UNIVERSITE DE GENEVE
Faculté des sciences

Section de Mathématiques

Optimization with Application I
Exercise Sheet 3 - Discussed on 23.10.2020

Exercise 1. Let $g : \mathbb{R}^N \to \mathbb{R}^M$ be an affine function given by $Ax+b$ and $f : \mathbb{R}^M \to \mathbb{R}$ a convex function.

1. Show that $f \circ g$ is convex.
2. If f is stricly convex, find a necessary and sufficient condition for $f \circ g$ to be striclty convex.

Exercise 2. Let $f : \mathbb{R}^N \to \mathbb{R}$ be a convex function. Show that the sets

$$\{x \in \mathbb{R}^N : f(x) < c\}$$

and

$$\{x \in \mathbb{R}^N : f(x) \leq c\}$$

are convex. Conclude that the set of minimizers of f is convex.

Exercise 3. Consider the model

$$Y_n = \mu + \epsilon_n, \quad n = 1, \dots N$$

where $\epsilon_n \sim \text{Laplace}(\lambda)$ are i.i.d. Give a closed form expression for the maximum likelihood estimator of (λ, μ) when $N = 2M+1$ is odd and all the data are distinct. Suggest an estimator when N is even and all data are distinct.

Exercise 4. Find a function $f : \mathbb{R}^N \to \mathbb{R}$ with the following properties :

1. f is differentiable.
2. $\mathbf{0}$ is a local minimum of f over all lines that pass through $\mathbf{0}$. i.e.

$$g(\alpha) := f(\alpha \mathbf{d})$$

   has a local minimum at $\alpha = 0$ for any $\mathbf{d} \in \mathbb{R}^N$.
3. $\mathbf{0}$ is not a local minimum of f.