UNIVERSITE DE GENEVE Section de Mathématiques
Faculté des sciences

Optimization with applications I
série 1

Exercise 1

Let us consider the parametric model

$$Y_n \overset{\text{i.i.d}}{\sim} \mathrm{N}(\mu, \sigma^2), \quad n = 1, \ldots, N,$$

with expectation μ and standard deviation $\sigma > 0$.

1. Write the likelihood function and the negative log-likelihood.
2. Find the maximum likelihood estimator of (μ, σ^2). Is the solution always in the feasible set, for any values of $y_1, \ldots, y_N$?
3. Find the maximum likelihood estimator of (μ, σ).
4. Compute the Hessian matrix at the point of maximum likelihood. Is it positive definite ?

Exercise 2

Let us consider the parametric model

$$Y_n \overset{\text{i.i.d}}{\sim} \text{Poisson}(\lambda), \quad n = 1, \ldots, N,$$

with expectation $\lambda > 0$. Find the maximum likelihood estimator of λ.

Exercise 3

Let us consider the following linear regression model :

$$Y_n = \alpha_0 + \alpha_1 x_{n1} + \ldots + \alpha_P x_{nP} + \epsilon_n, \quad n = 1, \ldots, N,$$

where $\epsilon_1, \ldots, \epsilon_N \overset{\text{i.i.d}}{\sim} \mathrm{N}(0, \sigma^2)$. We denote $\mathbf{Y} = (Y_1, \ldots, Y_N)^{\mathrm{T}}$ and

$$X = \begin{pmatrix} 1 & x_{11} & \ldots & x_{1P} \\ 1 & x_{21} & \ldots & x_{2P} \\ \vdots & \vdots & \ldots & \vdots \\ 1 & x_{N1} & \ldots & x_{NP} \end{pmatrix}.$$

a) What is the joint distribution of the $Z_n = Y_n - (\alpha_0 + \alpha_1 x_{n1} + \ldots + \alpha_P x_{nP})$?

b) Write the log-likelihood function of $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_P)^{\mathrm{T}}$ and σ^2.

c) Find the maximum likelihood estimator of $\boldsymbol{\alpha}$ and σ^2. Is it always well-defined ?

d) Repeat a), b) and c) when the noise (i.e. the ϵ_n) follow a Laplace distribution with parameter $\lambda > 0$, defined by the following probability density function :

$$f(x \mid \lambda) = \frac{\lambda}{2} e^{-\lambda |x|}$$