
Topics In Probability And Analysis
Exercise Sheet 10 - Discussed on 10.12.2020

Let's review some complex analysis.

Recall that we call a set K removable for a set of functions A on the domain Ω if

$$f \in \text{Hol}(\Omega \setminus K) \cap A \implies f \in \text{Hol}(\Omega).$$

Exercise 1. Show that the slit $[-1, 1]$ is not removable on the domain \mathbb{C} for bounded functions.

Exercise 2. Review the proof of the theorem showing that K is removable for $\text{Höl}(\alpha)$ if and only if $\mathcal{H}_{1+\alpha}(K) = 0$ for $0 < \alpha < 1$.

What goes wrong for $\alpha = 1$? Can you find a counterexample?

Exercise 3. In the lecture we constructed a measure μ supported on the set K via the Frostman lemma. We then constructed the function $f(z) = \frac{1}{z} * \mu = \int_K \frac{1}{z-u} d\mu(u)$.

Show that this is well defined for $z \notin K$.