

Report
Beam propagation method and absorption
boundary condition in 2-D

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1 Introduction

Beam propagation in media is of high interest for scientists and also for industrial purposes. In this report beam propagation will be simulated, problems will be discussed. The purpose of the project is to compare numerical method and analytical solution of beam propagation. Convergence of used numerical method is to be shown. For the simplicity simulation of beam propagation is done in free space, linear medium, at zero incident angle, so to fit the scope of mini-project, later on can be extended for more sophisticated cases.

1.1 Analytical solution

To better analytically understand propagation of beam in space we should solve Helmholtz Equation.

$$(\nabla^2 + k^2)E = 0$$

This equation generally holds for electromagnetic wave, hence for electric and magnetic field components, for simplicity we chose electric field E . Here k is wave number and is equal to $2\pi n/\lambda$, where λ is wavelength, n is refractive index of medium (here we assume it to be equal to 1). As assumption we consider that our wave propagates in z direction and electric field can be represented in next form.

$$E(x, y, z) = u(x, y, z)e^{-ik_0 z}$$

u is complex function which describes non-plane part of the beam. Substituting our ansatz into Helmholtz equation in cartesian coordinates and doing some simplifications we get.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - 2ik \frac{\partial u}{\partial z} + (k^2 - k_0^2)u = 0$$

Using paraxial approximation we see that third term is smaller than other, therefore can be neglected, as result we get following equation.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - 2ik \frac{\partial u}{\partial z} + (k^2 - k_0^2)u = 0 \quad (1)$$

Solving this differential equation we find u hence we find E . Solution for one dimensional case, where wave propagates only in x-z plane is,

$$E(x, z) = E_o \left(\frac{w_o}{w(z)} \right)^{\frac{1}{2}} \exp\left(\frac{x^2}{w(z)^2} \right) \exp\left(-i \left(kz + k \frac{x^2}{2R(z)} - \phi(z) \right) \right)$$

where $w(z)$ is beam radius, $w_o = w(0)$ is beam waist, E_o is electric field amplitude, $R(z)$ is beam curvature, ϕ is Gouy phase. Their explicit mathematical formula can be found in the table below. This electric field formula will be later used as reference to numerical solution.

Beam Radius	Beam Curvature	Gouy phase
$w(z) = w_o \sqrt{1 + \left(\frac{z\lambda}{\pi w_o^2}\right)^2}$	$R = z \left[1 + \left(\frac{\pi w_o^2}{z\lambda}\right)^2\right]$	$\phi(z) = \arctan\left(\frac{z\lambda}{\pi w_o^2}\right)$

1.2 Numerical solution

To simulate light beam propagation Beam Propagation Method (BPM) is used. BMP is discretization of formula (1), and can be rewritten in following way.

$$2ik_o \frac{u^{l+1} - u^l}{\Delta z} = \frac{1}{2} \left(L_h u^{l+1} + L_h u^l \right)$$

where L_h is spacial discretization operator for one dimensional case is equal to

$$L = \frac{\partial^2}{\partial x^2} + (k^2 - k_o^2)$$

L numerically will be defined by Thomas algorithm, also known as triagonal matrix algorithm.

$$L = \begin{bmatrix} -2 + (k^2 - k_o^2)\Delta x^2 & 1 & 0 & \dots & 0 \\ 1 & -2 + (k^2 - k_o^2)\Delta x^2 & 1 & \dots & 0 \\ 0 & 1 & -2 + (k^2 - k_o^2)\Delta x^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & -2 + (k^2 - k_o^2)\Delta x^2 \end{bmatrix}$$

From here doing simple mathematical rearrangement we can write previous equation in the following form.

$$u^{l+1} = \left(I - \frac{\Delta z}{4ik_o} L_h \right)^{-1} \left(I + \frac{\Delta z}{4ik_o} L_h \right) u^l$$

Here I is identity matrix, which has the same size as triagonal matrix L .

2 Results

3 Conclusion