UNIVERSITE DE GENEVE Section de Mathématiques
Faculté des sciences

Topics In Probability And Analysis
Exercise Sheet 5 - Discussed on 22.10.2020

We define $\text{Slice}_t := \{(x,y) : x - y = t\} \subset \mathbb{R}^2$.

Exercise 1. Show that for two sets A, B we have $\dim_M(A \times B \cap \text{Slice}_t) \leq (\dim_M(A \times B) - 1)_+$ for almost every t.

Exercise 2. Review the construction for $\dim(A \times B) > \dim(A) + \dim(B)$.

Show that if we only iterate the A step, we get

$$\dim(B) = 1,\ \dim(A) = \frac{1}{2},\ \dim(A \times B) = \frac{3}{2}.$$

Furthermore show that for $K_j = L_j = K\ \forall j$ we get

$$\dim(A) = \frac{3}{4} = \dim(B), \quad \dim(A \times B) = \frac{3}{4}.$$

Exercise 3. Give explicit formulas for the contractions that have the Koch Snowflake and the Sierpinski Carpet as attractors.

Exercise 4. Let $\text{CPT}(X) = \{K \subset X : K \text{ compact }, K \neq \emptyset\}$ and $d_H(K, L) = \inf\{\epsilon > 0 : K \subset L_\epsilon, L \subset K_\epsilon\}$ the Hausdorff distance.

Show that d_H is a metric on $\text{CPT}(X)$.