UNIVERSITE DE GENEVE
Faculté des sciences

Section de Mathématiques

Spectral Geometry - SS 2022
Sheet 5 - Discussed on 30.03.2022

Obligatory exercises are marked with a $(*)$. We ask you to solve these and be ready to present your solutions during the exercise session. Let me know one day in advance which ones you were able to solve and I will randomly assign some of you to present your solutions. You will get points for the exercises you announced as solved with possible deductions if your presentation is lacking.

Exercise 1 (*)**.** To finish the proof of Li-Yau theorem from class let's show the following. Let $\int f = k$ and $0 \le f \le M$. Then

$$\int f(\xi)|\xi|^2 \ge C_d k^{\frac{d+2}{d}} M^{-\frac{2}{d}} \tag{1}$$

and find C_d.

Exercise 2. Show that the LHS of (1) is minimal when f is the step function $f = \chi_{B_R(0)} M$ for appropriately chosen R.

Exercise 3 (*)**.** Prove Polya's conjecture $\lambda_k(\Omega) \ge C_d \left(\frac{k}{|\Omega|}\right)^{\frac{2}{d}}$ for tiling domains. Here C_d is the same constant as appears in Weyl's law.