UNIVERSITE DE GENEVE
Faculté des sciences

Section de Mathématiques

Optimization with Application I
Exercise Sheet 6 - Discussed on 11.12.2020

We will have a look at a classification problem. We are given a data set $(x_i, y_i)_{i=1}^n$ with $x_i \in \mathbb{R}^p$ and $y_i \in \{-1, 1\}$. These are points in p-dimensional space that each have a class ± 1 associated with them. We will call the set of points in class 1 $A = \{x_i : \ y_i = 1\}$ and $B = \{x_i : \ y_i = -1\}$.

A hyperplane is given by a vector $w \in \mathbb{R}^p$ and an offset $b \in \mathbb{R}$ via $\{x : < x, w > -b = 0\}$.

Exercise 1. Assume that there exists a hyperplane separating A and B.

1. We want to find the hyperplane H that has a maximum margin on both sides. That is we want to find the w, b such that $\min_{x \in A} \operatorname{dist}(x, H) = \min_{x \in B} \operatorname{dist}(x, H)$ is maximal. Formulate this as a constrained optimization problem.
2. Solve this problem using Lagrange multipliers. (You may need to expand on the cases we have considered, since your constraint will be an inequality.)
3. How would you now classify a new point $x \in \mathbb{R}^p$?

Exercise 2. Let $p = 2$ and $A \subset \mathbb{D}$, $B \subset \mathbb{C} \setminus \mathbb{D}$, where $\mathbb{D} = \{x : ||x||_2 \leq 1\}$. Find a mapping $\phi : \mathbb{R}^2 \to \mathbb{R}^3$ such that $\phi(A)$ is separable from $\phi(B)$ by a hyperplane.

What changes in the formulas from Exercise 1 when considering such a mapping first ?