UNIVERSITE DE GENEVE Section de Mathématiques
Faculté des sciences

Spectral Geometry - SS 2022
Sheet 9 - Discussed on 11.05.2022

Obligatory exercises are marked with a $(*)$. We ask you to solve these and be ready to present your solutions during the exercise session. Let me know one day in advance which ones you were able to solve and I will randomly assign some of you to present your solutions. You will get points for the exercises you announced as solved with possible deductions if your presentation is lacking.

Exercise 1 (Reading assignment)**.** Read and reproduce the proof of Lemma 1.8 in Zeev Rudnick's paper posted on moodle.

Exercise 2. Let $\varphi_\lambda = \sum_{|k^2|=\lambda} c_k e^{ikx}$ be an eigenfunction of the laplacian on the torus Π^2 with $||\varphi_\lambda||_2 = 1$.

1. Show that for any fixed $a > 0$ we have

$$\int_{[0,a]^2} |\varphi_\lambda|^2 \leq 4a^2 + o(1)$$

   as $\lambda \to \infty$.

2. Quantify $o(1)$ in terms of a and λ.

3. Show that any weak limit of measures $|\varphi_\lambda|^2 dx$ as $\lambda \to \infty$ is absolutely continuous wrt the Lebesgue measure.