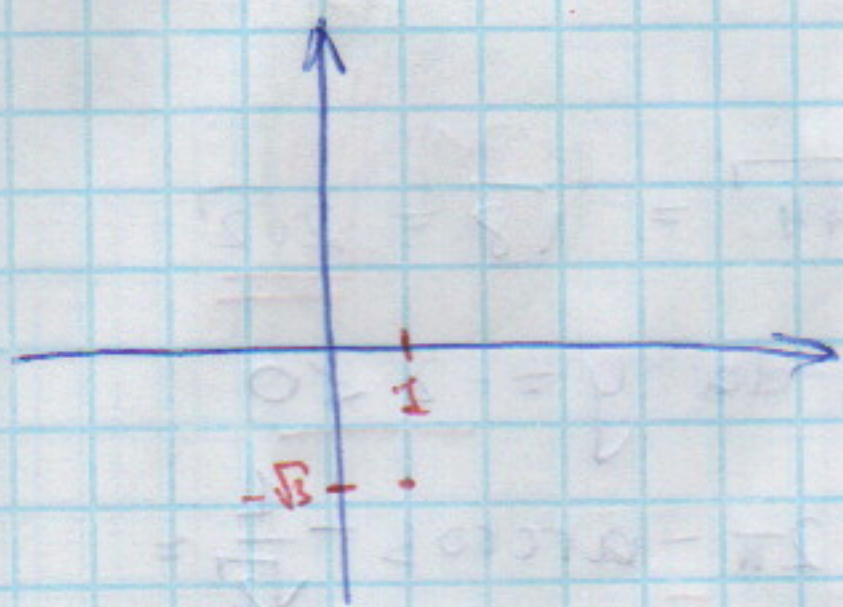
$\varphi = \arccos \frac{\frac{\sqrt{3}}{2}}{1} = \arccos \frac{\sqrt{3}}{2} = \underline{\underline{\frac{\pi}{6}}}$

$z_3 = r e^{i\varphi} = 1 \cdot e^{i \frac{\pi}{6}} = \underline{e^{i \frac{\pi}{6}}}$





d)  $z_4 = 1 - \sqrt{3}i$   
 $\parallel$   
 $(1; -\sqrt{3})$



in Polardarstellung:

1)  $r = \sqrt{1+3} = \underline{\underline{2}}$

2)  $\varphi = 2\pi - \arccos \frac{x}{|z|}$ , da  $y = -\sqrt{3} < 0$

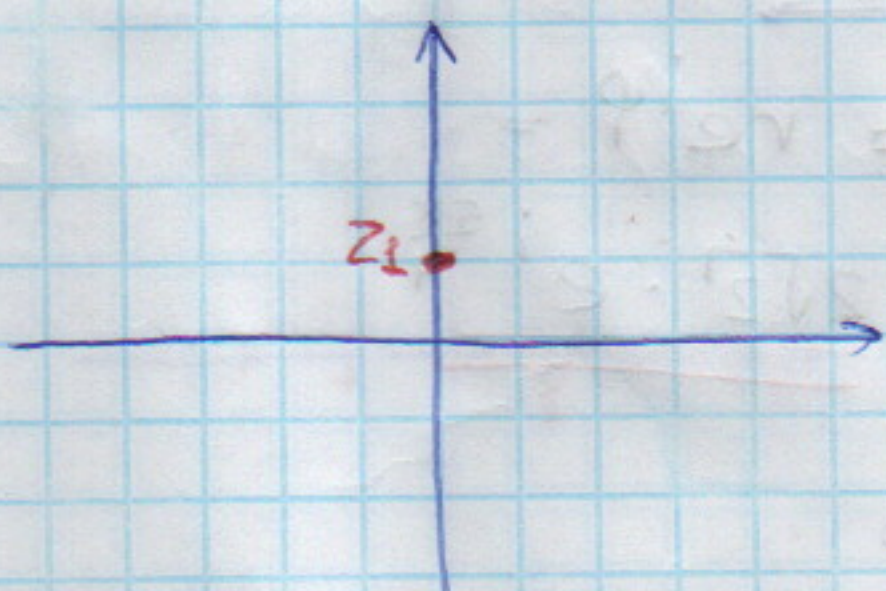
$\varphi = 2\pi - \arccos \frac{x}{|z|} = 2\pi - \arccos \frac{1}{2} =$

$= \frac{2\pi}{3} \cdot 3 - \frac{\pi}{3} = \underline{\underline{\frac{5\pi}{3}}}$

$z_4 = re^{i\varphi} = 2 \cdot e^{i\frac{5\pi}{3}}$

#### Aufgabe 4

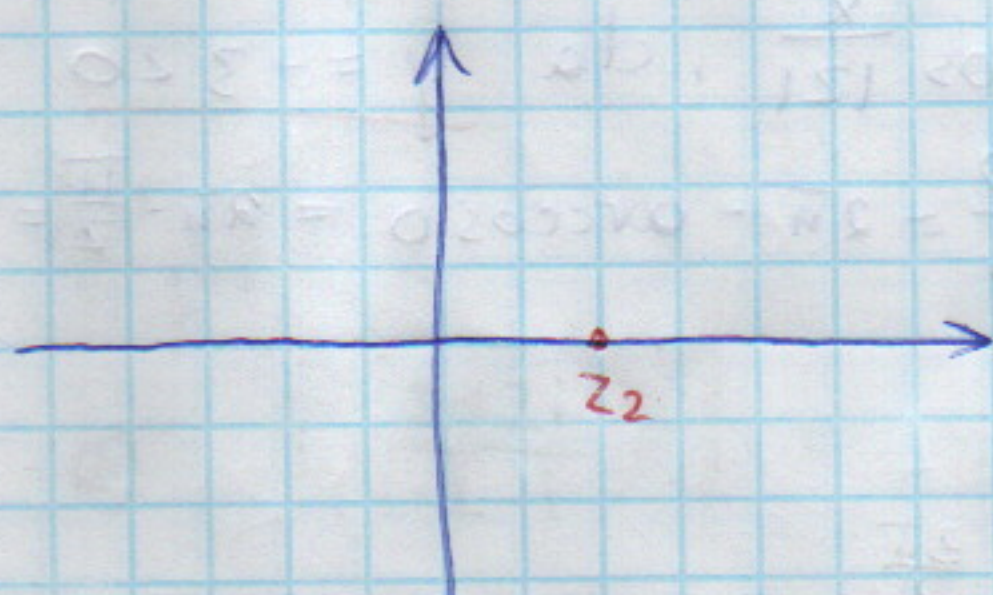
a)  $z_1 = e^{i\frac{\pi}{2}}$ , also  $r=1$ ;  $\varphi = \frac{\pi}{2}$



$z_1 = (0 + 1i) = i$

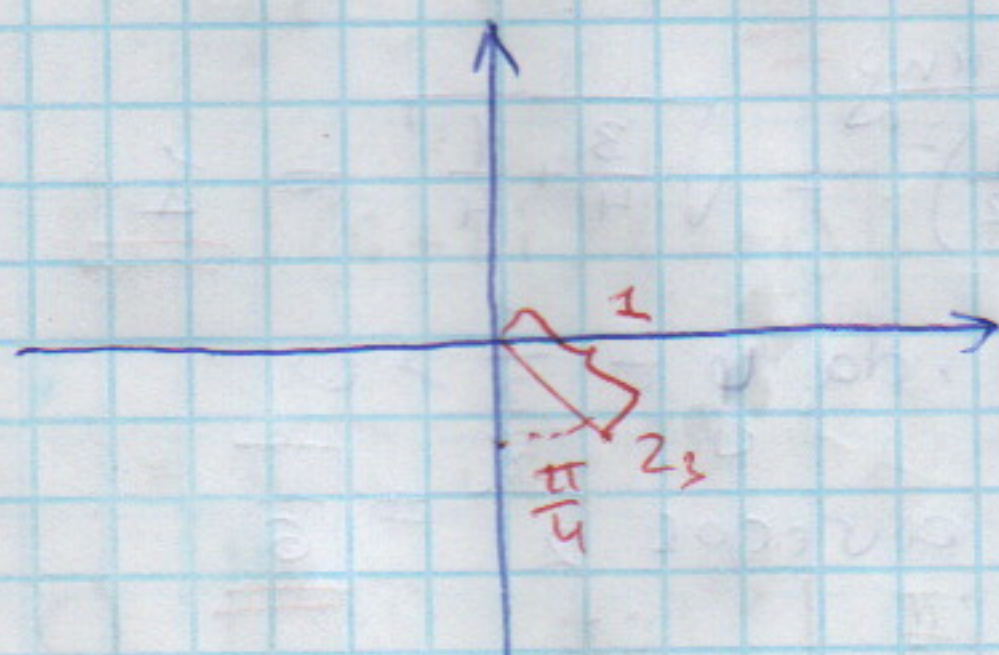
$z_1 = 0 + 1 \cdot i$

b)  $z_2 = 2e^{2\pi i}$ , also  $r=2$ ;  $\varphi = 2\pi$



$z_2 = (2 + 0i) = 2 + 0 \cdot i$

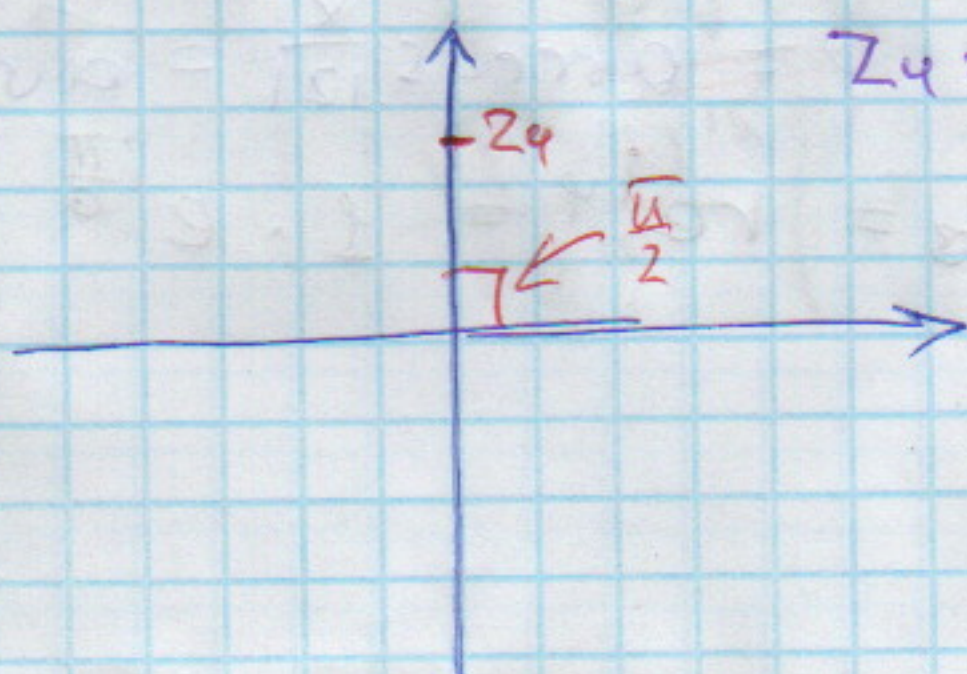
c)  $z_3 = e^{i\frac{15\pi}{4}}$



$z_3 = r \cos \varphi + i r \sin \varphi = \frac{\sqrt{2}}{2} + i \left(-\frac{\sqrt{2}}{2}\right) =$

$= \frac{\sqrt{2}}{2} - i \frac{\sqrt{2}}{2}$

d)  $z_4 = 3e^{-i\frac{3\pi}{2}}$



$z_4 = r \cos \varphi + i r \sin \varphi =$

$= 3 \cdot 0 + i \cdot 3 \cdot (-1) =$

$= -3i$