UNIVERSITE DE GENEVE — Section de Mathématiques
Faculté des sciences

Topics In Probability And Analysis
Exercise Sheet 2 - Discussed on 01.10.2020

Exercise 1 (4-adic boxes)**.** Consider the set $K \subset [0,1]^2$ given by the following construction. $K_0 = [0,1]^2$ and K_{n+1} is K_n with each square replaced by 4 squares of side length $\frac{1}{4^n}$ filling out their corners. Then $K = \bigcap_n K_n$.

Show that $\dim_M(K) = 1$.

Exercise 2 (Von Koch snowflake)**.** Let K be the von Koch snowflake as desribed in class with 4^n intervals of length $\frac{1}{3^n}$ at the n-th generation. Show that

$$\dim(K) = \dim_M(K) = \frac{\log(4)}{\log(3)}$$

Exercise 3. Show that any non-constant continuous curve γ has $\dim(\gamma) \geq 1$.

Exercise 4. Construct a set $S \subseteq \mathbb{N}$ such that

$$\overline{d}(S) = 1, \text{ and } \underline{d}(S) = 0.$$

Exercise 5. Show that for $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ the number of 2-adic intervals I_n needed to cover X_A build the Fibonacci sequence and calculate the spectral radius of A. Deduce the Minkowski dimension of X_A.

Exercise 6. Recall that for a matrix $b \times b$ matrix A with entries in $\{0,1\}$ we define $Y_A = \{(x,y) \in [0,1]^2 : A_{y_n,x_n} = 1 \ \forall n\}$ where x_n, y_n are the expansions in the basis of b. Let $||A||$ denote the number of ones in A. Show that

$$\dim_M(Y_A) = \frac{\log(||A||)}{\log(b)}.$$