SW6

Integer Linear Programming

e.g. $\max\{c^T x : x \in P \cap \mathbb{Z}^n\}$ with $P = \{x \in \mathbb{R}^n : Ax \leq b\}$

→ Mixed Integer LP (MIP) is, when some variables are integer and others are not.

It is not possible to just run a linear programming algorithm and then round up or down.

Problem 1: Rounding the LP solution may yield a non-feasible solution.

Problem 2: The optimal LP solution is maybe far away from the optimal ILP solution.

Relaxations:

(i) Enlarge the solution space: $S \to S'$ with $S \subseteq S'$ (e.g. by removing a constraint.)

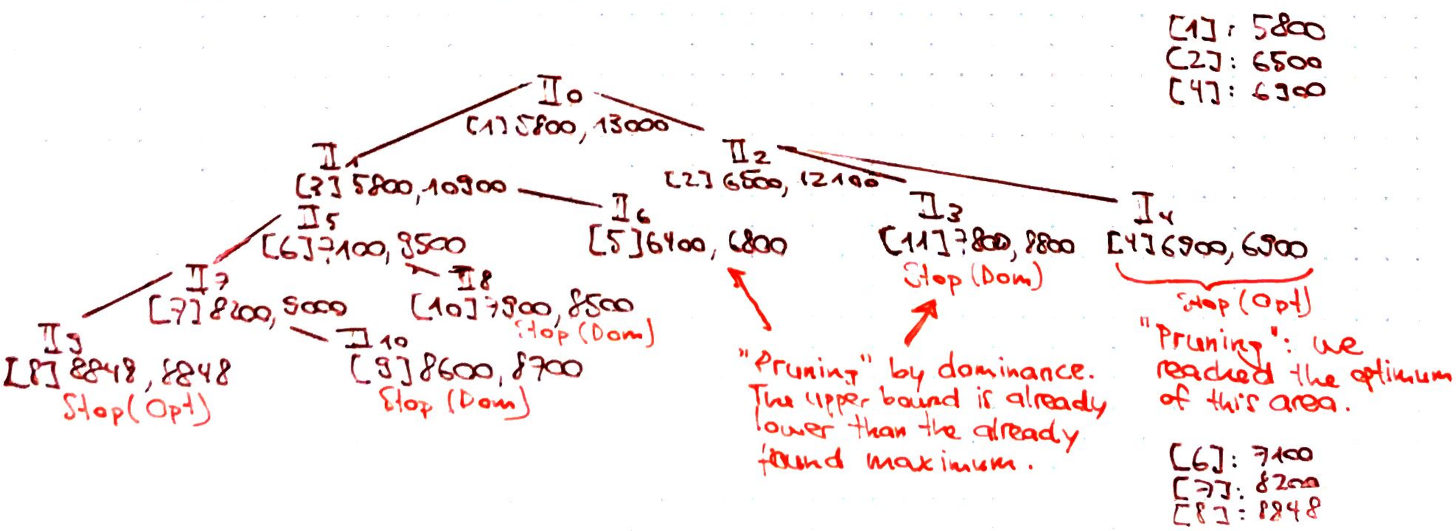
(ii) increase objective function: $f(x) \to f'(x)$ with $f'(x) \geq f(x)$ for $x \in S$

→ $f'(x)$ needs to be higher or equal to $f(x)$ within the solutionspace.

Branch-and-Bound Method (1 of 2 methods to solve ILP or MIP)

1) Split solution space iteratively into smaller subspaces ("Branch")
2) Calculate upper bound ("Bound") - e.g. Relaxation
3) Calculate a feasible solution → Lower Bound

Mount Everest Example. Helicopters search for the most top mountain in Himmalaya from top and the sherpas from the bottom.



Knapsack - Problem

- Items have a volume a and a benefit c , $j \in J$
- The knapsack has a capacity b

$\Rightarrow \max\{c^T x : a^T x \leq b, x \in \{0,1\}^J\}$

// maximise the benefit of the knapsack. x is 0, if the item is not put into the knapsack. The total benefit is the sum of all benefits · x.

LP Solution easy ("upper bound"):

(1) Sort items by decreasing benefit per volume: $\frac{c_j}{a_j}$

(2) Choose items in this order. Last item fractionally.

Feasible solution easy: Round down the last fractional item.

But this is not the final solution, but the solution of the first problem (first node in the "Branch and Bound" method). After doing this step, create two new branches. One with x = 0 for the fractional item and one with x = 1 for the fractional item.

Good example on slides chapter 4 slide 12.