UNIVERSITE DE GENEVE
Faculté des sciences

Section de Mathématiques

Topics In Probability And Analysis
Exercise Sheet 6 - Discussed on 29.10.2020

Exercise 1. Let $f_1(x) = \frac{x}{2}$ and $f_2(x) = \frac{2x+1}{3}$. Show that these contractions as functions on $[0, 1]$ do not satisfy the open set condition.

Exercise 2. Let (X, d) be a complete metric space with a finite set of contractions $\{f_j\}$ satisfying the open set condition. Show that the attractor K is given by

$$K = \{y : \exists \text{ series } (\sigma^k) \text{ with } f_{\sigma^k}(x) \to y\}$$

the set of limit points under application of the f's. Show that these limits are well defined i.e. do not depend on the choice of x.

Exercise 3. We call a set $K \subset [0, 1]$ homogeneous if it is invariant under the shift operator $T_b(x) = bx \mod 1$. Show that the following sets are homogeneous :

1. $K_D := \{\sum x_n b^{-n} : x_n \in D\}$ where $D \subset \{0, \ldots, b-1\}$.
2. $K_A = \{\sum x_n b^{-n} : \forall\, i \quad A_{x_i x_{i+1}} = 1\}$ where A is a $b \times b$ matrix with entries in $\{0, 1\}$.