

## SW7 ##

## Cutting Plane Method:

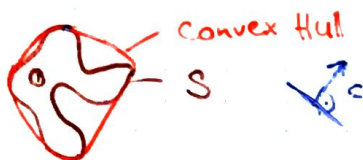
Let  $P$  and  $P'$  be two formulations for ILP  $\Pi: \max\{C^T x : x \in P \cap \mathbb{Z}^n\}$   
 $P'$  is called a better formulation for  $\Pi$  if  $P' \subseteq P$  // Wenn  $P'$  komplett in  $P$  abgebildet werden kann.

What is the best formulation?  $\rightarrow$  the convex hull, which is the tightest polyhedron around the integer points.

Problem here: It is hard or impossible to find the description of the convex hull.

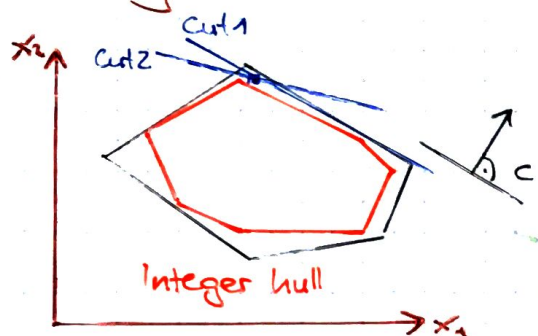
Behavior Convex Hull:

Convex hull = Integer hull



$\rightarrow$  The optimum of the convex hull is always a feasible solution in the solution space.

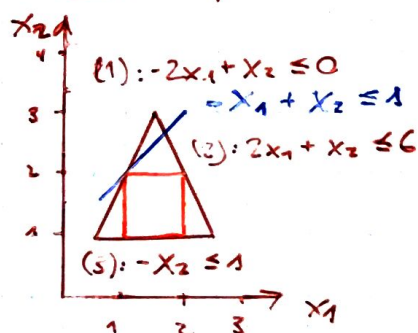
## Cutting Plane Method



We are searching cutting planes, which cut the original polyhedron but not the integer hull.

$$\text{Cut 1: } \{x \in \mathbb{R}^n : \alpha x \leq \beta\}$$

## Gomory-Chvatal-Cut:



Combine inequalities:  $\frac{3}{4} \cdot (1) + \frac{1}{4} \cdot (2)$

$$\Rightarrow = (-\frac{3}{4} 2x_1 + \frac{3}{4} x_2) + (\frac{1}{4} 2x_1 + \frac{1}{4} x_2) \leq \frac{3}{4} 0 + \frac{1}{4} 6$$

$$= -x_1 + x_2 \leq 1\frac{1}{2} = -x_1 + x_2 \leq 1$$

$1\frac{1}{2}$  kann gekürzt werden, da wir nur integer suchen.

Matrix notation:  $A = \begin{bmatrix} -2 & 1 \\ 2 & 1 \\ 0 & -1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 6 \\ -1 \end{bmatrix}$

$\cdot u = (\frac{3}{4}, \frac{1}{4}, 0)^T \geq 0$

$\cdot \alpha := u^T A = (-1, 1)^T$

G-C-cut:  $\alpha^T x \leq \beta$

$\cdot \beta := u^T b = 1\frac{1}{2}$

// The coefficients for the combination can only be found by guessing

The coefficients don't need to be 1 in total, but they need to result a non-integer on the right hand side of the inequality.