Incomplete Information in Production Networks

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Abstract

I develop a model of production networks in which firms choose inputs under incomplete information about sectoral productivity and aggregate demand disturbances. As in the rational expectations framework of Lucas (1972), firms use input prices as endogenous signals to infer broader economic conditions. Theoretically, I show that the effect of sectoral productivity shocks on aggregate output depends on the interaction between the economy's input-output structure and firm-level uncertainty. Specifically, shocks to upstream sectors have a larger impact on aggregate output during periods of high productivity uncertainty, while shocks to downstream sectors have a larger effect on output during periods of high demand uncertainty. When calibrated to historical U.S. data, the model generates a state-dependent measure of sectoral importance that diverges significantly from traditional metrics, particularly in economic downturns. On aggregate, the interaction between uncertainty and the U.S. economy's network structure dampened the impact of productivity shocks on aggregate output by 50% during Covid (a period of high measured demand uncertainty) relative to the Dot-Com bubble (a period of high measured productivity uncertainty) or non-crisis periods.

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1 Introduction

Many firms operate in complex supply chains that are vulnerable to a variety of microeconomic disturbances (*e.g.*, bankruptcies, regulatory changes, transportation disruptions, natural disasters, shifts in household demand, *etc.*). Recent work argues that these disturbances can propagate through the economy's input-output linkages, affect the production decisions of other firms, and create large macroeconomic fluctuations. In this networked view of the production process, the macroeconomic impact of microeconomic shocks critically hinges on how firms' input choices respond to shocks.

However, firms often have to make input choices without full or current information about the large number of disturbances that are occurring throughout the economy.[1] This uncertainty at the micro level can distort how firms respond to shocks, affect a firm's trading partners, and have important consequences for how shocks aggregate to economy-wide disturbances. Although incomplete information is arguably an important feature of firm production, the prevailing assumption in the resurgent literature on production networks is that firms possess perfect knowledge of the large vector of shocks hitting the economy. This motivates a central question: How do microeconomic shocks aggregate to economy-wide disturbances when firms are imperfectly informed about macroeconomic conditions?

In this paper, I address this question by embedding incomplete information in an otherwise standard general equilibrium model of production networks. In the model, firms buy inputs while being imperfectly informed about the vector of sectoral productivity disturbances and shocks to aggregate household demand. Although firms do not observe these shocks directly, they can condition their input purchases on the price of each input. As in the classic literature on rational expectations (Lucas, 1972, 1975), firms infer whether a change in input prices reflects a real disturbance, due to sectoral productivity shocks, or a nominal one, due to changes in aggregate demand. This price-based inference drives how much firms adjust their input purchases in response to cost changes and ultimately shapes the impact of sectoral productivity disturbances on aggregate output.

Theoretically, I characterize how the impact of a sectoral productivity shock on aggregate output is shaped by the sector's position in the production network and firms' uncertainty about underlying shocks. My main theoretical finding is that sectoral productivity shocks in upstream sectors (sectors that supply to many other sectors but use few inputs) have large effects on aggregate output when uncertainty about productivity shocks is high. In contrast, productivity shocks in downstream sectors have a large impact on aggregate output when

[1] A long-standing literature in macroeconomics argues that firms operate under informational frictions. Moreover, a recent empirical literature provides direct evidence that firms have a limited understanding of macroeconomic conditions (*e.g.*, Gennaioli et al., 2016; Coibion et al., 2020; or Kumar et al., 2023).

demand uncertainty is high. Intuitively, relative to the complete information benchmark, firms must forecast the demand of their customers when choosing their input quantities, which in turn depends on the demand and uncertainty of their customers' customers, and so forth. Firms adjust their inputs less in response to input price changes when they perceive input prices to be driven by nominal disturbances, as they forecast that lower prices are associated with lower customer demand. The level of firms' input responsiveness (which hinges on firms' uncertainty) has crucial implications for how sectoral productivity shocks cause aggregate fluctuations and can give rise to measures of sectoral importance that deviate substantially from complete information economies.

In a quantitative exercise, I calibrate the model to the US input-output network and historical measures of sectoral productivity and aggregate demand uncertainty. I find that incomplete information substantially changes the impact of sectoral productivity shocks on aggregate output relative to the complete information benchmark, particularly during economic downturns. At an aggregate level, I find that the interaction between uncertainty and the US input-output structure dampened the impact of aggregate productivity shocks on output by 50% during Covid (a time of high measured demand uncertainty) relative to the Dot-Com Bubble (a time of high measured productivity uncertainty) or non-crisis periods. More generally, by generating state-dependent measures of sectoral importance, the results provide a framework for designing bailouts or industrial policies that account for the role of uncertainty in firm decision-making.

Model. To study the macroeconomic implications of incomplete information in production networks, I embed production networks in a general equilibrium monetary macroeconomic model. Firm-to-firm linkages follow standard microfoundations, as in Acemoglu et al. (2012). Firms make static production decisions, but face time-varying volatility with respect to the two kinds of fundamental shocks that drive the economy: sectoral productivity shocks and an aggregate demand shock.

At each point in time, past productivity and aggregate demand shocks are common knowledge, but firms choose inputs under uncertainty about the contemporaneous realizations of these shocks. When firms purchase inputs, they therefore face some level of uncertainty about their costs and revenues. However, motivated by the classical literature on rational expectations (Lucas, 1972), I assume that firms can observe information conveyed in local transactions with their suppliers. Thus, firms can choose inputs based on each input's price, but these input choices are not perfectly contingent on a firm's own demand, productivity, or the productivity of other sectors. As in the ubiquitous "cost-plus" contracts that often mediate transactions (Bajari and Tadelis, 2001), this allows input choices to be cost-contingent, but not measurable with respect to the entire vector of shocks that hits the economy.

Incomplete Information at the Firm Level. Under incomplete information, firms choose inputs to maximize their expected profits, conditional on the price of each input. First, I characterize firms' input responsiveness, defined as the elasticity of their input demand with respect to the price of that input. I show that a firm's input responsiveness depends on how informative input prices are about its demand. Intuitively, firms reduce their input purchases when prices increase. However, if higher input prices signal greater demand, firms are less responsive to price changes in anticipation of higher future sales.

This hedging behavior, driven by incomplete information, alters how firms respond to changes in their costs. Hence, as in the "island" model of Lucas (1972), firms try to infer whether price movements result from real disturbances (due to a productivity shock of their supplier), or nominal disturbances (due to changes in aggregate household demand). Formally, I show that input responsiveness increases with a firm's perceived covariance between its revenues and input prices, but decreases with the unconditional volatility of its input prices. Thus, these second moments regarding the joint stochastic process of firms' revenues and costs determine how firms respond to input price fluctuations.

Incomplete Information in General Equilibrium. Next, I show that in general equilibrium, firms' input responsiveness, the network structure of the economy, and the exogenous volatilities of sectoral productivity and aggregate demand shocks endogenously determine the stochastic properties of firms' revenues and prices. In particular, I show that the solution to the equilibrium is characterized through a system of functional equations, as one needs to relate the realization of sectoral prices and revenues to each possible state of the economy. To solve this system, and thus study how sectoral productivity shocks affect aggregate output, I employ a two-step approach.

First, I exogenously *fix* firms' input responsiveness. I show that the magnitude of how much firms adjust their input quantities to changes in input prices has large implications for how sectoral productivity disturbances aggregate over input-output linkages to affect output. To do this, I derive an index, which I call the economy's Adjusted-by-Uncertainty Domar Index (AUDI), that relates the aggregate impact of any sectoral productivity disturbance to the level of firms' input responsiveness across the production network of the economy. Concretely, I demonstrate that the AUDI weight of any sector is given by a measure of how central that sector is in the production network, where its connections to distant sectors are penalized by an aggregator of input responsiveness. In particular, I show that AUDI is closely linked to the extensively studied concept of "alpha centrality" in the literature on social networks (Katz, 1953; Bonacich and Lloyd, 2001), which measures a sector's centrality when its higher-order linkages (*i.e.*, the linkages of its customers, its customers' customers, and so on) are discounted.

I show that the magnitude of firms' input responsiveness has especially large effects on the aggregate impact of productivity shocks in upstream sectors, whose output travels through many linkages before reaching final consumption. This is because the aggregate impact of a productivity shock compounds across input-output linkages according to the magnitude of firms' input responsiveness. Thus, as firms become on average more responsive to changes in their input prices in terms of their input quantities, the aggregate impact of sectoral productivity shocks from upstream sectors increases. In contrast, as average input responsiveness declines, downstream sectors (which sell their products directly to households) become more influential in shaping the dynamics of aggregate output. In the limit in which firms' input quantities become entirely unresponsive to changes in input prices, productivity shocks in non-final good producing sectors do not affect household consumption.

I then study the determinants of *optimal* input responsiveness in general equilibrium as a function of the exogenous volatilities of productivity and aggregate demand shocks that drive the economy. I find that input responsiveness is increasing in the relative volatility of productivity shocks to aggregate demand shocks. Hence, upstream sectors become important sources of macroeconomic risk in times of relatively high productivity uncertainty, while downstream sectors become more influential in times of relatively high aggregate demand uncertainty. Intuitively, when the volatility of aggregate demand increases, firms perceive all input price changes to be nominal. Thus, they forecast greater nominal revenues in response to greater nominal input prices and reduce their input responsiveness. Hence, firms' latent uncertainty about underlying economic shocks is a central determinant of how microeconomic shocks aggregate to economy-wide fluctuations.

A key implication of my results is that the economic analyst needs to account for the entire structure of the economic network and its interaction with uncertainty to understand the aggregate consequences of microeconomic shocks. This distinguishes my theory from frictionless economies, where the impact of a sectoral shock on output is directly inferred from that sector's Domar weight, defined as its revenue-share of GDP (Hulten, 1978). Moreover, incomplete information qualitatively alters productivity shocks' aggregate effect, not just in magnitude but also in sign. For example, if firms are negatively responsive to input price changes (so that input quantities fall when prices fall), then temporary sectoral productivity shocks can have a negative effect on aggregate output. Thus, even under fixed production network structures, measures of sectoral importance are not immutable objects, but depend on the underlying nature of uncertainty in the economy.

Quantitative Analysis. In the final part of the paper, I quantify the economic relevance of my results by calibrating my model to the US input-output network structure and historical measures of sectoral productivity and aggregate demand uncertainty. To do this, I estimate a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model on the time series for sectoral productivity and aggregate demand disturbances in the US economy. This approach provides a parsimonious way to proxy for the time-variation in the different sources of uncertainty faced by US firms. I then use my model-implied estimates relating uncertainty to optimal input responsiveness to construct time-varying AUDI weights for each sector in the US economy.

This exercise reveals substantial state-dependence in how microeconomic disturbances aggregate over the US input-output network. The AUDI weights of sectors increased during the Dot-Com Bubble (a time of relatively high measured productivity uncertainty), but decreased during Covid (a time of relatively high measured aggregate demand uncertainty). Consequently, according to the model, shocks to many downstream sectors (*e.g.*, food or hospital services) became more influential in shaping the dynamics of output and prices during Covid. In contrast, productivity shocks in upstream sectors with many interconnections (*e.g.*, credit intermediation or administrative and support services) became relatively less important during this period.

In general, I find that incomplete information predicts substantially different measures of sectoral importance than what is implied by the benchmark assumption of complete information. In particular, Domar weights are particularly poor at approximating the aggregate impact of sectoral productivity shocks exactly when these shocks are most volatile, which coincides with US economic downturns. For example, a 1% productivity shock in the credit intermediation sector — which is a large sector and would increase real GDP by 0.16% under complete information — results in a real GDP contraction during Covid. Thus, the results caution against using Domar weights as a sufficient statistic when designing industrial policy or bailouts in order to promote aggregate output in times of crises.

Finally, I consider how the interaction between the network structure of the economy and time-varying uncertainty affects the impact of *aggregate* productivity shocks on output. I find that the impact of aggregate productivity shocks on output was 50% lower in the aftermath of Covid relative to non-crisis periods, as this was a period characterized by especially high demand uncertainty. This result is driven by shifts in firms' input responsiveness due to the changing nature of uncertainty regarding supply and demand shocks. Moreover, I find that this state-dependence is important to account for the overall volatility of real GDP growth in my sample period. For example, counterfactually assuming that firms operate under complete information would overshoot the volatility of GDP growth by 54% relative to the data. Overall, the results underscore the importance of considering the interaction between uncertainty and input-output linkages in understanding aggregate fluctuations.

Literature. This paper contributes to the growing literature on production networks.[2] A number of papers show how production networks affect macroeconomic fluctuations in the presence of distortions (Jones, 2011; Liu, 2019; Baqaee and Farhi, 2020; Bigio and La'o, 2020); large shocks (Baqaee and Farhi, 2019; Dew-Becker, 2023); dynamics (Atalay, 2017; Dew-Becker, 2023; Liu and Tsyvinski, 2024); nominal rigidities (Basu, 1994; Nakamura and Steinsson, 2010; Pasten et al., 2017, 2020; Ghassibe, 2021b; Baqaee and Farhi, 2022; La'O and Tahbaz-Salehi, 2022; Rubbo, 2023); or entry/exit and link formation (Baqaee, 2018; Kevin, 2018; Acemoglu and Azar, 2020; Taschereau-Dumouchel, 2020; Ghassibe, 2021a; Dhyne et al., 2023; Acemoglu and Tahbaz-Salehi, 2024). These papers assume firms choose inputs under complete information about shocks. In contrast, I develop a model in which firms choose inputs under incomplete information, and thus cannot perfectly condition their input choices on the large vector of shocks that may be hitting the economy. The aggregate impact of sectoral productivity shocks in this setting is shaped by the interaction of macroeconomic uncertainty with the economy's production network structure.[3]

The closest analyses to mine are performed by Bui et al. (2022) and Pellet and Tahbaz-Salehi (2023), who embed quantity rigidities in a model of input-output networks. These works assume that firms make input commitments based on the observation of exogenous public and private signals. The key departure from these models is that I allow firms to condition their input choices on input prices, and thus on the local terms of trade with their suppliers. By doing so, my model is robust to the "re-contracting critique" of Grossman (1989), in which firms would re-contract with their suppliers upon observation of input prices. This modeling departure is important, as the key object that determines the aggregate impact of sectoral shocks in my analysis is how firm's respond to fluctuations in input prices.

For this reason, my work also borrows from the classical literature on rational expectations, as it features the shared methodological premise that agents act on what they learn from endogenous market outcomes (Lucas, 1972, 1975; Grossman and Stiglitz, 1980). By allowing firms to learn from the interactions with their suppliers in input markets, the inference problem that links firms' uncertainty to their input choice arises without reference to the physically separated "islands" of Lucas (1972). Instead, firms use input prices to make optimal forecasts about their potential revenues. Related works by Hellwig and Venkateswaran (2009, 2014) and Flynn et al. (2023) study how firms use market information to set optimal prices under monopolistic competition. In contrast, this paper emphasizes the role of firm-to-firm interactions in shaping how microeconomic shocks result in aggregate fluctuations.

[2] See Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) for reviews.

[3] Work by Kopytov et al. (2022) studies how uncertainty shapes the structure of the input-output network through technology choice. In contrast, I study how uncertainty shapes firms' input choices taking their production technology and the network structure as given.

Outline. The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents the theoretical results. Section 4 presents the quantitative findings when the model is calibrated to the US economy. Section 5 concludes.

2 Incomplete Information in Production Networks

In this section, I embed production networks in a general equilibrium monetary macroeconomic model. The model follows standard microfoundations for modeling input-output linkages (*e.g.*, Jones, 2011; Acemoglu et al., 2012), and only deviates from the literature in assuming that inputs are chosen under incomplete information about shock realizations. Time is discrete and infinite and indexed by $t \in \mathbb{N}$. The economy consists of a representative household and $N \in \mathbb{N}^+$ sectors with input-output linkages, indexed by $n \in \mathcal{N} = \{1, \ldots, N\}$. In each sector $n \in \mathcal{N}$, a competitive representative firm produces a good that can be either consumed or used as an input by other sectors.

2.1 Households

The representative household has standard (Golosov and Lucas, 2007) expected discounted utility preferences with a discount factor $\beta \in (0, 1)$ and per-period utility defined over a consumption aggregate $\mathcal{C}_t$, real money balances $M_t/\mathcal{P}_t$, and total labor supplied L_t:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\ln \mathcal{C}_t + \ln \frac{M_t}{\mathcal{P}_t} - L_t \right) \right] \tag{1}$$

The consumption aggregator $\mathcal{C}_t$ is defined by:

$$\mathcal{C}_t = \prod_{n \in \mathcal{N}} C_{nt}^{\gamma_n} \tag{2}$$

where C_{nt} is total consumption of sector n and the consumption shares $\gamma_n \geq 0$ are positive constants that sum to one: $\sum_{n \in \mathcal{N}} \gamma_n = 1$. When this creates no confusion, I use the terms *real GDP* and *aggregate consumption* interchangeably. To ease notation, I also define the $|\mathcal{N}|$-sized vectors of consumption shares $\gamma = [\gamma_n]$.

Households can save in either money or risk-free one-period bonds B_t (in zero net supply) that pay an interest rate of $(1 + i_t)$. The household owns the firms in the economy, which earn total profits Π_t.[4] Thus, the household faces the following budget constraint:

$$M_t + B_t + \mathcal{P}_t \mathcal{C}_t = M_{t-1} + (1 + i_{t-1})B_{t-1} + w_t L_t + \Pi_t \tag{3}$$

where $\mathcal{P}_t$ is the dual price index to $\mathcal{C}_t$ and w_t is the nominal wage. The aggregate money supply follows an exogenous random walk with drift μ_M and time-dependent volatility σ_t^M:

$$\log M_t = \log M_{t-1} + \mu_M + \delta \sigma_t^M \varepsilon_t^M \tag{4}$$

where the money innovation is an IID random variable that follows $\varepsilon_t^M \sim \mathrm{N}(0,1)$. The constant $\delta > 0$ is a strictly positive scalar that parameterizes the total amount of *uncertainty* in the economy, which will be useful in obtaining some linearized results later on. Furthermore, so that interest rates remain strictly positive, I assume that $\frac{1}{2}(\delta\sigma_t^M)^2 \leq \mu_M$ for all $t \in \mathbb{N}$.

I interpret an increase in the money supply as an *aggregate demand shock*. The major benefit of this specification of household utility is that we can study these demand shocks tractably. As I will show later, fluctuations in aggregate demand will induce fluctuations in prices. These price movements will force firms to hedge on whether movements in their input costs are driven by nominal disturbances or real disturbances (due to changes in the productivity of other sectors).

Finally, I assume that wages are determined according to the following equation:

$$w_t = (w_{t-1})^{\chi}(w_t^*)^{1-\chi} \tag{5}$$

where $\chi \in [0,1)$ parameterizes aggregate wage rigidities and w_t^* denotes the frictionless nominal wage rate, *i.e.* the wage rate that would prevail when $\chi = 0$ (to be determined in equilibrium). Households therefore supply sufficient labor to meet firms' labor demand. This specification of the real wage rate allows the model to parsimoniously capture the cyclicality of nominal wages, which is important determining the effect of an aggregate demand shock on real GDP. Note also that the environment with frictionless wages is nested under $\chi = 0$.

[4]Firms earn zero profits in expectation, but can have non-zero profits with positive probability (even though they are competitive) due to incomplete information, the structure of which will be explained shortly.

2.2 Intermediate Good Producers

Sector $n \in \mathcal{N}$ produces using labor and goods from other sector. Each sector produces output Q_{nt} using Cobb-Douglas technologies with constant returns to scale:

$$Q_{nt} = c_n z_{nt} \left(L_{nt}\right)^{\alpha_{nl}} \prod_{n' \in \mathcal{N}} X_{nn't}^{\alpha_{nn'}} \tag{6}$$

where c_n is a normalizing constant,[5] L_{nt} denotes the amount of labor purchased by firms in sector n at time t, and $X_{nn't}$ denotes the amount of inputs purchased by sector n from sector n' at time t. I assume that the Cobb-Douglas coefficients satisfy $\alpha_{nl} > 0$ and $\alpha_{nn'} \geq 0$. The scalar z_{nt} is a sector-specific technology shifter that follows an AR(1) process with time-varying volatility $\delta\sigma_{nt}^{z}$ given by:

$$\log z_{nt} = \rho_n^z \log z_{nt-1} + \delta\sigma_{nt}^{z}\varepsilon_{nt}^{z} \tag{7}$$

where $\varepsilon_{nt} \sim \mathrm{N}(0,1)$ and ρ_n^A is a sector-specific autocorrelation coefficient for the productivity process. I allow the ε_{nt}^A to be potentially correlated across sectors. Profits in sector $n \in \mathcal{N}$ are therefore given by:

$$\Pi_{nt} = P_{nt}Q_{nt} - w_t L_{nt} - \sum_{n' \in \mathcal{N}} P_{n't}X_{nn't} \tag{8}$$

where P_{nt} denotes the equilibrium price good n at time t. For ease of notation, I let $P_t = [P_{nt}]$ denote the vector of sectoral prices. I also define the economy's *input-output matrix* as $\mathbf{A} = [\alpha_{nn'}]$, which captures the interconnections between different sectors.

2.3 Production under Informational Frictions

At the beginning of time t, firms observe past shocks ($\{z_{nt-1}\}_{n\in\mathcal{N}}$ and M_{t-1}) and choose how much of each input to purchase. However, they face incomplete information about the contemporaneous realizations of these supply and demand disturbances. Consequently, firms face some level of uncertainty about their profits when making their input choices. Firms therefore choose inputs $X_{nn't}$ to maximize their expected real, risk-adjusted profits:

$$X_{nn't}(\mathcal{I}_{nn't}) = \arg\max_{X_{nn't}} \mathbb{E}\left[\frac{1}{\mathcal{P}_t\mathcal{C}_t}\Pi_{nt}\middle|\mathcal{I}_{nn't}\right] \quad \text{for all} \quad n, n' \in \mathcal{N} \tag{9}$$

[5] c_n is defined as: $c_n = \left(\alpha_{nl}^{-\alpha_{nl}} \prod_{n' \in \mathcal{N}} \alpha_{nn'}^{-\alpha_{nn'}}\right)$

where $\mathcal{I}_{nn't}$ is a *buyer-input* specific information set (to be described shortly), and $(\mathcal{P}_t\mathcal{C}_t)^{-1}$ is the households' stochastic discount factor.

This formulation emphasizes that firms can only condition their input choices on the relevant information set. For example, under complete information, $\mathcal{I}_{nn't}$ contains the set $\{\{z_{nt}\}_{n\in\mathcal{N}}, M_t\}$, so that firms can condition their input choices on the full vector of supply and demand disturbances in the economy. In practice, however, firms often have a limited ability to make input choices that are contingent on this rich set of variables, either due to inattention (Mankiw and Reis, 2002), organizational constraints (Simon, 2013), or contracting frictions (Taylor, 1980). This more general formulation formalizes the notion that firms might only be able to condition their input choices on a restricted set of variables.

Motivated by the classical literature on rational expectations, I assume that a key variable that enters firms' buyer-input specific information set is the price of the input that they purchase. As in the "island model" of Lucas (1972), this captures the fact that firms can learn from local transactions with their suppliers. Hence, in addition to past aggregate demand and productivity shocks (which are common knowledge), firms can condition their demand for each input on its price. Formally, I maintain the following assumption.

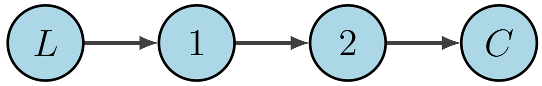
Assumption 1 (Locality). $\mathcal{I}_{nn't} = \{P_{n't}, \{z_{nt-1}\}_{n\in\mathcal{N}}, M_{t-1}\}$ *for all* $n, n' \in \mathcal{N}$

The interpretation of this assumption is as follows: firms can make price-contingent input plans based on that input's price, but cannot condition their choice perfectly based on the realization of their own demand, productivity, or the productivity of other sectors (or other input prices). Intuitively, input choices depend on the terms of trade between a buyer and their supplier, but not on the terms of trade of transactions happening elsewhere in the economy. Nevertheless, as input prices will fluctuate in response to shocks, some information on the current state of the economy is transmitted to agents through these input prices. Although this assumption is theoretically motivated, I later show that the measurability restriction that this informational structure implements is an empirically relevant feature of many firm-to-firm transactions.

Finally, in order to preserve market clearing, I assume that firms' labor inputs can adjust to realized demand conditions. Hence, labor is to be interpreted as labor utilization, hours worked, or any other input that can adjust to realized demand. However, the exact interpretation of this input is not important. Instead, what is important is that there exists a single input that can adjust to realized demand to ensure that markets clear. The firm therefore adjusts labor frictionlessly to maximize its profits, taking the price of its inputs, its previous input purchases, and demand as given:

$$\max_{L_{nt}} \Pi_{nt} \tag{10}$$

Figure 1: A Simple Supply Chain Economy



Note: Firms in Sector 1 purchase labor and sell to firms in Sector 2. Firms in Sector 2 buy inputs from Sector 1 under incomplete information about their productivity and household demand.

A Simple Supply Chain Example. In order to further understand what Assumption 1 represents, the following example may be useful. Consider a simple supply chain economy, depicted in Figure 1. Sector 1 hires labor to create an input (machine parts, raw materials, energy, information services, etc.). This input is then sold to Sector 2, which creates a final output for household consumption (finished products or services, data analysis, reports, etc.). When Sector 2 purchases from Sector 1, it does so under some uncertainty regarding what final household demand for its product will be. After Sector 2 faces the prices charged by its supplier and purchases inputs on its optimal demand schedule (Equation 9), it sells the final product to households, and realizes it profits.

A Contracting Interpretation. The key friction in this model is that firms cannot change the quantity of their input purchases, after having observed the realization of their demand or productivity. In this sense, it is equivalent to think of the informational friction in this model as an outcome generated by incomplete contracting. Suppose, for example, that all transactions in the economy were mediated through complete contingent contracts which specified the quantity produced for every possible shock realization z_{nt} and M_t. The allocations that would emerge in such a setting would be equivalent to those of a complete information economy, as in a standard Arrow-Debreu competitive equilibrium. In contrast, in this model, the terms of trade of the contract only depend on the price charged by the supplier — formally, the demand function given by Equation 9 is measurable only with respect to the per-unit price of the input, but not with respect to every shock in the economy or the terms of trade of other transactions in the economy.

Of course, complete contingent contracts that implement the full information allocation are not observed in practice due to their complexity and challenges of enforcement (Battigalli and Maggi, 2002). In the example of Figure 1, why would firms in Sector 1 conclude a contract for delivery that is contingent on final household demand for Sector 2's product, an outcome that only Sector 2 can observe? In this contracting interpretation, posted prices are observable by *both* seller and customer, and thus allocations depend on the price charged.[6]

[6]This notion is further explored in work by Radner (1982), who studies equilibrium under uncertainty. He writes: "In practice, since traders make contracts with other traders, and not with an abstract "market", delivery will be contingent upon information that is common to the two traders in question [...]"

In Appendix B.1, I show that the allocations generated by the model are equivalent to those generated by simple contracts of a *cost-contingent* nature which are negotiated bilaterally by suppliers and their customers ex-ante. These "cost-plus" contracts are commonly observed in practice (Hiller and Tollison, 1978; Crocker and Reynolds, 1993; Banerjee and Duflo, 2000; Bajari and Tadelis, 2001) and implement the same measurability constraints as the demand function of Equation 9.

Learning From Additional Prices. Under Assumption 1, firms condition their input demand on each input's price, but not on other input prices. This is reasonable given that inputs are often negotiated bilaterally between buyer and seller (as argued in the contracting interpretation above), or given that firm managers may only be able to observe local terms of trade when purchasing particular inputs. Nevertheless, I later show that the key qualitative predictions of the theory are preserved when one allows for learning from additional prices.

2.4 Rational Expectations Equilibrium

The equilibrium concept is that of a standard rational expectations equilibrium. Firms implement input plans according to the demand function given by Equation 9; total labor demanded for each industry satisfies Equation 10; households make their consumption and savings decisions; and markets clear. Formally, I define an equilibrium as follows.

Definition 1 (REE)**.** *An equilibrium is a collection of variables*

$$\{\{\{X_{nn't}\}_{n'\in\mathcal{N}}\}, P_{nt}, C_{nt}, L_{nt}, Q_{nt}, z_{nt}\}_{n\in\mathcal{N}}, \mathcal{C}_t, \mathcal{P}_t, L_t, w_t, M_t, B_t\}_{t\in\mathbb{N}}, \quad \textit{such that:}$$

1. (Input Optimality) *Firms choose input plans to maximize their expected profits given their available information according to Equation 9.*
2. (Labor Optimality) *Firms choose labor to maximize their profits according to Equation 10, taking the price of inputs, their productivity, and their own demand as given.*
3. (Household Optimality) *The household maximizes their expected discounted utility 1 subject to their budget constraint 3, taking prices and the nominal rate as given.*
4. (Rational Expectations) *Expectations are consistent variables' laws of motion.*
5. (Market Clearing) *The markets for goods and labor clear:*

$$Q_{nt} = C_{nt} + \sum_{n'\in\mathcal{N}} X_{n'nt}, \quad L_t = \sum_{n'\in\mathcal{N}} L_{n't} \tag{11}$$

   and the markets for money balances and bonds clear.

3 Theory: Sectoral Disturbances and Fluctuations

In this section, I describe how sectoral productivity and aggregate nominal demand disturbances shape aggregate fluctuations under incomplete information. I proceed in three steps. First, I show that the joint stochastic process of firms' revenues and prices determines input responsiveness, or the elasticity of input purchases to their price. Second, I derive an index of sectoral importance that links a sector's position in the input-output network to the level of input responsiveness in the economy. I show that this index is closely linked to the concept of a network's "alpha centrality", which measures the centrality of a sector when connections to distant sectors are penalized by an aggregator of firms' input responsiveness. Third, I describe how this index depends on firms' relative uncertainties about sectoral productivity to aggregate demand disturbances. Overall, incomplete information implies that the impact of productivity and aggregate demand shocks on real GDP is state-dependent and determined by the interaction of uncertainty with production network structure.

3.1 The Complete Information Benchmark

Before analyzing how incomplete information affects macroeconomic fluctuations, it is useful to analyze equilibrium allocations under the prevalent benchmark in the literature where firms make production decisions under complete information about shocks. In the complete information economy, firms face no uncertainty. Hence, they choose inputs so that costs are a constant fraction of realized revenues R_{nt}:

$$X_{nn't} = \alpha_{nn'} \frac{R_{nt}}{P_{n't}} \quad \text{and} \quad L_{nt} = \alpha_{nl} \frac{R_{nt}}{w_t} \tag{12}$$

where $R_{nt} = P_{nt} \times Q_{nt}$. These demand functions are in terms of endogenous variables. However, we may use the following Lemma to express household wages and nominal expenditures in terms of exogenous variables at time t, (which holds under any information structure).

Lemma 1. *Nominal expenditures and flexible wages satisfy*

$$\mathcal{P}_t \mathcal{C}_t = \iota_t M_t \quad \textit{and} \quad w_t^* = \iota_t M_t \tag{13}$$

where $\iota_t := \frac{i_t}{1+i_t} > 0$ *follows a deterministic sequence.*

Proof. See Appendix A.1 □

Thus, nominal wages and expenditures are simply proportional to the total stock of money balances in the economy. The household's problem therefore collapses to a static one given knowledge of the demand shocks M_t. Intuitively, the presence of money in the utility function gives rise to an additional intertemporal trade-off between savings and consumption, thereby allowing us to express nominal expenditures in terms of the total money supply. I now present a first result that links aggregate output to shocks under complete information.

Proposition 1. *In the complete information economy, real GDP is given by:*

$$\log \mathcal{C}_t = cons + \chi \log M_t + \sum_{n \in \mathcal{N}} \lambda_{nt} \log z_{nt} \tag{14}$$

where the constant is independent of time t shocks and $\lambda_t = [\lambda_{nt}]$ is the vector of Domar weights, defined as:

$$\lambda_t = \frac{R_t}{\mathcal{P}_t \mathcal{C}_t} = \left[\gamma'(\mathbf{I} - \mathbf{A})^{-1}\right]' \tag{15}$$

Proof. See Appendix A.2. □

Proposition 1 describes how real GDP depends on aggregate demand and sectoral productivity shocks under complete information. This reveals two observations. First, the effect of an aggregate demand shock on output is independent of the network structure of the economy and is given by the overall level of nominal wage rigidities. In particular, when $\chi = 0$, we recover monetary neutrality. Second, the effect of a sectoral productivity shock on output is given by that sector's *Domar weight*, defined as the sector's revenue share to nominal GDP. In equilibrium, this revenue share is the product of household expenditure shares γ with the economy's inverse *Leontief matrix* $\mathbf{L}$, defined as:

$$\mathbf{L}^{-1} = (\mathbf{I} - \mathbf{A})^{-1} = \sum_{k=0}^{\infty} \mathbf{A}^k \tag{16}$$

Domar weights thus combine household preferences (through γ) with the economy's input-output structure to give rise to a measure of sectoral importance for the complete information economy. Intuitively, an increase in sectoral productivity reduces the corresponding sector's price. That sector's customers respond by purchasing more inputs, and this reduces their price, as well as their customers' price, and so on. These higher-order linkages are captured through the power sum of the input-output matrix $\mathbf{A}$. The total effect of a productivity shock on output thus amounts to aggregating a sector's direct and indirect linkages to all other sectors, and weighting these by household expenditure shares.

Proposition 1 emphasizes that, under complete information, knowing the revenue shares of each sector are sufficient to characterize the dynamics of output. In other words, two identical economies that feature the same Domar weights will also feature identical dynamics for real GDP. This is a benchmark result in the literature on production networks: the first-order impact of sectoral productivity shocks on real GDP in frictionless economies is given by that sector's Domar weight — an observation commonly referred to as Hulten's Theorem (Hulten, 1978).[7] Second, by construction, the stochastic properties of either revenues or prices do not affect firms' decisions and thus the impact of these shocks on output. In the next section, I show that these observations are no longer true when firms face incomplete information about shocks and must choose inputs under uncertainty.

3.2 Production under Incomplete Information

I now describe firms' optimal input choices under incomplete information.

Proposition 2. *Firms' optimal choice of inputs and labor satisfies:*

$$X_{nn't} = \alpha_{nn'} \frac{\mathbb{E}_t\left[(M_t)^{-1} R_{nt} | P_{n't}\right]}{\mathbb{E}_t\left[(M_t)^{-1} P_{n't} | P_{n't}\right]} \quad \textit{and} \quad L_{nt} = \alpha_{nl} \frac{R_{nt}}{w_t} \tag{17}$$

Proof. See Appendix A.3. □

Thus, relative to complete information, firms choose inputs to maximize their real, risk-adjusted profits, conditional on the price of that input (as $\iota_t M_t$ is the households' nominal stochastic discount factor). Recall that under complete information, inputs purchased from other firms depend on both input costs and *realized* revenues. In the incomplete information economy, the quantity of a firm's input purchases depend on a firm's *expectation* about its revenues. If a firm forecasts higher revenues, it increases its input purchases, all else constant. Moreover, a firm's beliefs about its potential revenues might depend on its input costs, to the extent that revenues and input prices covary.

Input prices therefore play a dual role in formulating firms' optimal demand schedule under incomplete information. First, an increase in input prices directly increases a firms' costs and reduces the optimal input quantity purchased. Second, an increase in input prices might signal to firm that its demand (and thus its revenues) will change. Both of these forces determine how responsive firms' input purchases are to input price changes. As I will show shortly, this change in firm-level responsiveness due to incomplete information shapes how productivity and demand disturbances affect aggregate output.

[7] Even in economies that feature distortions due to wedges, as in Baqaee and Farhi (2020), Hulten's Theorem applies if the production function is Cobb-Douglas. Under incomplete information, the aggregate impact of sectoral shocks is different than sectoral Domar weights even in the presence of Cobb-Douglas technology.

Prices and Revenues in General Equilibrium. Proposition 2 demonstrated that firms' optimal input choices depend on the joint stochastic properties of their revenues and prices. I now characterize how these endogenous objects are determined in general equilibrium.

Proposition 3. *Equilibrium prices $\{P_{nt}\}_{n\in\mathcal{N}}$ and revenues $\{R_{nt}\}_{n\in\mathcal{N}}$ satisfy the following system of equations:*

$$R_{nt} = \tilde{c}_{nt} z_{nt} P_{nt} \left(\frac{R_{nt}}{M_t^{1-\chi}}\right)^{\alpha_{nl}} \prod_{n'\in\mathcal{N}} \left(\frac{\mathbb{E}_t\left[M_t^{-1} R_{nt} \middle| P_{n't}\right]}{\mathbb{E}_t\left[M_t^{-1} P_{n't} \middle| P_{n't}\right]}\right)^{\alpha_{nn'}} \tag{18}$$

$$R_{nt} = \gamma_n \iota_t M_t + \sum_{n'\in\mathcal{N}} \alpha_{n'n} P_{nt} \frac{\mathbb{E}_t\left[M_t^{-1} R_{n't} \middle| P_{nt}\right]}{\mathbb{E}_t\left[M_t^{-1} P_{nt} \middle| P_{nt}\right]} \tag{19}$$

for all $n \in \mathcal{N}$, where $\tilde{c}_{nt} := \left(\iota_t^{1-\chi} w_{t-1}^{\chi}\right)^{\alpha_{nl}}$ is independent of time t shocks.

Proof. See Appendix A.4 □

Proposition 3 derives a system of *functional equations* that relates realized prices and revenues to exogenous shocks, firms' expectations, and the input-output structure of the economy. To understand this Proposition, note that Equation 18 is derived by substituting firms' optimal input choices into their production function. Hence, this equation states that a firm's revenues must be equal to its output, multiplied by its price. Equation 19 is derived by substituting firms' input choices into market clearing — a firm's revenues must be consistent with its sales to households and sales to other downstream firms.

These equations are illustrative in understanding how a sectoral productivity shock can aggregate to create economy-wide fluctuations. Consider a productivity shock in sector $n \in \mathcal{N}$. This directly increases the output of that sector through Equation 18. The new equilibrium price must be consistent with market clearing (Equation 19), which depends partly on how that sector's customers respond to changes in input prices in terms of their input purchases. Of course, the resulting equilibrium price shapes sector n's revenues and thus how firms in that sector optimally choose inputs to begin with.

This interdependence reveals *feedback loops*. Concretely, fluctuations in prices and revenues depend on firms' optimal input choices. However, these input choices primitively depend on firms' expectations about the co-movement between revenues and prices. As in Lucas (1972) or Grossman and Stiglitz (1980), optimal choices partly depend on the stochastic properties of endogenous objects. Thus, relative to complete information, the key challenge in this economy is to solve for how firms' expectations depend on the network structure of the economy and the exogenous volatilities of underlying disturbances.

3.3 Example: A Simple Supply Chain Economy

In order to illustrate how firms' production decisions under incomplete information affects output, consider the simple supply chain economy of Figure 1. In this example, Sector 1 is an upstream sector that purchases labor and sells an intermediate product to Sector 2. Firms in Sector 2 then sell a final product to households. Thus, this economy can be described through two simple equations. First, from Equation 18, we can write Sector 1's price as:

$$\log P_{1t} = -\log \tilde{c}_{1t} - \log z_{1t} + (1-\chi)\log M_t \tag{20}$$

As Sector 1's only input is labor (and this adjusts frictionlessly to demand by assumption), the price of Sector 1 is simply equal to the marginal cost of production. Second, as Sector 2 sells directly to households, real GDP in this economy is simply equal to Sector 2's input purchases. Using Sector 2's optimal input demand, we obtain:

$$\underbrace{\log \mathcal{C}_t}_{\text{real GDP}} = \underbrace{\log \mathbb{E}_t[M_t^{-1}(\iota_t M_t)|P_{1t}] - \log \mathbb{E}_t[M_t^{-1} P_{1t}|P_{1t}]}_{\text{input purchases conditional on price}} \tag{21}$$

where we have used the fact that Sector 2's revenues are simply equal to total household expenditures ($R_{2t} = \iota_t M_t$). Hence, the effect of shocks on consumption depends on how firms in Sector 2 respond to input price movements. In particular, this effect depends on how well firms are able to forecast changes in their sales through changes in their costs. The following proposition derives firms' optimal input demand function by solving for the conditional expectation in Equation 21.

Proposition 4 (Macroeconomic Outcomes in the Simple Supply Chain)**.** *Real GDP in the simple supply chain economy is given by:*

$$\log \mathcal{C}_t = cons + \frac{\left(\sigma_{1t}^z/\sigma_t^M\right)^2 - \chi(1-\chi)}{\left(\sigma_{1t}^z/\sigma_t^M\right)^2 + (1-\chi)^2}\left(\log z_{1t} - \chi \log M_t\right) \tag{22}$$

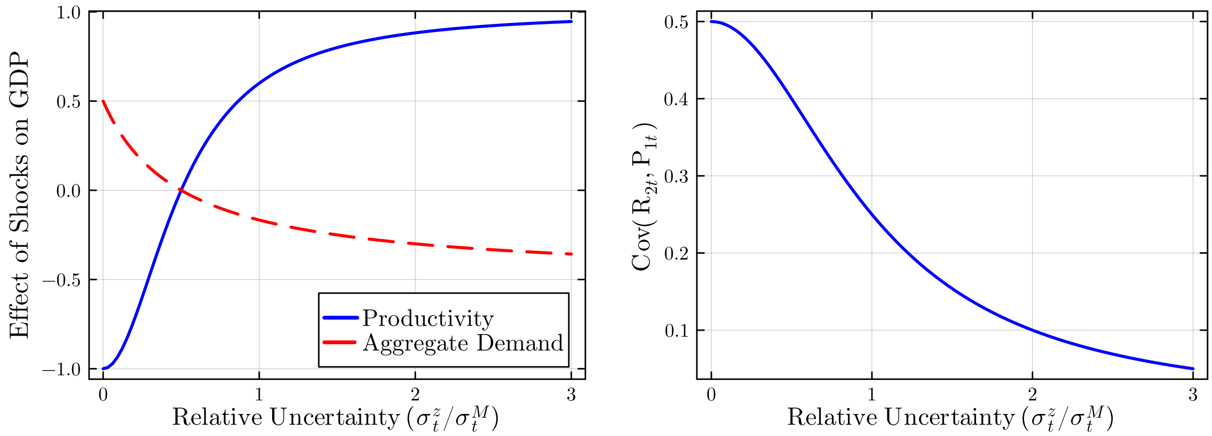
where the constant is independent of time t shocks.

Proof. See Appendix A.5. □

Proposition 4 shows that the impact of Sector 1's productivity shock or an aggregate demand shock on output depends on firms' *relative* uncertainty about these shocks (σ_{1t}^z/σ_t^M). Figure 2a illustrates this result graphically. When productivity uncertainty rises, the impact of productivity shocks on output also increases. In contrast, as aggregate demand uncertainty rises, the impact of a demand shock on output decreases.

Figure 2: How Uncertainty Shapes the Dynamics of GDP, Revenues, and Prices

(a) Effect of Shocks on GDP **(b)** Covariance of Revenues and Input Price



Note: Left panel: the effect of a sectoral productivity shock and an aggregate demand shock on real GDP at time t as a function of relative uncertainty about productivity and demand: $\frac{\mathrm{d}\log \mathrm{GDP}_t}{\mathrm{d}\log x}$, for $x \in \{z_t, M_t\}$. Right panel: the covariance between Sector 2's revenues and input price as a function of relative uncertainty, keeping total uncertainty constant, $(\sigma_t^z)^2 + (\sigma_t^M)^2 = 1$. See Figure 1 for a graphical depiction of the supply chain economy. Parameterization: $\chi = 0.5$.

These effects are tightly linked to the equilibrium stochastic properties of firms' revenues and input costs. Figure 2b plots the equilibrium covariance between Sector 2's revenues and its input price as a function of relative uncertainty. As relative uncertainty about aggregate demand increases, firms in Sector 2 perceive all fluctuations in their input price to be generated by demand disturbances. Hence, these firms forecast that their revenues will increase when they observe higher input prices. As a consequence, firms hedge against higher input costs by responding less in terms of their input purchases. Productivity shocks in Sector 1 therefore have a smaller effect on aggregate consumption when aggregate demand uncertainty increases.

Conversely, as productivity uncertainty increases, input prices become less correlated with household expenditures. Firms in Sector 2 therefore become more responsive to input price changes in terms of their input purchases. These firms respond to the inflationary pressures of aggregate demand shocks by reducing their inputs more aggressively, and this attenuates (and can even flip the sign of) the impact of aggregate demand shocks on output.

Takeaways from this Simple Example. This exercise emphasizes three key findings. First, the aggregate impact of sectoral productivity shocks is qualitatively different from the complete information benchmark and the foundational theorem of Hulten (1978) in which the effect of setoral shocks on GDP is equal to the corresponding sector's Domar weight.

Indeed, in this example, both sectors have a Domar weight of unity as their revenues are simply equal to final household expenditures. Yet, the impact of a productivity shock in Sector 1 can substantially differ from unity, and can even flip sign, depending on uncertainty.[8]

Second, the effect of productivity and aggregate demand shocks on output is shaped by firms' *relative* uncertainties about these disturbances. In the same spirit as Lucas (1972), firms attempt to "disentangle" real movements (due to productivity shocks) from nominal movements (due to aggregate demand shocks) in their input prices when making production decisions. This uncertainty dictates how responsive input choices are to prices. Moreover, as these effects are independent of *total* uncertainty, even small levels of uncertainty (in which case $\delta \to 0$) can have a large effect on the aggregate impact of shocks.

Third, the effect of sectoral disturbances on aggregate output depends on the *interaction* between the production network structure of the economy and uncertainty. Indeed, it is straightforward to see that a sectoral productivity disturbance in Sector 2 would pass-through to output one-to-one, independent of underlying uncertainty. The macroeconomic impact of a sectoral productivity disturbance thus crucially depends on that sector's position in the economy's input-output network.

In more general networks, firms must make inferences about the demand of their downstream customers, which may depend on the demand (and thus the uncertainty) of their downstream customers, and so on. This gives rise to a rich interaction between the network structure of the economy and uncertainty in shaping aggregate output. I next turn to analyze how uncertainty shapes sectoral importance in general input-output structures.

3.4 Sectoral Importance under Incomplete Information

In order to characterize how sectoral productivity shocks affect aggregate output, I employ a two-step approach that allows me to decouple the feedback between uncertainty and input responsiveness embodied in Proposition 3. First, I solve for the dynamics of real GDP by *fixing* firms' demand schedules for inputs. I then solve for firms' *optimal* demand schedules by making sure these are consistent with the model-implied equilibrium laws of motion for prices and revenues. To keep the analysis tractable, I linearize the market clearing equation 19 around the complete information economy by taking $\delta \to 0$ (where recall that δ parameterizes the overall level of uncertainty about aggregate shocks). Note that there is no need to linearize Equation 18, as firms' production functions are already log-linear. Hence,

[8] The notion that temporary TFP shocks can be contractionary is not merely a theoretical curiosity, but has empirical relevance. For example, Basu et al. (2006) find that that technology shocks lower input use and output on impact using an "augmented-growth-accounting" approach, while Angeletos and La'O (2010) document this phenomenon in an RBC model with dispersed information.

the analysis preserves firms' global best response functions for their production choices.[9]

Output Dynamics under Fixed Responsiveness. I begin by defining two matrices that are central to the analysis:

$$\mathbf{\Omega}_t := [\omega_{nn't}] = \left[-\frac{\mathrm{d}\log X_{nn't}}{\mathrm{d}\log P_{n't}} \right] \quad \text{and} \quad \mathbf{S} := [s_{nn'}] = \left[\alpha_{n'n} \frac{\lambda_{n'}}{\lambda_n} \right] \tag{23}$$

The matrix $\mathbf{\Omega}_t$ is the economy's *responsiveness matrix*, which contains the (negative) elasticity of firms' input demand to its own input. The matrix $\mathbf{S}$ is the *sales matrix*. The elements $s_{nn'}$ denote the share of revenues in sector n from selling to n' in the complete information economy. I define the remainder of its row sums $s_{nc} = 1 - \sum_n s_{nn'}$ as the sales share to households (*i.e.* the fraction of sales going to final consumption). I now state the first Theorem of this section. I use the short-hand notation $\mathrm{diag}(x_n)$ to denote a diagonal matrix with elements x_n.

Theorem 1. *Real GDP under fixed responsiveness is given by:*

$$\log \mathcal{C}_t = cons + \epsilon(\mathbf{\Omega}_t)' \log z_t + \left[1 - \epsilon(\mathbf{\Omega}_t)' \times \mathrm{diag}\big((1-\chi)\alpha_{nl} + (1-\alpha_{nl})s_{nc}\big) \right] \log M_t \tag{24}$$

where the constant is independent of time t shocks and $\epsilon(\mathbf{\Omega}_t)$, the economy's Augmented-by-Uncertainty Domar Index (AUDI) is given by:

$$\epsilon(\mathbf{\Omega}_t) = \gamma' \Bigg[\underbrace{\mathbf{I} - \mathbf{A} \odot \mathbf{\Omega}_t}_{\substack{\text{responsiveness-adjusted} \\ \text{Leontief matrix}}} \underbrace{- \mathrm{diag}\Bigg((1-\alpha_{nl}) \times \sum_{n \in \mathcal{N}} s_{nn'}(1 - \omega_{n'nt}) \Bigg)}_{\text{demand impact matrix}} \Bigg]^{-1} \tag{25}$$

whenever the matrix in brackets is invertible.[10]

Proof. See Appendix A.6. □

Theorem 1 describes how real GDP responds to sectoral and aggregate demand shocks under a given level of input responsiveness. In order to understand this result, consider first how prices respond to a productivity shock. First, a sectoral productivity shock directly increases its sector's output. In equilibrium, that sector's output price must change to equate supply and demand. The magnitude of this change is given by how responsive that sector's customers are to input prices in terms of their purchases, as well as how effectively

[9]There was no need to linearize in the simple supply chain example considered previously as the market clearing equation was already log-linear in revenues and prices. This is a general feature of "vertical" production network economies, which will be studied later.

[10]The assumption of invertibility is without loss of generality, as the underlying volatilities σ^z_{nt} and σ^M_t are drawn from smooth distributions and the mapping from these volatilities to responsiveness is continuous.

firms in that sector can respond to demand conditions. This force is captured by the diagonal "demand impact matrix" in $\epsilon(\mathbf{\Omega}_t)$, which contains the input responsiveness of each sector's customers weighted by their sales share ($\sum s_{nn'}(1-\omega_{n'nt})$), as well as one minus the sector's labor share in production $(1-\alpha_{nl})$.

This change in equilibrium prices has spillovers to other sectors' output through input-output linkages. The magnitude of these spillovers is mediated by the Hadamard product of input-output linkages $\mathbf{A}$ with the responsiveness matrix $\mathbf{\Omega}_t$. This increase in output then yields an additional round of spillovers. Assuming $\mathbf{A} \odot \mathbf{\Omega_t}$ has a principal eigenvalue less than unity in absolute value, we can write this channel as:

$$[\mathbf{I} - \mathbf{A} \odot \mathbf{\Omega}_t]^{-1} = \mathbf{I} + \mathbf{A} \odot \mathbf{\Omega}_t + (\mathbf{A} \odot \mathbf{\Omega}_t)^2 + \ldots$$

Thus, the matrix $\mathbf{I}-\mathbf{A}\odot\mathbf{\Omega}_t$ is the standard Leontief matrix, adjusted for input responsiveness. Multiplying these output changes by household expenditure shares (γ) then yields the final effect of a sectoral disturbance on real GDP.

Finally, a similar propagation mechanism underscores how aggregate demand shocks affect real GDP. Absent any changes in prices, an increase in money balances increases household consumption one-to-one. However, this shock will also have some inflationary effects on prices. First, aggregate demand directly increases firms' prices because it increases wages (the term $(1-\chi)\alpha_{nl}$) and firms' revenues through their sales to households (as captured by the term $(1-\alpha_{nl})s_{nc}$). Firms respond to this price *increase* by reducing inputs according to $\epsilon(\mathbf{\Omega}_t)$, as in the previous discussion regarding a productivity shock. The total change in final household consumption is thus given by $1 - \epsilon(\mathbf{\Omega}_t)' \times \text{diag}((1-\chi)\alpha_{nl} + (1-\alpha_{nl})s_{nc})$.

The Economic Interpretation of AUDI. Theorem 1 highlighted that the effect of macroeconomic shocks on real GDP is mediated by a single statistic, which is the economy's Adjusted-by-Uncertainty Domar Index (AUDI). I refer to elements of this index as *AUDI weights*. This index gives us a well-defined measure of sectoral importance, as its weights measure how a sectoral productivity shock in one sector affects real GDP. By definition, as a sector's AUDI weight increases, productivity disturbances in that sector have a larger effect on output. Moreover, it is straightforward to see that as the AUDI weight of any sector increases, the impact of an aggregate demand shock on output decreases.

A key implication of incomplete information is that measures of sectoral importance are state-dependent and shaped by the overall amount of uncertainty in the economy, as mediated through the responsiveness matrix $\mathbf{\Omega}_t$. Sectors that have a high AUDI weight (and are thus systemically "important") when there is a high level of responsiveness might not be systemically important when responsiveness is low. Thus, to understand *which* sectors are important and *when*, it is critical to understand their placement within the network structure of the economy.

To interpret how AUDI relates to a sector's position in the production network structure and how this depends on input responsiveness $\mathbf{\Omega}_t$, I study the simple but illustrative case in which all firms have a common level of responsiveness, $\omega_{nn't} = \omega_t^*$. We then obtain the following result.

Proposition 5. *Suppose $\omega_{nn't} = \omega_t^*$ for all $n, n' \in \mathcal{N}$. Then, one can express* AUDI *as:*

$$\epsilon(\omega_t^*)' = \gamma' \left[\sum_{k=0}^{\infty} \big(\underbrace{\omega_t^* \times (\mathbf{I} - \text{diag}(b_{nt}))^{-1}}_{\textit{discounting weights}} \big)^k \mathbf{A}^k \right] (\mathbf{I} - \text{diag}(b_{nt}))^{-1} \tag{26}$$

where the elements b_{nt} of the diagonal matrix are defined as

$$b_{nt} = (1 - \omega_t^*)(1 - \alpha_{nl})(1 - s_{nc}) \tag{27}$$

If $\omega_t^* \leq 1$, the matrix $\omega_t^* \times (\mathbf{I} - \text{diag}(b_{nt}))^{-1}$ has elements weakly less than unity. Moreover, all elements are strictly increasing in ω_t^*.

Proof. See Appendix A.7. □

Proposition 5 shows that AUDI has a simple economic interpretation: a sector's importance is simply the discounted sum of its higher-order linkages, as captured by the power sum of the matrix $\mathbf{A}$. These higher-order linkages are discounted at a rate given by the diagonal matrix $\omega^* \times (\mathbf{I} - \text{diag}(b_{nt}))^{-1}$. This matrix reflects how much a firm responds to input price changes in terms of its input quantities (ω_t^*), as well as how effectively the firm can adjust its output in response to changes in demand from its customers (as captured by the elements b_{nt}). Because this matrix is diagonal, one can interpret incomplete information as attaching different relative weights to these higher-order linkages.

Furthermore, observe that these discounting weights are increasing in firms' common assumed responsiveness ω_t^*. Intuitively, higher-order linkages become more important in propagating shocks through the economy as firms become more responsive to input prices in terms of their input purchases. In the extreme case in which firms are entirely unresponsive to price changes ($\omega_t^* = 0$), output only responds to productivity disturbances of the most *downstream* firms — that is, for firms that sell directly to the consumer. This contrasts sharply to the complete information benchmark, in which the effect of a sectoral productivity disturbances on real GDP is equal to that sector's Domar weight, irrespective of the sector's position in the production network.

Away from a common responsiveness ω_t^*, one cannot transparently characterize AUDI as in Proposition 5 and the more general Theorem 1 applies. Nevertheless, I show later that a common responsiveness arises endogenously for all firms when one source of macroeconomic uncertainty dominates. The following corollary summarizes how AUDI weights depend on input responsiveness and its implications for real GDP under this simplification.

Proposition 6. *Assume a common firm-level responsiveness $\omega_{nn't} = \omega_t^*$ and suppose that $\alpha_{nn} \geq (1-\alpha_{nl})(1-s_{nc})$ for all $n \in \mathcal{N}$. Then,*

1. *The* AUDI *weights for each sector are weakly increasing in ω_t^*.*
2. *The effect of a productivity shock in sector $n \in \mathcal{N}$ on real GDP $(\mathrm{d}\log \mathcal{C}_t / \mathrm{d}\log z_{nt})$ is weakly increasing in ω_t^*.*
3. *The effect of an aggregate demand shock on real GDP $(\mathrm{d}\log \mathcal{C}_t / \mathrm{d}\log M_t)$ is weakly decreasing in ω_t^*.*

Proof. See Appendix A.8. □

Connection to Alpha Centrality. The above analysis implies that AUDI is related to an extensively studied topic in the social network literature, which is the concept of a node's *alpha centrality* (Katz, 1953). The alpha centrality (AC) of a sector for a scalar $\alpha \in [0,1]$ is defined as follows:

$$AC = \gamma'(\mathbf{I} - \alpha \mathbf{A})^{-1} = \gamma' \sum_{k=0}^{\infty} \alpha^k \mathbf{A}^k \tag{28}$$

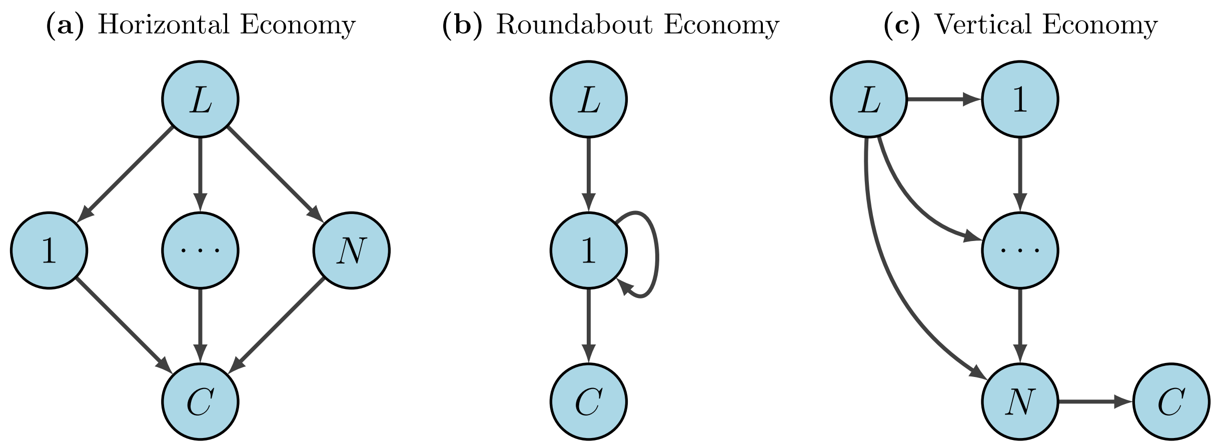
Intuitively, alpha centrality captures how "central" a sector is in an economy, where connections to distant sectors are penalized by an attenuation factor α. The relevant attenuation factor in our framework is not a scalar, but is captured through the diagonal discounting weights in Proposition 5. However, if we assume that all sectors have a common labor share $\alpha_{nl} = \alpha_l$ and a common revenue share of consumption $s_{nc} = s_c$, we can indeed write this attenuation factor in terms of a scalar $\alpha(\omega_t^*)$:

$$\alpha(\omega_t^*) \equiv \frac{\omega_t^*}{1 - (1-\alpha_l)(1-s_c)(1-\omega_t^*)} \tag{29}$$

where it is easy to check that $\alpha(\omega^*) \in [0,1]$ if and only if $\omega^* \in [0,1]$.[11] The systemic importance of a sector under incomplete information is therefore given by its alpha centrality (up to a scaling factor), where the alpha centrality parameter is determined endogenously by the production network structure of the economy and the nature of firms' uncertainty.

[11] Of course, there is no *a priori* economic restriction that $\alpha(\omega^*) \in [0,1]$. If $\alpha(\omega^*) > 1$, the vector $\epsilon(\omega^*)'$ is still well defined as long as the infinite sum in Proposition 5 converges.

Figure 3: Simple Network Structures



3.5 Examples: Sectoral Importance and Input Responsiveness

In this section, I use Theorem 1 to illustrate how sectoral importance is shaped by input responsiveness in three simple network structures.

Example 1: A Horizontal Economy. Consider first a horizontal production network economy, depicted in Figure 3a. Here, labor is the only factor of production for all sectors. Since there are no input-output linkages, the presence of incomplete information is irrelevant. The effect of a sectoral productivity shock on real GDP is independent of input responsiveness and equal to that sector's Domar weight (its revenue share in household expenditures, γ_n), as in the complete information economy.

Corollary 1. *In the horizontal production network economy, real GDP is given by*

$$\log \mathcal{C}_t = cons + \sum_{n \in \mathcal{N}} \gamma_n \log z_{nt} + \chi \log M_t \tag{30}$$

where the constant is independent of time t variables.

Proof. See Appendix A.9. $\square$

Example 2: A Roundabout Economy. Consider the roundabout production network, depicted in Figure 3b. Firms in sector 1 purchase labor (with input share $1-\alpha_1$) and inputs from their own sector (with input share α_1). The dynamics of real GDP can be analyzed through a scalar ω_t^*, which is the elasticity of input purchases to price changes in the firm's own sector.

Corollary 2. *In the roundabout economy, real GDP is given by:*

$$\log \mathcal{C}_t = cons + \left(\frac{1}{1+\alpha_1(1-\omega_t^*)} \times \frac{1}{1-\alpha_1}\right)\log z_{1t} + \left(\frac{\chi - \alpha_1\omega_t^*}{1+\alpha_1(1-\omega_t^*)}\right)\log M_t \tag{31}$$

where the constant is independent of time t shocks.

Proof. See Appendix A.10 □

This corollary emphasizes that the sectoral importance in the roundabout economy depends on how much weight is attached to the sector's own input share, where this weight is mediated by the responsiveness parameter ω_t^*. To see this, observe that we may write AUDI in this economy as:

$$\epsilon(\omega_t^*) = \frac{1}{1-\alpha_1^2(1-\omega_t^*)}\sum_{k=0}^{\infty}\underbrace{\frac{\omega_t^*}{1-\alpha_1^2(1-\omega_t^*)}}_{\text{discounting weight}}\alpha_1 = \left(\frac{1}{1+\alpha_1(1-\omega_t^*)}\right)\times \underbrace{\frac{1}{1-\alpha_1}}_{\text{Domar Weight}} \tag{32}$$

This representation makes it clear that input responsiveness weights the economy's higher-order linkages according to ω_t^*. Whenever $\omega_t^* \in (0,1)$, the AUDI weight of the roundabout sector is equal to an attenuation factor that multiplies its complete information Domar weight. Moreover, it is straightforward to see that the impact of productivity shocks on output is increasing in ω_t^*, while the impact of aggregate demand shocks on output is decreasing in ω_t^*, in line with Proposition 6.

Example 3: A Vertical Economy. Finally, consider a vertical supply chain economy, as in Figure 3c. I denote the labor share of sector n as $1-\alpha_n$ and the input share of its upstream supplier α_n. I also let ω_{nt} denote the input responsiveness for sector n's input.

Corollary 3. *Real GDP in the vertical supply chain economy is given by:*

$$\log \mathcal{C}_t = cons + \sum_{n\in\mathcal{N}}\epsilon_n(\mathbf{\Omega}_t)\log z_{nt} + \left[(1-\alpha_N)\chi - \sum_{k=1}^{N-1}\epsilon_{N-k}(\mathbf{\Omega}_t)(1-\alpha_{N-k}(1-\chi))\right]\log M_t$$

where the constant is independent of time t shocks and the AUDI weight of sector n satisfies the following recursion, with $\epsilon_N(\mathbf{\Omega_t}) = 1$:

$$\epsilon_n(\mathbf{\Omega}_t) = \frac{\alpha_{n+1}\omega_{nt}}{1-\alpha_n(1-\omega_{nt})}\times \epsilon_{n+1}(\mathbf{\Omega}_t) \quad \textit{for} \quad n\in\{1,\dots,N-1\} \tag{33}$$

Proof. See Appendix A.11. □

Corollary 3 states that the sectoral importance of a sector is shaped by the input responsiveness of all sectors that are *downstream* to it. In particular, the sectoral importance of sector N, the most downstream sector, is equal to unity and independent of input responsiveness. This formalizes the prediction of Section 3.3, which showed that incomplete information does not affect sectoral importance for the most downstream sectors. Moreover, it is straightforward to see that if firms in sector k become entirely unresponsive to input prices in terms of their input purchases, $\omega_{kt-1} = 0$, then sectoral productivity disturbances in its upstream suppliers ($n < k$) have no effect on output.

3.6 Responsiveness and the Role of Uncertainty

The previous discussion highlighted that the macroeconomic implications of both sectoral productivity disturbances and aggregate nominal disturbances depend on firms' responsiveness to input price changes. In this section, I characterize how the responsiveness matrix $\mathbf{\Omega}_t$ is shaped by the exogenous volatilities of underlying disturbances. I begin by relating input responsiveness to the second moments of sectoral revenues and input prices.

Lemma 2. *In all log-linear equilibria, input responsiveness is given by:*

$$\mathbf{\Omega}_t = [\omega_{nn't}] = \left[1 - \frac{\text{Cov}(\log R_{nt}, \log P_{n't})}{\text{Var}(\log P_{n't})}\right] \tag{34}$$

where the variance and covariance are conditional on the shocks $\{z_{nt-1}\}_{n\in\mathcal{N}}$ *and* M_{t-1}*.*

Proof. See Appendix A.12. □

This Lemma is a consequence of the assumed log-normality of shocks and firms' Cobb-Douglas technologies, which implies that equilibrium revenues and prices are jointly log-normal whenever $\mathbf{\Omega}_t$ is independent of the realization of prices.[12] This Lemma can be understood through the following "OLS" interpretation: optimal responsiveness is the coefficient that emerges when one regresses the optimal *ex-post* choice of inputs $X^*_{nn't}$ on its input price:

$$\log X^*_{nn't} = cons - \underbrace{\left[1 - \frac{\text{Cov}(\log R_{nt}, \log P_{n't})}{\text{Var}(\log P_{n't})}\right]}_{\text{OLS coefficient}} \times \log P_{n't} + \underbrace{\log X^*_{nn't} - \mathbb{E}_t[\log X^*_{nn't} | \log P_{n't}]}_{\text{error term}}$$

[12]This qualification is why, in principle, there could exist other equilibria in which $\mathbf{\Omega}_t$ takes other functional forms. Another functional form for the responsiveness matrix could imply that revenues and prices are no longer log-normal. The resultant joint stochastic process could then make the assumed functional form for responsiveness self-fulfilling. Although I cannot rule out equilibria that are not log-normal, I have not encountered such equilibria numerically.

where recall that under complete information, optimal input expenditures are a constant fraction of revenues $P_{n't}X^*_{nn't} = \alpha_{nn'}R_{n't}$. Thus, if input prices increase, a firm decreases its inputs because they become more expensive. If higher input prices and revenues covary positively, it purchases more inputs in anticipation of higher demand. Both this cost component and inference component shape firms' optimal responsiveness to input prices.

Solution under Dominant Uncertainty Limits. I now use Lemma 2 to characterize firms' responsiveness as either productivity or demand uncertainty becomes large. To this end, I define the *uncertainty ratio* u_{nt} of productivity to demand in sector n as:

$$u_{nt} = \frac{\sigma^z_{nt}}{\sigma^M_t} \tag{35}$$

The ratios $\{u_{nt}\}_{n\in\mathcal{N}}$ parameterize the extent of prior uncertainty about productivity shocks *relative* to demand shocks. I now establish firms' *optimal* input responsiveness as either source of uncertainty dominates.

Theorem 2. *If $u_{nt} \to 0$ for all $n \in \mathcal{N}$, then:*

$$\omega_{nn't} = -\frac{\chi}{1-\chi} \quad \textit{and} \quad \frac{\mathrm{d}\log \mathcal{C}_t}{\mathrm{d}\log M_t} = \chi \tag{36}$$

Suppose further that the matrix $\mathbf{A}$ is irreducible. If $u_{nt} \to \infty$ for some $n \in \mathcal{N}$, then:

$$\omega_{nn't} = 1 \quad \textit{and} \quad \frac{\mathrm{d}\log \mathcal{C}_t}{\mathrm{d}\log z_t}' = \gamma'(\mathbf{I}-\mathbf{A})^{-1} \tag{37}$$

Proof. See Appendix A.13. □

Theorem 2 shows that the impact of demand shocks on real GDP is equal to the full information benchmark as uncertainty about demand becomes dominant. Intuitively, as prior uncertainty about the money supply increases, firms believe that prices are driven by nominal disturbances. For this reason, firms expect their revenues and prices to co-move positively. This force induces firms to become less responsive to input price changes in terms of their input purchases. From Proposition 5, this lowers the importance of higher-order linkages in propagating shocks. Observe that firms are not at all responsive to input prices ($\omega_{nn't} = 0$) when there is no wage rigidity. In this case, nominal disturbances have no effect on output and we recover monetary neutrality.

When uncertainty about productivity shocks becomes dominant, the effect of a sectoral productivity shock on consumption is given by that sector's complete information Domar weight. Intuitively, as prior uncertainty about productivity shocks increase, firms believe that prices are driven by *real* disturbances. For this reason, firms expect their revenues to remain unchanged when faced with higher input prices. This force induces firms' input responsiveness to increase, thereby increasing the importance of higher-order linkages in propagating shocks. Finally, the irreducibility condition on the input-output matrix $\mathbf{A}$ ensures that all input prices respond to any given sectoral disturbance. Away from irreducibility, the second statement of Theorem 2 is true if and only if the uncertainty ratio of all sectors becomes large, so that $u_{nt} \to \infty$ for all $n \in \mathcal{N}$. Thus, the relative volatilities of productivity and aggregate demand are critical in shaping real GDP dynamics through their effect on input responsiveness, which is the key statistic that governs the aggregate impact of shocks on the economy.

The Role of Relative Uncertainty. An implication of Theorem 2 is that *total* uncertainty does not matter for equilibrium shock transmission. Rather, it is only the *relative* uncertainty between different kinds of shocks that determines firms' input responsiveness. This generalizes a key prediction of the simple supply chain economy in Section 3.3.

Corollary 4. *In equilibrium, $\mathbf{\Omega}_t$ is a function only of the relative uncertainty ratios $\{u_{nt}\}_{n\in\mathcal{N}}$, and does not additionally depend on total uncertainty $\{\sigma^z_{nt}\}_{n\in\mathcal{N}}$, σ^M_t.*

Proof. See Appendix A.14. □

This result states that small amounts of uncertainty can have big effects. Concretely, even vanishingly small amounts of uncertainty imply that the equilibrium can differ substantially from the complete information benchmark. As such, the basic mechanism of the model which suggests firms use input prices to form inferences about their revenues is preserved in environments of low uncertainty.

The Roundabout Economy Revisited. Equilibrium responsiveness and dynamics can be solved in closed form for the roundabout economy in Figure 3b. Because a firm's input price in this network is also its output price, optimal responsiveness is given by:

$$\omega^*_t = 1 - \frac{\mathrm{Cov}(\log R_{nt}, \log P_{nt})}{\mathrm{Var}(\log P_{nt})} \tag{38}$$

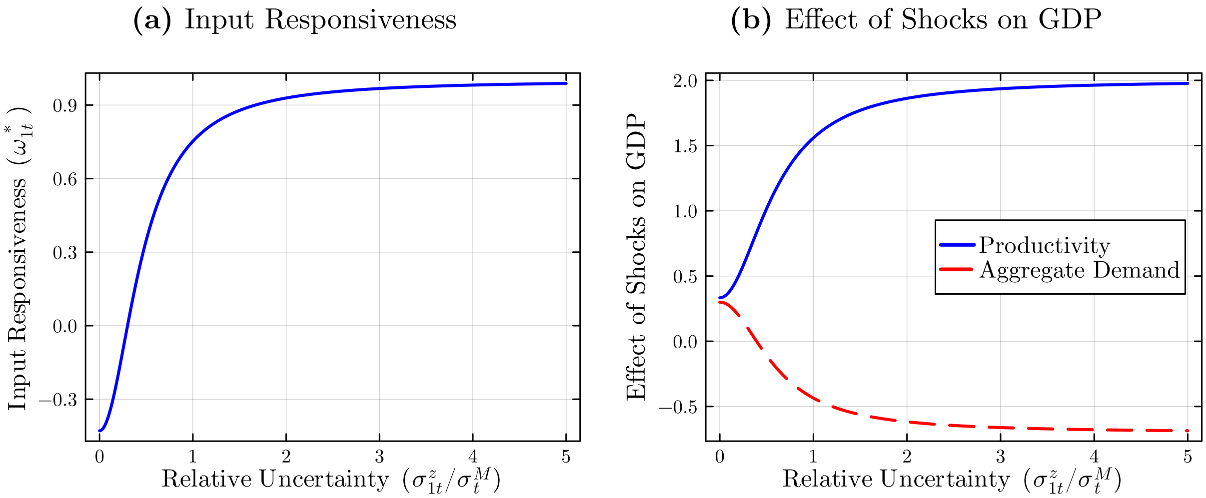
We can solve for this fixed point using the dynamics for revenues and prices (Proposition 3).

Proposition 7. *Optimal responsiveness ω^*_t in the roundabout economy is given by:*

$$\omega^*_t = 1 - \frac{1}{1 + \frac{(1-\alpha_1)^{-2}}{1+\alpha_1-\chi} u^2_{1t} - \chi} \tag{39}$$

Proof. See Appendix A.15. □

Figure 4: Responsiveness and the Effect of Shocks on Real GDP



Note: Left panel: This figure plots the optimal responsiveness in a roundabout economy given by Proposition 7. Right panel: the associated pass-through of a productivity and aggregate demand shock to real GDP, $\frac{\mathrm{d}\log \mathrm{GDP}_t}{\mathrm{d}\log x}$, for $x \in \{z_{1t}, M_t\}$, as a function of relative uncertainty about productivity to aggregate demand shocks. Parameterization: $(\alpha_1, \chi) = (0.5, 0.3)$.

Proposition 7 shows that there exists a unique firm responsiveness to input prices that is only a function of relative uncertainty in the economy. Figure 4 plots firms' optimal responsiveness as a function of relative uncertainty (left panel) and the associated dynamics for real GDP in response to a productivity and demand shock (right panel). As productivity uncertainty increases, firm's respond one-to-one to input price changes and the pass-through of productivity shocks to output converges to the complete information benchmark, while the pass-through of demand shocks to output decreases.

Implications for the State-Dependent Effects of Shocks. An implication of Theorem 2 is that shocks that feature a large uncertainty have a larger pass-through to real GDP. For example, higher productivity uncertainty raises firms' input responsiveness. In light of Proposition 6, this increases the pass-through of productivity shocks to output and increases firms' AUDI weights. Conversely, higher demand uncertainty makes firms less responsive and increases the pass-through of demand shocks to output. Thus, it is exactly when uncertainty about an underlying shock is highest that the shock has the greatest impact on macroeconomic dynamics.

Moreover, these results suggest that Domar weights can function as particularly poor measures of sectoral importance in times of high demand uncertainty. When demand uncertainty is high, firms respond less to input price changes relative to complete information. As we have seen, this implies that the AUDI weights of upstream sectors can decrease relative to downstream sectors, even if all sectors in the economy have the same Domar weight. In general, the effect of a sectoral productivity shock on real GDP depends on the *kind* of uncertainty that firms face (real or nominal) and its position in the production network structure of the economy.

3.7 Discussion and Extensions

Conditioning on Further Input Prices. Under Assumption 1, firms can condition their quantity of an input demanded on its price, but not on the prices of other inputs. This assumption is motivated both by the rational expectations literature, in which input choices depend on local terms of trade, as well as the contracting interpretation of Section 2, in which input deliveries are often negotiated bilaterally between a buyer and a seller.

Away from this assumption, one would have to analyze how firms' demand functions are shaped by a vector of endogenous public signals, which would render analyzing the equilibrium intractable and not quantitatively implementable (as the state space scales exponentially in the number of observed variables). Thus, the buyer-input specific information set of Assumption 1 allows me to tractably characterize the equilibrium while preserving the key feature that firms may be learning from local economic conditions. Nevertheless, even in this more general setting, the response of aggregate output to shocks is shaped by a qualitatively similar notion of input responsiveness. In Appendix B.2, I show that real GDP dynamics are identical to Theorem 2 under high aggregate demand uncertainty when firms can condition their demand functions on any arbitrary subset of prices in the economy.

Extension to Monopolistic Competition. None of the results hinge on the assumption of price-taking. In Appendix B.3, I show that the theoretical results are identical in a context where firms operate monopolistically and the government levies a subsidy to undo the monopolistic distortion.

Allowing for More Flexible Inputs. I have assumed that labor can be scaled up or down in accordance to realized demand. The fact that only labor can be adjusted frictionlessly is not essential for the main results. In Appendix B.4, I allow any arbitrary subset $\mathcal{S}^{CI} \subset \mathcal{N}$ of inputs to be chosen under complete information, while its complement $\mathcal{S}^{II} = \mathcal{N}/\mathcal{S}^{CI}$ is chosen under incomplete information.

Interim Public Signals. In order to highlight the informational role of prices in shaping macroeconomic fluctuations, I have abstracted from additional sources of information that firms might receive when choosing their inputs. However, the model can accommodate exogenous interim public signals about contemporaneous shock realizations, as modelled in Angeletos et al. (2016) or La'O and Tahbaz-Salehi (2022). As is well known, the only additional implication of interim public signals is that firms respond to shocks beyond the information that may be conveyed through prices. For this reason, the main theoretical results are unchanged to this modification. This extension is explored in Appendix B.5.

4 Quantitative Analysis: AUDI in the US

In this section, I study the model's implications for the effect of productivity and aggregate demand shocks on aggregate output when calibrated to the input-output structure and historical measures of uncertainty in the US economy. First, I find that the impact of aggregate productivity shocks on output is 50% lower during Covid (a time of relatively high measured demand uncertainty) relative to the Dot-Com Bubble (a time of relatively high measured productivity uncertainty) or non-crisis periods. Second, I show that this state-dependence is linked to time-variation in estimated AUDI weights: the sectoral importance of downstream sectors increases during times of relatively high demand uncertainty, while the sectoral importance of upstream sectors increases when productivity uncertainty is high. Overall, Domar weights can be poor measures of sectoral importance, particularly during recessions. Hence, the results suggest that policymakers should account for uncertainty when designing bailouts or industrial policy to promote output during crises.

4.1 Data

My analysis relies on three sources of data. First, I use the 2022 input-output tables constructed by the Bureau of Economic Analysis (BEA), which provides information on the intermediate input expenditures of each industry, contribution to final uses, and total employee compensation. Second, I use the March 2024 release of the BEA/BLS Integrated Production Level Accounts (ILPA) which contains data on industry-level productivities at an annual frequency over the 1987-2023 period. I use this data to obtain measures of the time-varying productivity covariance matrix across industries. Finally, I use data on nominal GDP to obtain measures of the time-variation in demand uncertainty. Recall that nominal GDP in my model is given by $\iota_t M_t$, where $\iota_t = i_t/(1+i_t)$ (c.f. Lemma 1). A key benefit of this approach is that it captures nominal demand uncertainty generated by policy (as captured through M_t) as well as demand uncertainty generated through changes in household behavior (as captured through the endogenous velocity term ι_t), without separately estimating a money growth rule.

I merge the BEA input-output data with the ILPA data at the 3-digit NAICS industry level, while excluding industries that correspond to federal, state, and local governments. As the ILPA data is slightly more aggregated than the BEA data, I attribute the productivity of each BEA industry to its more aggregated counterpart, thereby obtaining a matched data set of 66 industries.

4.2 Calibration

I interpret each period as a quarter. I calibrate the input-output matrix **A** and labor expenditures $\{\alpha_{nl}\}_{n\in\mathcal{N}}$ of each industry in the linearized economy to match the intermediate good expenditure shares and compensation of employees in the BEA input-output data. I also calibrate the final consumption shares γ to match the corresponding final consumption expenditures in the data.[13] Finally, I set $\chi = 0.915$ to match the estimated covariance between the log-changes in nominal GDP and wages over the sample period.

Next, I use the ILPA data to calculate the implied productivity variance-covariance matrix over the 1987-2023 period. Concretely, I residualize the logarithm of productivity for each sector on its lag following Equation 7. This gives me estimates of the productivity residuals $\{\hat{u}^z_{nt}\}_{n\in\mathcal{N}}$ at the annual frequency. I use these residuals to obtain estimates of time-varying productivity uncertainty using a multivariate GARCH model. In particular, letting Z_t denote the vector of residuals $\{\hat{u}^z_{nt}\}_{n\in\mathcal{N}}$, I model

$$Z_t \sim N(0, \Sigma_t), \qquad \Sigma_t = D_t^{\frac{1}{2}} C D_t^{\frac{1}{2}} \tag{40}$$

where D_t is a diagonal matrix of time-varying variances, and C is a static matrix of correlations. I assume that each diagonal element of D_t, denoted as $\sigma^2_{i,t}$, evolves as:

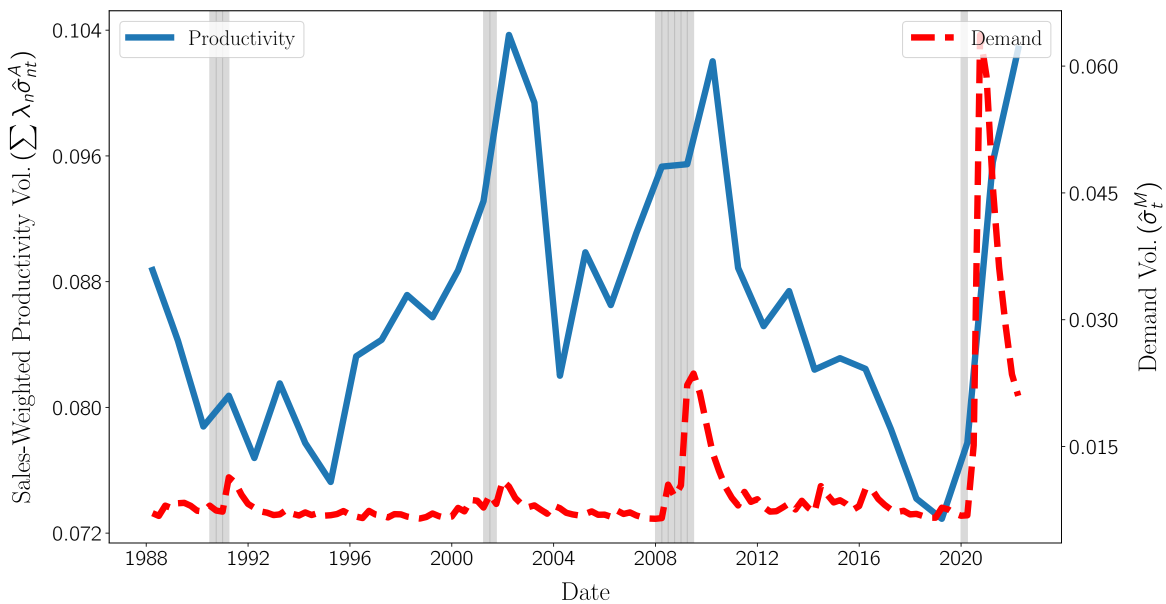
$$\sigma^2_{i,t} = s_i + \alpha_i \hat{u}^2_{i,t-1} + \beta_i \sigma^2_{i,t-1} \tag{41}$$

with unknown constant s_i and coefficients (α_i, β_i). Formally, this is a GARCH(1,1) model with constant conditional correlations (Bollerslev, 1990). I estimate all of the parameters via joint maximum likelihood. As the ILPA data is only available at an annual frequency, I assume that the variance is evenly distributed across all quarters.[14] Finally, log-differencing and demeaning the nominal GDP measure yields estimates of the demand residuals $\hat{u}^M_t$

[13] Away from complete information ($\delta = 0$), the model-implied input expenditure shares of each industry are time-varying and a function of uncertainty. However, this time-variation is quantitatively small and does not meaningfully change the calibrated input-output matrix **A**.

[14] Linearly interpolating the productivity data between quarters, as in La'O and Tahbaz-Salehi (2022), gives quantitatively similar results.

Figure 5: Estimated Time-Varying Productivity and Demand Volatilities



Note: This figure plots the estimated average demand volatility $\hat{\sigma}^M_t$ (red dashed line) and average productivity volatility weighted by sales-share to GDP $\sum_{n\in\mathcal{N}} \lambda_n \hat{\sigma}^z_{nt}$ (blue solid line) over the sample period, as estimated by the CCC GARCH model, described in Equations 40 and 41. Shaded regions correspond to NBER recession dates.

at the quarterly frequency. I similarly use a GARCH(1,1) process to obtain estimates of time-varying demand uncertainty $\hat{\sigma}^M_t$.

In light of Theorem 2, it is only *relative* uncertainty that matters for the determinants of firms' optimal demand schedules. For this reason, my goal is to capture broad trends in relative uncertainty about the economy's underlying shocks. The GARCH approach provides one parsimonious way of doing so.[15] In my estimated model, uncertainty is high when there is a large prediction error in the variables ($\alpha_i > 0$) and if uncertainty was high previously ($\beta_i > 0$). The assumption that the matrix C is time-invariant imposes all covariances to move in proportion to the variances, thereby ruling out the possibility that the correlation structure among productivity and demand shocks is changing over time. This makes estimation of the time-variation in volatility feasible by significantly reducing the number of estimated parameters and improving the convergence of the maximum likelihood algorithm.

Figure 5 plots the estimated average demand volatility $\hat{\sigma}^M_t$ and average sales-weighted productivity volatility $\sum_{n\in\mathcal{N}} \lambda_n \hat{\sigma}^z_{nt}$ over the sample period. First, note that both series fea-

[15]There are, of course, many possible statistical models to capture time-varying volatility. Latent-state models that allow volatility to be directly affected by contemporaneous shocks obtain qualitatively and quantitatively similar predictions to GARCH models (Jurado et al., 2015).

ture elevated volatility at times of NBER recession dates. This is consistent with the findings of Jurado et al. (2015), who find periods of heightened uncertainty during recessions in a model of stochastic volatility, and with Bloom et al. (2018), who find a negative correlation of TFP dispersion with the cycle. While these works focus on how total volatility moves with the cycle, my analysis suggests that changes in the *relative* volatilities of these series are also important to understand macroeconomic dynamics. For example, my GARCH specification shows that the Dot-Com bubble was characterized by large productivity uncertainty, but relatively little demand uncertainty. In contrast, the Financial Crisis and Covid were characterized by both high demand productivity uncertainty.

Second, away from these times of elevated uncertainty, it is productivity uncertainty that dominates demand uncertainty in "normal times", defined as non-recessionary periods. This is consistent with the interpretation that "granular" fluctuations that occur at the sector-level are relatively more volatile than aggregate productivity disturbances (Gabaix, 2011; Acemoglu et al., 2016).[16] Indeed Figure 9 in the Appendix shows that the conditional volatility of aggregate TFP measures are an order of magnitude lower relative to their disaggregated counterparts. A key implication of my theory is that uncertainty at the "micro" level matters for the dynamics of output and prices, which cannot be recovered separately from aggregated "macro" data.

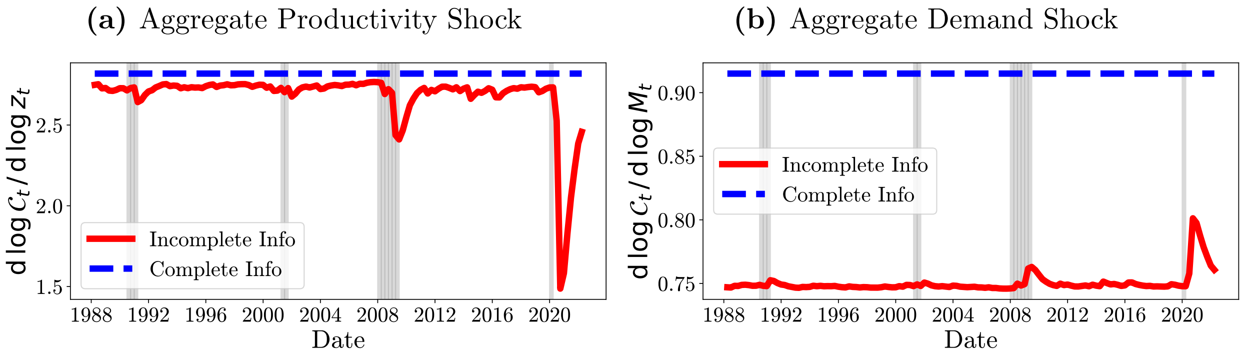
Solving the Model. The GARCH model provides estimates of the time-variation in uncertainty given by our estimates of the covariance matrix $\hat{\Sigma}_t$. With this estimate in hand, we can solve for the linearized dynamics of the system obtained in Theorem 1. The key challenge is to ensure that firm level responsiveness (as defined in Proposition 2) is consistent with the equilibrium stochastic properties of revenues and prices. Since the choice of firm-level responsiveness shapes the stochastic properties of these objects, it is necessary to solve for industry-level prices, revenues, and firms' optimal responsiveness to input prices jointly. This requires solving for 4488 endogenous variables for each estimate of underlying uncertainty $\hat{\Sigma}_t$.[17]

I implement a three-step procedure to solve for the dynamics of the system. First, I solve for the linearized equilibrium dynamics by guessing a firm-level responsiveness matrix $\mathbf{\Omega}_t$. Second, I calculate the resulting covariance matrix of revenues and prices for each value of $\hat{\Sigma}_t$ and the implied optimal responsiveness matrix $\mathbf{\Omega}_t^*$ using Equation 34. Third, I solve for the

[16]For example, Acemoglu et al. (2016) show that the contribution of sectoral volatility to aggregate output volatility is proportional to the square of that sector's Domar index. The relative contribution of each sector to volatility in my framework is proportional to the square of their augmented-by-uncertainty Domar index, which is endogenous to economy-wide volatility.

[17]There are 2×66 total sectoral-level prices and revenues and 66^2 demand schedules, as each sector implements a demand schedule for each other sector.

Figure 6: The Historical Impact of Aggregate Shocks on Output



Note: This figure plots the pass-through of an aggregate productivity shock (left panel) and an aggregate nominal demand disturbance (right panel) to real GDP over the period (1988Q1-2023Q1). Shaded areas indicate NBER recession dates.

matrix $\mathbf{\Omega}_t$ that obtains the roots of the residual $\mathbf{\Omega}_t^* - \mathbf{\Omega}_t = 0$ through the Newton-Raphson method. The responsiveness matrix $\mathbf{\Omega}_t$ is then a solution of the model, as it is consistent with the model-implied stochastic properties of prices and revenues.

Having obtained the matrix of equilibrium firm-level responsiveness, we can estimate our uncertainty-augmented Domar indices (AUDI) as:

$$\epsilon(\mathbf{\Omega}_t)' = \gamma' \left[\mathbf{I} - \mathbf{A} \odot \mathbf{\Omega}_t - \text{diag}\left((1 - \alpha_{nl}) \times \sum_{n \in \mathcal{N}} s_{nn'}(1 - \omega_{n'nt})\right)\right]^{-1} \tag{42}$$

which measures the macroeconomic impact of sectoral productivity shocks. Note that these measures primitively depend on firm-level responsiveness $\mathbf{\Omega}_t$, which is a function of the underlying uncertainty in the economy $(\hat{\Sigma}_t, \hat{\sigma}_t^M)$.

4.3 Results: Sectoral Importance and Uncertainty

I now assess how my estimated measures of sectoral importance shape the impact of shocks on (real) GDP over time. A useful counterfactual to gauge the relevance of these results is to compare them to the benchmark case of complete information. On average, I find that the impact of an aggregate productivity shock on real GDP is lower by only 4% relative to the counterfactual economy with complete information. The intuition for this result is straightforward. As seen in Figure 5, sectoral-level productivity uncertainty is generally large relative to demand uncertainty. Firms thus perceive most of the variation in their input prices to be attributed to real rather than nominal disturbances. According to Theorem 2,

Table 1: Standard Deviation of Output Growth (1987-2023)

Data	*Model*	*Complete Info*	*Fixed Responsiveness*
0.0119	0.0114	0.0148	0.009

Note: This table depicts the standard deviation of output growth for the sample period in the data, model, and the complete information benchmark. The fixed responsiveness column indicates the standard deviation of output that arises when firms' estimated responsiveness ($\hat{\boldsymbol{\Omega}}_t$) is time-invariant and fixed at its post-Covid average.

the macroeconomic impact of sectoral productivity shocks therefore closely aligns with the complete information benchmark.

However, the macroeconomic impact of an aggregate shock changes substantially in times of high demand volatility, such as the financial crisis or in the aftermath of the Covid pandemic. The elasticity of output to aggregate productivity decreases by 14% during the financial crisis, and by 46% during Covid. Thus, incomplete information matters most when firms are less able to distinguish whether changes in their costs are driven by demand or supply disturbances. During these times, firms respond less to changes in their input prices to hedge against uncertainty in their revenues. Moreover, this hedging behavior is especially important for shaping output dynamics in times of high volatility. These results are illustrated graphically in Figure 6a.

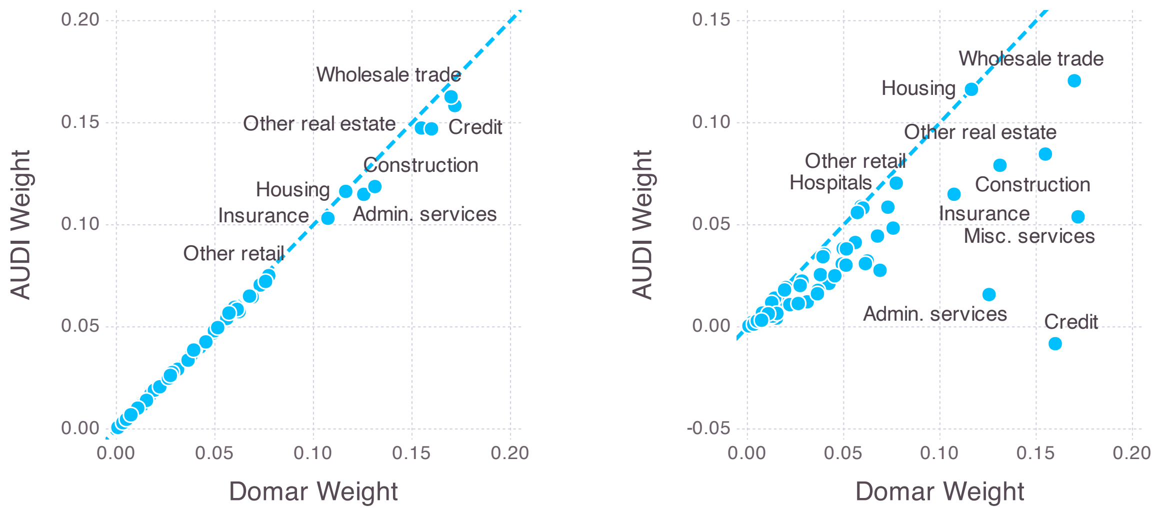
The theory also has implications for how aggregate nominal demand disturbances affect output over time. Times of relatively high productivity uncertainty are associated with a low pass-through of demand shocks to output. Conversely, output responds more to nominal demand disturbances as the relative volatility of demand shocks increases. These predictions are shown graphically in Figure 6b.

Time-Varying Volatility and the "Great Moderation". A large body of literature argues that smaller shock sizes and changes in the conduct of monetary policy can account for the volatility decline in US output in the post-Volcker era (the "Great Moderation").[18] The results in this section suggest that changes in the *relative* volatilities of underlying disturbances can also create meaningful time-variation in macroeconomic time series. For example, more stable monetary policy (interpreted via a reduction in σ_t^M) directly affects output volatility for two reasons. First, shock sizes are smaller. Second, relatively lower demand volatility increases firms' responsiveness to input price changes, which further reduces the impact of demand shocks on output (but increases the impact of productivity shocks).

[18]See, for example, Clarida et al. (2000), Justiniano and Primiceri (2008), and Galí and Gambetti (2009).

Figure 7: Sectoral Importance and its Relationship to Size

(a) Normal Times **(b)** Covid



Note: These figures plot the economy's AUDI weights as a function of their revenues share of GDP (Domar weight) in normal times and Covid (2020 Q1-Q4). The responsiveness matrix in normal times is calculated as $\sum_{t\in\mathcal{T}}\frac{1}{|\mathcal{T}|}\hat{\mathbf{\Omega}}_t$, where $\mathcal{T}$ includes all non-recessionary quarters prior to 2020.

Table 1 shows that this responsiveness channel is critical for the model to match the volatility of output growth in 1987-2023. For example, counterfactually fixing firm-level responsiveness to an "as if" scenario in which demand disturbances are twice as volatile over the sample period reduces implied output volatility by 20%. Hence, shifts in the relative volatilities of underlying economic disturbances — through their effect on agents' policy functions — may be an important, but understudied, feature in accounting for the stochastic volatility of many macroeconomic variables.

Sectoral Importance in Times of High Volatility. The preceding results demonstrated that the responsiveness of output to productivity shocks has historically displayed large variations. This fact reflects that the importance of different input-output linkages in shaping macroeconomic fluctuations is time-varying. But which sectors are most important in shaping the dynamics of real GDP in volatile times?

Figure 7 plots the percentage impact of a sectoral productivity shock on output (which is given by that sector's AUDI weight) as a function of the sector's revenue share of GDP (or its corresponding Domar weight). Quantitatively, the model predicts that AUDI weights are generally always lower than Domar weights (as seen by the fact that almost all sectors lie beneath the dashed forty-five degree line). Moreover, there is a strong positive correlation between a sector's AUDI weight and its Domar weight in normal times (Figure 7a). This rationalizes why the effect of an *aggregate* productivity shock on output is close to the complete information benchmark, on average.

However, Domar weights are a poor predictor of sectoral importance in times of high demand volatility, such as Covid (Figure 7b). In particular, large upstream sectors which are traditionally important in normal times (such as credit intermediation, or administrative and support services) are two of the least systemically important sectors during Covid. The sectors with the highest AUDI weights during Covid are relatively more downstream sectors, such as hospitals, retail, and housing services. Consequently, incomplete information delivers an elasticity of output to sectoral productivity shocks that diverges significantly from the foundational prediction of Hulten's Theorem, which states that this elasticity should be equal to the sector's sales share of GDP (Hulten, 1978).

In order to shed intuition on these findings, Figure 8 plots the economy's input demand matrices, defined by the input share of each input multiplied by responsiveness ($\mathbf{A} \odot \mathbf{\Omega}_t$), during normal times and Covid. Sectors are sorted by the upstreamness measure of Antràs et al. (2012), which captures the average number of rounds it takes for sectoral output to reach the final consumer.[19] Sorting sectors by upstreamness reveals a hierarchical structure in production: there is a clear order in production in which downstream sectors purchase from upstream ones (but not the reverse). This is because the matrix is dense below the diagonal, but sparse above it.[20]

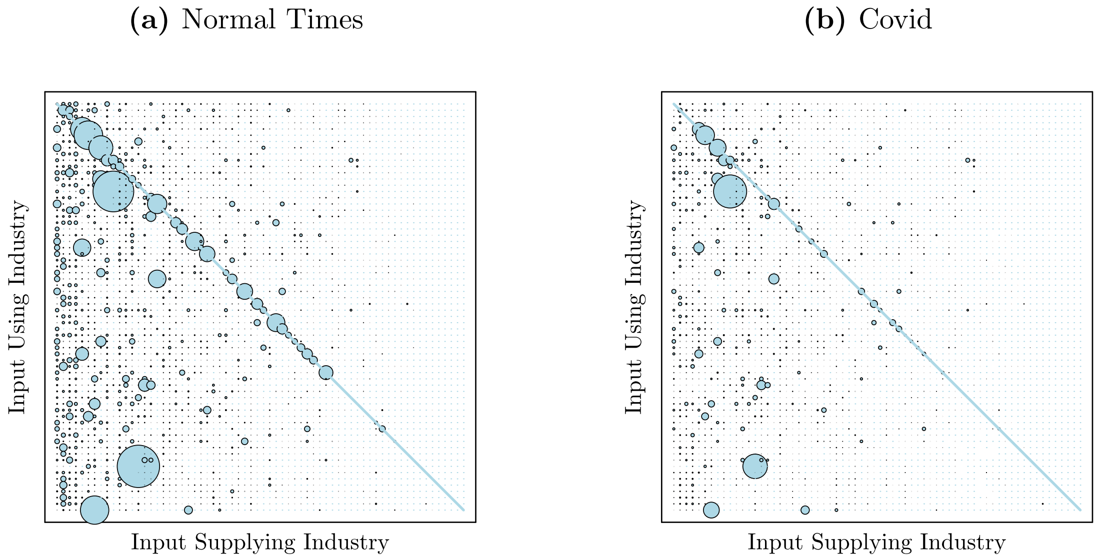
In normal times (Figure 8a), upstream sectors supply their inputs to many downstream firms, and these firms are responsive to input price changes. Hence, they have a high AUDI weight. During Covid, however, the economy's input demand matrix becomes more sparse (Figure 8b), as firms reduce their optimal input demand elasticities in response to heightened demand uncertainty. Consequently, productivity disturbances in upstream sectors have a lower impact on final household consumption. In contrast, productivity disturbances from downstream sectors travel through fewer linkages until they reach the final consumer. Thus, their AUDI weight increases relative to upstream sectors.

Table 2 shows how sectoral importance changes over different recessionary episodes. The Dot-Com bubble was characterized by high productivity uncertainty, and therefore features high AUDI weights. The financial crisis and Covid were characterized by relatively high demand uncertainty, and thus sectors' AUDI weights decreased. Moreover, upstream sectors generally feature the greatest variation in their AUDI weight, precisely because their alpha

[19] Formally, upstreamness is defined as $U = (\mathbf{I} - \mathbf{A}')^{-1}\mathbf{1}$, where $\mathbf{1}$ denotes a vector of ones.

[20] Liu and Tsyvinski (2024) also note this hierarchical production structure for the US economy. Liu (2019) demonstrates that this hierarchical structure is also a feature of the Chinese and South Korean economies.

Figure 8: The Economy's Input Demand Matrices in Different Time Periods: $\mathbf{A} \odot \mathbf{\Omega}_t$



Note: These figures depict the economy's input demand matrices, defined as the Hadamard product between the input-output matrix and responsiveness, $\mathbf{A} \odot \mathbf{\Omega}_t$. The responsiveness matrix in normal times is calculated as the average responsiveness $\sum_{t\in\mathcal{T}} \frac{1}{|\mathcal{T}|}\hat{\mathbf{\Omega}}_t$, where $\mathcal{T}$ includes all non-recessionary quarters prior to 2020.

centrality in the production network is changing over time. Housing, for example, is a sector that does not enter as an input into any other industry. For this reason, its AUDI weight is stable and the sector increases in relative importance in times of high demand uncertainty.

Industrial Policy: Which Sectors and When. Governments often undertake bailouts or industrial policies in order to promote aggregate output expansion. These policies have been increasingly used by advanced economies to navigate times of crisis. For example, during the financial crisis, the US government undertook financial sector support through, *inter alia*, its Troubled Asset Relief Program (TALP) and Capital Purchase Program (CPP). It also promoted stability in the real estate sector via its Home Affordable Modification Program (HAMP). During Covid, the government explicitly designed policies that targeted the health care sector (CARES Act), education (ESSER Act), and telecommunications (BEAD program).

In all cases, these selective interventions were temporary measures designed to navigate the economy through volatile macroeconomic conditions. A key benefit of the AUDI weights is that they take the state of the economy into account when measuring sectoral importance. Upstream sectors with many linkages should be subsidized in times of high productivity uncertainty. In contrast, downstream sectors that produce goods and services

Table 2: Time-Variation in Sectoral Importance for the Ten Largest Sectors

Sector	% Change in AUDI: $\epsilon(\hat{\mathbf{\Omega}}_t)/\epsilon(\hat{\mathbf{\Omega}}_{Normal})-1$			Upstreamness
	Dot Com	*Financial Crisis*	*Covid*	
Miscellaneous services	0.01	-0.14	-0.66	8.67
Wholesale trade	0.00	-0.10	-0.26	9.63
Credit intermediation	0.03	-0.08	-1.06	6.60
Other real estate	0.00	-0.11	-0.43	9.36
Construction	0.00	-0.13	-0.33	2.71
Admin. & support services	0.01	-0.23	-0.86	9.16
Housing	0.00	0.00	0.00	1.00
Insurance carriers	0.01	-0.10	-0.37	7.03
Other retail	0.00	-0.04	-0.06	1.63
Chemical products	0.00	-0.10	-0.33	7.63

Note: This table shows changes in a sector's AUDI weight over time, alongside its corresponding upstreamness measure. Upstreamness is the average distance to final use, defined as in Antràs et al. (2012). Industry indices have been shortened for readability. "Miscellaneous services" corresponds to "Miscellaneous professional, scientific, and technical services", "Credit intermediation" corresponds to "Federal Reserve banks, credit intermediation, and related activities", "Insurance carriers" corresponds to "Insurance carriers and related activities".

for final household consumption should be subsidized in times of high demand uncertainty — precisely because the alpha centrality of these sectors is relatively high during these times. Consequently, the model provides a rationale for promoting the health care, education, and telecommunications sectors during Covid *in addition* to the positive health-related externalities that these sectors may have provided during the pandemic. Conversely, it provides a rationale for *not* promoting traditionally important sectors (such as finance and insurance) when demand uncertainty is relatively high.

5 Conclusion

A key assumption in many models of production networks is to assume that firms operate under complete information of the large vector of shocks that hits the economy. In this paper, I study how sectoral productivity disturbances aggregate to economy-wide fluctuations when firms choose inputs under incomplete information about shocks. The uncertainty generated by incomplete information affects how firms' input purchases respond to cost changes, generates spillovers through input-output linkages, and has large implications for the macroeconomic impact of microeconomic shocks.

Theoretically, I show that the impact of sectoral productivity shocks on real GDP is shaped by the interaction between uncertainty and the economy's production network structure. Productivity shocks in upstream sectors have large effects on aggregate output when relative uncertainty about productivity to aggregate demand shocks is high. In contrast, shocks to downstream sectors become more influential in shaping the dynamics of output when aggregate demand uncertainty is high. Quantitatively, I find that incomplete information generates measures of sectoral importance that diverge significantly from traditional metrics (such as Domar weights), especially during economic downturns. Thus, the results emphasize the importance of accounting for economic uncertainty when designing industrial interventions during times of crises.

This study is only a first exploration within the context of how incomplete information shapes the microeconomic origins of aggregate fluctuations. I highlight two important implications of my analysis that I leave open for future research. First, I have documented that firms' input demand elasticity to input prices is a key determinant in shaping the aggregate impact of sectoral productivity shocks. Future work that directly measures these input elasticities at a granular level would be valuable in disciplining a large class of production networks models in which firms interact with their suppliers through markets. Second, this work has abstracted from the normative implications of incomplete information and the effect of shocks on household welfare. Studying optimal industrial policy and monetary stabilization rules in this context would be an interesting research avenue for the future.

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A Omitted Proofs

A.1 Proof of Lemma 1

Proof. We use the households' first-order condition to derive an expression for the real stochastic discount factor $(\mathcal{P}_t\mathcal{C}_t)^{-1}$ in terms of the money supply M_t. From the intratemporal Euler equation for consumption demand *vs.* labor supply, we can obtain an expression for the *frictionless* wage rate w_t^*:

$$w_t^* = \mathcal{P}_t\mathcal{C}_t \tag{43}$$

From Equation 5, the wage rate therefore satisfies the recursion:

$$w_t = (w_{t-1})^{\chi}(\mathcal{P}_t\mathcal{C}_t)^{1-\chi} \tag{44}$$

From the intertemporal Euler equation between consumption and money today, the cost of holding an additional dollar today equals the benefit of holding an additional dollar today plus the value of an additional dollar tomorrow:

$$\frac{1}{\mathcal{P}_t\mathcal{C}_t} = \frac{1}{M_t} + \beta\mathbb{E}_t\left[\frac{1}{\mathcal{P}_{t+1}\mathcal{C}_{t+1}}\right] \tag{45}$$

Further, from the intertemporal choice between bonds, the cost of saving an additional dollar today equals the nominal interest rate $1+i_t$ times the value of an additional dollar tomorrow:

$$\frac{1}{\mathcal{P}_t\mathcal{C}_t} = \beta(1+i_t)\mathbb{E}_t\left[\frac{1}{\mathcal{P}_{t+1}\mathcal{C}_{t+1}}\right] \tag{46}$$

Combining these two equations, we obtain that aggregate consumption follows:

$$\mathcal{P}_t\mathcal{C}_t = \iota_t M_t \tag{47}$$

where $\iota_t = i_t/(1+i_t)$. This equation implies that nominal expenditures are proportional to money balances. Note further that the Cobb-Douglas aggregator over final sectoral goods in household preferences implies that expenditure shares are constant:

$$P_{nt}C_{nt} = \gamma_n\mathcal{P}_t\mathcal{C}_t = \gamma_n\iota_t M_t \tag{48}$$

Finally, the nominal interest rate adjusts to clear the bond market. Substituting Equation

47 back into Equation 46, we obtain a recursion that interest rates must satisfy:

$$\frac{1+i_t}{i_t} = 1 + \beta \mathbb{E}_t \left[\frac{1+i_{t+1}}{i_{t+1}} \frac{M_t}{M_{t+1}} \right] \tag{49}$$

As money follows a random walk, solving this equation forward and employing the household's transversality condition, we obtain that:

$$\frac{1+i_t}{i_t} = 1 + \beta \exp\left\{ -\mu_M + \frac{1}{2}(\sigma^M_{t+1})^2 \right\} \left(1 + \sum_{i=1}^{\infty} \prod_{j=1}^{i} \beta \exp\left\{ -\mu_M + \frac{1}{2}(\sigma^M_{t+j+1})^2 \right\} \right) \tag{50}$$

which is deterministic, but depends on the full future path of monetary volatility.

□

A.2 Proof of Proposition 1

Proof. Substituting firms' demand functions into their production function, we obtain

$$Q_{nt} = z_{nt} \left(\frac{R_{nt}}{w_t} \right)^{\alpha_{nl}} \prod_{n' \in \mathcal{N}} \left(\frac{R_{nt}}{P_{n't}} \right)^{\alpha_{nn'}}$$

Multiplying by sectoral prices yields

$$R_{nt} = z_{nt} P_{nt} \left(\frac{R_{nt}}{w_t} \right)^{\alpha_{nl}} \prod_{n' \in \mathcal{N}} \left(\frac{R_{nt}}{P_{n't}} \right)^{\alpha_{nn'}}$$

Taking logarithms and cancelling revenues in this expression, we can write in in matrix form:

$$(\mathbf{I} - \mathbf{A}) \log P_t = -\log z_t + \operatorname{diag}(\alpha_{nl}) \log w_t$$

where we use the short-hand notation $\operatorname{diag}(x_n)$ to denote a diagonal matrix with elements x_n and $\log w_t$ is short-hand for $\log w_t \times \mathbf{1}$, where $\mathbf{1}$ is a $|\mathcal{N}|$ vector of ones. Since $1 - \sum_{n' \in \mathcal{N}} \alpha_{nn'} = \alpha_{nl}$ for all $n \in \mathcal{N}$, we can rewrite the above expression as:

$$(\mathbf{I} - \mathbf{A}) \log P_t = -\log z_t + (\mathbf{I} - \mathbf{A}) \log w_t$$

Using Lemma 1 and the fact that $\log w_t = \chi \log w_{t-1} + (1-\chi) \log w^*_t$ then yields the desired expression. Note that the invertibility of the matrix $\mathbf{I} - \mathbf{A}$ follows from the fact that it is an M-matrix, as its row-sums are positive and strictly less than unity (Johnson, 1982). □

A.3 Proof of Proposition 2

Proof. Firms maximize their risk-adjusted profits. They therefore solve:

$$\max_{X_{nn't}} = \mathbb{E}[(\mathcal{P}_t\mathcal{C}_t)^{-1}\Pi_{nt}\big|\mathcal{I}_{nn't}] \tag{51}$$

where $\mathcal{I}_{nn't}$ denotes the firm n's information set when purchasing inputs from sector n'. We have the following first-order condition:

$$\mathbb{E}\left[(\mathcal{P}_t\mathcal{C}_t)^{-1}\left(\alpha_{nn'}P_{nt}c_n z_{nt}\left(L_{nt}\right)^{\alpha_{nl}}\prod_{k\in\mathcal{N}/n'}X_{nk't}^{\alpha_{nk'}}X_{nn't}^{\alpha_{nn'}-1}-P_{n't}\right)\Bigg|\mathcal{I}_{nn't}\right]=0$$

which yields

$$\mathbb{E}\left[(\mathcal{P}_t\mathcal{C}_t)^{-1}\left(\alpha_{nn'}P_{nt}c_n z_{nt}\left(L_{nt}\right)^{\alpha_{nl}}\prod_{k\in\mathcal{N}/n'}X_{nk't}^{\alpha_{nk'}}X_{nn't}^{\alpha_{nn'}-1}\right)\Bigg|\mathcal{I}_{nn't}\right]=\mathbb{E}\left[(\mathcal{P}_t\mathcal{C}_t)^{-1}P_{n't}\big|\mathcal{I}_{nn't}\right]$$

Since $X_{nn't}$ is measurable with respect to $\mathcal{I}_{nn't}$, we can multiply both sides of this equation by $X_{nn'}$ to obtain

$$\mathbb{E}\left[(\mathcal{P}_t\mathcal{C}_t)^{-1}\left(\alpha_{nn'}P_{nt}c_n z_{nt}\left(L_{nt}\right)^{\alpha_{nl}}\prod_{n'\in\mathcal{N}}X_{nn't}^{\alpha_{nn'}}\right)\Bigg|\mathcal{I}_{nn't}\right]=X_{nn't}\mathbb{E}\left[(\mathcal{P}_t\mathcal{C}_t)^{-1}P_{n't}\big|\mathcal{I}_{nn't}\right]$$

We can write this as:

$$\mathbb{E}\left[(\mathcal{P}_t\mathcal{C}_t)^{-1}R_{nt}|\mathcal{I}_{nn't}\right]=X_{nn't}\mathbb{E}[(\mathcal{P}_t\mathcal{C}_t)^{-1}P_{n't}|\mathcal{I}_{nn't}] \tag{52}$$

where $R_{nt} = P_{nt}Q_{nt}$ denotes sectoral revenues. Using the fact that $(\mathcal{P}_t\mathcal{C}_t) = \iota_t M_t$ from Lemma 1, dividing by $\mathbb{E}[(\mathcal{P}_t\mathcal{C}_t)^{-1}P_{n't}|\mathcal{I}_{nn't}]$, and letting $\mathcal{I}_{nn't} = P_{n't}$ then yields the result. The demand function for labor is derived by solving:

$$\max_{L_{nt}}\Pi_{nt}$$

Simple manipulations then yield the result. This completes the proof. □

A.4 Proof of Proposition 3

Proof. The demand function for inputs is given by

$$X_{nn't} = \alpha_{nn'} \frac{\mathbb{E}_t\left[(P_t C_t)^{-1} R_{nt} \middle| P_{n't}\right]}{\mathbb{E}_t\left[(P_t C_t)^{-1} P_{n't} \middle| P_{n't}\right]} \tag{53}$$

Substituting into the firm's production function 6 and multiplying by P_{nt} yields:

$$R_{nt} = z_{nt} P_{nt} \left(\frac{R_{nt}}{w_{nt}}\right)^{\alpha_{nl}} \prod_{n' \in \mathcal{N}} \left(\frac{\mathbb{E}_t\left[(P_t C_t)^{-1} R_{nt} \middle| P_{n't}\right]}{\mathbb{E}_t\left[(P_t C_t)^{-1} P_{n't} \middle| P_{n't}\right]}\right)^{\alpha_{nn'}} \tag{54}$$

We can combine market clearing with firm's optimal demand schedules for inputs (Equation 53 to obtain:

$$R_{nt} = P_{nt} C_{nt} + \sum_{n' \in \mathcal{N}} \alpha_{n'n} P_{nt} \frac{\mathbb{E}_t\left[(P_t C_t)^{-1} R_{n't} \middle| P_{nt}\right]}{\mathbb{E}_t\left[(P_t C_t)^{-1} P_{nt} \middle| P_{nt}\right]} \tag{55}$$

We can then combine equations 54 and 55 with the expression for nominal expenditures 47 and 48 to obtain a characterization of revenues and prices in terms of exogenous variables:

$$R_{nt} = \tilde{c}_{nt} z_{nt} P_{nt} \left(\frac{R_{nt}}{M_t^{1-\chi}}\right)^{\alpha_{nl}} \prod_{n' \in \mathcal{N}} \left(\frac{\mathbb{E}_t\left[M_t^{-1} R_{nt} \middle| P_{n't}\right]}{\mathbb{E}_t\left[M_t^{-1} P_{n't} \middle| P_{n't}\right]}\right)^{\alpha_{nn'}} \tag{56}$$

$$R_{nt} = \gamma_n \iota_t M_t + \sum_{n' : n \in \mathcal{N}} \alpha_{n'n} P_{nt} \frac{\mathbb{E}_t\left[M_t^{-1} R_{n't} \middle| P_{nt}\right]}{\mathbb{E}_t\left[M_t^{-1} P_{nt} \middle| P_{nt}\right]} \tag{57}$$

where $\tilde{c}_{nt} = (\iota_t)^{\alpha_{nl}(1-\chi)} w_{t-1}^{\alpha_{nl}\chi}$, w_{t-1}^{χ} is independent of time t shocks, and ι_t follows a deterministic sequence. This proves the claim. □

A.5 Proof of Proposition 4

Proof. Observe that $\log P_{1t}$ is log-normal, being the product of log-normal variables. Using properties of conditional expectations for log-normal variables, we have that

$$\log \mathcal{C}_t = cons + \frac{\text{Cov}(\log M_t, \log P_{1t})}{\text{Var}(\log P_{1t})} \log P_{1t} - \log P_{1t}$$

Using the derivation of $\log P_{1t}$ in Equation 20, we have:

$$\log \mathcal{C}_t = cons + \frac{(1-\chi)(\sigma_t^M)^2}{(1-\chi)^2(\sigma_t^M)^2 + (\sigma_{1t}^z)^2} \log P_{1t} - \log P_{1t}$$

Collecting the terms involving $\log P_{1t}$ and dividing by $(\sigma_t^M)^2$ then yields the claim. $\square$

A.6 Proof of Theorem 1

We begin by proving the following Lemma. To simplify notation, we let $\hat{x} := \log x$.

Lemma 3. *Around, $\delta \to 0$, the dynamics of the economy are characterized by the following system of equations:*

$$(1-\alpha_{nl})\hat{R}_{nt} = cons + \hat{z}_{nt} + \hat{P}_{nt} - \alpha_{nl}(1-\chi)(\hat{M}_t) - \sum_{n' \in \mathcal{N}} \alpha_{nn'}\omega_{nn't}\hat{P}_{n't} \tag{58}$$

$$\hat{R}_{nt} = s_{nc}\hat{M} + \sum_{n' \in \mathcal{N}} s_{nn'}(1-\omega_{n'nt})\hat{P}_{nt} \tag{59}$$

where the constant is independent of time t shocks.

Proof. By definition of $\mathbf{\Omega}_t$, firms use log-linear demand schedules:

$$X_{nn't} = \tilde{\omega}_{nn't}P_{n't}^{-\omega_{nn't}} \tag{60}$$

where $\omega_{nn't}$ can potentially depend on the realization of $P_{n't}$, but $\tilde{\omega}_{nn't}$ is independent of time t shocks. For firms' flexible inputs, we have:

$$X_{nn't} = \alpha_{nn'}\frac{R_{nt}}{P_{n't}} \tag{61}$$

Substituting Equations 60 and 61 into the production function 6, taking logarithms and re-arranging then yields Equation 58 directly. To derive Equation 59, we may substitute the demand for rigid inputs into the market clearing equation 11. This yields:

$$R_{nt} = \gamma_n \iota_t M_t + P_{nt}\sum_{n' \in \mathcal{N}} \tilde{\omega}_{n'nt}P_{nt}^{-\omega_{n'nt}} \tag{62}$$

Log-linearizing this expression yields around $\delta = 0$ yields

$$\hat{R}_{nt} = cons + s_{nc}\hat{M}_t + \sum_{n' \in \mathcal{N}} s_{nn'}(1-\omega_{nn't})\hat{P}_{nt} \tag{63}$$

where

$$s_{nc} = \frac{\gamma_n}{\sum_{n' \in \mathcal{N}} \alpha_{n'n}\lambda_{n'} + \gamma_n} = \frac{\gamma_n}{\lambda_n} \quad \text{and} \quad s_{nn'} = \frac{\alpha_{n'n}\lambda_{n'}}{\sum_{n' \in \mathcal{N}} \alpha_{n'n}\lambda_{n'} + \gamma_n} = \frac{\alpha_{n'n}\lambda_{n'}}{\lambda_n} \tag{64}$$

and where λ_n are the full-information Domar weights given by:

$$\lambda = (\mathbf{I} - \mathbf{A}')^{-1}\gamma \tag{65}$$

□

We can now prove the Theorem.

Proof. For ease of notation, define $\mathbf{A}(\omega_t) = \mathbf{A} \odot \mathbf{\Omega}_t$ and $\mathbf{S}(\omega_t) = [s_{nn'}(1 - \omega_{n'nt})]$. Observe that Equation 59 can be written in matrix form as:

$$\hat{R}_t = cons + (\mathbf{I} - \mathbf{S})\hat{M}_t + \operatorname{diag}(\mathbf{S}(\omega_t)\mathbf{1})\hat{P}_t \tag{66}$$

Next, observe that Equation 58 can be expressed as:

$$\operatorname{diag}(\mathbf{A1})\hat{R}_t = cons + \hat{z}_t + (\mathbf{I} - \mathbf{A}(\omega_t))\hat{P}_t - (1-\chi)(\mathbf{I} - \mathbf{A})\hat{M}_t \tag{67}$$

We may substitute for $\hat{R}_t$ to obtain:

$$\begin{aligned} \operatorname{diag}(\mathbf{A1})\left[(\mathbf{I} - \mathbf{S})\hat{M} + \operatorname{diag}(\mathbf{S}(\omega_t)\mathbf{1})\hat{P}_t\right] = \\ cons + \hat{z}_t + (\mathbf{I} - \mathbf{A}(\omega_t))\hat{P}_t - (1-\chi)(\mathbf{I} - \mathbf{A})\hat{M}_t \end{aligned} \tag{68}$$

Rearranging the above expression then yields:

$$\begin{aligned} \left[\mathbf{I} - \mathbf{A}(\omega_t) - \operatorname{diag}((1-\alpha_{nl}) \times \sum_{n \in \mathcal{N}} s_{nn'}(1 - \omega_{nn't}))\right]\hat{P}_t = \\ cons - \hat{z}_t + \operatorname{diag}((1-\alpha_{nl})s_{nc} + (1-\chi)\alpha_{nl})\hat{M}_t \end{aligned} \tag{69}$$

and note that, by assumption, the matrix multiplying $\hat{P}_t$ is invertible. We can also express as 66 as

$$\hat{R}_t = cons + \operatorname{diag}(s_{nc})\hat{M}_t + \operatorname{diag}\left(\sum_{n' \in \mathcal{N}} s_{nn'}(1 - \omega_{n'nt})\right)\hat{P}_t \tag{70}$$

Finally, note that real GDP is given by

$$\hat{\mathcal{C}}_t = \log(\iota_t M_t) - \hat{\mathcal{P}}_t \tag{71}$$

where the ideal price index is given by

$$\hat{\mathcal{P}}_t = \gamma' \hat{P}_t \tag{72}$$

Substituting for $\hat{P}_t$ from 69 into the real GDP equation 71 then yields the desired result. $\square$

A.7 Proof of Proposition 5

Proof. We let

$$\mathbf{D}(\omega_t^*) = \text{diag}\left((1-\alpha_{nl}) \times \sum_{n \in \mathcal{N}} s_{nn'}(1-\omega_t^*) \right)$$

We first note that we can write

$$\mathbf{I} - \omega_t^* \mathbf{A} - \mathbf{D}(\omega_t^*) = (\mathbf{I} - \mathbf{D}(\omega_t^*)) \left(\mathbf{I} - (\mathbf{I} - \mathbf{D}(\omega_t^*))^{-1} \omega_t^* \mathbf{A} \right) \tag{73}$$

where $\mathbf{I} - \mathbf{D}(\omega^*)$ is invertible for $\omega_t^* \in [0,1]$ given the assumption that $\sum_{n' \in \mathcal{N}} \alpha_{nn'} < 1$ and $\sum_{n' \in \mathcal{N}} s_{nn'} \leq 1$.

Next, we show that $\mathbf{I} - (\mathbf{I} - \mathbf{D}(\omega^*))^{-1} \omega_t^* \mathbf{A}$ is an M-matrix. To this end, observe that the diagonal elements of this matrix are given by

$$1 - \frac{\omega_t^* \alpha_{nn}}{1 - (1-\omega_t^*) \sum_{n' \in \mathcal{N}} \alpha_{nn'} \sum_{n' \in \mathcal{N}} s_{nn'}} \tag{74}$$

We therefore require that

$$1 - \omega_t^* \alpha_{nn} - (1-\omega_t^*) \sum_{n' \in \mathcal{N}} \alpha_{nn'} \sum_{n' \in \mathcal{N}} s_{nn'} > 0 \tag{75}$$

Note that this expression is strictly positive for $\omega_t^* = 0$ and $\omega_t^* = 1$ (given the assumption that $\sum_{n' \in \mathcal{N}} \alpha_{nn'} < 1$ and $\sum_{n' \in \mathcal{N}} s_{nn'} \leq 1$). Since this is a linear function of ω_t^* it is therefore strictly positive for all $\omega^* \in [0,1]$.

Next, observe that all elements of $(\mathbf{I} - \mathbf{D}(\omega_t^*))^{-1} \omega_t^* \mathbf{A}$ are weakly positive. Moreover, the sum of each row of this matrix is given by

$$\frac{\omega_t^* \sum_{n' \in \mathcal{N}} \alpha_{nn'}}{1 - (1-\omega_t^*) \sum_{n' \in \mathcal{N}} \alpha_{nn'} \sum_{n' \in \mathcal{N}} s_{nn'}} < 1 \tag{76}$$

By the Gershgorin Circle Theorem, the norm of the principal eigenvalue of this matrix is less than unity. Hence, $(\mathbf{I} - \mathbf{D}(\omega_t^*))^{-1} \omega_t^* \mathbf{A}$ is an M-matrix. Its inverse can therefore be written as the power sum:

$$\left[\mathbf{I} - (\mathbf{I} - \mathbf{D}(\omega_t^*))^{-1}\mathbf{A}(\omega_t^*)\right]^{-1} = \sum_{k=0}^{\infty} \left(\omega_t^*(\mathbf{I} - \mathbf{D}(\omega_t^*))^{-1}\mathbf{A}\right)^k \tag{77}$$

Moreover, the matrix $\omega_t^*(\mathbf{I} - \mathbf{D}(\omega_t^*))^{-1}$ is diagonal with elements given by

$$\frac{\omega_t^*}{1 - (1 - \omega_t^*)x_n} \tag{78}$$

for $x_n < 1$. This ratio is therefore increasing for all $\omega_t^* \in \mathbb{R}$. Hence, the elements of this matrix are increasing in ω_t^*. This completes the proof. □

A.8 Proof of Proposition 6

Proof. We let

$$\mathbf{D}(\omega_t^*) = \text{diag}\left((1 - \alpha_{nl}) \times \sum_{n \in \mathcal{N}} s_{nn'}(1 - \omega_t^*)\right)$$

Observe that statements two and three are immediate if we can show that AUDI weights are element-wise increasing in ω_t^*. In order to show that the AUDI weights are element-wise increasing, it suffices to show that the inverse of $\mathbf{E} \equiv \mathbf{I} - \mathbf{A} \odot \mathbf{\Omega}_t - \mathbf{D}(\mathbf{\Omega}_t)$ is element-wise increasing.

First, observe that $\mathbf{E}$ is an M-matrix (Johnson, 1982) for $\omega^* \in [0, 1]$, since the sum of its rows are strictly positive and strictly less than unity for $\omega* \in [0, 1]$. Hence, by Gershgorin Circle Theorem, the norm of the principal eigenvalue of this matrix is less than unity.

Since $\mathbf{E}$ is an M-matrix, we can write its inverse as:

$$\mathbf{E}^{-1} = \sum_{k=0}^{\infty} (\mathbf{A} \odot \mathbf{\Omega}_t - \mathbf{D}(\mathbf{\Omega}_t))^k \tag{79}$$

It thus suffices to show that $\mathbf{A} \odot \mathbf{\Omega}_t - \mathbf{D}(\mathbf{\Omega}_t)$ is element-wise increasing in ω^*. All off-diagonal elements of the matrix are clearly increasing in ω^*. All diagonal elements are increasing in ω^* if and only if $\alpha_{nn} \geq (1 - \alpha_{nl})(1 - s_{nc})$. The result then follows. □

A.9 Proof of Corollary 1

Proof. The total revenues of each sector are given by $P_{nt}C_{nt} = \gamma_n \mathcal{P}_t \mathcal{C}_t$. Hence, the Domar weight for each sector is given by γ_n. The result then follows from Proposition 1. □

A.10 Proof of Corollary 2

Proof. The result follows by noting that $s_{11} = \alpha_1$ in the roundabout network economy. Substituting for s_{11} into the definition of $\epsilon(\boldsymbol{\Omega}_t)$ in Proposition 5 yields the AUDI weight of the roundabout sector. The effect of productivity and demand on output follows from Theorem 1. $\square$

A.11 Proof of Corollary 3

Proof. We use Lemma 3 of the Appendix to derive the equilibrium dynamics of the system. For ease of notation, we denote $\hat{x} = \log x$. Using Equations 58 and 59 for sector 1, we obtain the following:

$$\hat{P}_{1t} = cons + -\hat{z}_{1t} + (1-\chi)\hat{M}_t \tag{80}$$

For sector $n \in \{2, \dots, N-1\}$, we obtain:

$$\hat{R}_{nt} = cons + (1-\omega_{nt})\hat{P}_{nt} \tag{81}$$

which implies that we can solve for prices as:

$$\hat{P}_{nt} = cons + \frac{1}{1-\alpha_n(1-\omega_{nt})}\left[\hat{z}_{nt} - (1-\alpha_n)(1-\chi)\hat{M} - \alpha_n\omega_{n-1t}\hat{P}_{n-1t}\right] \tag{82}$$

For sector N, we have:

$$\hat{P}_{Nt} = cons + \alpha_N\hat{M}_t - \hat{z}_{Nt} + (1-\alpha_N)(1-\chi)\hat{M}_t + \alpha_N\omega_{N-1t}\hat{P}_{N-1t} \tag{83}$$

Moreover, from Corollary 1, we have:

$$\hat{\mathcal{P}}_t + \hat{\mathcal{C}}_t = \hat{P}_{Nt} + \hat{\mathcal{C}}_t = cons + \hat{M}_t \tag{84}$$

This implies that $\mathcal{C}_t$ is given by:

$$\hat{\mathcal{C}}_t = cons + \chi(1-\alpha_N)\hat{M}_t + \hat{z}_{Nt} - \alpha_N\omega_{N-1t}\hat{P}_{N-1t} \tag{85}$$

Observe that the AUDI weight of sector N is equal to unity, as claimed. Solving this difference equation backwards using $\hat{P}_1$ as a terminal condition then yields the AUDI weight recursion. The effect of productivity shocks on output follows by the definition of AUDI weights. The effect of an aggregate demand shock on output then follows by straightforward calculation. $\square$

A.12 Proof of Lemma 2

Proof. First, the expression for revenues follows directly from Theorem 1 and is derived in Appendix A.6 (Equation 70). Next, note that given the assumption of log-normality for the shocks $\{z_{nt}\}_{n\in\mathcal{N}}$ and M_t, prices and revenues are also jointly log-normally distributed under the assumption that $\mathbf{\Omega}_t$ is independent of P_t. We will now verify that an equilibrium in which revenues and prices are log-normally distributed exists, with an optimal responsiveness matrix $\mathbf{\Omega}_t$ that is independent of shock realizations.

Under the assumption of log-normality, the demand schedule for inputs chosen under incomplete information satisfies

$$X_{nn't} = \alpha_{nn'} \frac{\exp\left\{\mu_{R_{nt}|P_{n't}} + \frac{1}{2}\sigma^2_{\mathcal{P}_t\mathcal{C}_t|P_{n't}} + \frac{1}{2}\sigma^2_{R_{nt}|P_{n't}} + \sigma_{\mathcal{P}_t\mathcal{C}_t,R_{nt}|P_{n't}}\right\}}{\exp\left\{\mu_{P_{n't}|P_{n't}} + \frac{1}{2}\sigma^2_{\mathcal{P}_t\mathcal{C}_t|P_{n't}} + \frac{1}{2}\sigma_{P_{n't}|P_{n't}} + \sigma_{\mathcal{P}_t\mathcal{C}_t,P_{n't}|P_{n't}}\right\}} \tag{86}$$

where $\sigma_{X,Y|Z} = \mathrm{Cov}(\log X, \log Y \,|\, \log Z)$ and $\mu_{X|Z} = \mathbb{E}[\log \hat{X} \,|\, \log Z]$. Using standard Gaussian formulas for conditional expectations, we have:

$$\mu_{R_{nt}|P_{n't}} = \mu_{R_{nt}} + \frac{\sigma_{R_{nt},P_{n't}}}{\sigma^2_{P_{n't}}}(\log P_{n't} - \mu_{P_{n't}}) \tag{87}$$

$$\mu_{P_{n't}|P_{n't}} = \log \hat{P}_{n't} \tag{88}$$

All other variances and covariances are independent of $\log P_{n't}$. Hence, we have that

$$-\frac{\log X_{nn't}}{\log P_{n't}} = 1 - \frac{\mathrm{Cov}(\log R_{nt}, \log P_{n't})}{\mathrm{Var}(\log P_{n't})} \tag{89}$$

which is independent of the realization of prices P_{nt}. This verifies the initial conjecture of a responsiveness matrix independent of P_t with log-normally distributed prices and revenues.

□

A.13 Proof of Theorem 2

Proof. I first prove the first statement of the Theorem. I let $\hat{x} = \log x$ for notational simplicity. From Equations 58 and 59, the dynamics of the economy are given by:

$$\left(\sum_{n'\in\mathcal{N}} \alpha_{nn'}\right) \hat{R}_{nt} = cons + \hat{z}_{nt} + \hat{P}_{nt} - \alpha_{nl}(1-\chi)\hat{M}_t + \sum_{n'\in\mathcal{N}} \alpha_{nn'}\mathbb{E}[\hat{R}_{nt}|\hat{P}_{n't}] \tag{90}$$

$$\hat{R}_{nt} = cons + s_{nc}\hat{M}_t + \sum_{n' \in \mathcal{N}} s_{nn'}\mathbb{E}[\hat{R}_{n't}|\hat{P}_{nt}] \tag{91}$$

I now suppress dependence on time indices for notational simplicity. We guess that there exists a solution to the above system of equations in which $\hat{R}_{nt}$ is linear in shocks:

$$\hat{R}_n = cons + \chi^R_{nM}\hat{M} + \sum_{n' \in \mathcal{N}} \chi^R_{nn'}\hat{z}_{n'} \tag{92}$$

for scalars $\{\chi^R_{nn'}\}_{n' \in \mathcal{N}}$ and χ^R_{nM}. We similarly guess that prices are linear in shocks:

$$\hat{P}_n = cons + \chi^P_{nM}\hat{M} + \sum_{n' \in \mathcal{N}} \chi^P_{nn'}\hat{z}_{n'} \tag{93}$$

We further assume that $\chi^P_{nM} > 0$ for all $n \in \mathcal{N}$, a guess that will be verified later. Observe that due to the log-normality of the aggregate shocks, we have that:

$$\mathbb{E}[\hat{R}_n|\hat{P}_{\tilde{n}}] = \frac{\text{Cov}(\hat{R}_n, \hat{P}_{\tilde{n}})}{\text{Var}(\hat{P}_{\tilde{n}})}\hat{P}_{\tilde{n}} \tag{94}$$

for $n, \tilde{n} \in \mathcal{N}$. Further, observe that if $u_n \to 0$, then:

$$\mathbb{E}[\hat{R}_n|\hat{P}_{\tilde{n}}] = \frac{\chi^R_{nM}}{\chi^P_{\tilde{n}M}}P_{\tilde{n}} \tag{95}$$

We substitute our guess in Equation 91 and collect coefficients on $\hat{M}$:

$$\chi^R_{nM} = s_{nc} + \sum_{n' \in \mathcal{N}} s_{nn'}\chi^R_{n'M} \tag{96}$$

We can rewrite this in matrix form as:

$$\chi^R_M = (\mathbf{I} - \mathbf{S})1 + \mathbf{S}\chi^R_M \tag{97}$$

$$\chi^R_M = (\mathbf{I} - \mathbf{S})^{-1}(\mathbf{I} - \mathbf{S})1 = 1 \tag{98}$$

where χ^R_M is the N-sized vector of χ^R_{nM}. Hence, $\chi^R_{nM} = 1$ for all $n \in \mathcal{N}$ is the unique solution to this equation when $u_{nt} \to 0$. We now need to solve for χ^P_{nM} to obtain the firms' demand schedules. Using Equation 90 and χ^R_{nM}, we obtain:

$$0 = \chi^P_{nM} - \alpha_{nl}(1 - \chi) - \sum_{n' \in \mathcal{N}} \alpha_{nn'}\chi^P_{n'M} \tag{99}$$

Solving this system of equations yields:

$$\chi_M^P = 1 - \chi \tag{100}$$

where χ_M^P is the N-sized vector of χ_{nM}^P. Note this vector has strictly positive elements given the assumption that $\chi < 1$. Hence, using the definition of a demand schedule, we have: $\omega_{nn'} = 1 - \frac{1}{\chi_{n'M}^P} = -\frac{\chi}{1-\chi}$. The linearity of revenues and prices then follows from Theorem 1. Finally, note that

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{M}_t} = 1 - \gamma' \hat{P}_t = 1 - (1-\chi) = \chi \tag{101}$$

This proves the first statement of the Theorem.

We now prove the second statement of the Theorem. We again guess that there exists a solution to the system of equations 90 and 91:

$$\hat{R}_n = cons + \chi_{nM}^R \hat{M} + \sum_{n' \in \mathcal{N}} \chi_{nn'}^R \hat{z}_{n'} \tag{102}$$

for scalars $\{\chi_{nn'}^R\}_{n' \in \mathcal{N}}$ and χ_{nM}^R. We similarly guess that prices are linear in shocks:

$$\hat{P}_n = cons + \chi_{nM}^P \hat{M} + \sum_{n' \in \mathcal{N}} \chi_{nn'}^P \hat{z}_{n'} \tag{103}$$

We now guess that $\chi_{nn'}^R = 0$ for all $n, n' \in \mathcal{N}$. Using the log-linearity of the shocks, this implies that:

$$\mathbb{E}[\hat{R}_n | \hat{P}_{\tilde{n}}] = \frac{\chi_{nM}^R \chi_{\tilde{n}M}^P \sigma_M^2}{\mathrm{Var}\left(\sum_{n' \in \mathcal{N}} \chi_{\tilde{n}n}^P z_{n'}\right) + \chi_{\tilde{n}M} \hat{M}} \tag{104}$$

Note that if $u_n \to \infty$, $\mathbb{E}[\hat{R}_n | \hat{P}_{\tilde{n}}] = 0$ if $\chi_{\tilde{n}n}^P > 0$, a condition which we will verify momentarily. Using Proposition 2, we observe that $\mathbb{E}[\hat{R}_n | \hat{P}_{\tilde{n}}] = 0$ is equivalent to $\omega_{nn'} = 1$ for all $n, n' \in \mathcal{N}$. Furthermore, recall that revenues are given by Equation 66, which can be written as:

$$\hat{R}_t = [\mathbf{I} - \mathrm{diag}(b_1, \ldots, b_N)] \hat{M}_t + \mathrm{diag}(c_1, \ldots, c_N) \hat{P}_t \tag{105}$$

where $b_n = \sum_{n' \in \mathcal{N}} s_{nn'}$ and $c_n = \sum_{n' \in \mathcal{N}} s_{nn'}(1 - \omega_{n'nt})$. If $\omega_{nn't} = 1$, revenues are indeed independent of productivity shocks. This verifies the guess that $\chi_{nn'}^R = 0$.

We now need to verify that $\chi_{n'n}^P > 0$, which will prove that $\mathbb{E}[\hat{R}_n | \hat{P}_n] = 0$. From Theorem 1, the elasticity of output to productivity shocks is given by:

$$(\mathbf{I} - \mathbf{A})^{-1} = \sum_{k=0}^{\infty} \mathbf{A}^k \tag{106}$$

Hence, the pass-through of a productivity shock from sector n to all other sectors is non-zero if, for each n', there exists some finite $k \in \mathbb{N}$ such that $[\mathbf{A}^k]_{nn'} > 0$. By Theorem 8.3.5 in Meyer (2023), this condition is satisfied if and only if the matrix $\mathbf{A}$ is irreducible. Finally, note that

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{z}_t}' = -\gamma' \hat{P}_t = \gamma'(\mathbf{I} - \mathbf{A})^{-1} \tag{107}$$

This proves the second statement of the Theorem. □

A.14 Proof of Corollary 4

Proof. From Lemma 2, one can express prices $\log P_t$ and revenues $\log R_t$ as a linear combination of shocks in any log-linear equilibrium. Hence, optimal responsiveness is a ratio of linear combinations of underlying shock variances. One can therefore divide both numerator and denominator by a constant and leave underlying responsiveness unchanged. Hence, one can always normalize one variance without affecting equilibrium responsiveness. □

A.15 Proof of Proposition 7

Proof. Substituting the first-order response of revenues into Equation 38, we obtain:

$$\omega^* = 1 - \alpha_{nn}(1 - \omega^*_{nt}) - (1 - \alpha_{nn}) \frac{\mathrm{Cov}(\log M_t, \log P_{nt})}{\mathrm{Var}(\log P_{nt})} \tag{108}$$

From Corollary 2, we have that

$$\log P_{nt} = cons + \frac{1}{1 + \alpha_{nn}(1 - \omega_{nt})} \left[-(1 - \alpha_{nn})^{-1} \hat{z}_{nt} + (1 + \alpha_{nn} - \chi) \log M_t \right] \tag{109}$$

Substituting for $\log P_{nt}$ into Equation 108 yields the desired expression after straightforward calculations. □

B Model Extensions

B.1 Equivalence to Bilateral Contracting

The model in the main text assumed that input purchases are made under incomplete information in a spot market. Below, I show that the informational structure in the main text arises endogenously in a model in which firms negotiate input purchases bilaterally ex-ante. Formally, these *cost-contingent* bilateral contracts implement demand schedules that are equivalent to Proposition 2.

Primitives. To make the model amenable to this contracting interpretation, I assume that there exists a continuum of identical firms in each sector n, indexed by $i \in [0,1]$. Firms in each sector sell to a *market maker* in their own sector, who aggregates production in its respective sector. This market maker then contracts with firms in other sectors. Technology remains otherwise unchanged and labor continues to be purchased in spot markets.

Contracting under Uncertainty. A firm $i \in [0,1]$ in sector n negotiates bilaterally with a market maker in sector n to purchase inputs $X_{in,n',t}$. As all firms are symmetric, the cost of market maker n to produce each input is simply by given the common price charged by all firms in that sector, P_{nt}.

Prior to the realization of $\{z_{nt}\}_{n\in\mathcal{N}}$ and M_t, the firm $i \in [0,1]$ and the market maker negotiate a cost-contingent transfer $\tau_{in,n't}(P_{n't})$ as well as cost-contingent input delivery $X_{in,n',t}(P_{n't})$ via Nash bargaining. The firm and supplier therefore choose transfers and input deliveries to maximize the generalized joint surplus:

$$\left(\mathbb{E}_t\left[\frac{1}{\mathcal{P}_t\mathcal{C}_t}\left(\tilde{\Pi}_{in,t}(\mathbf{P}_t)\right)\right]\right)^{\beta}\left(\mathbb{E}_t\left[\frac{1}{\mathcal{P}_t\mathcal{C}_t}\left(\tau_{in,n',t}(P_{n't}) - P_{n't}X_{in,n',t}(P_{n't})\right)\right]\right)^{1-\beta} \tag{110}$$

where β parameterizes the firm's bargaining power. The expectation conditions on the past shocks $\{z_{nt-1}\}_{n\in\mathcal{N}}$ and M_{t-1}. The variable $\tilde{\Pi}_{in,t}$ denotes the firm's nominal profits post-transfer:

$$\tilde{\Pi}_{in,t}(\mathbf{P}_t) = P_{nt}\left[Q_{in,t}(\mathbf{P}_t)\right] - w_t L^d_{in,t} - \sum_{k\in\mathcal{N}} \tau_{in,k,t}(P_{k't}) \tag{111}$$

Note that profits depend on the entire vector of costs $\mathbf{P}_t$ through input deliveries (thereby changing the quantity produced $Q_{in,t}$) and transfer payments made to final good producers. However, the contract between firm i in sector n and the market maker in sector n' is only contingent on the market maker's costs (and not the on the costs of other sectors). This is an intuitive theoretical and practical restriction, which I discuss further below: the terms of trade of the contract do not depend on the costs of *other* suppliers.

We can solve this contracting problem for the *functions* $X_{in,n',t}(P_{n't})$ and $\tau_{in,n',t}(P_{n',t})$ using variational methods. The proposition below characterizes the optimal cost-contingent input deliveries:

Proposition 8. *The optimal contract satisfies*

$$X_{in,n',t}(P_{n't}) = \alpha_{nn'}\frac{\mathbb{E}_t[(\mathcal{P}_t\mathcal{C}_t)^{-1}R_{nt}|P_{n't}]}{\mathbb{E}_t[(\mathcal{P}_t\mathcal{C}_t)^{-1}P_{n't}|P_{n't}]} \tag{112}$$

Proof. See Appendix C.1 □

Hence, this contracting interpretation implements the same demand function as in Proposition 2. This formally shows that negotiating input-deliveries on an ex-ante basis is equivalent to purchasing inputs in a spot market under the informational structure considered in the main text. Intuitively, the optimal contract delivers the optimal input choice upon observation of $P_{n't}$ on an *ex-post* basis. This result can be understood through the following "re-contracting" principle: if the optimal contract did not satisfy Proposition 8, both parties would want to recontract upon observation of the market maker's costs $P_{n't}$. Moreover, observe that the allocations are independent of firms' bargaining power. This is because the Nash bargaining solution is constrained efficient in the Pareto sense. The transfer function, however, *will* depend on bargaining weights, but characterizing the transfer function only affects the distribution of profits and is not needed to solve for real allocations.

Discussion of Contracting Assumptions. The contingency assumption on contracts in this section are based on theoretical and empirical grounds. The preceding discussion assumed that the contract is contingent on the supplier's costs, but not on other stochastic variables (such as the supplier's revenues).[21] This assumption is meant to capture the idea that the supplier's costs can be verified at the time of input delivery, while other random variables (such as the firm's revenues from the use of those inputs) are still uncertain. Of course, *in theory*, there is nothing that prevents the firm and the supplier from specifying a fully contingent contract ex-ante which would allow the parties to implement the perfect information allocation ex-post. Such contracts, however, are complex and costly to write (Battigalli and Maggi, 2002) and are therefore not commonly observed in practice. Indeed, the "vast majority" of contracts in the building and construction industry, for example, are "simple" contracts of a *cost-contingent* nature (Bajari and Tadelis, 2001). Moreoever, the literature suggests that simple cost-contingent contracts are widely used in other sectors, such as air force engine procurement (Crocker and Reynolds, 1993), defense (Hiller and Tollison, 1978), or the Indian software industry (Banerjee and Duflo, 2000).

B.2 Conditioning on Additional Prices

In this subsection, I consider equilibrium dynamics when firms can condition their input choices on additional input prices. Concretely, suppose firms also observe a subset of input prices $\mathcal{P}_n \subset \mathcal{N}$ when making their input choices. We let $\mathcal{I}_{nt}$ denote the set $\{P_{nt}\}_{n \in \mathcal{P}_n}$. Following the analysis in the main text, it is straightforward to compute firms' optimal choice of inputs:

$$X_{nn't} = \alpha_{nn'} \frac{\mathbb{E}_t[(M_t)^{-1} R_{n't} | P_{n't}, \mathcal{I}_{nt}]}{\mathbb{E}_t[(M_t)^{-1} P_{n't} | P_{n't}, \mathcal{I}_{nt}]} \tag{113}$$

[21] Note that the contract can be contingent on the firm's and the supplier's *beliefs* about these variables.

Similarly, following Proposition 3, we can express equilibrium revenues and prices as the system of equations:

$$R_{nt} = \tilde{c}_{nt} z_{nt} P_{nt} \left(\frac{R_{nt}}{M_t^{1-\chi_n}} \right)^{\alpha_{nl}} \prod_{n' \in \mathcal{N}} \left(\frac{\mathbb{E}_t \left[M_t^{-1} R_{nt} \middle| P_{n't}, \mathcal{I}_{nt} \right]}{\mathbb{E}_t \left[M_t^{-1} P_{n't} \middle| P_{n't}, \mathcal{I}_{nt} \right]} \right)^{\alpha_{nn'}} \tag{114}$$

$$R_{nt} = \gamma_n \iota_t M_t + \sum_{n' \in \mathcal{N}} \alpha_{n'n} P_{nt} \frac{\mathbb{E}_t \left[M_t^{-1} R_{n't} \middle| P_{n't}, \mathcal{I}_{n't} \right]}{\mathbb{E}_t \left[M_t^{-1} P_{nt} \middle| P_{n't}, \mathcal{I}_{n't} \right]} \tag{115}$$

where $\tilde{c}_t = (\iota_t)^{\alpha_{nl}(1-\chi)} (w_{t-1})^{\alpha_{nl}\chi}$.

The challenge with this formulation is that firms choose inputs on the basis of a *vector* of endogenous public signals. The informativeness of this signal is determined by firms' actions, which in turn depend on the realization of the signal to begin with. Nevertheless, we may obtain a variant of Theorem 2 in this more general setting. The following Proposition shows that the dynamics for real GDP in this economy are equivalent to the economy in the main text when aggregate demand uncertainty becomes large.

Proposition 9. *As aggregate demand uncertainty becomes large, so that $u_{nt} \to 0$ for all $n \in \mathcal{N}$, the stochastic process for real GDP is equivalent to an economy in which a firms' input-specific information set only includes the price of that input.*

Proof. See Appendix C.3. □

This proposition shows that the dynamics of real GDP derived in Theorem 2 generalize to settings that feature richer information structures. Of course, for simple network structures, such as the roundabout economy of Figure 3b, the results in the main text are without loss of generality because the firm is already conditioning its purchases on the price of all inputs in the economy.

B.3 Extension to Monopolistic Competition

I now show that the model is isomorphic to one with monopolistic competition in which the government levies an ad-valorem subsidy to undo the monopolistic distortion. Hence, the price formulation of the market structure is immaterial for the main results.

Primitives. I only change the model's market structure, and leave the household side of the economy and informational structure unchanged. The model follows standard micro-foundations in modeling production networks with firm market power (e.g. Basu, 1994; Afrouzi and Bhattarai, 2023; La'O and Tahbaz-Salehi, 2022). There exist N sectors with input-output linkages, indexed by $n \in \mathcal{N} = \{1, \ldots, N\}$. In each sector $n \in \mathcal{N}$, a continuum of intermediate good producers indexed by $i \in [0,1]$ use labor and aggregate goods from other sectors to produce an intermediate good under monopolistic competition. They sell these goods to aggregate good producers within the same sector, who aggregate these intermediate varieties into a final sectoral good. In turn, aggregate good producers sell these goods to households and other intermediate good producers.

Aggregate Good Producers. Competitive aggregate good producers purchase a continuum of intermediate varieties $i \in [0,1]$ from its own sector n, and produces output Q_{nt} using a CES production function. They sell these sectoral goods to intermediate good producers and households. The profit maximization problem of the firm is:

$$\max_{\{x_{in,t}\}_{i\in[0,1]}} P_{nt}Q_{nt} - \int_{i\in[0,1]} p_{in,t}x_{in,t}\mathrm{d}i \quad \text{s.t.} \quad Q_{nt} = \left(\int_{in\in[0,1]} x_{in,t}^{\frac{\eta_n-1}{\eta_n}}\,\mathrm{d}i\right)^{\frac{\eta_n}{\eta_n-1}} \tag{116}$$

$x_{in,t}$ is the amount of variety in purchased at a given price of $p_{in,t}$, P_{nt} is the aggregate good's production price, and $\eta_n > 1$ is the elasticity of substitution between varieties for sector n. Note that this specification defines a standard iso-elastic demand function for each intermediate good given by:

$$x_{in,t} = \left(\frac{p_{in,t}}{P_{nt}}\right)^{-\eta_n} Q_{nt} \quad \text{where} \quad P_{nt} = \left(\int_{i\in[0,1]} p_{in,t}^{1-\eta_n}\mathrm{d}i\right)^{\frac{1}{1-\eta_n}} \tag{117}$$

Given that aggregate good producers are perfectly competitive under constant returns to scale, they earn zero profits and have zero value added. The purpose of these aggregator firms is therefore to define a unified final good for each industry.

Intermediate Good Producers. There exist a continuum of $i \in [0,1]$ intermediate good producers in sector $n \in \mathcal{N}$. Each intermediate good producer i in sector n purchases aggregate sectoral goods $X_{in,n',t}$ from sectors $n' \in \mathcal{N}$ at a price of $P_{n't}$ as well as labor $L_{in,t}$ at the prevailing wage rate w_t to produce an intermediate good $x_{in,t}$. It then sells this intermediate good monopolistically at a price of $p_{in,t}$ to a final good producer in its own sector. Intermediate good producers in sector n produce with Cobb-Douglas technology under constant returns to scale:

$$q_{in,t} = c_n z_{nt} \left(L_{in,t}\right)^{\alpha_{nl}} \prod_{n'\in\mathcal{N}} X_{in,n',t}^{\alpha_{nn'}} \tag{118}$$

I also assume that the government imposes an ad-valorem subsidy τ_n to all intermediate good producers in sector n in proportion to their monopolistic mark-up. The firm's profits are therefore given by:

$$\Pi_{in,t} = (1+\tau_n) p_{in,t} q_{in,t} - w_{nt} L_{in,t} - \sum_{n' \in \mathcal{N}} P_{n't} X_{in,n',t} \tag{119}$$

Intermediate Producers' Demand Functions. Intermediate good producers choose inputs under incomplete information shock realizations, but can condition each input choice on the price of that input. If we can show that monopolistic competition implements the same demand functions as in the perfect competition economy, the theoretical analysis in the main text remains unchanged. The following Proposition shows that this indeed the case.

Proposition 10. *Each firm $i \in [0,1]$ in sector $n \in \mathcal{N}$ implements a common demand function for inputs in sector $n' \in \mathcal{N}$. These demand functions satisfy:*

$$X_{in,n',t} = \alpha_{nn'} \frac{\mathbb{E}_t\left[(M_t)^{-1} R_{nt} | P_{n't}\right]}{\mathbb{E}_t\left[(M_t)^{-1} P_{n't} | P_{n't}\right]} \quad \textit{and} \quad L_{nt} = \alpha_{nl} \frac{R_{nt}}{w_t} \tag{120}$$

where the expectation conditions on $t-1$ shock realizations, and $R_{nt} = P_{nt} \times Q_{nt}$ are sectoral revenues.

Proof. See Appendix C.2. □

Proposition 10 relies on two features of the monopolistic structure. First, the government levies an ad-valorem subsidy, which undoes the monopolistic distortion. This ensures that production is at its constrained efficient levels. Second, all intermediate good producers within each sector are homogeneous. Thus, they behave symmetrically, charge the same price, and choose the same number of each inputs. This ensures that the demand functions of each sector are common across all firms. Consequently, the conclusions of my analysis do not hinge on the assumptions pertaining to market structure considered in the main text.

B.4 Arbitrary Flexible Inputs

The main text assumed that labor is an input that adjusts to realized demand conditions, and can thus frictionlessly be chosen to maximize profits under complete information. This section generalizes some of the results in the main text to the case in which an arbitrary subset of inputs is chosen under complete information. Concretely, let $\mathcal{S}^{CI} \subset \mathcal{N}$ denote the subset of inputs chosen under complete information, and suppose that the remaining inputs $\mathcal{S}^{II} = \mathcal{N}/\mathcal{S}^{CI}$ are chosen under incomplete information.

Characterizing Prices with Arbitrary Flexible Inputs. First, note that the demand schedule for flexible inputs is given by:

$$X_{in,n',t} = \alpha_{nn'} \frac{R_{nt}}{P_{n't}} \tag{121}$$

for $n' \in \mathcal{S}^{CI}$. We therefore obtain the following alternative to Proposition 3.

Proposition 11. *Equilibrium prices $\{P_{nt}\}_{n\in\mathcal{N}}$ and revenues $\{R_{nt}\}_{n\in\mathcal{N}}$ satisfy the following system of equations:*

$$R_{nt} = \tilde{c}_{nt} z_{nt} P_{nt} \left(\frac{R_{nt}}{M_t^{1-\chi_n}}\right)^{\alpha_{nl}} \prod_{n'\in\mathcal{S}^{II}} \left(\frac{\mathbb{E}_t\left[M_t^{-1} R_{nt} \middle| P_{n't}\right]}{\mathbb{E}_t\left[M_t^{-1} P_{n't} \middle| P_{n't}\right]}\right)^{\alpha_{nn'}} \prod_{n'\in\mathcal{S}^{CI}} \left(\frac{R_{nt}}{P_{n't}}\right)^{\alpha_{nn'}} \tag{122}$$

$$R_{nt} = \gamma_n \iota_t M_t + \sum_{n'\in\mathcal{S}^{II}} \alpha_{n'n} P_{nt} \frac{\mathbb{E}_t\left[M_t^{-1} R_{n't} \middle| P_{nt}\right]}{\mathbb{E}_t\left[M_t^{-1} P_{nt} \middle| P_{nt}\right]} + \sum_{n'\in\mathcal{S}^{CI}} \alpha_{n'n} R_{n't} \tag{123}$$

where $\tilde{c}_t = (\iota_t)^{\alpha_{nl}(1-\chi)}(w_{t-1})^{\alpha_{nl}\chi}$.

Proof. Follows from the proof of Proposition 3. □

We can also characterize the joint stochastic properties of revenues and prices, holding responsiveness fixed. To this end, define the rigid revenue share matrix as:

$$\mathbf{S}^r \equiv [s_{nn'} \times \mathbb{1}\{n \in \mathcal{S}^{II}\}] \tag{124}$$

We also define the flexible revenue share matrix as:

$$\mathbf{S}^f \equiv [s_{nn'} \times \mathbb{1}\{n \in \mathcal{S}^{CI}\}] \tag{125}$$

We can also similarly define the *rigid* input-output matrix as $\mathbf{A}^r = [\alpha_{nn'} \times \mathbb{1}\{n' \in \mathcal{S}^{II}\}]$ and the flexible input out matrix $\mathbf{A}^f = \mathbf{A} - \mathbf{A}^r$. Finally, we define the *demand-adjusted* input-output matrix and revenue-share matrix as $\mathbf{A}^r(\omega_t) = [\alpha_{nn'} \times \omega_{nn't} \times \mathbb{1}\{n' \in \mathcal{S}^{II}\}]$ and $\mathbf{S}(\omega_t) = [s_{nn'} \times (1-\omega_{n'nt}) \times \mathbb{1}\{n \in \mathcal{S}^{II}\}]$.

Proposition 12. *Assume $\sum_{n'\in\mathcal{N}} s_{nn'} \times \mathbb{1}\{n \in \mathcal{S}^{CI}\} < 1$. Then, the first-order response of demand and productivity shocks to prices and revenues is given by:*

$$\mathbf{Z}(\omega_t)\hat{P}_t = cons - \hat{z}_t + [(1-\chi)(\mathbf{I}-\mathbf{A}) + \mathrm{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I} - (\mathbf{I}-\mathbf{S}^f)^{-1}\mathrm{diag}(\mathbf{S}^r\mathbf{1}))]\hat{M}_t \tag{126}$$

$$\hat{R}_t = cons + (\mathbf{I} - (\mathbf{I}-\mathbf{S}^f)^{-1}\mathbf{S}^r)\hat{M}_t + (\mathbf{I}-\mathbf{S}^f)^{-1}\mathrm{diag}(\mathbf{S}^r(\omega_t)\mathbf{1}))\hat{P}_t \tag{127}$$

where $\hat{x}_t = \log x_t$ *the matrix* $\mathbf{Z}(\omega_t)$ *is given by*

$$\mathbf{Z}(\omega_t) = \underbrace{\mathbf{I} - \mathbf{A}^r(\omega_t) - \mathbf{A}^f}_{\substack{\text{demand-adjusted} \\ \text{Leontief matrix}}} - \underbrace{\mathrm{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I} - \mathbf{S}^f)^{-1}\mathrm{diag}(\mathbf{S}^r(\omega_t)\mathbf{1})}_{\substack{\text{revenue impact} \\ \text{matrix}}} \tag{128}$$

Proof. The result follows from the same steps taken in Theorem 1. □

AUDI Weights for General Flexible Inputs. For a common responsiveness parameter $\omega_{nn't} = \omega^*$, we can write the matrix $\mathbf{Z}(\omega^*)$ as

$$\mathbf{Z}(\omega^*) = \mathbf{I} - \omega^*\mathbf{A} - (1-\omega^*)\left(\mathbf{A}^f + \mathrm{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I} - \mathbf{S}^f)^{-1}\mathrm{diag}(\mathbf{S}^r\mathbf{1})\right) \tag{129}$$

Under the assumption that $\mathbf{Z}(\omega^*)$ is an M-matrix, we can write the effect of a productivity shock on prices as the inverse of this matrix given by:

$$\sum_{k=0}^{\infty}\left[\omega^*\mathbf{A} + (1-\omega^*)\left(\mathbf{A}^f + \mathrm{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I} - \mathbf{S}^f)^{-1}\mathrm{diag}(\mathbf{S}^r\mathbf{1})\right)\right]^k \tag{130}$$

In this more general set-up, we see that the effect of productivity shocks to output is given by a convex combination of the economy's input-output matrix and demand impact matrix. When $\omega^* = 1$, we recover the complete information result that the effect of productivity shocks on prices is given by the Leontief inverse matrix $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$.

B.5 Interim Public Signals

In this section, I show how the model can accommodate an interim public signals. Concretely, I assume that all firms in the economy receive a public signal given by

$$\log s_t^p = \log M_t + \varepsilon_t^p \tag{131}$$

where $\varepsilon^p \sim \mathrm{N}(0, (\sigma_t^p)^2)$. I define the noise to signal ratio of aggregate demand to this public signal as

$$\kappa_t = \frac{(\sigma_t^M)^2}{(\sigma_t^M)^2 + (\sigma_t^p)^2} \tag{132}$$

Hence, as the precision of this signal increases, the signal to noise ratio tends to one. Under this information structure, firms choose inputs according to:

$$X_{nn't} = \alpha_{nn'}\frac{\mathbb{E}_t[(M_t)^{-1}R_{nt}|s_t^p, P_{n't}]}{\mathbb{E}_t[(M_t)^{-1}P_{n't}|s_t^p, P_{n't}]} \tag{133}$$

I now establish how this extension nests the complete information economy considered in the main text and the benchmark complete information economy.

Proposition 13. *The following statements are true.*

1. *When $\kappa_t = 1$, there exists an equilibrium in which the dynamics of real GDP are equal to the dynamics of the complete information economy.*

2. *When $\kappa_t = 0$, there exists an equilibrium in which the dynamics of real GDP are equal to the dynamics of the incomplete information economy considered in the main text.*

Proof. See Appendix C.4. □

Sectoral Importance under Fixed Responsiveness. I now derive a sectoral measure of importance under this new information structure. As in the main text, I assume that firms fix their input responsiveness according to the matrix $\mathbf{\Omega}_t$. I then show how $\mathbf{\Omega}_t$ depends on the economy's stochastic structure for revenues, prices, and signals. We obtain the following proposition, which forms the counterpart of Theorem 1 when firms receive interim public signals.

Proposition 14. *In the unique log-linear equilibrium of the economy, real GDP under fixed responsiveness is given by:*

$$\log \mathcal{C}_t = cons + \epsilon(\mathbf{\Omega}_t)' \log z_t + [1 - \gamma' \mathrm{diag}(a_n)] \log M_t - \gamma' \mathrm{diag}(b_n) \log s_t^p \tag{134}$$

where the constant is independent of time t shocks and $\epsilon(\mathbf{\Omega_t})$ is the economy's Augmented-by-Uncertainty Domar Index (AUDI), derived in Theorem 1. Moreover, a_n and b_n satisfy:

$$\begin{aligned} a_n = (1-\chi) - &(1-\kappa_t)\left[(1-\alpha_{nl})(1-\chi-s_{nc})\right] \\ &- \kappa_t \left[(1-\chi)\left(\sum_{n' \in \mathcal{N}} \alpha_{nn'}\omega_{nn't} + \alpha_{nl} \sum_{n' \in \mathcal{N}} s_{nn'}(1-\omega_{n'nt})\right)\right] \end{aligned} \tag{135}$$

$$\begin{aligned} b_n = \kappa_t \Bigg[&(1-\chi-s_{nc})(1-\alpha_{nl}) - (1-\chi)\sum_{n' \in \mathcal{N}} \alpha_{nn'}\omega_{nn't} \\ &- (1-\chi)(1-\alpha_{nl})\sum_{n' \in \mathcal{N}} s_{nn'}(1-\omega_{n'nt})\Bigg] \end{aligned} \tag{136}$$

Proof. See Appendix C.5. □

This proposition reveals three observations. First, *conditional* on the same input responsiveness matrix, the impact of sectoral disturbances on output are identical to those in the main text. Second, the effect of an aggregate demand shock on output now depends on the signal-to-noise ratio κ_t. When $\kappa_t = 0$, it is straightforward to verify that the impact of an aggregate demand shock on output is identical to Theorem 1. Finally, whenever $\kappa_t \neq 0$, aggregate output responds to movements in this public signal, as it is informative about aggregate demand disturbances.

A corollary of this result is that the connection to alpha centrality and the AUDI weights of the economy extends to richer information structures. Hence, the basic insight that incomplete information changes the relative importance of a sector's higher-order linkages in propagating macroeconomic shocks is preserved in these more general settings.

Optimal Responsiveness in General Equilibrium. I now analyze how firms' optimal input responsiveness is determined in this setting. The following Lemma is the counterpart to Lemma 2 when firms receive interim public signals.

Lemma 4. *There exists an equilibrium in which input responsiveness is given by:*

$$\boldsymbol{\Omega}_t = [\omega_{nn't}] = \left[1 - \frac{\mathrm{Cov}(\log R_{nt}, \log P_{n't} | s_t^p)}{\mathrm{Var}(\log P_{n't} | s_t^p)}\right] \tag{137}$$

$$= \left[1 - \frac{\mathrm{Cov}(\log R_{nt}, \log P_{n't}) - \frac{\mathrm{Cov}(\log R_{nt}, \log s_t^p)\mathrm{Cov}(\log P_{n't}, \log s_t^p)}{\mathrm{Var}(\log s_t^p)}}{\mathrm{Var}(P_{n't}) - \frac{\mathrm{Cov}^2(\log P_{n't}, \log s_t^p)}{\mathrm{Var}(\log s_t^p)}}\right] \tag{138}$$

Proof. See Appendix C.6 □

Lemma 4 shows that input responsiveness is determined by firms' *conditional expected* covariances between revenues and prices, and the conditional expected variance of input prices. This result also lends itself to an OLS interpretation. In particular, optimal responsiveness is the coefficient that emerges when one runs an OLS regression of optimal ex-post input choices $X^*_{nn't}$ on its input price and the public signal:

$$\log X^*_{nn't} = \beta_0 + \beta_1 \log R_{n't} + \beta_2 \sigma_t^p + \text{error} \tag{139}$$

The coefficient β_1 is thus the OLS coefficient after one "partials out" the predictive power of the public signal on input choices.

Takeaways from More General Information Structures. This section shows that the model can be extended to accommodate interim public signals. Crucially, these richer informational structures preserve the key economics of the model: incomplete information shapes input responsiveness which mediates the effect of shocks on aggregate output. Relative to the model in the main text, the key difference is that input responsiveness is shaped by firms' expected covariance between their revenues and input prices, where this expectation is conditioned by the realization of public signals.

C Additional Proofs

C.1 Proof of Proposition 8

Proof. We first note that all firms in each sector continue to be homogeneous. Hence, we can write firms' profits as

$$\tilde{\Pi}_{in,t}(\mathbf{P}_t) = R_{nt} - w_t L_{in,t} - \sum_{n' \in \mathcal{N}} \tau_{in,n',t}(P_{n't}) \tag{140}$$

where $R_{nt} = P_{nt}Q_{nt}$ are sector-level revenues.

Suppose now that a given contract $\{\tau^*_{in,n',t}(P_{n't}), X^*_{in,n',t}(P_{n't})\}$ is optimal. Consider a variation $\tilde{\tau}_{in,n',t}(P_{n't}) = \tau^*_{in,n't}(P_{n't}) + \varepsilon h(P_{n't})$ for some $\varepsilon > 0$. The logarithm of the joint surplus generated from this variation is given by

$$\begin{aligned} J(\varepsilon, h) = {} & \beta \log \mathbb{E}_t \left(\frac{1}{\mathcal{P}_t \mathcal{C}_t} (\tilde{\Pi}_{in,t}(\mathbf{P}_t) - \varepsilon h(P_{n't})) - O_f \right) \\ & + (1-\beta) \mathbb{E}_t \left(\frac{1}{\mathcal{P}_t \mathcal{C}_t} (\tau^*_{in,n',t}(P_{n't}) + \varepsilon h(P_{n't}) - P_{n't} X^*_{in,n',t}(P_{n't})) - O_s \right) \end{aligned} \tag{141}$$

A necessary condition for optimality is that $J_\varepsilon(\varepsilon, h)$ achieves its maximum at $\varepsilon = 0$ for all $h(P_{nt})$. Hence, we have the following necessary first-order condition:

$$\frac{-\beta}{\mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} (\tilde{\Pi}_{in,t}(\mathbf{P}_t)) - O_f \right]} + \frac{(1-\beta)}{\mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} (\tau^*_{in,n',t}(P_{n't}) - P_{n't} X^*_{in,n',t}(P_{n't})) - O_s \right]} = 0 \tag{142}$$

which holds for all $h(P_{nt})$. We can also consider an identical variation for the input delivery $\tilde{X}_{in,n't} = X^*_{in,n',t}(P_{n't}) + \varepsilon h_x(P_{nt})$. The necessary first-order condition that is obtained from this variation is:

$$\frac{-\beta \mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} \alpha_{nn'} R_{nt} h_x(P_{n't}) \right]}{\mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} (\tilde{\Pi}_{in,t}(\mathbf{P}_t)) - O_f \right]} + \frac{(1-\beta) \mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} P_{n't} X^*_{in,n',t}(P_{n't}) h_x(P_{n't}) \right]}{\mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} (\tau^*_{in,n',t}(P_{n't}) - P_{n't} X^*_{in,n',t}(P_{n't})) - O_s \right]} = 0 \tag{143}$$

Combining the two first-order conditions yields the necessary condition

$$\mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} \alpha_{nn'} R_{nt} h_x(P_{n't}) \right] = \mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} P_{n't} X^*_{in,n',t}(P_{n't}) h_x(P_{n't}) \right] \tag{144}$$

This is true for any function $h_x(P_{n't})$. We can therefore consider the Dirac function $\delta_{\hat{P}_{n't}}$ for some $\hat{P}_{n't} \in \mathbb{R}_{++}$. We then obtain

$$\mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} \alpha_{nn'} R_{nt} \middle| \hat{P}_{n't} \right] = \mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} P_{n't} X^*_{in,n',t}(P_{n't}) \middle| \hat{P}_{n't} \right] \tag{145}$$

Since $X^*_{in,n',t}$ is adapted to $P_{n't}$ by assumption, we obtain

$$X^*_{in,n',t}(\hat{P}_{n't}) = \frac{\mathbb{E}_t \left[(\mathcal{P}_t \mathcal{C}_t) R_{nt} | \hat{P}_{n't} \right]}{\mathbb{E}_t \left[(\mathcal{P}_t \mathcal{C}_t) P_{n't} | \hat{P}_{n't} \right]} \tag{146}$$

This completes the proof. $\square$

C.2 Proof of Proposition 10

Proof. Substituting the producer's demand for intermediate inputs 117, as well as the intermediate good producer's production function 6 into their profits 119 and taking a first-order condition with respect to $X_{in,n',t}$, we obtain:

$$X_{in,n',t} = \alpha_{nn'} \frac{\mathbb{E}_{in,t} \left[(\mathcal{P}_t \mathcal{C}_t)^{-1} P_{nt} Q_{nt}^{\frac{1}{\eta_n}} q_{in,t}^{\frac{\eta_n - 1}{\eta_n}} \middle| P_{n't} \right]}{\mathbb{E}_{in,t} \left[(\mathcal{P}_t \mathcal{C}_t)^{-1} P_{n't} | P_{n't} \right]} \quad \text{for} \quad n' \in \mathcal{N} \tag{147}$$

Since firms in each sector are homogeneous, we have $q_{in,t} = Q_{nt}$. Moreover, since past shocks $\{z_{nt-1}\}_{n \in \mathcal{N}}$ and M_{t-1} are common knowledge to all firms in the economy, we have $\mathbb{E}_{in,t} = \mathbb{E}_t$. The optimal input purchased is therefore proportional to industry-level revenues over the price of that input

$$X_{in,n',t} = \alpha_{nn'} \frac{\mathbb{E}_t [(\mathcal{P}_t \mathcal{C}_t)^{-1} R_{nt} | P_{n't}]}{\mathbb{E}_t [(\mathcal{P}_t \mathcal{C}_t)^{-1} P_{n't} | P_{n't}]} \tag{148}$$

The derivation of the demand function for labor follows by removing the expectation operator from the above equation. $\square$

C.3 Proof of Proposition 9

Proof. A sufficient condition for the dynamics of the two economies to be identical is that for the demand function

$$X_{nn't} = cons + \omega^1_{nn't}P_{n't} + \sum_{n'\in\mathcal{P}_n} \omega^k_{nn't}P_{n't}$$

we have $\omega^k_{nn't} = 0$ for $k \neq 1$ and $\omega_{nn't} = -\frac{\chi}{1-\chi}$.

I now prove that this is an equilibrium. We proceed as in Theorem 2 by guessing that there exists a solution to revenues and prices that is linear in shocks (where we suppress the dependence on time indices for notational simplicity).

$$\hat{R}_n = cons + \chi^R_{nM}\hat{M} + \sum_{n'\in\mathcal{N}} \chi^R_{nn'}\hat{z}_{n'} \tag{149}$$

for scalars $\{\chi^R_{nn'}\}_{n'\in\mathcal{N}}$ and χ^R_{nM}. We similarly guess that prices are linear in shocks:

$$\hat{P}_n = cons + \chi^P_{nM}\hat{M} + \sum_{n'\in\mathcal{N}} \chi^P_{nn'}\hat{z}_{n'} \tag{150}$$

Observe that due to the log-normality of the aggregate shocks, we have that:

$$\mathbb{E}[\hat{R}_n|\hat{P}_{n'}, \mathcal{I}_n] = [\sigma_{R_n,P_{n'}}, \sigma_{R_n,\mathbf{P}_n}]\mathbf{\Sigma}^{-1}[P_{n'}, \mathbf{P}_n]' \tag{151}$$

where $\mathbf{P}_n$ is the vector $[P_{n'}]_{n'\in\mathcal{P}_n}$ and $\mathbf{\Sigma}$ is the variance-covariance matrix of the stacked vector $[P_{n'}, \mathbf{P}_n]'$. However, observe that all elements of the variance-covariance matrix $\mathbf{\Sigma}$ converge to $[\chi^P_{nM} \times \chi^P_{kM}]$ for some $n, k \in \mathcal{N}$ as $u_{nt} \to 0$. All elements will be identical, and hence the matrix will be singular, if χ^P_{nM} is the same across all $n \in \mathcal{N}$. Suppose this is the case. In this case, we have that:

$$\mathbb{E}[\hat{R}_n|\hat{P}_{n'}, \mathcal{I}_n] = \frac{\chi^R_{nM}}{\chi^P_{n'M}}P_{n'} \tag{152}$$

as $P_{n'}$ is collinear with $\mathcal{I}_n$. We now proceed in similar steps to Theorem 2. Substituting the above expression into Equation 91 and collecting coefficients on $\hat{M}$ yields:

$$\chi^R_{nM} = s_{nc} + \sum_{n'\in\mathcal{N}} s_{nn'}\chi^R_{n'M} \tag{153}$$

We can rewrite this in matrix form as:

$$\chi_M^R = (\mathbf{I} - \mathbf{S})1 + \mathbf{S}\chi_M^R \tag{154}$$

$$\chi_M^R = (\mathbf{I} - \mathbf{S})^{-1}(\mathbf{I} - \mathbf{S})1 = 1 \tag{155}$$

where χ_M^R is the N-sized vector of χ_{nM}^R. Hence, $\chi_{nM}^R = 1$ for all $n \in \mathcal{N}$ is the unique solution to this equation when $u_{nt} \to 0$. We now need to verify that χ_{nM}^P is common across all firms. Using Equation 90 and χ_{nM}^R, we obtain:

$$0 = \chi_{nM}^P - \alpha_{nl}(1 - \chi) - \sum_{n' \in \mathcal{N}} \alpha_{nn'}\chi_{n'M}^P \tag{156}$$

Solving this system of equations yields:

$$\chi_M^P = 1 - \chi \tag{157}$$

which verifies our initial guess and gives us that our initial demand schedule is: $\omega_{nn'}^1 = 1 - \frac{1}{\chi_{n'M}^P} = -\frac{\chi}{1-\chi}$ with $\omega_{nn'}^k = 0$ for $k \neq 1$. This completes the proof. □

C.4 Proof of Proposition 13

Proof. The second statement is trivial, as $\kappa_t = 0$ implies that the signal reveals no information about aggregate demand. Thus, firms disregard the signal.

When $\kappa_t = 1$, the public signal perfectly reveals the aggregate demand shock. Assume that the dynamics for real GDP and revenues are equal to the complete information economy, given in Proposition 1. In this case, firms can perfectly infer their revenues from the aggregate demand shock, as revenues are independent of sectoral productivity shocks. Hence, we can write firms' input choices as:

$$X_{nn't} = \alpha_{nn'} \frac{\mathbb{E}_t[(M_t)^{-1}R_{nt}|s_t^p, P_{n't}]}{\mathbb{E}_t[(M_t)^{-1}P_{n't}|s_t^p, P_{n't}]} = \alpha_{nn't}\frac{R_{nt}}{P_{n't}} \tag{158}$$

which are simply the complete information demand functions. This completes the proof. □

C.5 Proof of Proposition 14

Proof. Under the assumption that firms use log-linear demand schedules with responsiveness parameter $\omega_{nn't}$, we have:

$$X_{nn't} = \tilde{\omega}_{nn't}P_{n't}^{-\omega_{nn't}} \tag{159}$$

by definition. Firm's optimal choice of $\tilde{\omega}_{nn't}$ given a fixed responsiveness parameter of $\omega_{nn't}$ is given by:

$$\tilde{\omega}_{nn't} = \alpha_{nn'} \frac{\mathbb{E}_t[(M_t)^{-1} R_{nt} | s_t^p]}{\mathbb{E}_t[(M_t)^{-1} P_{n't}^{1-\omega_{nn't}} | s_t^p]} \tag{160}$$

We now use Equations 59 and 58 in the proof of Theorem 1. If $\tilde{\omega}_{nn't}$ is a log-linear function of s_t^p, then revenues and prices are log-linear as well. We conjecture the log-linearity of these variables and later verify the log-linearity of $\tilde{\omega}_{nn't}$. Since revenues and prices are log-linear, we can exploit the conditional expectation properties of log-normal random variables to write input choices under fixed responsiveness as

$$X_{nn't} = \phi_n^R \log s_t^p - \phi_{n'}^P (1 - \omega_{nn't}) \log s_t^p - \omega_{nn't} \log P_{n't} \tag{161}$$

for some constants ϕ_n^R and $\phi_{n'}^P$. Linearizing the market clearing equation by following the same steps as Theorem 1 and substituting in for $X_{nn't}$ yields

$$\hat{R}_{nt} = cons + s_{nc} \hat{M}_t + \sum_{n' \in \mathcal{N}} s_{nn'} \Big((1 - \omega_{n'nt}) \hat{P}_{nt} + \phi_{n'}^R \hat{s}_t^p - \phi_n^P \hat{s}_P (1 - \omega_{n'nt}) \Big) \tag{162}$$

where we use the short-hand notation $\hat{x} = \log x$. Similarly, substituting for $X_{nn't}$ into firms' production functions and taking logs yields

$$\begin{aligned} \hat{R}_{nt} =& cons + \hat{z}_{nt} + \hat{P}_{nt} + \alpha_{nl}(\hat{R}_{nt} - (1-\chi)\hat{M}_t) - \sum_{n' \in \mathcal{N}} \alpha_{nn'} \omega_{nn't} \hat{P}_{n't} \\ &+ \sum_{n' \in \mathcal{N}} \alpha_{nn'} \left(\phi_n^R \hat{s}_t^p - \phi_{n'}^P (1 - \omega_{nn't} \hat{s}_t^p) \right) \end{aligned} \tag{163}$$

In matrix form, these equations are

$$\begin{aligned} \operatorname{diag}(1 - \alpha_{nl}) \hat{R}_t = \hat{z}_t + \hat{P}_t - (1-\chi)\hat{M}_t - (\mathbf{A} \odot \mathbf{\Omega}_t) \hat{P}_t \\ + \operatorname{diag}(1 - \alpha_{nl}) \operatorname{diag}(\phi_n^R) \hat{s}_t^P - \mathbf{A} \odot (\mathbf{1} - \mathbf{\Omega}_t) \hat{s}_t^P \end{aligned} \tag{164}$$

and

$$\begin{aligned} \hat{R}_t = \operatorname{diag}(s_{nc}) \hat{M}_t + \operatorname{diag}\left(\sum_{n' \in \mathcal{N}} s_{nn'} (1 - \omega_{n'nt}) \right) \hat{P}_t \\ + \mathbf{S} \operatorname{diag}(\phi_n^R) \hat{s}_t^P - \operatorname{diag}\left(\sum_{n' \in \mathcal{N}} s_{nn'} (1 - \omega_{n'nt}) \right) \operatorname{diag}(\phi_n^P) \hat{s}_t^P \end{aligned} \tag{165}$$

where $\mathbf{1}$ is and $N \times N$ matrix of ones. We now solve for the vector of coefficients ϕ^P and ϕ^R. In particular, these must satisfy the consistency condition:

$$\mathbb{E}_t[\hat{R}_t|\hat{s}_t^P] = \phi^R \odot \hat{s}_t^P \quad \text{and} \quad \mathbb{E}_t[\hat{P}_t|\hat{s}_t^P] = \phi^P \odot \hat{s}_t^P$$

We first solve the first conditional expectation and collect coefficients on $\hat{s}_t^P$. This yields

$$\phi^R = \kappa \text{diag}(s_{nc}) + \text{diag}\left(\sum_{n' \in \mathcal{N}} s_{nn'}(1-\omega_{n'nt})\right)\phi^P + \mathbf{S}\phi^R - \text{diag}\left(\sum_{n' \in \mathcal{N}} s_{nn'}(1-\omega_{n'nt})\right)\phi^P$$

which upon simplification yields

$$\phi_n^R = \kappa_t \quad \text{for all } n \in \mathcal{N}$$

Similarly, we can solve the second expectation term to obtain

$$\phi_n^P = \kappa_t(1-\chi) \quad \text{for all } n \in \mathcal{N}$$

Substituting these derived coefficients into 164 and 165, and using the fact that

$$\hat{\mathcal{C}}_t = \hat{M}_t - \gamma' \hat{P}_t$$

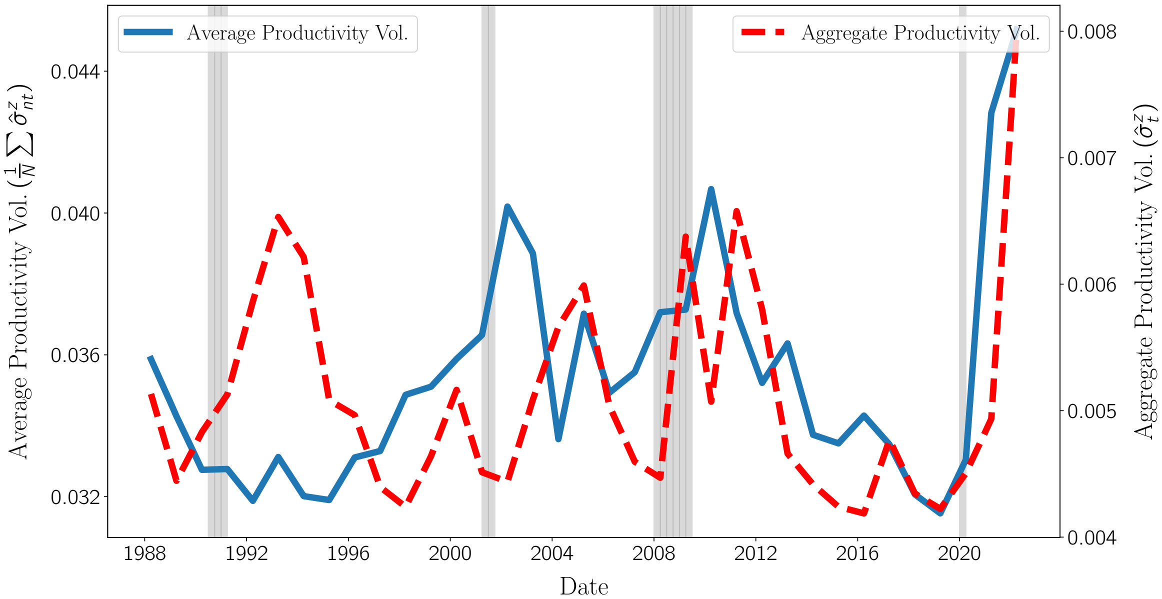
then yields the desired result. Note also that this gives us the unique log-linear equilibrium of the economy. □

C.6 Proof of Lemma 4

Proof. Under the presumed functional form for input responsiveness, revenues, prices, and the public are jointly log-normally distributed from Proposition 14. The formula then follows from Lemma 2 and the conditional expectation properties of log-normal random variables. □

D Additional Figures

Figure 9: Aggregate Productivity Volatility Over Time



Note: This figure plots the volatility of average sectoral productivity over time, alongside the volatility of aggregate productivity. Aggregate productivity is obtained from the BLS's website and its volatility is estimated using a GARCH(1,1) model, as described in the main text.