Exploring electric and magnetic forces using computer simulations

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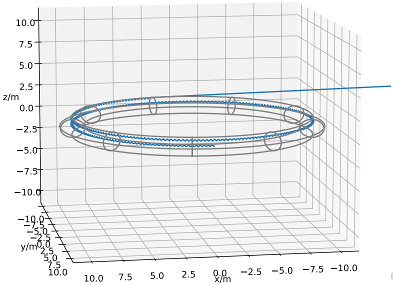
Student Colloquium in Physics and Astronomy, Aarhus University

March 2021



Wellcome

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- Todays plan

Introduction (3 minutes)

Theory and background (13 minutes)

Steering particles with $\vec{B}$ fields (14 min)

Introducing Electric fields (10 min)

Conclusion and Questions(5 min)

Introduction, what and why

- (Classical) Charged particles in Electric and Magnetic fields

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- How can magnetic fields steer and collect particles

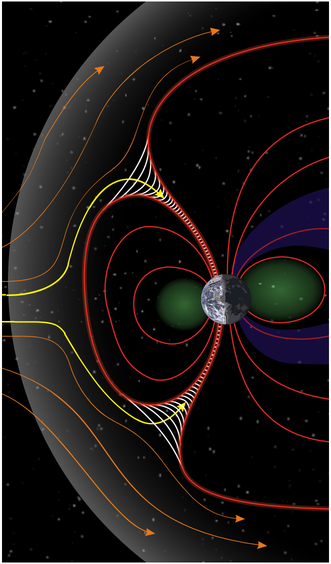
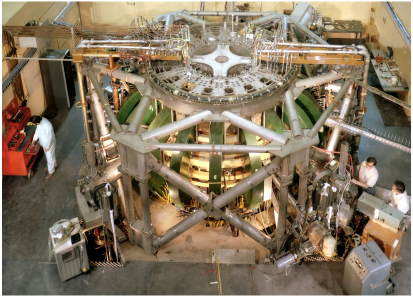


Illustration originally from Nasa. Published on wikipedia, in Public Domain

- (Classical) Charged particles in Electric and Magnetic fields
- How can magnetic fields steer and collect particles
- Real world examples:
  - Magnetic traps: "Tokamak" style fusion reactors.



Princeton Large Torus in 1975, image in Public Domain

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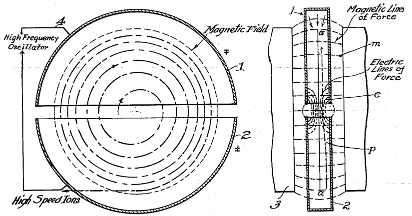

Ernest O. Lawrence, 1934, U.S. Patent 1,948,384; image in Public Domain

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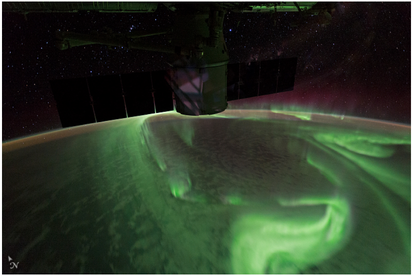
  - The Aurora.



Nasa: Aurora Australis, Indian Ocean from ISS in 2017; image in Public domain

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- Simulations are not experiments!

Theory and background (13 minutes)

Theory: Electric and magnetic fields

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- Only 1 particle! so pre-programmed depending on the setup.

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$$\vec{E}(\vec{r}, t) = \nabla\phi(\vec{r}, t)$$
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- Hamiltonian equation of motion

$$\dot{r}_i = \frac{\partial \mathcal{H}}{\partial p_i},$$
$$\mathcal{H} = \frac{\vec{p}^2}{2m} + V(\vec{r}) \rightarrow \frac{(\vec{p}^2 - q\vec{A}(\vec{r}, t))}{2m} + q\phi(\vec{r}, t) + V(\vec{r}).$$

The ODE implementation

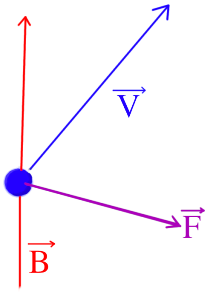
```
auto ODE = [...](const state_type Data, state_type &
    dDatadt, const double t)
{
    //Extract position and velocity from data
    ...

    //Get current force
    vec F = Charge*(Fields.get_Efield(pos,t)+
        cross(velocity,Fields.get_Bfield(pos,t)));

    vec dVdt = F*Inv_mass;  //get acceleration

    //Save derivative of data
    ...
};
```

Known results, cyclotron motion $\vec{B}$ fields



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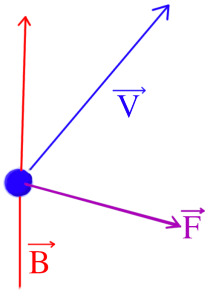
- Magnetic forces do no work:

$$dW_{\vec{B}} = \vec{F}_B \cdot d\vec{r} \propto (\vec{v} \times \vec{B}) \cdot \vec{v} = 0.$$



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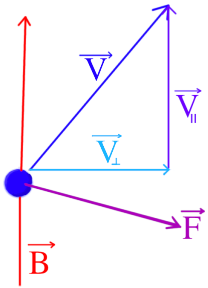
$$|\vec{F}_B| = |q(\vec{v} \times \vec{B})| = |qv_\perp B|.$$



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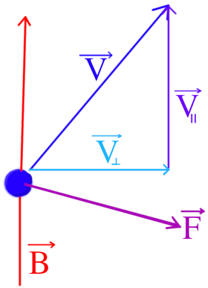

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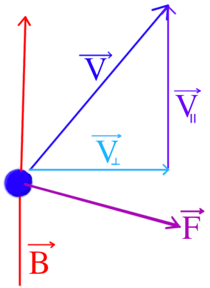
- $(\vec{v} = \vec{v}_\perp + \vec{v}_\parallel)$:

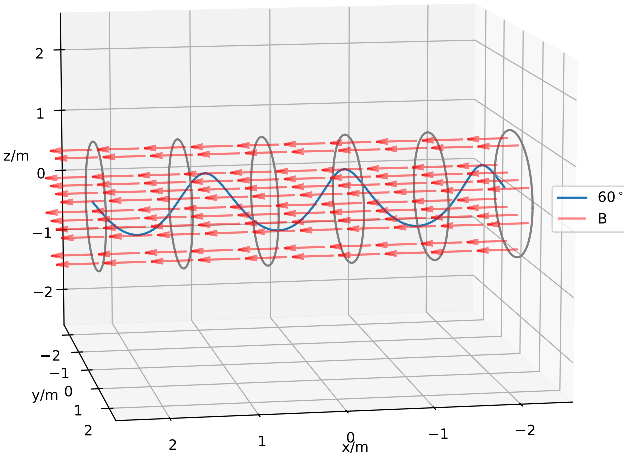
$$|\vec{F}_B| = |q(\vec{v} \times \vec{B})| = |q v_\perp B|.$$

- Same as Centripetal force: Cyclotron motion
- Cyclotron radius and frequency:

$$R = \frac{v_\perp m}{|q| B} \quad \omega_c = \frac{|q| B}{m}.$$



Protons in a Solenoid, 3D plots



- Solenoid with $N = 1000$ turns per m, $I = 5\,\mathrm{A}$, $r = 1\,\mathrm{m}$, $|\vec{B}| \approx 6\,\mathrm{mT}$.
- Proton with $E_{kin} = 1\,\mathrm{MeV/c^2}$ ($|v| \approx 3{,}195 \times 10^5\,\mathrm{m/s}$)

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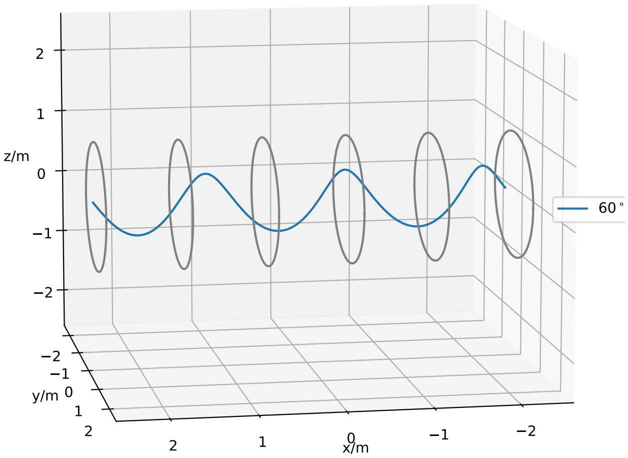

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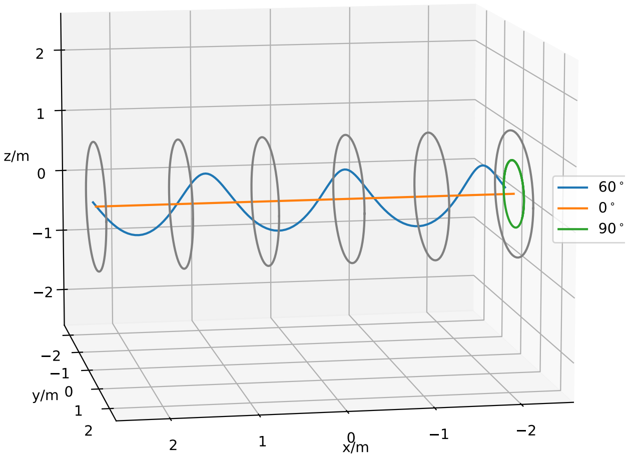

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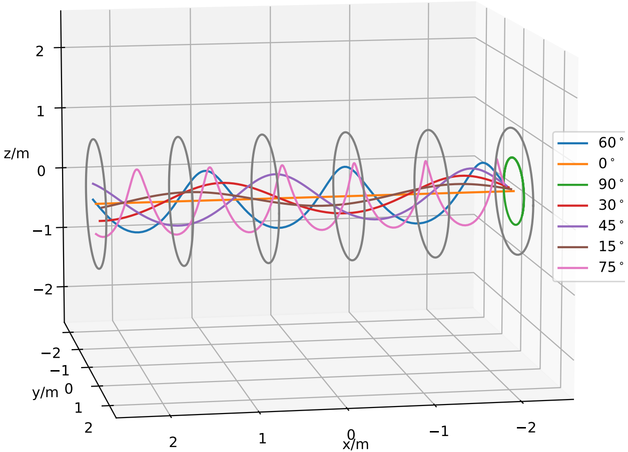

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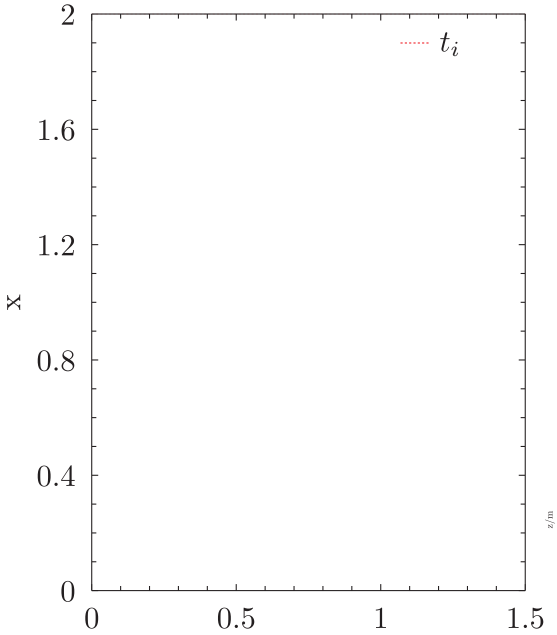

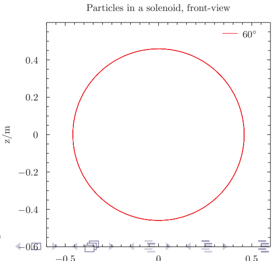

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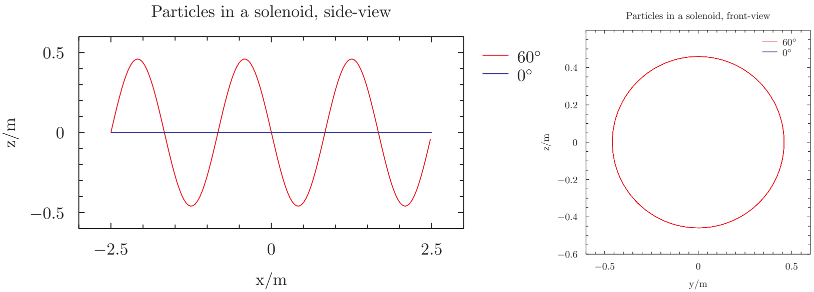

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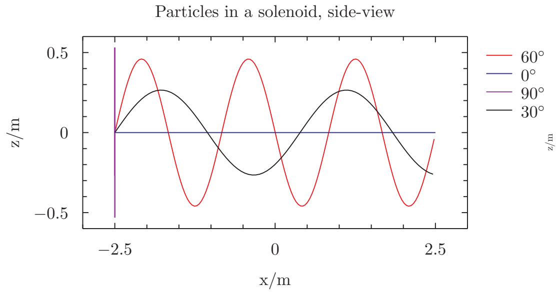

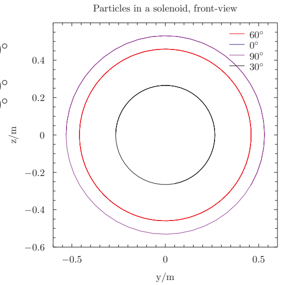

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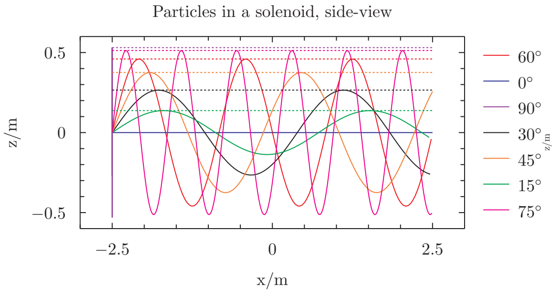

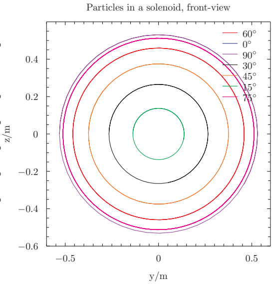

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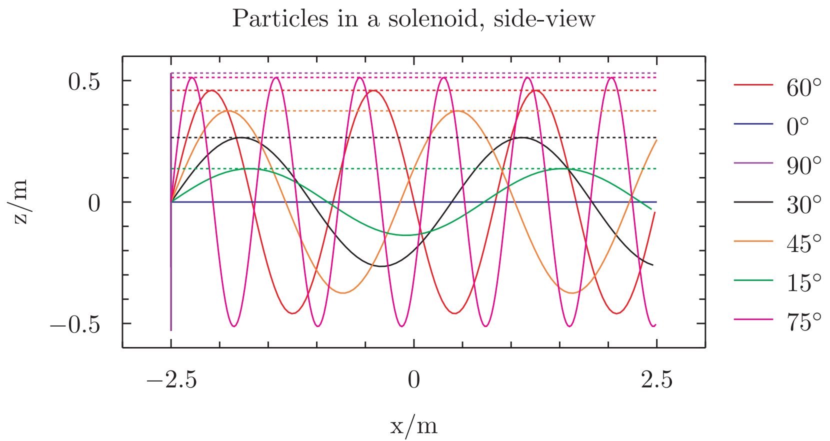

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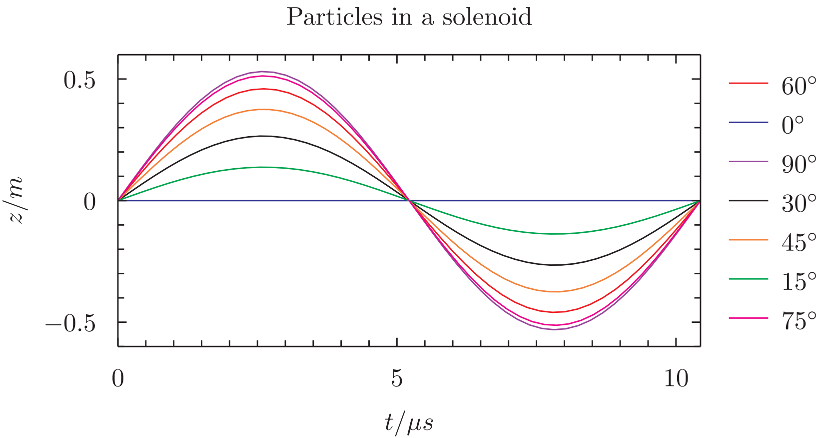

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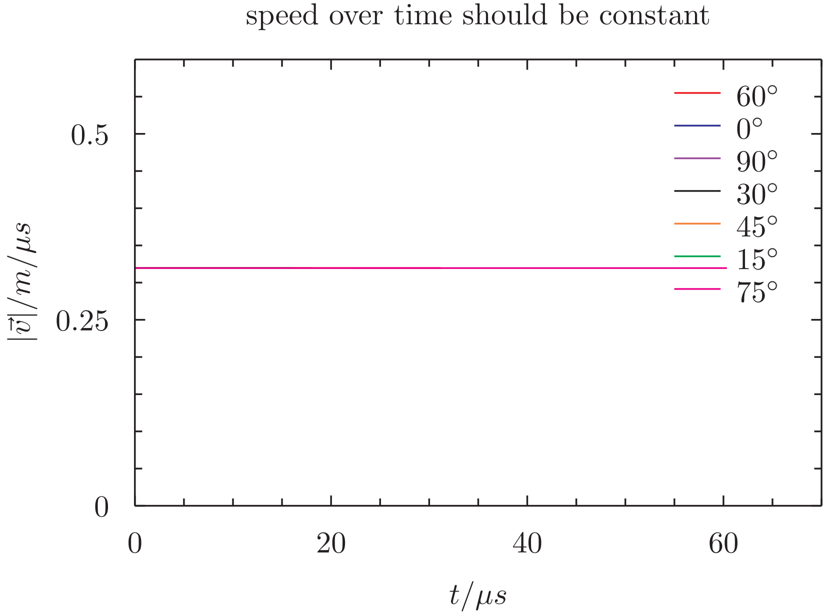

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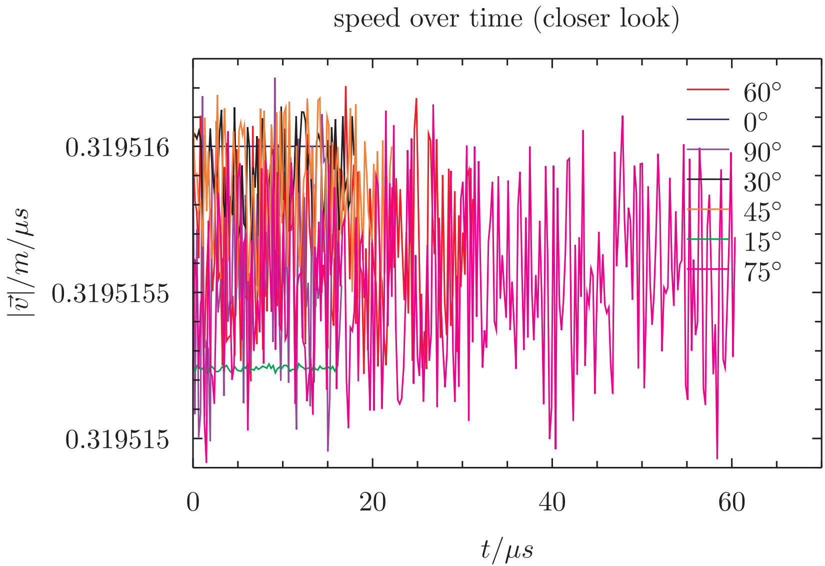
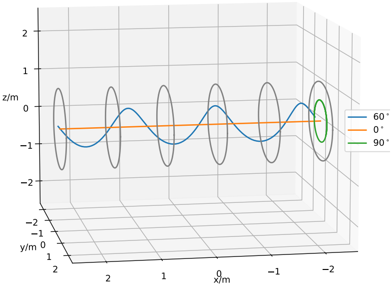


Steering particles with $\vec{B}$ fields (14 min)

how can $\vec{B}$ field steer particles

- The parallel velocity is untouched.



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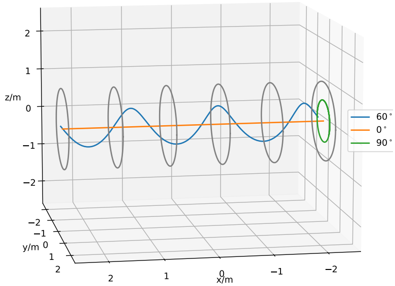

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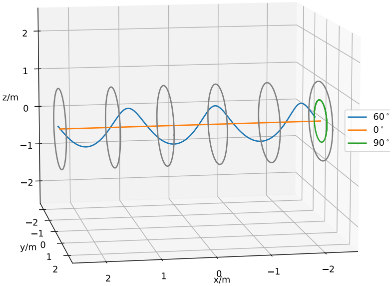

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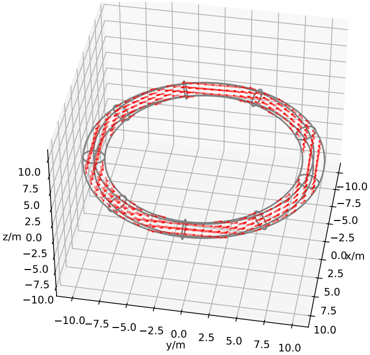
- Example torus:



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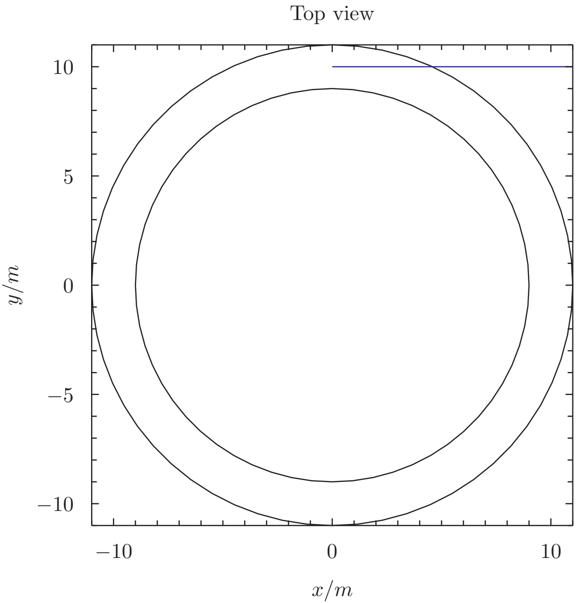
how can $\vec{B}$ field steer particles



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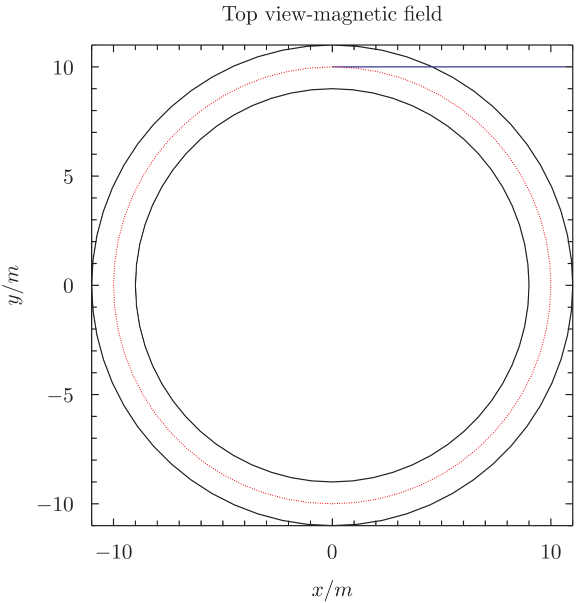
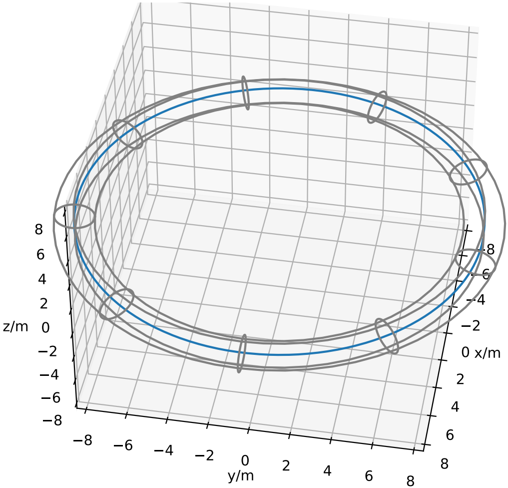


Containing protons in a Torus



Proton starting at $(0, 10, 0)$, $\vec{v}_0 = (0.3, 0, 0)\mathrm{m}/\mu\mathrm{s}$. $|B| \approx 60\,\mathrm{mT}$

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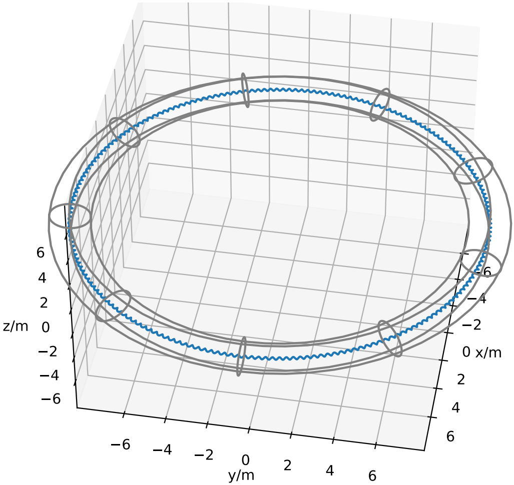
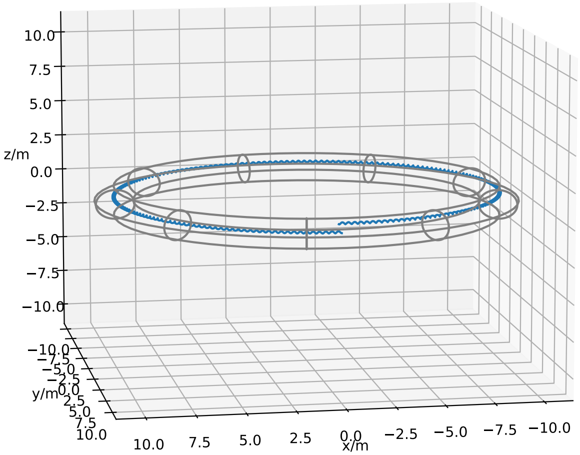


Proton

starting at $(0, 10, 0)$, $\vec{v}_0 = (0.3, 0.4, 0)\mathrm{m}/\mu\mathrm{s}$.

Instability, Error?



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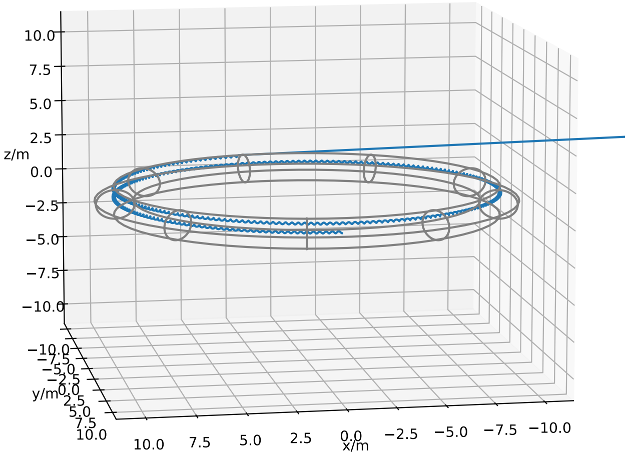
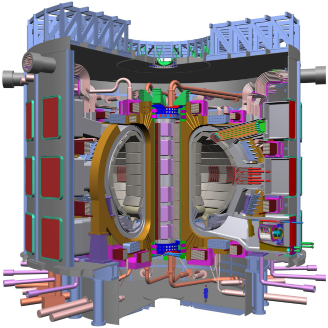


Not a torus

- "Tokamak" style reactors are NOT toruses.



Iter: International Thermonuclear Experimental Reactor, Image published by U.S. Department of Energy, placed in Public Domain

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- "Tokamak" style reactors are NOT toruses.
- Remember:

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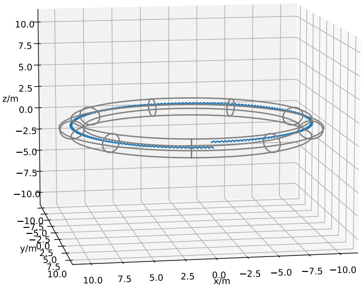

Not a torus

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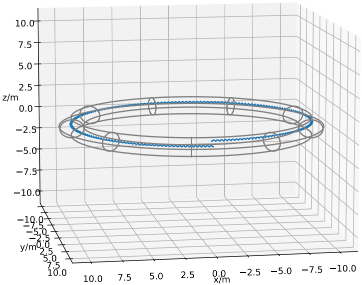
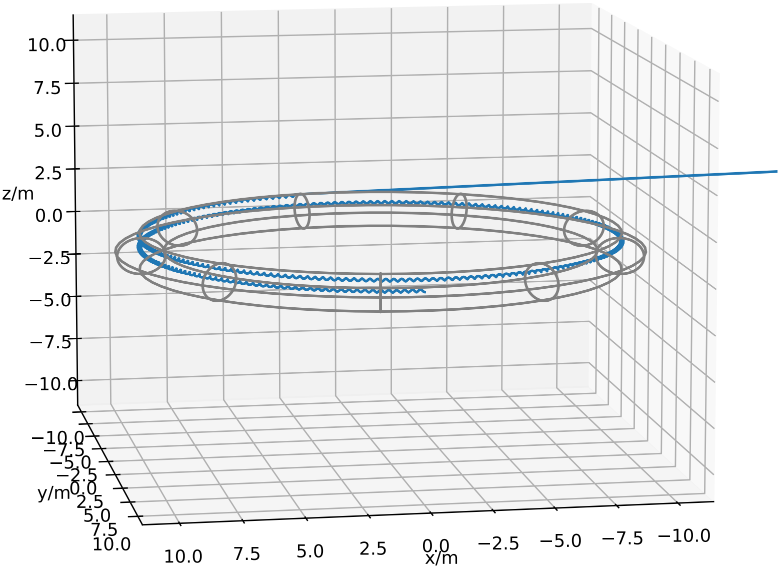
- Does lead to drift.



Solving the problem



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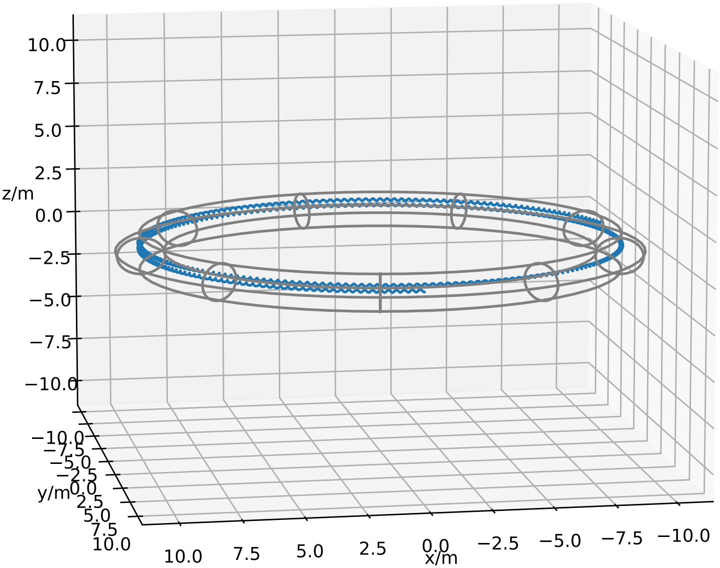

- Soviet Stamp from 1987 showing concept of Tokamak, Image in Public Domain
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- More stabilization and control is needed.

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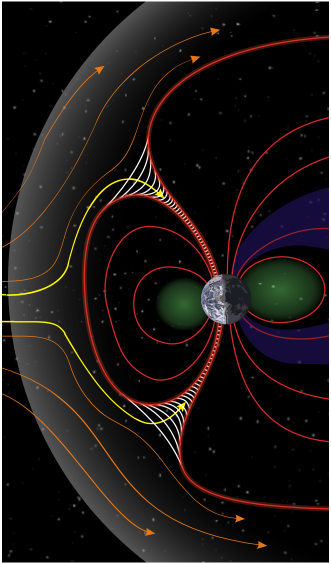
- Need to consider “current” of the plasma.
- More stabilization and control is needed.
- Particles do follow magnetic fields!

Another example, Aurora

- Solar wind (ions) vs. Earth Magnetic field.
- Illustration originally from Nasa. Published on wikipedia, in Public Domain (Cropped to fit page)



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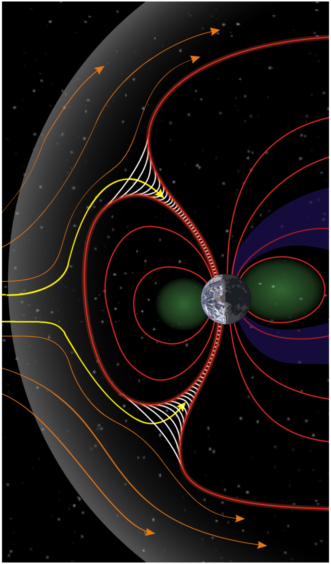

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- Here is someone elses simulation
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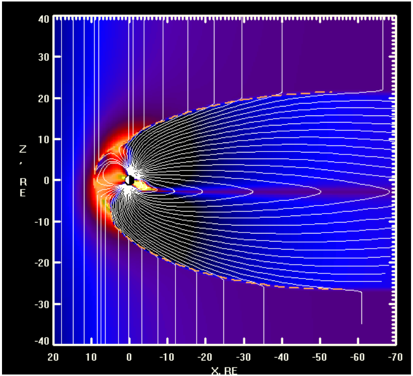
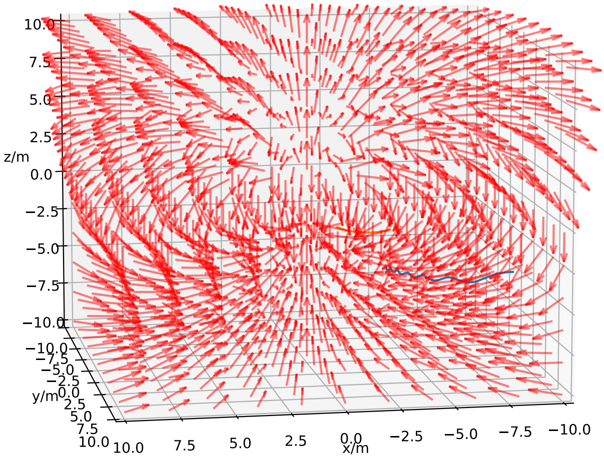


Frame from animation of the Earth magnetic field during increased activity, by N. A. Tsyganenko, Published under the GNU General Public License V. 3

Aurora, bad simulation



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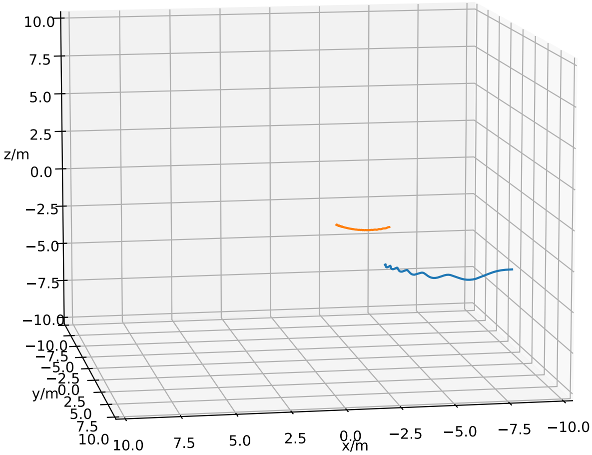

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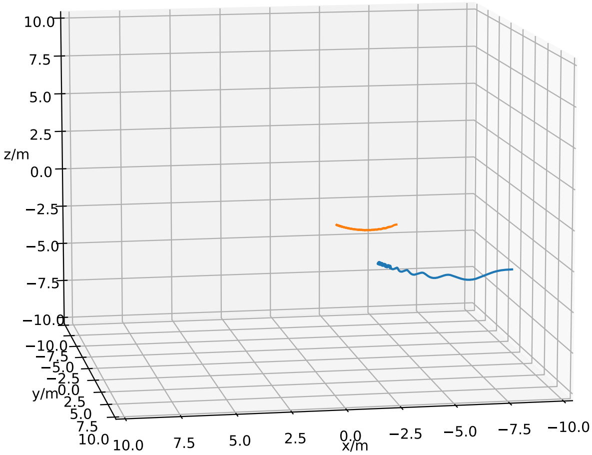

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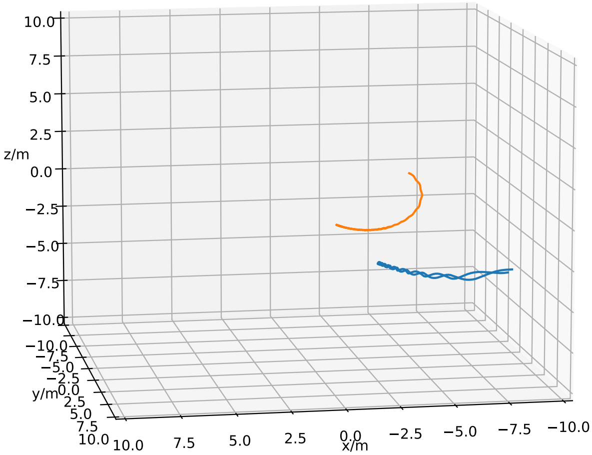
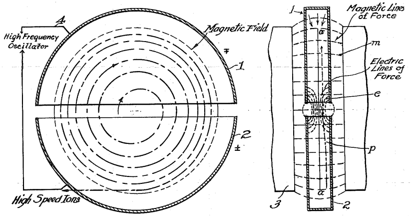


Introducing Electric fields (10 min)

Electric fields

- Electric forces do work.



Ernest O. Lawrence, 1934, U.S. Patent 1,948,384; image in Public Domain.

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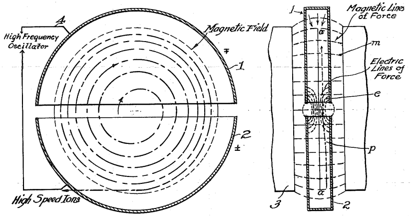

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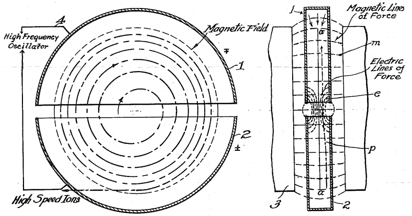
- Practical example, the Cyclotron.



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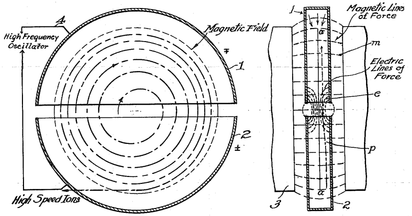
- Electric forces do work.
- Can be used in particle accelerators.
- Practical example, the Cyclotron.
- Single gab, oscillating field.
- Uses Cyclotron frequency



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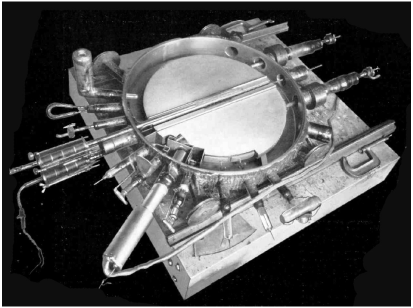
Simulating the Cyclotron

- I test same $\vec{B}$ as before (around $6\,\mathrm{mT}$), so $T \approx 10\,\mu\mathrm{s}$



Photography of Lawrence's 27-inch proof of concept Cyclotron, in Public Domain.

Simulating the Cyclotron

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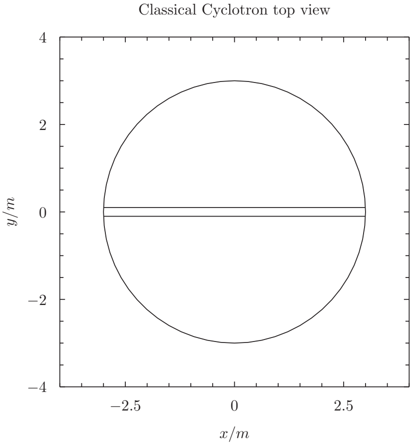
- $E_{max} = 1\,kV/m$ (For display purposes), gab 20 cm, radius 3 m.



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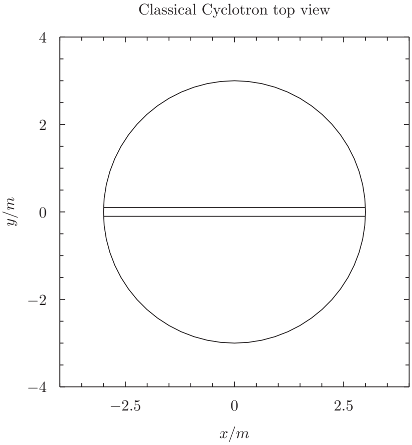
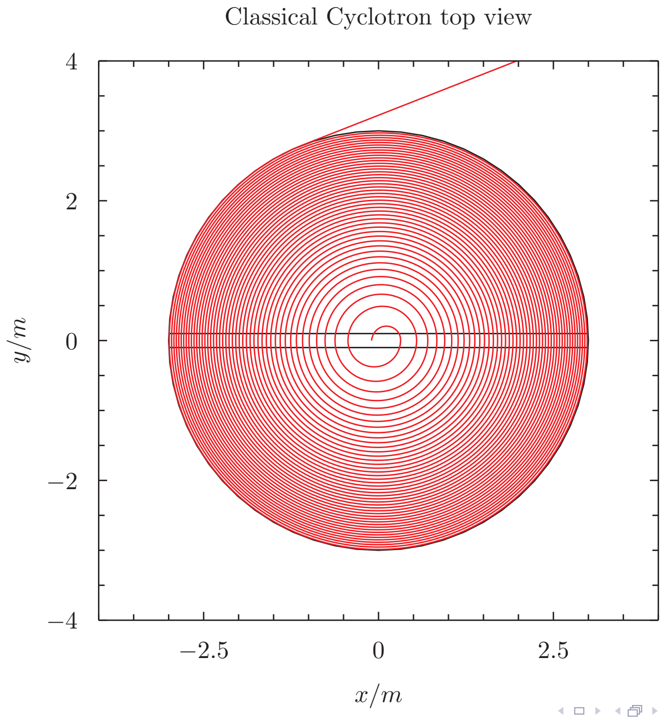
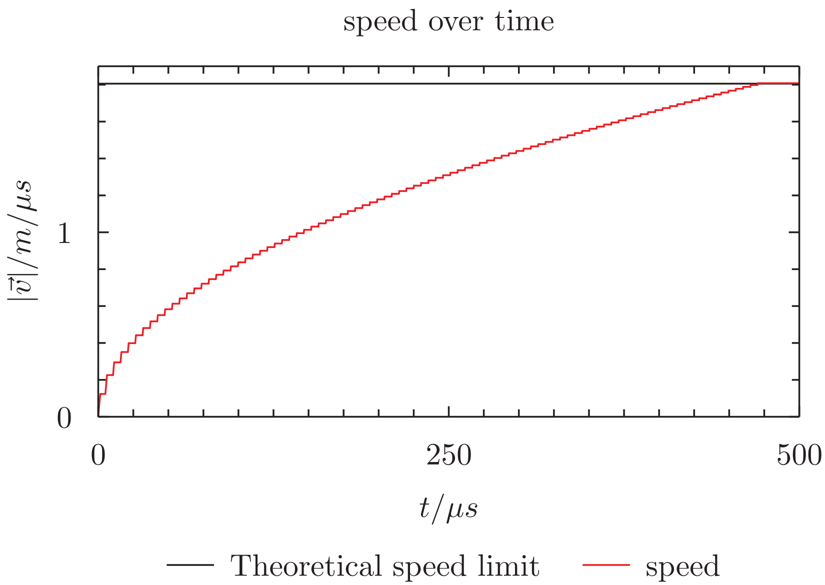
- $E_{max} = 1\,kV/m$ (For display purposes), gab 20 cm, radius 3 m.
- Energy depends only on the radius and $\vec{B}$ field:

$$\frac{R|q|B}{m} = v_\perp$$





Classical Cyclotron top view
4
2
0
−2
−4
y/m
−2.5
0
2.5
x/m



speed over time
$|\vec{v}|/m/\mu s$
0
1
0
250
500
$t/\mu s$
Theoretical speed limit
speed

Limitations of the Simulation

- Key points changed by relativity. $\vec{v}m \to \gamma\vec{v}m$, for $\gamma = 1/\sqrt{1 - v^2/c^2}$.

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- Cyclotron needs to be synchronized.
- Single particles is a limitations.
- Simulating the Electric and Magnetic field would be better.

Questions