UNIVERSITE DE GENEVE Section de Mathématiques
Faculté des sciences

Optimization with Application I
Exercise Sheet 4 - Discussed on 13.11.2020

Exercise 1. Let X be an $N \times P$ matrix with $P > N$ of rank N and $y \in \mathbb{R}^N$. Using a Lagrange multiplier $\lambda \in \mathbb{R}^N$ show that

$$\min_{\alpha \in \mathbb{R}^P} ||\alpha||_2^2 \text{ s.t. } y = X\alpha$$

has a closed form expression. Comment on the dimension of the matrix that needs to be inverted.

Exercise 2. Consider the problem of minimizing

$$f(x, y) = x + y$$

under the constraints

$$h_1(x, y) = (x - 1)^2 + y^2 - 1 = 0$$

and

$$h_2(x, y) = (x - 2)^2 + y^2 - 4 = 0.$$

Draw the level lines of the cost function and the constraint sets. Identify the unique feasible solution.

Show that this point is not a regular point and that there exists no Lagrange multipliers for this problem.

Exercise 3. Use the Lagrange multiplier theorem to solve $\min f(x)$ subject to $h(x) = 0$ with

1. $f(x) = ||x||_2^2, h(x) = \sum_{p=1}^P x_p - 1$
2. $f(x) = \sum_{p=1}^P x_p, h(x) = ||x||_2^2 - 1.$