UNIVERSITE DE GENEVE
Faculté des sciences

Section de Mathématiques

Spectral Geometry - SS 2022
Sheet 4 - Discussed on 23.03.2022

Obligatory exercises are marked with a $(*)$. We ask you to solve these and be ready to present your solutions during the exercise session. Let me know one day in advance which ones you were able to solve and I will randomly assign some of you to present your solutions. You will get points for the exercises you announced as solved with possible deductions if your presentation is lacking.

Exercise 1. Work through one of the proofs of the isoperimetric inequality found here.

Exercise 2 $(*)$**.** Let D be a simply connected bounded domain in $\mathbb{R}^2$ and let $\lambda = \lambda_1(D)$ be the smallest eigenvalue of $-\Delta$ with Dirichlet boundary conditions. Show that there exist universal constants c, C such that

1. one can inscribe a ball of size $\frac{c}{\sqrt{\lambda}}$ in D.
2. It is impossible to inscribe a ball of size $\frac{C}{\sqrt{\lambda}}$ in D.