UNIVERSITE DE GENEVE
Faculté des sciences

Section de Mathématiques

Topics In Probability And Analysis
Exercise Sheet 7 - Discussed on 05.11.2020

We call a set $H \subset \mathbb{N}$ Poincaré if $\forall S \subset \mathbb{N}$ with $\bar{d}(S) > 0$ we have

$$(S - S) \cap H \neq \emptyset$$

Exercise 1. Let $H \subset \mathbb{N}$ be finite. We denote by

$$K_H = \{x = \sum x_n 2^{-n} : \ x_n x_{n+h} = 0 \text{ if } h \in H\}.$$

Show that $\dim(K_H) = \dim_M(K_H)$. How can we calculate $\dim(K_H)$?

Exercise 2. For the following sets check whether they are Poincaré or not :

1. $k\mathbb{N}$
2. $2\mathbb{N} + 1$
3. $\{n^2\}_n$
4. $\{n^2 + 1\}_n$