UNIVERSITE DE GENEVE
Faculté des sciences

Section de Mathématiques

Topics In Probability And Analysis
Exercise Sheet 9 - Discussed on 02.12.2020

Exercise 1. Let's consider the Whitney decomposition in one dimension.

1. Let $\{Q_j\}$ be the Whitney decomposition of an interval I. Check that $\sum_j |Q_j|^s$ is comparable to $|I|^s$ for any $s \in (0, 1]$.
2. Conclude how one can compute the Whitney dimension for compact sets $K \subset \mathbb{R}$ without calculating the Whitney decomposition.
3. Where does this go wrong in higher dimensions ?
4. Calculate the Whitney dimension of the Cantor set K directly.

Exercise 2. Write the Whitney dimension in terms of

$$\inf\{\beta : \int_{\text{outside of } K} \text{dist}(x, K)^\beta d\text{Vol}(x) < \infty\}.$$