UNIVERSITE DE GENEVE Section de Mathématiques
Faculté des sciences

Optimization with Applications I
TP 2

Exercise 1. Implement the Newton algorithm used to find roe minimum of a function. It should be a function with the signature

```
def function newton(f_der, f_der2, x_0, epsi=1.e-6, max_steps=1000):
        # your code here
    if not converged:
        raise Exception("does not converge")
    else:
        return (zero it converged to)
```

Here *f_der* and *f_der2* should be the function's first and second derivatives calculated analytically.

Test it on multiple functions with multiple starting values and find good examples of convergence and non-convergence.

Exercise 2. Replace the input of the derivative in the algorithm by the first week's numerical derivative, so the new newton has the signature

```
def function num_newton(f_der, x_0, epsi=1.e-6, max_steps=1000):
        # your code here
    if not converged:
        raise Exception("does not converge")
    else:
        return (zero it converged to)
```

Try to reuse the code from the first exercise.

Can you find an example that converges for the analytical derivative but not for the numerical ?

Exercise 3. Implement the golden section algorithm to find a minimum of a given unimodal function f over the interval $[a, b]$.

```
def function golden_section(f, a, b, eps=1.e-6):
    # your code here
    return minimum
```

Exercise 4. We want to perform linear regression like in Exercise 3 on TD1. In order to produce nice pictures we will do this in dimension 2.

Pick some parameters α_0, α_1 and a variance σ^2. Draw $n = 50$ points $\{x_i\}_{i=1}^n$ at random from some interval you choose. Produce the values $y_i = \alpha_0 + \alpha_1 x_i + \epsilon_i$ where $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$.

Now we only treat $\{y_i\}$ as input and want to find the maximum likelihood $\alpha_0, \alpha_1, \sigma^2$. Find these values and plot the x, y plot as well as the line defined by α_0, α.

How does your result compare to the actual variables that created the data ?

Important : Every function and exercise must be tested. Plug in some values for which you know the correct answers and compare the output of your function.