UNIVERSITE DE GENEVE
Faculté des sciences

Section de Mathématiques

Topics In Probability And Analysis
Exercise Sheet 8 - Discussed on 26.11.2020

Exercise 1. We define the packing number as

$$N_p(K,\epsilon) = \max\{\#z_j \in K : \ |z_j - z_i| > \epsilon \ \forall i \neq j\}.$$

Show that

$$N(K,4\epsilon) \leq N_p(K,\epsilon) \leq N(K,\epsilon)$$

and therefore the packing dimension and the Minkowski dimension agree.

Exercise 2. Let $K \subset \mathbb{R}^d$. Denote by K^ϵ the ϵ-neighborhood of K. Show that

$$\overline{\dim}_M(K) = d + \overline{\lim_{\epsilon \to 0}} \frac{\log(\mathrm{Vol}(K^\epsilon))}{\log(\frac{1}{\epsilon})}.$$