Student Colloquium, Simulating particles in a solenoids

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1 Progress this week

This week, I applied my simulation to particles traveling in a straight or curved solenoid.

I also made some updates to how the data is displayed (particle path includes time-stamp points and names, and I also draw visual aides, in this case rings, to indicate the solonoid) and saved (not all data-points are saved, significantly reducing file-size and improving the performance of the python plots).

2 Particles in a solenoid

The magnetic field of a straight solenoid is as simple as it is possible to be.

If a solenoid has n turns of wire per unit length, where each wire carries current I, and the solenoid is wrapped counterclockwise around the $\vec{\hat{n}}$ with radius R, as on figure 1, then the field a distance r from the center is

$$\vec{B}(r) = \vec{\hat{n}} \begin{cases} \mu_0 I n & r < R \\ 0 & r > R \end{cases}. \tag{1}$$

This can be shown using Ampere's law, as illustrated on figure 1. Ampere's law states that the field around the loops A and B is $\oint B \cdot d\vec{l} = \mu_0 I_{encl}$. here the current through loop A is $I_{encl,A} = nlI$ and the current through loop B is 0. Alongside the symmetry of the setup, this allows us to find the field.

This field is easy to set up in the simulation, and in the first example I use a solenoid along the x-axis with 1000 turns per meter, with a current of 5 A and a radius of 1,0 m (for reasons which will become apparent later). This gives us a field inside the solenoid:

$$\vec{B}_{inside} \approx \vec{x} 6{,}28\,\mathrm{mT}. \tag{2}$$

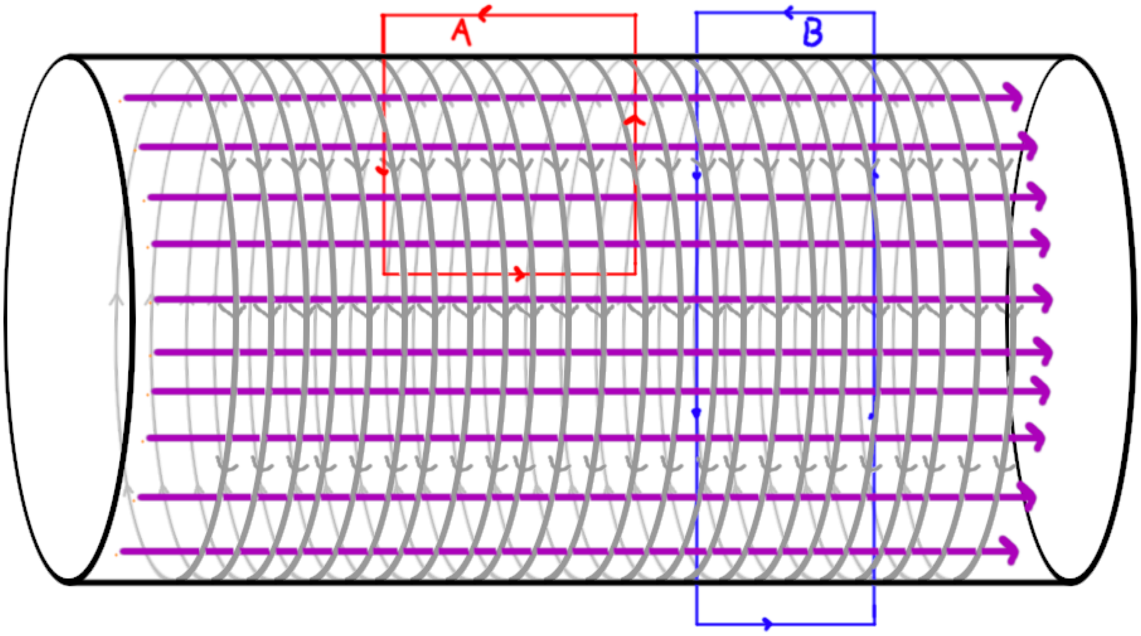

Figure 1: Sketch of a solenoid (black), with 2 imaginary loops (red and blue), which can be used to calculate the field (purple).

In the simulation, I will look at a proton, with charge $q_p = e = 1{,}60 \times 10^{-19}\,\mathrm{C}$ and mass $m_p = 1{,}67 \times 10^{-27}\,\mathrm{kg}$.

Regardless of the velocity of the particle the "cyclotron frequency" is:

$$\omega = \frac{q_p B}{m_p} \approx 6{,}02 \times 10^5\,\mathrm{Hz} \tag{3}$$

so a particle will make one full rotation each $1/\omega \approx 1{,}66 \times 10^{-5}\,\mathrm{s}$.

Then we should pick what velocity the particle in the simulation is traveling at, let us for instance say we have a particle with a kinetic energy of $1\,\mathrm{MeV/c^2}$, sticking with classical physics for now, that equates to $|v| = 3{,}195 \times 10^5\,\mathrm{m/s}$ (still so much less than the speed of light I do feel comfortable using non-relativistic physics). We know that the Cyclotron radius (aka. Larmor-, or gyroradius) , i.e. the radius of the cyclotron path is $r_c = m_p v_\perp / B q_p$. So if a particle with this velocity is traveling entirely perpendicular to the field it will trace out a circle with radius:

$$r_c = \frac{m_p v_\perp}{B q_p} \approx 0{,}531\,\mathrm{m} \tag{4}$$

Hence why I picked the radius of the Solenoid to be 1,0 m

The precision of `double` in C++ is $2^{-53} \approx 10^{-15}$ (on my computer at least, specified as `DBL_MANT_DIG` in `float.h`). So as far as the simulation is concerned $q_p = 0$ and $m_p = 0$.

One option is to use arbitrary precision floating point numbers (e.g. [1]), which have potentially infinite precision, at the cost of having undefined data size. However that would make it harder to save and load data as binary.

Instead, I just pick our base time, mass, charge and distance unit to be around 1 at the scales we are working with, I choose, micro-seconds, atomic mass units, elementary charge (obviously) and meters. Under this system, the relevant characteristics are roughly (the constants and conversions are used with higher precision than shown here):

Variable	SI units	Simulation units
kg	$1\,\mathrm{kg}$	$6{,}022 \times 10^{26}\,\mathrm{u}$
C	$1\,\mathrm{C}$	$6{,}242 \times 10^{18}\,\mathrm{e}$
s	$1\,\mathrm{s}$	$1 \times 10^{6}\,\mathrm{\mu s}$
T	$1\,\mathrm{kg/(s\,C)}$	$96{,}49\,\mathrm{u/(\mu s\,e)}$
m_p	$1{,}67 \times 10^{-27}\,\mathrm{kg}$	$1{,}01\,\mathrm{u}$
q_p	$1{,}60 \times 10^{-19}\,\mathrm{C}$	$1{,}0\,\mathrm{e}$
v	$3{,}20 \times 10^{5}\,\mathrm{m/s}$	$0{,}32\,\mathrm{m/\mu s}$
μ_0	$1{,}26 \times 10^{-6}\,\mathrm{N/A^2}$	$1{,}94 \times 10^{-17}\,\mathrm{u\,m/e^2}$
$\vec{B}_{inside}$	$\vec{x}6{,}28\,\mathrm{mT}$	$\vec{x}0{,}61\,\mathrm{u/(\mu s\,e)}$
ω	$6{,}02 \times 10^{5}/\mathrm{s}$	$0{,}602/\mathrm{\mu s}$
r_c	$0{,}53\,\mathrm{m}$	$0{,}53\,\mathrm{m}$