http://www.columbia.edu/~njr2121/1-s2.0-004579069190035X-main.pdf

In order to apply the Newton–Raphson method, first from (11) we define six scalar functions:

$$f_i(\mathbf{a}) = (\bar{x}_i + u_i)^2 + (\bar{y}_i + v_i)^2 + (\bar{z}_i + w_i)^2 - l_i^2 = 0, \tag{18}$$

for $i = 1, 2, \ldots, 6$, where the vector $\mathbf{a}$ is defined as:

$$\mathbf{a} = [a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6]^T = [x \quad y \quad z \quad \alpha \quad \beta \quad \gamma]^T \tag{19}$$

and then employ the following algorithm [20] to solve for $\mathbf{a}$:

Newton–Raphson Algorithm

Step 1 Select an initial guess $\mathbf{a}$.
Step 2 Compute the elements r_{ij} of ${}^B_P\mathbf{R}$ using (17) for $i, j = 1, 2, 3$.
Step 3 Compute $\bar{x}_i$, $\bar{y}_i$ $\bar{z}_i$ using (7) and u_i, v_i, w_i using (9) for $i = 1, 2, \ldots, 6$.
Step 4 Compute $f_i(\mathbf{a})$ and $A_{ij} = \dfrac{\partial f_i}{\partial a_j}$ using (18) for i, $j = 1, 2, \ldots, 6$.
Step 5 Compute $B_i = -f_i(\mathbf{a})$ for $i = 1, 2, \ldots, 6$. If $\sum_{j=1}^{6} |B_j| <$ tolf (tolerance), stop and select $\mathbf{a}$ as the solution.
Step 6 Solve $\sum_{j=1}^{6} A_{ij}\delta a_j = B_i$ for δa_j for i, $j = 1, 2, \ldots, 6$ using LU decomposition. If $\sum_{j=1}^{6} \delta a_j <$ tola (tolerance), stop and select $\mathbf{a}$ as the solution.
Step 7 Select $\mathbf{a}^{new} = \mathbf{a} + \delta\mathbf{a}$ and repeat Steps $1 - 7$.

Step 2 Compute the elements r_{ij} of ${}^B_P\mathbf{R}$ using (17) for $i, j = 1, 2, 3$.

$$ {}^B_P\mathbf{R} = \mathbf{R}_{RPY} = \begin{bmatrix} c\alpha\, c\beta & c\alpha\, s\beta\, s\gamma - s\alpha\, c\gamma & c\alpha\, s\beta\, c\gamma + s\alpha\, s\gamma \\ s\alpha\, c\beta & s\alpha\, s\beta\, s\gamma + c\alpha\, c\gamma & s\alpha\, s\beta\, c\gamma - c\alpha\, s\gamma \\ -s\beta & c\beta\, s\gamma & c\beta\, c\gamma \end{bmatrix}. \tag{17}$$

*$c\alpha \equiv \cos\alpha$ and $s\alpha \equiv \sin\alpha$.

Step 3 Compute $\bar{x}_i$, $\bar{y}_i$ $\bar{z}_i$ using (7) and u_i, v_i, w_i using (9) for $i = 1, 2, \ldots, 6$.

$$= \begin{bmatrix} x - b_{ix} \\ y - b_{iy} \\ z - b_{iz} \end{bmatrix} = \begin{bmatrix} x - b_{ix} \\ y - b_{iy} \\ z \end{bmatrix} = \begin{bmatrix} \bar{x}_i \\ \bar{y}_i \\ \bar{z}_i \end{bmatrix}, \tag{7}$$

ed vector of ${}^B\mathbf{d}$ and

$$ {}^B\mathbf{p}_i = {}^B_P\mathbf{R} \quad {}^P\mathbf{P}_i \tag{8}$$

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} p_{ix} \\ p_{iy} \\ p_{iz} \end{bmatrix} = \begin{bmatrix} r_{11}p_{ix} + r_{12}p_{iy} \\ r_{21}p_{ix} + r_{22}P_{iy} \\ r_{31}p_{ix} + r_{32}p_{iy} \end{bmatrix} = \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix}, \tag{9}$$

Step 4 Compute $f_i(\mathbf{a})$ and $A_{ij} = \dfrac{\partial f_i}{\partial a_j}$ using (18) for i, $j = 1, 2, \ldots, 6$.

$$f_i(\mathbf{a}) = (\bar{x}_i + u_i)^2 + (\bar{y}_i + v_i)^2 + (\bar{z}_i + w_i)^2 - l_i^2 = 0\,, \tag{18}$$

Step 5 Compute $B_i = -f_i(\mathbf{a})$ for $i = 1, 2, \ldots, 6$. If $\sum_{j=1}^{6} |B_j| <$ tolf (tolerance), stop and select $\mathbf{a}$ as the solution.

Step 6 Solve $\sum_{j=1}^{6} A_{ij}\delta a_j = B_i$ for δa_j for i, $j = 1, 2, \ldots, 6$ using LU decomposition. If $\sum_{j=1}^{6} \delta a_j <$ tola (tolerance), stop and select $\mathbf{a}$ as the solution.

Step 7 Select $\mathbf{a}^{\text{new}} = \mathbf{a} + \delta\mathbf{a}$ and repeat Steps $1 - 7$.