SW3

Termonologie:

$x^* \in S$ such that $f(x^*) \geq f(x)$ for all $x \in S$

↳ Optimal Solution (x with function $f(x^*)$)

$f(x^*)$

→ Optimum (real value)

Definition of Solution Space:

- Functional constraints:

  e.g. $S = \{x \in \mathbb{R} \times \mathbb{Z} \times \{0,1\} : x_1 + x_2 + x_3 = 1, x \geq 0\}$

  i.e. $x = (x_1, x_2, x_3)^T$ and $x_1 \in \mathbb{R}$, $x_2 \in \mathbb{Z}$, $x_3 \in \{0,1\}$

- Non-Functional constraints:

  $S = \{x \in \mathbb{Z}^n : x$ is a permutation of $1 \dots n\}$

  → Hard for solvers, impossible

Problems and Problem instances:

$\max\{\sum_{j=1}^{n} c_j x_j : \sum_{j=1}^{n} a_j x_j \leq b, x \in \mathbb{R}^n\}$ ← This is the Problem.

The Problem Instance is with actual parameters.

Neighborhood:

Neighborhood: $N: S \to P(S)$ where $S \subseteq \mathbb{R}^n$, $P(S)$ Powerset of S

$x \mapsto N(x)$

Neighborhood Solution: $N(x) \subseteq S$

$P(S)$ is a set of all subsets of S. if $S = \{1,2,3\}$

$P(S) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,3\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$

A neighborhood is a set of subset of the powerset, where the subsets are neighbors of x. For instance, if the neighborhood of 1 is 1 and 3:

$N(1) = \{1, 3\}$

If we have a feasible solution $x \in S$, we search in the neighborhood for a better solution. If we don't find a better solution in the neighborhood, we have a local optimum.

You repeat the search in neighborhoods, till you have a maximum.
That's called Principle of Local Search Methods or
Trajectory-Based Search.

Boundaries / Interior Points:

Interior Point: Point within the boundary, NOT on the boundary

Boundary Point: Point on the boundary.

closed: all boundary points of S are in S

open: all points are interior

> How is it possible that something is not closed?

Theorem of Weierstrasse:

Let $S \subseteq \mathbb{R}^n$ non-empty, closed, bounded $f: S$ continuous.

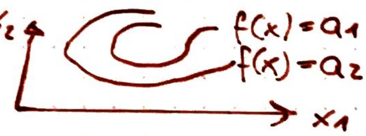
Level Sets:

Graph of a function $f: S \mapsto \mathbb{R}$:

$H = \{(x, f(x)) : x \in S\}$ simple an additional dimension $f(x)$

Level Sets for level a:

$L_a = \{x \in S : f(x) = a\}$

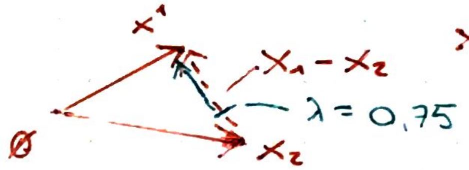


In particular: Linear Functions

Convex Optimization:

Convex combination of $x^1, x^2 \in \mathbb{R}^n$: $\Rightarrow x^n$, n is the index

$$x = \lambda x^1 + (1-\lambda)x^2 \quad \text{for some } \lambda \in \mathbb{R} \text{ with } 0 \leq \lambda \leq 1$$
$$= x^2 + \lambda(x^1 - x^2)$$



$$x = x_2 + \lambda(x_1 - x_2), \quad 0 \leq \lambda \leq 1$$

Convex Combination of more points is also possible

$x^1, x^2, x^3, x^4 \in \mathbb{R}$

$$x = \sum_{i=1}^{k} \lambda_i x^i \qquad \sum_{i=1}^{k} \lambda_i = 1 \leftarrow \text{Bedingung}$$

Convex set $S \subseteq \mathbb{R}^n$