Housing Market Dynamics under Segmentation: Theory and Evidence

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Abstract

I empirically show that different types of houses experience substantially different rates of return over nation-wide housing cycles, even within narrow geographical areas. In order to understand the implications of this rate of return heterogeneity for the propagation of macroeconomic shocks, I develop a dynamic assignment model in which housing is segmented by various quality tiers. I show that the model's unique equilibrium features unidirectional propagation of shocks within the housing market – changes in households' valuation of low-end homes spill over upwards in the quality ladder, making them important drivers of average house price changes. However, shocks do not "trickle down": changes in the valuation of high-end homes leave the prices of other quality tiers unaffected. I analytically characterize the strength of these pricing spill-overs and examine how they shape the cross-sectional response of house prices to key macroeconomic drivers of housing markets, including credit and interest rates. Unlike the homogeneous housing benchmark, I show that segmentation can amplify the response of aggregate consumption expenditures to changes in house prices from macroeconomic shocks by creating an unequal incidence of capital gains. I test the key theoretical prediction of unidirectional shock propagation using plausibly exogenous variation in shocks that are unequally distributed within housing markets: an expansion in subprime credit and fluctuations in stock market wealth. Consistent with the theory, shocks that affect high-income households only induce changes in the prices of high-end homes, whereas shocks that affect low-income households affect house prices in the entire market.

JEL Codes: E21, E32, G51, R21, R31

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1 Introduction

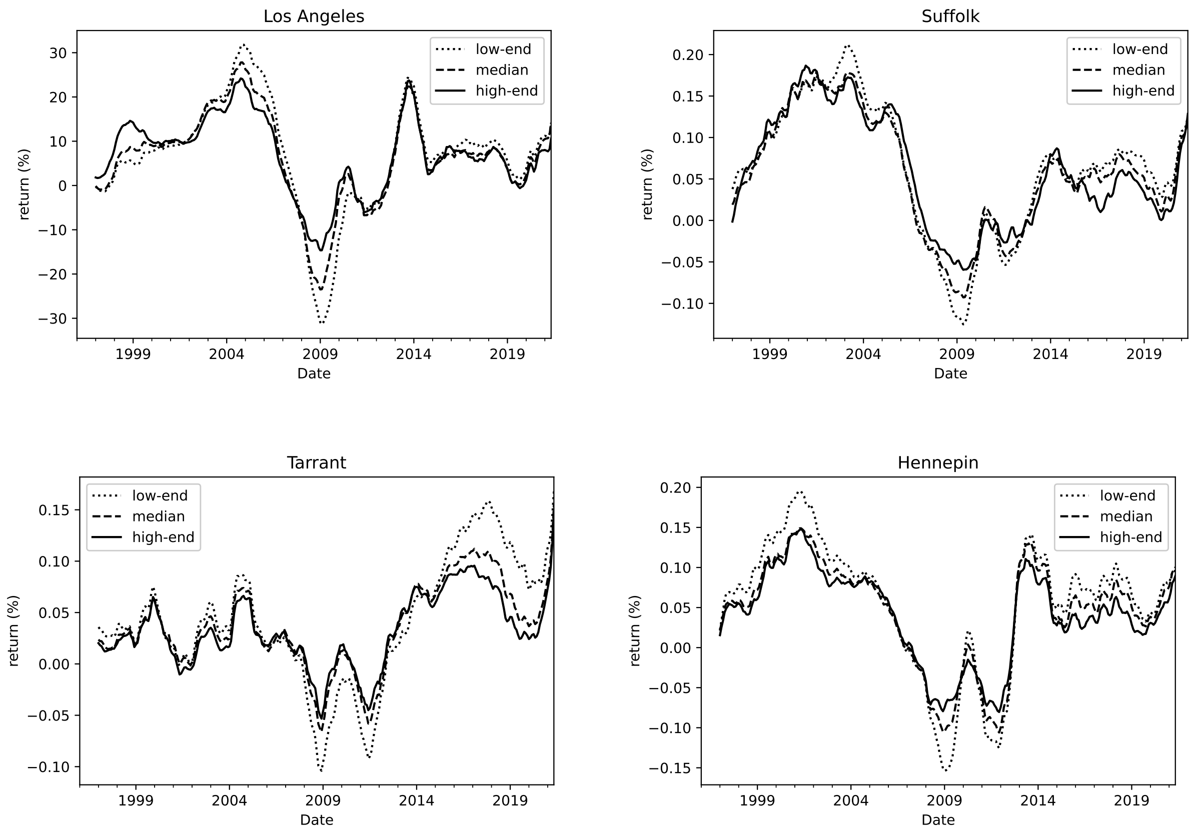
Housing markets are segmented: even within a narrow geographical region, there is substantial heterogeneity in relative price changes for different types of houses. Figure 1 illustrates this fact. It plots the annualized capital gains of houses for counties in various metro areas within the United States from 1990-2021, disaggregated by their initial price, alongside the county median. In Los Angeles, for example, cheap (low-end) houses depreciated approximately three times as much as expensive (high-end) homes (in percentage terms) in the year 2009. As I will show later, this disparity in relative rates of return is a robust feature for local housing markets across the entire United States. This basic fact implies that we cannot descriptively represent the housing market as a single "market".

Segmented markets and the return heterogeneity they induce can have important macroeconomic consequences. Shocks that affect house prices can lead to large distributional effects on household wealth through capital gain variation. Moreover, the unequal incidence of housing wealth effects can in turn be an important driver for other macroeconomic aggregates, such as consumption expenditures. Although the urban economics literature has long recognized segmentation as an important feature of housing markets (Goodman and Thibodeau, 1998; Bourassa et al., 1999; Watkins, 2001), macroeconomic models with a housing sector typically model housing as a commodity that can be bought and sold at a common per unit price from a homogeneous stock. Consequently, capital gains in these models are constant across all households, even if these households are heterogeneous.

In order to understand the implications of this rate of return heterogeneity for the propagation of macroeconomic shocks, I develop an analytically tractable, dynamic assignment model in which housing is segmented by various quality tiers. I show that the presence of housing market segmentation substantially alters the response of house prices to shocks relative to the homogeneous housing benchmark. In the unique equilibrium of the model, shocks propagate within the housing market *unidirectionally* – changes in households' valuations of low-end homes spill-over upwards in the house quality ladder, making them important drivers of average house price changes. I characterize the strength of these spill-overs analytically, and examine how they shape the response of the entire *cross-section* of house prices to changes in economic conditions. The model also gives rise to an amplification channel for the response of house price changes to aggregate consumption expenditure through the heterogeneous incidence of capital gains.

I operationalize this model empirically to measure the importance of housing market segmentation in the data. In the data, shocks that affect high-income households only induce price changes in high-end homes, consistent with the unidirectional propagation of shocks. In contrast, shocks that affect low-income households affect house price in the entire market through pricing spill-overs. These tests empirically reject the predictions of the homogeneous housing benchmark, and suggest that the presence of segmentation is an important to understand how macroeconomic shocks affect the housing market.

Figure 1



Note: Annualized rates of return for counties in various metro areas (Greater Los Angeles, New York-Newark-Jersey City, Dallas-Fort Worth-Arlington, Minneapolis-Saint Paul). House price data from Zillow Research (single family homes index). The low-end (high-end) series plots average annualized returns for zip codes in the bottom (top) quartile of the ZIP code price distribution within the given county in the year 1999. The median series represents the median house price value within a given a county.

Model. In the model, there exists a continuum of indivisible houses that provide different flows of housing services to households. In equilibrium, households match to houses of varying qualities according to the net present value of their income. Because houses are priced competitively, this gives rise to a continuum of pricing equations, in which each household is a marginal investor for the house that they purchase. These pricing equations relate households' willingness to pay for inframarginal quality units to their marginal rate of substitution between consumption and housing services. Households' marginal rate of substitution, in turn, depends on aggregate economic conditions, such as the level of interest rates or the availability of credit. Heterogeneity in housing capital gains is then a consequence of heterogeneous variations in marginal rates of substitution in response to different economic shocks.

Theoretical Results. The existence of housing market segmentation gives rise to distinct asset pricing implications relative to the homogeneous housing benchmark. First, I show that certain "localized" shocks that affect only a subset of households can spill-over into neighboring quality tiers by affecting the willingness to pay of neighboring households along the income distribution.

This gives rise to a mechanism in which shocks that affect the prices of low-end homes "trickle up" to the valuations of high-end homes. However, shocks that affect the prices of high-end homes cannot "trickle down" the quality ladder. This is a consequence of the positive assortative matching between housing qualities and income in the equilibrium assignment. Intuitively, the price of a given quality tier is determined by the willingness to pay for additional housing services by households that reside strictly *below* that tier. For this reason, the preferences of wealthy households – that reside in high-end homes – will not affect the prices of lower quality segments. However, the preferences of low-income households do matter for the prices of homes occupied by high-income households.

Second, I show that two important macroeconomic forces in the housing market operate on different ends of the wealth and housing quality distribution. Changes in credit conditions have the largest effects on the prices of low-end homes, by affecting the willingness to pay of credit constrained, low-income households. In contrast, interest rate changes, as well as changes in the perceptions of future house values, have the largest effects on the prices of high-end homes. This is because future house values are mostly pay-off relevant for unconstrained, high-income households through their effect on intertemporal consumption smoothing. As such, the cross-sectional distribution of capital gains is sensitive to the nature of the shock that hits the economy. Nevertheless, the presence of pricing spill-overs implies that credit shocks (and other "low-end" shocks) can have a large impact on prices over the whole quality ladder. Even if few households are credit constrained in equilibrium, the average house price index is sensitive to changes in credit conditions.

Third, I demonstrate that market segmentation amplifies various aggregate shocks relative to the homogeneous housing benchmark. Intuitively, this is because market segmentation creates a positive covariance between capital gain incidence (which are wealth effects in the model) and marginal propensities to consume (MPCs). In contrast, in the absence of market segmentation, the housing market gives rise to a form of dampening for *all* aggregate fluctuations. This is because capital gains contrivedly become a decreasing function of liquid assets, which establishes a strictly negative covariance between wealth effects and MPCs. This introduces redistributive effects which would otherwise be absent in the homogeneous housing benchmark.

Empirical Analysis. Finally, I turn to empirically testing the key predictions of the theory. To do so, I exploit cross-sectional variation in the incidence of various shocks across local housing markets in the United States. In the first test, I isolate plausibly exogenous variation in mortgage lending to create a proxy for a localized shock to *low-end* households. I do so by leveraging banks' heterogeneous exposure to the rise in the private label mortgage securitization market (the "PLS market") over 2002-2005 (Mian and Sufi, 2022). I show that this credit shock is almost entirely localized within low-end segments of the housing market and that it leads to monotonicity in housing capital gains across the quality ladder: low-end homes appreciate more than high-end homes, as predicted by the theory. However, I also provide evidence for the existence of "trickle up" forces, which cause an economically significant increase in the prices of homes at the top end of the quality distribution.

In the second test, I confirm the absence of “trickle down” effects by isolating plausibly exogenous variation in stock market wealth changes (Chodorow-Reich et al., 2021). I show that stock market wealth is primarily held in high-end segments of the house quality distribution. These high-end shocks cause a statistically and economically significant increase on the prices of high-end homes, but have no effect on lower quality tiers. Moreover, the results are robust to local general equilibrium spillovers, modeled as in Guren et al. (2021) or Nakamura and Steinsson (2018). Consequently, the data suggests that there is strong *directionality* in the propagation of shocks within the housing market. Taken together, both the theory and data highlight the importance of housing market segmentation in understanding the propagation of macroeconomic shocks to house prices, and *vice versa*.

Relation to Existing Literature. This paper lies at the intersection of several strands of literature. First, this paper relates to a large empirical and theoretical literature on the effect of macroeconomic shocks on house price movements (Cox and Ludvigson, 2021; Favilukis et al., 2017; Garriga and Hedlund, 2020; Greenwald, 2018; Greenwald and Guren, 2021; Kaplan et al., 2020; Kiyotaki et al., 2011; Mian and Sufi, 2022; Piazzesi and Schneider, 2016). This literature primarily characterizes how shocks affect average house prices *across* different housing markets. In contrast, I analyze the propagation of shocks to house prices in a framework that features segmentation *within* a housing market. I demonstrate that within-market segmentation gives rise to distinct, testable asset pricing and macroeconomic predictions which I provide evidence for in the data.[1]

Second, this paper contributes to the literature on within-city house price dynamics. Ortalo-Magne and Rady (2006) construct a life-cycle model of the housing market in which households live for four periods and show that the prices of flats display more volatility than houses. My model features a simpler life-cycle specification (in which households only live for two periods), but permits a more transparent characterization of the cross-section of capital gains in response to different shocks. I primarily focus on how *demand-side* changes affect house prices. A related work by Guerrieri et al. (2013) demonstrates how *supply-side* effects due to gentrification can cause the prices of low-end homes tend to appreciate more in MSAs during nation-wide housing booms.

Määttänen and Terviö (2014) also consider an assignment model with varying house quality types. They show that house price changes due to changes in the income distribution spill over upwards in the housing quality ladder under no-trade equilibria in a uniform fashion. As such, high-end homes appreciate the most in response to income growth. I also show that shocks trickle up when housing transactions are realized (thereby permitting a notion of a capital gain). However, these spill-overs are *not* uniform in my framework. This allows for richer capital gain heterogeneity that informs the general equilibrium response of house prices to aggregate shocks. Moreover, the framework of Määttänen and Terviö (being static) cannot analyze how credit frictions, interest rates, or perceptions regarding *future* house values affect capital gains and propagate onto the rest

[1] Greenwald and Guren (2021) analyze a distinct form of segmentation in housing markets between rental and owner-occupied housing. In contrast, I analyze how segmentation in owner-occupied housing that arises through indivisibility and heterogeneity affects housing market dynamics.

of the economy.

Of particular relevance is the work of Landvoigt et al. (2015). They construct a model with a continuum of housing quality tiers and quantify it using micro-data on San Diego County during the 2000-2005 boom. They show that the increase in the availability of cheap credit, along with a change in the housing quality distribution over this period can jointly generate the cross-sectional capital gains observed in the data. Their work is primarily quantitative in nature. In contrast, I provide theoretical results on the cross-sectional distribution of capital gains to a wide range of economic shocks, which I test empirically in the data.

Finally, this work relates to papers by Mian et al. (2013), Mian and Sufi (2014), Berger et al. (2018), and Guren et al. (2021), which analyze the effect of house price changes on consumption expenditures. My work offers a complementary mechanism for the propagation of house price changes to the rest of the macroeconomy through heterogeneity in capital gain incidence.[2] Moreover, most theoretical work in this literature examines the consumption response to house price changes in partial equilibrium. In contrast, I emphasize the importance of general equilibrium forces in shaping the transmission mechanism of housing wealth effects through the cross-sectional incidence of capital gains. Hence, the *type* of underlying shock is crucial in determining the response of aggregate consumption expenditures through the housing market.

Outline. The rest of the paper proceeds as follows. Section 2 documents additional motivating facts regarding housing market segmentation. Section 3 sets up the model. Sections 4 and 5 present the main theoretical results. Section 6 tests the key predictions of the theory. Section 7 concludes.

2 Facts About Housing Market Segmentation

I first provide quantitative evidence for the premise of this paper – housing market segmentation. To this end, I analyze the rates of return of different types of houses within various local housing markets across the United States. My definition of a local housing market is that of a county: this is consistent with prior studies on the within-market variation in housing prices (Guerrieri et al., 2013; Landvoigt et al., 2015), as well as empirical work that quantifies the effect of net worth changes on local employment (Mian and Sufi, 2014).[3]

2.1 Data

My measure of house prices comes from the Zillow zip-code level price indices. The Zillow index is a hedonically adjusted price index. It uses detailed information on properties, collected from public

[2]The redistributive effects of house price changes and its role in shaping aggregate consumption expenditures is also emphasized in Kiyotaki et al. (2011).

[3]The results are quantitatively similar when using MSAs as the market definition. There are more sophisticated methods to identify housing markets using the search behaviour of households as in Piazzesi et al. (2020) or a variant of the stochastic block model used in Nimczik (2017). Doing so requires data on the search behavior of households for houses, which is challenging to obtain and beyond the scope of this paper.

records, to create a seasonally adjusted measure of the typical home value in a given region.[4] I restrict the analysis to the years 2000-2020. Many zip codes are missing prior to the year 2000, which makes it difficult to characterize the cross-sectional evolution of house prices preceding this time period. In order to construct weighted averages of house price growth, I compute total number of houses by county and zip code from the American Community Survey (ACS). I use the ACS five-year survey whose corresponding year is closest to the year in which I disaggregate houses into different quality tiers.

2.2 Methodology

I treat each county as a common housing market that is segmented in various quality tiers. I follow Landvoigt et al. (2015) in assuming that market prices approximately reflect a one-dimensional index of housing quality. The reasoning for this is that any changes in the payoff relevant characteristics of the house, such as land, location, and structure, should be incorporated into house valuations in competitive equilibrium. Moreover, in the theoretical model presented in the next section, I show that price is a sufficient statistic for a given housing type's quality ranking. Consequently, I sort zip codes in a particular county into different quality quantiles according to their price.[5]

2.3 Cross-Sectional Patterns

The national house price cycle can be trisected into the 2000-2006 boom, the 2006-2012 bust, and its subsequent recovery. Hence, it is natural to analyze cross-sectional distribution of house prices within each time frame separately.

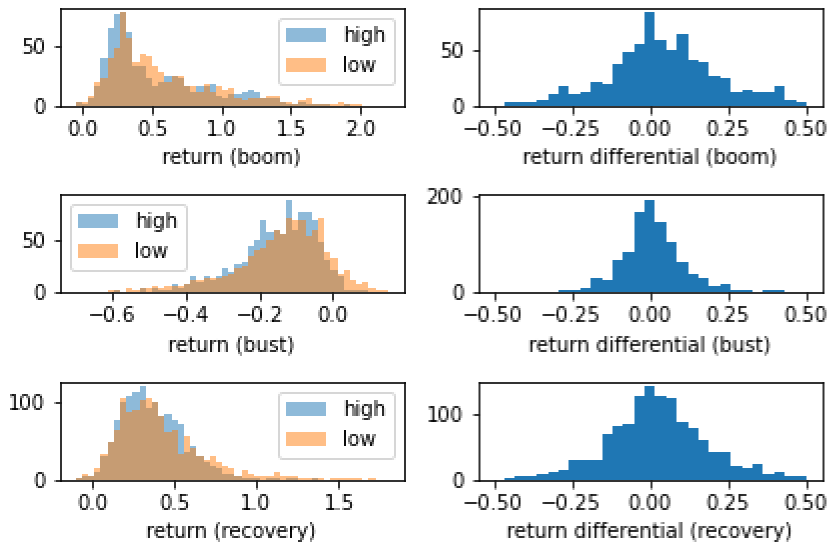
The left column of Figure 2 plots the average growth rates of housing prices within the top and bottom quality tier quartiles for all counties and each time period under consideration. The right column of Figure 2 plots the differences between the bottom and top quality quartiles *within* each county. Two observations immediately stand out. First, there is significant variation in the across county growth rate of house prices, even in the same relative quality tier. Second, there is significant variation in the *intra*-county relative growth rates of various quality tiers.

Moreover, in the first and third rows of Figure 2, where the national housing market was trending upwards (boom and recovery), the distribution of low quality growth rates places more mass on its right tail relative to its high quality counterpart. Hence, in periods of national house price appreciation, low quality housing tends to appreciate more on average relative to high-end housing. This observation is consistent with the results in Guerrieri et al. (2013), who demonstrate that houses with lower initial prices experience larger capital gains during nationwide housing increases.

[4] My measure comes from the ZHVI Single-Family Homes Time Series index in `https://www.zillow.com/research/data/`. I use data on single family homes to maintain consistency with my theoretical model in Section 4.

[5] When looking at changes in prices experienced by houses of a given quality over a particular time frame, the year in which houses are sorted into quality tiers is somewhat arbitrary. However, the relative ranking of median house prices across zip codes within counties is very persistent. As such, all the empirical results are insensitive to the year in which zip codes are sorted into different quality quartiles.

Figure 2: Heterogeneity in Within-Market Housing Returns



Note: Relative rates of return for all counties during the boom (first row), bust (middle row), and recovery (third row). The first column plots the histograms of cumulative returns for the bottom and top quartile of house qualities for the corresponding time period. The second column plots the return difference of the bottom quartile relative to the top quartile. Counties with an average house price change of less than 2% (in absolute terms) have been dropped from the sample.

For counties that experienced a county-wide average house price growth of more than 10% during the boom (recovery) low-end housing appreciated by 4.6% (4.9%) more than high-end housing, on average. However, the reverse observation is true during the bust: high-end homes *depreciated* by 1.4% more on average.

Although these numbers might appear small, a closer inspection of the data reveals substantial housing market segmentation. For example, across the 862 counties in the sample during the boom, 47% experienced an absolute difference in the returns of the low-end relative to high-end homes of more than 10%. Of these counties, 66% saw low-end homes appreciate more, with an *average* difference across quality tiers of 25%. These numbers are clearly non-negligible. Counties during the bust and recovery experienced a similar disconnect in their rates of return (See Table 1). These features suggest that one cannot characterize the housing market as a single "market". Hence, in order to understand what moves house prices, we must understand what moves different quality segments within the housing market.

3 A Model of Segmented Markets

To study the key drivers of heterogeneity in capital gains, as well as its role in amplifying macroeconomic shocks, I present a tractable macroeconomic model with a housing sector. The model embeds two key features of housing markets that will be crucial in generating heterogeneous capital gains across different types of houses, (i) *segmentation* across houses of varying quality and (ii) *indivisibility* of housing units.

Table 1

	return differential > 10%	
	percentage of counties	average return differential
boom (2000-06)	47%	25%
bust (2006-12)	23%	19%
recovery (2012-19)	43%	24%

Note: This table depicts summary statistics for return differences across counties in the United States. Return differential is defined as the average absolute return between the lowest and highest quality quartiles within a given county over the corresponding period.

Agents and Preferences. Time is discrete and indexed by $t \in \mathbb{N}$. The economy is populated by an overlapping generations of households. Each period, a mass $m > 1$ of households is born and all households live for two periods. Household age is indexed by $j \in \{0, 1\}$ and will be referred to interchangedly as "young" and "old".

Households have perfect foresight and there is no aggregate uncertainty. They derive utility over consumption streams of a numeraire good and housing. I assume that households purchase housing when they are young, and sell on to the next generation's young when they are old. Payoffs are given by:

$$\mathcal{U}\left(\{c_{t+j,j}\}_{j=0}^{1}, h_{t,1}\right) = \log(c_{t,0}) + \beta \log(c_{t+1,1}) + \chi \log(h_{t,1}) \tag{1}$$

where $c_{t,j}$ and $h_{t,j}$ denote the consumption of the numeraire good and housing quality purchased at time t and age j, respectively. The parameter $\chi > 0$ is a utility weight on housing services and $0 < \beta < 1$ is the household's discount factor. Finally, I assume that households incur a proportional transaction cost $\delta \in (0, 1)$ from selling a house.

Housing Market. The housing market consists of a collection of indivisible housing units. Houses are heterogeneous and are indexed in terms of their quality $h \in \mathbb{R}^+$. This one-dimensional index can be thought of as an appropriately weighted aggregator of the various amenities households care about in a house (including location, number of bathrooms, quality of neighboring schools, etc.). House quality in any given period is distributed according to a strictly increasing cumulative distribution function $H(\cdot)$ and where the support of $H(\cdot)$ is $[\underline{h}, \bar{h}] \subset \mathbb{R}^+$. Each house can accommodate at most one household, who must be an owner-occupier.

Households can purchase at most one house of a given quality. A house of quality h at time t trades at a price $q_t(h)$ in a competitive market. Note that this gives rise to a price *function* – a price for each house quality tier. Households have the option to not become home-owners, in which case they receive an exogenous outside flow of housing services of quality $0 < b < \underline{h}$ at zero cost. This assumption ensures that household utility is well-defined if households choose not to enter into the housing market. I normalize the total quantity of housing units available for purchase to unity. Since $m > 1$, at least some households will opt for this outside option in equilibrium.

Credit Frictions. Households can participate in credit markets in their youth by either lending or borrowing at a gross interest rate of $R > 1$. However, the presence of credit frictions implies that households are subject to a loan-to-value constraint

$$d_{t,1} \leq (1-\theta)q_{t+1}(h_{t,1}) \tag{2}$$

where $d_{t,1}$ is the household's choice of debt at time t, and θ is a proportional down-payment required by the household.[6] Note that this constraint implies that non-home owning households cannot borrow at all with respect to their second-period income.

In order to isolate the direct effect of various changes in macroeconomic conditions on the cross-section of house prices, I assume a small open economy and take R to be exogenous. However, in the following analysis, I also characterize how changes in rates affect house prices in order to inform the full general equilibrium effect.

Endowments. Households are born without any initial wealth, but are heterogeneous in their first period income. In particular, each household receives income y at age 0, which is distributed according to a strictly increasing cumulative distribution function $F(\cdot)$ with support $[\underline{y}, \bar{y}] \subset \mathbb{R}^+$. In the second period, households receive a common income stream $y_1 > 0$. This single margin of heterogeneity is a tractable way to generate sorting between households and house quality type, but is not necessary to obtain the main results. In Appendix B.1, I demonstrate how the model can be extended to incorporate household heterogeneity in old income as well as income risk.

3.1 Equilibrium

We are now in a position to state the household problem. A household with income y that is born at time t takes the pricing functions $\{q_t(h)\}_t^{t+1}$ as given, and solves

$$\sup_{\{c_{t+j,j}\}_{j=0}^{1},\, h_{t,1},\, d_{t,1}} \mathcal{U}\left(\{c_{t+j,j}\}_{j=0}^{1}, h_{t,1}\right) \tag{3}$$

$$\text{s.t. } c_{t,0} + q_t(h_{t,1}) = y + d_{t,1} \tag{4}$$

$$c_{t+1,1} = y_1 - Rd_{t,1} + (1-\delta)q_{t+1}(h_{t,1}) \tag{5}$$

$$d_{t,1} \leq (1-\theta)q_t(h_{t,1}) \tag{6}$$

where (4)-(5) are the household's first and second-period budget constraint, respectively, and (6) is the loan-to-value constraint described above.

In the following analysis, I focus on the response of the economy to unanticipated changes in various parameters from a stationary equilibrium. A stationary competitive equilibrium in this

[6] This can be microfounded as in Kiyotaki and Moore (1997) or Iacoviello (2005). Borrowers can repudiate on their debt obligations while lenders can only repossess the borrower's assets by paying a proportional transaction cost $\theta q_{t+1}(h_{t,j})$. Therefore, lenders will never lend more than $(1-\theta)q_{t+1}(h_{t,j})$ in a subgame perfect equilibrium. With this microfoundation, the fact the down-payment is paid in proportion to *future* house prices is needed to avoid default in equilibrium following unanticipated shocks. However, this particular specification is inessential for the main results.

economy is defined as follows:

Definition 1. *Given a distribution of income earnings $F(\cdot)$, a distribution of house qualities $G(\cdot)$, and a gross interest rate R, a stationary equilibrium is a set of time-invariant individual decision functions $\{c_j^*\}_{j=0}^1$: $[\underline{y}, \bar{y}] \to \mathbb{R}^+$, d_1^*: $[\underline{y}, \bar{y}] \to \mathbb{R}$ and h_1^*: $[\underline{y}, \bar{y}] \to \{b\} \times [\underline{h}, \bar{h}]$, as well as a pricing function q: $[\underline{h}, \bar{h}] \to \mathbb{R}^+$, such that*

1. *The individual decision functions solve the household problem (3) - (6)*
2. *The market for each housing quality h clears*

$$H(h) = m \int_{\underline{y}}^{\bar{y}} 1[\underline{h} \leq h_1^*(y) \leq h] \mathrm{d}F(y), \quad \forall\, h \in [\underline{h}, \bar{h}] \tag{7}$$

3. *The housing pricing function is time-invariant $q_t(\cdot) = q(\cdot)$, $\forall\ t \in \mathbb{N}$*

Equation (7) ensures that the number of household that choose a housing quality less than h is equal to the total number of available houses below that quality. The key difference in this definition from the standard case with homogeneous housing is that there exist a *continuum* of market clearing conditions, one for each house quality tier.

3.2 Further Extensions

Housing Supply. In analyzing the response of house price changes to aggregate shocks, I intentionally abstract from housing *supply* movements to isolate *demand*-side factors. This can be viewed as a useful approximation over short-term horizons, when housing supply is inelastic. Moreover, declining housing markets are generally associated with prices that are below the minimum profitable production cost, which makes the supply schedule across all housing qualities inelastic (Glaeser and Gyourko, 2018). Appendix B.2 presents an extension of this model with free entry into a construction sector in which houses are demand-determined following contractionary shocks.

Rental Markets. The absence of rental markets is inessential to the main analysis. The model can be extended to allow for a fixed rental rate of housing, in which case the assumption $b < \underline{h}$ would reflect the observed segmentation between rental and owner-occupied markets (Garriga and Hedlund, 2020; Greenwald and Guren, 2021). Appendix B.3 presents an extension of the model in which households can purchase rental units from a rental stock.

Bequests. Appendix B.4 extends the baseline model to incorporate bequests from the old to the young. Bequests can introduce additional variation in marginal propensities to consume, thereby affecting the propagation of house price movements to aggregate consumption expenditures. This mechanism is studied in greater detail in Section 5.2.

4 Unidirectional Shock Propagation

I now characterize the main features of the stationary equilibrium. I show that it admits a unique, differentiable price function characterized by two implicit ordinary differential equations. Next, I show that the equilibrium exhibits *positive assortative matching* (PAM) between household income and house quality. Finally, I show that PAM implies that shocks to house prices can only propagate upwards across house quality tiers, but not downwards.

4.1 Equilibrium Characterization

The following proposition characterizes the pricing function through the households' first-order condition.

Proposition 1. *$q^*(\cdot)$ is strictly increasing and differentiable $\forall\ h \in (\underline{h}, \overline{h})$. For an unconstrained household with income y, the pricing function satisfies:*

$$\left(1 - \frac{1-\delta}{R}\right) \frac{\mathrm{d}}{\mathrm{d}h} q^*(h_1^*(y)) = \underbrace{\chi \frac{c_0^*(y)}{h_1^*(y)}}_{MRS} \tag{8}$$

where $c_0^*(y) = \frac{1}{1+\beta}\left(y + \frac{y_1}{R} - \frac{R+\delta-1}{R} q^*(h_1^*(y))\right)$. For a constrained household:

$$\left(1 - \frac{1-\delta}{R}\right) \frac{\mathrm{d}}{\mathrm{d}h} q^*(h_1^*(y)) = \chi \frac{1}{h} \left(\frac{x}{\tilde{c}_0^*(y)} + \frac{(1-x)\beta R}{\tilde{c}_1^*(y)}\right)^{-1} \tag{9}$$

where $x \equiv \frac{\theta R}{R+\delta-1}$, $\tilde{c}_0^*(y) = y - \theta q(h_1^*(y))$ and $\tilde{c}_1^*(y) = y_1 - [R(1-\theta) - (1-\delta)]\, q^*(h_0^*(y))$.

Proof. See Appendix A.1. □

Equation (8) demonstrates that the slope of the pricing function at a given house quality tier is dictated by the *marginal rate of substitution* (MRS) between consumption and housing services of the household that purchases that housing type in equilibrium. The willingness of households to sacrifice consumption goods for additional housing services therefore determines the *rate of change* of prices across the quality spectrum. This is contrast to the homogeneous housing stock case, in which the MRS of all households prices a common *per-unit* price of housing.

Equation (9) illustrates the impact of credit constraints on the pricing function. Relative to the unconstrained case, we see that what determines the rate of change of house prices is a *distorted* marginal rate of substitution: consumption enters the MRS through a linear combination of marginal utilities across young and old age. The constant $x \equiv \theta R/(R + \delta - 1)$ parameterizes the distribution of household liabilities across time. As θ increases, x increases, thereby making *current* marginal utility relatively more important in pricing houses. Clearly, if $\tilde{c}_1^*(y) = \beta R \tilde{c}_0^*(y)$, the household lies on its Euler equation and Equations (8) and (9) are identical.

Notably, the presence of market segmentation and indivisibilities imply that each household becomes a "marginal" investor for the house quality that it optimally chooses to purchase in equilibrium. In other words, changes in the MRS for a particular household will affect the rate of change of prices only at that households' neighboring quality tiers. Shocks that heterogeneously affect the MRS of households along the income distribution will therefore naturally give rise to heterogeneous capital gains along the house quality distribution. The next proposition characterizes the housing policy function $h_1^*(y)$.

Proposition 2. *The following two statements are true in any stationary equilibrium.*

1. *There exists a $y_p \in (\underline{y}, \bar{y})$ such that households with $y > y_p$ are homeowners.*
2. *The policy function $h_1^*(y)$ is strictly increasing in y for all $y > y_p$.*

Proof. See Appendix A.2. □

This result follows from the weak supermodularity in consumption and housing in the households' payoffs. It allows us to relate changes in the willingness to pay for housing services along the *income* distribution to changes in prices along the *quality* distribution. To see this, note that monotonicity in h_1^* implies that the market clearing condition (7) must now equate the mass of all home-owners below a given quality with the corresponding supply:

$$m\left(F(y(h)) - F(y_p)\right) = H(h), \quad \forall\, h > \underline{h} \tag{10}$$

or, equivalently,

$$y(h) = F^{-1}\left(\frac{1}{m}\left(H(h) + F(y_p)\right)\right), \quad \forall\, h > \underline{h} \tag{11}$$

which gives rise to the *inverse* matching function $y(\cdot) \equiv \left(h_1^*\right)^{-1}(\cdot)$ in closed form. As such, we can index household's policy functions through their *equilibrium* choice of housing. This observation will be extremely useful in characterizing how the pricing function will respond to certain *localized* shocks. Before doing so, I settle the question of equilibrium existence and uniqueness.

Proposition 3. *Assume $x < 1$. Then, a unique stationary equilibrium exists.*

Proof. See Appendix A.3. □

The assumption $x < 1$ ensures that constrained households' consumption is decreasing in house prices at old age. This suffices to uniquely pin down the initial condition $q(\underline{h})$. I maintain this assumption for the remaining analysis.

4.2 Price Response to Localized Shocks

This section shows that there is a sense in which shocks propagate unidirectionally in the housing market. I consider an unanticipated, permanent change to the income distribution $F(\cdot)$ to $\tilde{F}(\cdot)$. I denote all equilibrium variables following the shock with a tilde.

Theorem 1. *Suppose $F(\cdot)$ and $\tilde{F}(\cdot)$ satisfy $F(y) = \tilde{F}(y)\ \forall\, y < \tilde{y}$, where $\tilde{y} \in [\underline{y}, \bar{y}]$. Then, $\tilde{q}^*(h) = q^*(h)\ \forall\, h \leq h^*(\tilde{y})$.*

Proof. See Appendix A.4. □

This proposition states that shocks that affect richer households will have no effect on house prices below that income group. Intuitively, this is because price changes only flow in the same direction in which *incentives* bind. Increasing the income of a household will increase the demand for housing services *above* its original housing tier, but will leave the prices below that tier unaffected. Thus, shocks do not "trickle down". House prices at low quality tiers are invariant to distributional changes in the income of households that match to *better* houses. Crucially, the reverse reasoning does not apply. The next lemma considers a first-order stochastic shift of the income distribution for *low-income* households – *i.e.* low-income households become richer.

Lemma 1. *Suppose $\tilde{F}(y) \leq F(y)$ for all $y \in [\underline{y}, y_p]$ where $y_p \in [\underline{y}, \bar{y}]$ and where the inequality is strict for some open interval. Suppose further $\tilde{F}(y) = F(y)$ for all $y_p \in (y_p, \bar{y}]$. Then, $q(h) \leq \tilde{q}(h)$ for all $h \in [\underline{h}, \bar{h}]$. Moreover, $q^*(h) < \tilde{q}^*(h)$ for all $h \in (h_1^*(y_p), \bar{h}]$.*

Proof. See Appendix A.5. □

Hence, increasing the income of households below a particular income quantile will increase the prices of *all* homes. At its core, this is a non-trivial implication of PAM inherent within the competitive equilibrium. Intuitively, incentives for housing choice bind *upwards.* Absent any price changes, increasing the income of a particular household will induce it to demand better quality homes. However – due to PAM – richer households must still reside in the homes that now a less wealthy household is demanding. The only way these richer households can continue to reside in their original quality tiers is if they themselves bid more for housing. One therefore gets pricing spill-overs that occur from the bottom of the house quality distribution to the top.[7]

Hence, the presence of market segmentation implies that changes in marginal rates of substitution can "trickle up" along the quality ladder, but shocks can never "trickle down". In this sense, shocks to poorer households are most important in shaping the cross-section of housing returns because they affect the house prices of all qualities *above* them through spill-overs in quality tiers.

A variant of this "trickle up" effect is also present in Määttänen and Terviö (2014), who consider the effect of income inequality on house prices under no-trade equilibria, and in Terviö (2008), who considers the determinants of CEO pay.[8] This result stands in stark contrast to the predictions from models with a homogeneous housing stock, in which changes in the marginal rate of substitution of any given household will affect the house prices of all households in the economy. Moreover, this result is robust to the functional forms of the income and housing distribution, or the value of credit constraints and interest rates. The generalizability of this result begs for a data-driven test and motivates my empirical analysis in Section 6.

[7] The preceding results generalize to the case in which we consider changes in the housing distribution $G(h)$. As long as as the housing distribution remains unchanged for all h below some threshold $\tilde{h}$, prices are guaranteed to remain constant for all $h < \tilde{h}$.

[8] In both these works, the magnitude of these spill-over effects is constant across the quality spectrum. Hence, the top quantiles of the quality distribution under consideration *always* gain the most in response to shocks. In contrast, I demonstrate in the next section that spillovers in this model *dissipate* across the quality spectrum and map the rate at which they dissipate to model primitives for a wide class of matching functions.

5 Segmentation and Aggregate Shocks

In this section, I consider how changes to certain *aggregate* shocks affect the cross-sectional distribution of capital gains. Because these changes typically affect the MRS of all households in the economy, we cannot use the results in the previous section to characterize the distribution of capital gains. However, with segmented housing markets, some shocks invariably affect some marginal rates of substitution more than others. Consequently, price changes will be more pronounced for quality tiers bought by households whose MRS change more. The distribution of capital gains thus crucially depends on the *type* of shock under consideration.

5.1 The Price Response to Aggregate Shocks

Characterizing the distribution of capital gains for arbitrary functional forms is challenging because the pricing function is generally non-linear across quality tiers. This can induce non-monotonicities in constraint status, where wealthy households are constrained because the pricing function increases at a faster rate than their income. In order to characterize the effect of aggregate shocks to the pricing function in closed form, I maintain the following assumption:

Assumption 1. *$\beta R y_p > y_1$. Moreover, χ is not too large.*[9]

Assumption 1 is a sufficient condition that ensures that the borrowing constraint for wealthy households is not violated by bounding the marginal utility of housing. Intuitively, sufficiently small χ ensure that house prices and debt does not grow at a faster rate than income, thereby ensuring constraint status is monotone. We may now characterize the pricing function:

Proposition 4. *There exists a threshold $y_c \in [\underline{y}, \bar{y}]$ such that households with $y \geq y_c$ are not credit constrained. Moreover, $q^*(h)$ satisfies the following equation for all $h \geq h(y_c)$:*

$$q^*(h) \quad = \quad \underbrace{\frac{\chi}{\omega(1+\beta)} \int_{h_c}^{h} \left(\frac{\tilde{h}^{\frac{\chi}{1+\beta}-1}}{h^{\frac{\chi}{1+\beta}}} \right) y(\tilde{h}) d\tilde{h}}_{(1):\ \textit{slope}} + \underbrace{\frac{y_1}{R\omega} \left(1 - \left(\frac{h_c}{h} \right)^{\frac{\chi}{1+\beta}} \right)}_{(2):\ \textit{future income}} + \underbrace{\left(\frac{h_c}{h} \right)^{\frac{\chi}{1+\beta}} q^*(h_c)}_{(3):\ \textit{spill-overs}} \tag{12}$$

where $\omega \equiv \left(1 - \frac{1-\delta}{R}\right)$ and $h_c \equiv h(y_c)$.

Proof. See Appendix A.6. ∎

This result establishes three features of the effect of changes in economic fundamentals on the distribution of capital gains. First, changes in *future* income y_1 have a relatively higher impact on the prices of *high*-end homes through term (2). This is because changes in future income is mostly payoff relevant to wealthier households not on their credit constraint. The MRS of these households increases, and this induces spillovers upwards in quality.

Second, changes in credit conditions will mostly affect the prices of *low*-end homes. Note that θ does not directly appear in Equation (12) since credit is not pay-off relevant to unconstrained

[9] Appendix A.7 contains exact parametric restrictions in terms of model primitives.

households. However, credit still affects high-end home prices through changes in $q(h_c)$. Hence, the effects of credit are larger for low house qualities, and dissipate as one moves upwards the quality ladder. Note that these spill-overs dissipate at a rate that is proportional to $h^{-\frac{\chi}{1+\beta}}$. Intuitively, this is linked to the slackness in the housing market – when χ is high (or β is low), small changes in consumption have a large effect on the household's marginal rate of substitution. This makes it more difficult for households to accommodate pricing pressures from below.

Third, high-end homes are particularly sensitive to changes in interest rates or transaction costs. This is embodied through ω, which captures the proportional change in the net present value of liabilities of a household when purchasing a more expensive home. Whenever term (1) is increasing in house quality, changes in interest rates or transaction costs will have a *proportionately* larger effect on high-end homes.

The next result formalizes this discussion. In order to tractably characterize the effect of shocks on *percentage* changes, I assume that the matching function belongs to a linear class $y(h) = kh$ (for some $k > 0$), but note that all results pertaining to *absolute* price changes are independent of the precise form of the matching function.[10]

Theorem 2. *The pricing function responds as follows to permanent, unanticipated changes in parameters at time t:*

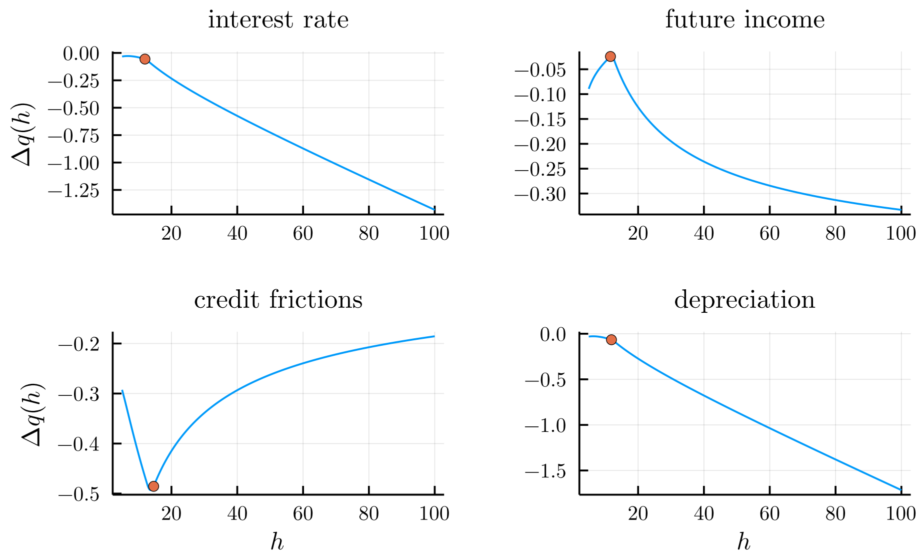
1. *Loan-to-value constraint (θ) — $\frac{\partial q_t(h)}{\partial\theta\partial h} > 0$ and $\frac{\partial \log q_t(h)}{\partial\theta\partial h} > 0$ for $h > h_c$. Moreover, $\lim_{h\to\infty} \frac{\partial \log q_t(h)}{\partial\theta} = 0$.*
2. *Interest rate (R) — $\frac{\partial q_t(h)}{\partial R\partial h} < 0$ for $h > h(y')$, where $y' > 0$ and $\lim_{h\to\infty} \frac{\partial \log q(h)}{\partial R} = -\frac{1-\delta}{R(R+\delta-1)}$*
3. *Transaction cost (δ) — $\frac{\partial q_t(h)}{\partial\delta\partial h} < 0$ for $h > h(y')$, where $y' > 0$ and $\lim_{h\to\infty} \frac{\partial \log q_t(h)}{\partial\delta} = -\frac{1}{R+\delta-1}$*
4. *Future income (y_1) — $\frac{\partial q_t(h)}{\partial y_1\partial h} > 0$ and $\frac{\partial \log q_t(h)}{\partial y_1\partial h} < 0$ for $h > h_c$. Moreover, $\lim_{h\to\infty} \frac{\partial \log q(h)}{\partial y_1} = 0$*

Proof. See Appendix A.7. □

This result highlights that the incidence of capital gains along the income distribution depends on the type of shock under consideration. Note that the result characterizes both *absolute* capital gains and percentage *returns* in response to different kinds of shocks. The top-left panel in Figure 3 and Figure 4 plot the change in the pricing function (in absolute and percentage terms, respectively) due to a 10% increase in the net interest rate. An interest rate change has little effect on the prices of low-end homes that constrained households match to. This is because changes in the interest rate affects the net present value of liabilities, which is something that is mostly pay-off relevant to unconstrained households. These losses accumulate as one moves up the quality ladder due to spillovers. As such, capital losses are monotonic in quality. A similar reasoning applies for changes in transaction costs and future income changes. However, because changes in future income do not *proportionately* affect the valuation of a house, percentage returns dissipate as one moves up the quality ladder.

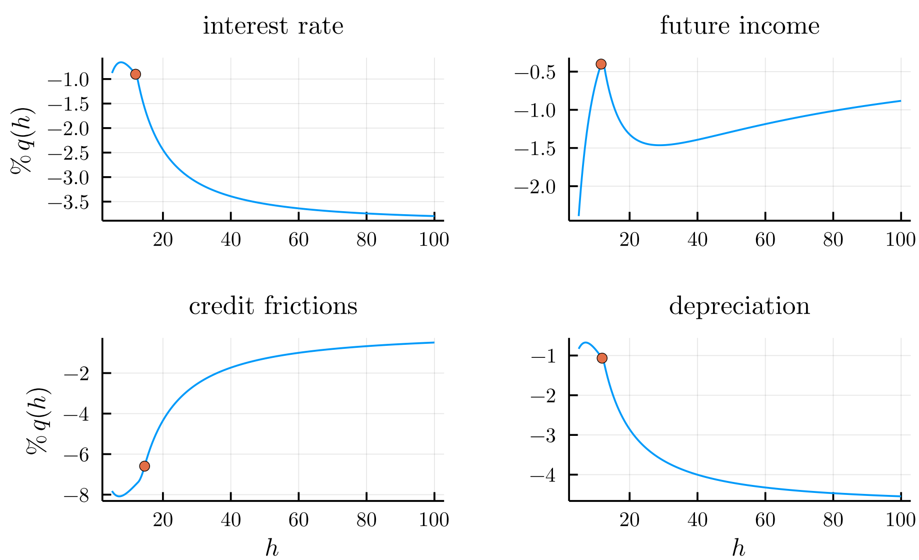
[10] This assumption ensures that the price-to-income ratio is asymptotically stable. Appendix ?? shows that linear matching functions obtain when the income and house quality distribution are Pareto with common shape parameters.

Figure 3: Absolute Price Differences across Quality Tiers



Note: Absolute price differences in response to various shocks. Each subplot depicts the capital gains of various quality tiers in response to a 10% contractionary shock. Parameters are $(\beta, k, R, \theta, y_1, \underline{h}, b, \underline{y}, \delta, m) = (1, 1, 1.1, 0.2, 1.0, 5, 1, 0.65, 0.1, 2)$. The distribution for housing and income are Pareto with a common shape parameter of two. The red dot marks the credit threshold h_c.

Figure 4: Percentage Price Changes across Quality Tiers



Note: Capital gains (in percent) in response to various shocks. Each subplot depicts the capital gains of various quality tiers in response to a 10% contractionary shock. The red dot marks the credit threshold h_c. See Figure 3 for model parameterization.

In contrast, a ten percent permanent increase in the loan-to-value ratio (bottom left) results in larger capital losses amongst low-end homes. Moreover, these capital losses are monotonic and decreasing for $h > h_c$. This is because constrained households are more sensitive to changes in the loan-to-value constraint relative to their wealthier counterparts. An increase in credit frictions magnifies the discrepancy between the marginal valuation of consumption across periods for constrained households. As a result, low-end homes experience greater capital losses, both in absolute and relative terms. All of these effects are absent in a model with a homogeneous housing stock, in which percentage returns are fixed to be constant.

Amplification of Credit Shocks. Observe that the effect of credit on house prices converges to zero as their quality increases, but prices still drop substantially for mid-range homes through spill-overs in quality tiers. Hence, market segmentation gives rise to a mechanism in which changes in credit can have a large affect on *average* house prices, even if relatively few households are credit constrained. This is contrast to the homogeneous housing stock case, in which case changes in credit can have a large effect on the per unit price of housing only many households are credit constrained (Greenwald and Guren, 2021). In this sense, market segmentation can amplify the effect of credit conditions on house price fluctuations.

Drivers of the Boom and Bust. The above analysis shows that different shocks have different predictions for the cross-sectional distribution of house prices, a result that is obscured when only looking at aggregate returns. Consequently, one can identify the key shocks that drive housing market returns by analyzing the distributional changes at the micro level. For example, a recent literature analyzes whether the boom-bust cycle of the Great Recession was driven by credit conditions or beliefs about house values (Cox and Ludvigson, 2021; Kaplan et al., 2020). If changes in credit conditions were the dominant force in the boom-bust cycle, we should expect low-end homes to have more volatile returns relative to mid-tier and high-end homes and *vice versa* for changes in beliefs (as captured by changes in transaction costs).

Illustrative Example. Theorem 2 characterizes the response of the pricing function above the credit constraint threshold h_c. In this section, I provide a simple parameterization which allows for comparative statics to the whole pricing function in closed form. I assume the income distribution and the house quality distribution is Pareto with a common shape parameter $\alpha > 0$:

$$F(y) = 1 - \left(\frac{\underline{y}}{y}\right)^{\alpha}, \quad G(h) = 1 - \left(\frac{\underline{h}}{h}\right)^{\alpha} \tag{13}$$

and cutoffs $\underline{y}$ and $\underline{h}$, respectively. Assuming $m = 1$ (so that all households are home-owners) and using (10), we obtain the following inverse matching function in terms of house qualities:

$$y(h) = \frac{\underline{y}}{\underline{h}} h \tag{14}$$

for all $h \geq \underline{h}$, which is linear in housing quality. In order to obtain the constrained pricing function in closed form, I also assume that borrowing constraints are maximally tight $\theta \approx 1$, that housing depreciates fully $\delta = 1$ and that households are perfectly patient $\beta = 1$. Finally, I take $\underline{h} \to 0$ and

$\underline{y} \to 0$ such that $\frac{\underline{y}}{\underline{h}} \to 1$,[11] This yields the following pricing ODE for constrained households:

$$q'(h) = \frac{\chi}{\theta}\frac{h - \theta q(h)}{h} \tag{15}$$

This is a separable ordinary differential equation, the solution of which is

$$q(h) = \frac{\chi}{(1+\chi)\theta}h \tag{16}$$

since the price of the house of lowest quality must be zero given that the poorest household has zero income and cannot borrow.

Hence, we can readily observe the impact of credit constraints along all households that will be credit constrained. In particular, in response to a temporary tightening in the borrowing constraint, the price change will be

$$\frac{d\ln q(h)}{d\theta} = -\frac{1}{\theta} < 0 \tag{17}$$

which is *independent* of the house quality h. Therefore, the model predicts that as long as households are credit constrained (and credit constraints are sufficiently tight), house price appreciation or depreciation in response to changes in credit conditions will be uniform across all such house qualities. What can be said about house qualities for which the matched households are not credit constrained? Letting h_c, y_c denote the lowest housing and income level above which households are not constrained,[12] one can show that the pricing ODE for an unconstrained household satisfies

$$q'(h) = \chi\frac{h + \frac{y_1}{R} - q(h)}{2h} \tag{18}$$

the unique solution of which is

$$q(h) = \frac{\chi}{2+\chi}h - \xi\left(\frac{1}{h}\right)^{\frac{\chi}{2}}\left(\frac{\theta y_1}{R}\right)^{\frac{\chi}{2}+1} + \frac{y_1}{R} \qquad \text{for} \qquad h > h_c \tag{19}$$

where $\xi \equiv \frac{\chi(1+\chi)^{\frac{\chi}{2}+1}}{2+\chi}$

In the limit as $h \to \infty$, pricing for high quality houses becomes linear, but is non-linear for lower-tier houses. Moreover, we can transparently see how different shocks affect the cross-sectional distribution of capital gains. A tightening in the borrowing constraint induces constant capital gains for housing qualities up to h_c, and decreasing capital gains thereafter, as predicted by Theorem 2.

Changes in y_1 will only affect house prices for households that can afford to tap into their future earnings. Indeed, an increase in y_1 leaves house prices for constrained households entirely unaffected, but raises house prices for higher quality tiers. Similarly, a decrease in the interest rate

[11] Formally, we can consider sequence of economies in which cut-offs of the income and housing distributions converge to zero at the same rate. Taking the limits of the cut-offs to zero is necessary in order to prevent indeterminacy in equilibrium. Otherwise, multiple equilibria would arise due to the presence of discrete choice. This is common in assignment models with indivisible assets due to the generic non-uniqueness of the core (Shapley and Shubik, 1977).

[12] Under the preceding assumptions, this is given by $y_c = \theta\frac{1+\chi}{R}y_1$.

increases the present value of income, which boosts house prices for all qualities greater than h^*. However, it does not change the prices of low-end homes because the owners of these homes are credit constrained: marginal changes in the future income of these households does not affect their willingness to pay in their first period.

5.2 Capital Gain Heterogeneity and Consumption

This section illustrates that the heterogeneous incidence can be an important driver of other macroeconomic aggregates, such as aggregate consumption expenditures. Intuitively, market segmentation allows larger capital losses to be experienced precisely by those households most important in driving aggregate consumption. In particular, the effect of house price changes on consumption will depend on how capital gains covary with a "distorted" marginal propensity to consume (MPC): a weighted difference in the MPC of young households, keeping their housing choice fixed, and the MPC of old households.

Concretely, let $\widetilde{MPC}_0(h)$ denote the additional consumption of a young household due to an incremental income increase, holding the housing policy choice fixed at h. Moreover, let $MPC_1(h)$ denote the marginal propensity to consume of an old household that has purchased a house quality of h. The next proposition characterizes the effect of house price changes on aggregate consumption expenditures.

Proposition 5. *Suppose the economy is in a stationary equilibrium at time t and consider an unanticipated, permanent change to $s \in \{\theta, R, \delta, y_1\}$. Then, the effect of house price changes on aggregate consumption dC_t is:*

$$dC_t = \underbrace{\mathbb{E}[\overline{MPC}(h)]\mathbb{E}[dq_t(h)]}_{\textit{homogeneous response}} + \underbrace{cov(\overline{MPC}(h), dq_t(h))}_{\textit{matching "multiplier"}} \tag{20}$$

where $\overline{MPC}(h) = (1-\delta)MPC_1(h) - \widetilde{MPC}_0(h)$ and the expectation is over house qualities.

Proof. See Appendix A.8. □

The key insight in proving this result is to notice that changes in house prices constitute a pure *negative* income effect to young, home-purchasing households, and a *positive* wealth effect to old, home-owning households. Hence, the aggregate effect on consumption expenditures depends on the relative MPCs of home-owners (adjusted for depreciation) to young purchasers, holding their housing policy function fixed. Intuitively, the presence of housing indivisibility and positive assortative matching implies that any given change in house prices will not affect the relative quality tier that a household will target in equilibrium. Hence, market segmentation implies that we can abstract from *substitution* effects that arise due to house price changes.[13]

[13] This is in contrast to models with a homogeneous housing stock, in which changes in house prices create substitution, income, and collateral effects for household consumption, making such a simple characterization generally intractable (Berger et al., 2018).

This proposition highlights the role of redistribution in driving aggregate consumption. Conditional on the *average* fall in house prices in an economy, the *covariance* of MPCs with capital gains can be an important force in shaping aggregate consumption expenditures. Modeling house prices in general equilibrium is therefore important to discipline the incidence of capital gains. This is once again in contrast to models with a homogeneous housing stock, in which *any* kind of shock generally creates a negative covariance of capital gains with MPCs.[14] Although prior work has emphasized the role of redistribution in shaping the transmission mechanism of monetary policy (Auclert, 2019; Bilbiie, 2018), the preceding discussion demonstrates that housing market segmenation is a *necessary* ingredient in order for redistribution to matter for the transmission of house price changes to the rest of the economy.

In Appendix B.4, I extend the model to allow for more realistic MPC heterogeneity through a *warm-glow* bequest motive, as modeled in De Nardi (2004). This extended model captures the empirically relevant feature that MPCs are declining in wealth (Kaplan and Violante, 2014), while leaving the main theoretical results unchanged. Whenever MPCs are declining in wealth, the homogeneous response and matching multiplier term in Proposition 5 work in the same direction in response to a credit supply shock, because credit supply shocks result in larger capital losses precisely for those households that have high MPCs. In contrast, other shocks, such as changes in interest rates or depreciation, induce capital losses that are mostly incident upon low MPC households. This suggests that the underlying shock affecting house prices can give rise to a "matching multiplier" (Patterson, 2023), an amplification channel that is a product of the model's equilibrium sorting.

6 Testing the Theory

In this section, I test the key theoretical finding of *unidirectional shock propagation*. First, I show that shocks to high-end quality tiers do not *trickle down* to lower quality tiers by exploiting plausibly exogenous variation to stock market wealth changes through a shift-share design. I show that households residing in low-end homes hold little stock market wealth, thereby making stock market wealth changes a valid shock to high-end tiers. Second, I show that shocks to low-end tiers *trickle up* to high-end tiers by exploiting plausibly exogenous variation in mortgage growth originating from an expansion of the private label securitization market (the "PLS" market). I show that this mortgage growth was mostly incident on low-end houses, making it a valid shock to test the model's trickle up mechanism.

[14] To see this, consider a similar model in which households can purchase housing at a common per unit price q. Any change in q will generate constant returns to housing across the entire wealth distribution. If poorer households have lower MPCs and purchase less housing, all shocks must generate a negative covariance in response to a per-unit increase in the price of housing:

$$cov(MPC(h), dq(h)) = cov(MPC(h), hdq) < 0 \tag{21}$$

6.1 Test I: Trickle Down of Stock Market Wealth Shock

The test for the absence of a trickle down effect exploits the regional heterogeneity in stock market wealth using a Bartik-style shift-share instrument, as in Chodorow-Reich et al. (2021). Intuitively, we may obtain exogenous variation in stock market wealth by taking advantage of the fact that changes in aggregate stock prices (the shifter) should affect housing markets with different levels of stock market wealth (the shares) differentially.

Data. The goal is to analyze the effect of changes in stock market wealth in a given housing market on the cross-sectional distribution of house prices across quality tiers. In order to do so, I collect data (at a monthly frequency) on house prices, stock market wealth, and stock market returns. I continue using Zillow zip code level price indices for single family homes as my measure of house prices, and define a housing market as a county as in Section 2.[15]

I follow the model in constructing quality tiers by sorting zip codes according to their relative price ranking within a county. Moreover, I obtain per-capita stock market wealth at the zip code level using the IRS Statistics of Income (SOI) data over the period 2005-2019 (the shares), and use returns on the S&P 500 as a measure of stock market returns (the shifter). Additional details regarding data collection can be found in Appendix D.1.

Methodology. I denote my main regressor as $Shock_{c,t} \equiv W_{c,t-1} \times R_{c,t-1}$, where $W_{c,t-1}$ is stock market wealth per capita in period $t-1$ in county c and $R_{c,t-1}$ is the return on the S&P 500 between $t-1$ and t. Motivated by the theoretical results presented in Section 4, I assume the following specification:

$$\triangle_h p_{z,c,t} = \sum_{q=1}^{4} \left[\beta_{q,h} \times 1_{\{z\in q\}} \times Shock_{c,t}\right] + \Gamma_h' X_{c,z,t} + \varepsilon_{z,c,t,h} \tag{22}$$

where $\triangle_h p_{z,c,t}$ is the absolute change in prices for a particular zip code z in county c between $t+h$ and $t-1$, $1_{\{.\}}$ is an indicator for whether a zip code belongs to a housing quartile q, $X_{c,z,t}$ is a vector of zip-county-time specific controls, and $\varepsilon_{z,c,t,h}$ is an idiosyncratic error component. The coefficients $\{\beta_q\}_{q=1}^4$ capture the heterogeneous response of house prices across quality quartiles to changes in total stock market wealth per capita in a given housing market.

The key assumption for identification in this framework is that high and low wealth counties are not heterogeneously affected by other aggregate variables that co-move with stock market returns, conditional on the controls. This condition mirrors the parallel trends assumption in a continuous difference-in-difference design with multiple treatments (Goldsmith-Pinkham et al., 2020). Chodorow-Reich et al. (2021) argue extensively that this assumption is satisfied in their specification that focuses on the response of labor market variables. Below, I provide further evidence for exogeneity of the shock by illustrating the absence of pre-trends prior to the treatment.

[15]The results are robust to defining a housing market as an MSA.

Table 2

quartile	average $\lvert Shock_{z,t}\rvert$ (per capita)
1	106.2
2	166.2
3	261.3
4	716.6

Note: Average (absolute value of the) stock market wealth shock across quality quartiles over 2005-2019. For example, a number of 106.2 for quartile 1 would imply that low-end ZIP codes experienced an average monthly return of 106.2 dollars per month over the period 2005-2019 on a per capita basis.

Results. I first begin by providing evidence that the shock is sufficiently targeted to high-end zip codes to make it a valid high-end shock. Table 2 shows the average value of the shock over 2005-2019 for different quality quartiles: households that reside in zip codes within the highest quality quartile hold the highest amount of stock market wealth. Moreover, they hold approximately seven times as much as stock market wealth, on average, than households residing in zip codes in the bottom quality quartile. Hence, the theory would predict that the effect of the shock on low-end homes should be small, since shocks cannot trickle down in the presence of segmentation.

Table 3 reports the effect of the shock on all quality quartiles over a 12 month horizon, for various specifications of Equation (22). In my preferred specification (final column), I non-parametrically control for time-invariant aggregate shocks that covary with stock market returns *and* the share of wealth within a given county by including county fixed effects. I also control for unobservable variables that might heterogeneously affect different quartiles depending on their initial level of stock market wealth by including quartile fixed effects. Furthermore, I include time fixed effects and clean up any residual correlation in the stock market wealth shock by including 12 lags of $S_{c,t}$. I also exploit only within state variation by including state×time fixed effects.[16]

The impact of a one dollar stock wealth increase per capita for a given county increases the prices of high-end homes by 1.27 dollars over a six-month horizon.[17] Moreover, the effect of the shock is statistically indistinguishable from zero for the bottom two quartiles, and is small (but in line with the direct effect of the shock) for the third quality quartile. These observations are entirely in line with Theorem 1: Shocks to high-end homes are contained within their own quality quartile and do not trickle down.

In order to show that the results are robust to different time horizons, Figure 5 plots the local projections of Equation (22). The shock occurs at period -1 and is equivalent to a one dollar per capita increase in the stock market wealth for a given county. Figure 5 provides further evidence

[16] I double cluster all standard errors at the county and time. Clustering by time soaks up any remaining serial correlation on stock market returns, while clustering by county allows households in the same county to experience common shocks.

[17] This does not imply an MPC that is greater than one for non-durable consumption because – as shown in Table 2 – the incidence of the treatment is unequally distributed across quality quartiles.

Table 3

	(1)	(2)	(3)	(4)
	$\triangle_{12}p_{z,c,t}$	$\triangle_{12}p_{z,c,t}$	$\triangle_{12}p_{z,c,t}$	$\triangle_{12}p_{z,c,t}$
β_1	0.453	0.214	0.357	0.0542
	(0.284)	(0.280)	(0.266)	(0.166)
β_2	0.441	0.339	0.401	0.206
	(0.237)	(0.230)	(0.228)	(0.162)
β_3	0.661*	0.591*	0.590*	0.377*
	(0.276)	(0.255)	(0.253)	(0.170)
β_4	1.672**	1.614**	1.456**	1.274**
	(0.488)	(0.469)	(0.474)	(0.432)
Lagged Returns	✓	✓	✓	✓
Time FE	✓	✓	✓	
County FE		✓	✓	✓
Quartile FE			✓	✓
State×Time FE				✓
N	2397225	2397225	2397225	2397225

Note: The table reports alternative specifications for the baseline regression (22). The dependent variable is the six-month change in the house price level for a given zip code. Robust standard errors double-clustered by county and time in parentheses. $*$ denotes significance at the 5% level, and $**$ at the 1% level.

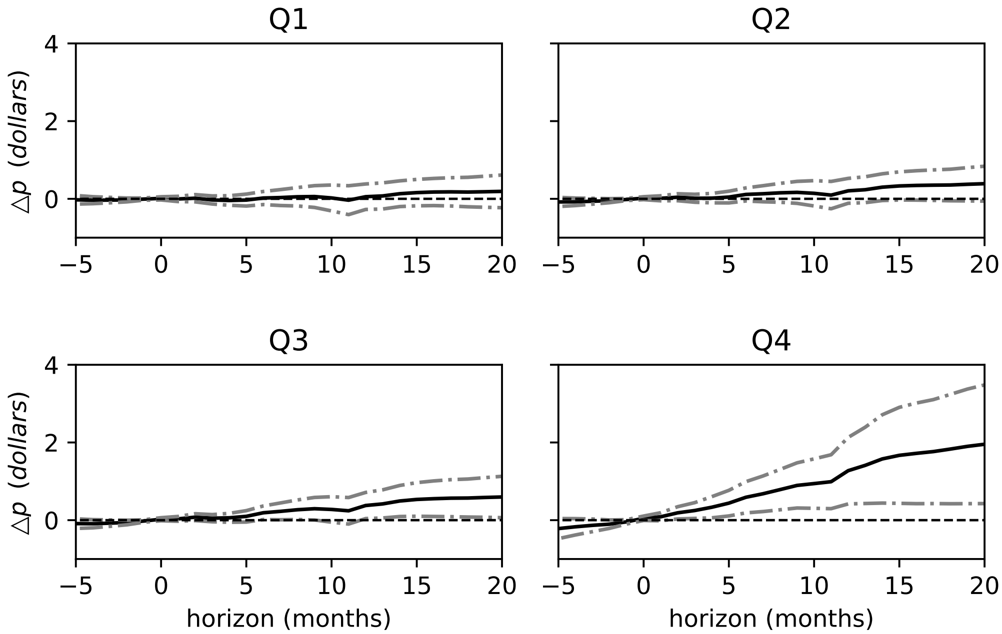
for exogeneity by showing the absence of pre-trends. The only pronounced increase in house prices occurs in the highest quality quartile, exactly as predicted by the theory. Note that the increase appears persistent – this is consistent with the interpretation that the stock market is perceived to be close to a random walk and mirrors the results of Chodorow-Reich et al. (2021). The price effects for lower quality quartiles are small and statistically indistinguishable from zero at long time horizons.

Additional General Equilibrium Spill-Overs. Note that the local projections validate the predictions of Theorem 1 in spite of any county-level general equilibrium effects likely to occur through the stock market wealth shock (e.g. through standard Keynesian reasoning), which have *not* been controlled for (Nakamura and Steinsson, 2018). Consequently, the results suggest that the model's mechanism is robust to the presence of additional county-level general equilibrium spill-overs.

6.2 Test 2: Trickle Up of a Sub-Prime Credit Shock

I next test for trickle up effects from low- to high-end quality tiers. I do so by isolating a subprime credit shock that was mostly incident on low-end zip codes. In particular, I exploit the differential city-level exposure to the sudden, pronounced increase to the PLS market in 2003 (Justiniano et al.,

Figure 5: Price Responses from a Stock Market Wealth Shock



Note: Local projections to column 4 of Equation (22). The grey dotted lines depict 95% confidence intervals, based on standard errors clustered two-way by county and time. The shock occurs at $h = -1$.

2019). I follow Mian and Sufi (2022) in hypothesizing that banks that relied more heavily on non-core-deposit financing were able to expand their mortgage lending more aggressively during this time period. Mian and Sufi (2022) argue that exposure to high non-core-liability (NCL) lenders provides a plausibly exogenous source of variation in mortgage credit supply across housing markets.

Data. Testing the trickle-up mechanism requires data on house prices, mortgage origination growth, and the liabilities of issuing institutions. For house price data, I continue using the publicly available zip code level Zillow series. I collect data on the flow of new mortgage loans originated in the years 2000-2010 through the "Home Mortgage Disclosure Act" (HMDA) data set. I obtain the non-core liabilities of each lender by using Call Report data and merge the two datasets using key provided by the Federal Reserve Board. Appendix D.2 provides further details on the data construction.

Methodology. I follow Mian and Sufi (2022) in constructing a county-level exposure to high-NCL lenders. Concretely, I construct a county-level average of the 2002 NCL ratios of mortgage lenders, where the average is weighted by a lender's amount of mortgage originations in 2002:

$$NCLShare_{c,2002} = \sum_b \omega_{c,b,2002} \times NCL_{b,2002} \tag{23}$$

where

$$\omega_{c,b,2002} = \frac{Originations_{c,b,2002}}{\sum_b Originations_{c,b,2002}} \tag{24}$$

Table 4

	% Change 2002-2005		
	Originations (1)	Price (2)	Income (3)
Quartile 1	7.303*	2.911**	-0.073**
	(3.014)	(0.974)	(0.028)
Quartile 2	5.868**	3.349**	-0.066
	(2.002)	(0.941)	(0.037)
Quartile 3	2.391	3.023**	-0.093*
	(1.484)	(0.878)	(0.043)
Quartile 4	0.378	2.298*	0.035
	(1.681)	(0.789)	(0.023)
Quartile FE	✓	✓	✓
N	9453	9135	9135

Note: The table reports the coefficients and p-values from from Equation (26) for the percentage growth in originations over 2002-2005 (column 1), prices (column 2) and income (column 3) for quality quartiles. Robust standard errors clustered at the county-level in parentheses. $*$ denotes significance at the 5% level and $**$ at the 1% level.

and where $NCL_{b,2002}$ denotes the non-core liabilities of bank b in 2002, and $Originations_{c,b,2002}$ denotes the amount of mortgage originations from bank b to county c in 2002.

I then regress the percentage change in house prices directly on the 2002 NCL ratio to analyze the impact of this subprime loan expansion on house prices.[18] I consider the following specification, which closely mirrors the one used for the stock-market wealth shock:

$$\triangle_{2005,2002} Price_{z,c} = \sum_{q=1}^{4} \left[\beta_q \times 1_{\{z \in q\}} \times NCLShare_{c,2002}\right] + \Gamma' X_{z,c} + \varepsilon_{z,c} \tag{25}$$

where $\triangle_{2005,2002} Price_{z,c}$ is the log-change in house prices in zip code z and county c over 2002-2005 and $\varepsilon_{z,c}$ reflects unmodeled determinants of price growth. The identifying assumption is that counties did not experience shocks post-treatment that were correlated with their level of the NCL share as of 2002. Mian and Sufi (2022) argue that this orthogonality assumption is satisfied. I provide further evidence for this below by showing lack of pre-trends.

Results. I first show that the additional supply in credit that is induced by being in a high NCL county is mostly incident on low-end ZIP codes within a county. This is necessary in order to ensure that price variation that is attributed to the trickle up effect that occurs in general equilibrium is not confounded with the direct effect of providing more credit to high-end ZIP codes. To this end,

[18]This approach is followed by Greenwald and Guren (2021) and Mian and Sufi (2022), who also regress outcomes directly on the instrument. This is sufficient to provide evidence for the trickle up mechanism, although it implies that we cannot interpret the coefficients in terms of units of credit. Appendix E shows that the trickle-up mechanisms is robust to a 2SLS procedure, although the coefficients are estimated with greater noise.

I run the following regression:

$$\triangle_{2005,2002} Orig_{z,c} = \sum_{q=1}^{4} \left[\gamma_q \times 1_{\{z\in q\}} \times NCLShare_{c,2002}\right] + \Gamma' X_{z,c} + \varepsilon_{z,c} \tag{26}$$

where $\triangle_{2005,2002} Orig_{z,c}$ denotes the log-change in mortgage originations. Column 1 of Table 4 depicts the coefficients γ_q for each quality quartile. These coefficients capture the expected increase in mortgage origination that occurs in a quality tier q when the NCL exposure of a given housing market increases from zero to one.

Table 4 shows that the treatment due to changes in the NCL share is almost entirely localized in low-end ZIP codes. An increase in the NCL share from zero to one results in an seven-fold increase in the amount of mortgages that are originated in the lowest quality quartile, but an imprecisely estimated 0.3-fold increase for the highest quality quartile. This is consistent with the notion that the vast majority of privately securitized loans were originated to buyers in low-end, and therefore low-income and low-credit score neighborhoods.[19]

Having shown that the treatment is targeted to low-end zip codes, I next turn my attention to its effect on house prices across the quality spectrum. Column 2 of Table 4 depicts the coefficients β_q from this regression. Changing the county-wide NCL ratio from zero to one increases the prices of homes in the bottom two quartiles of zip codes by an additional 290% and 335%, respectively. Moreover, there are significant trickle-up effects: a unit increase in a county's NCL ratio causes a 230% increase in house prices for the highest quality quartile. This is in spite of the relatively few (albeit imprecisely estimated) originations to these high-end homes. These results are consistent with Theorem 2 in that $\beta_1 > \beta_4 > 0$. Low quality homes do appreciate more in response to a credit expansion than high-end homes, but high-end homes appreciate through spill-overs that dissipate across the quality ladder.

The main threat to a causal interpretation of these findings is that counties with a high NCL ratio are also impacted by other covariates that affect house prices and co-move with local mortgage growth. In order to provide further evidence for the exogeneity of the NCL-based credit shock, I employ an event-study approach and run the following regression separately for each housing quality quartile:

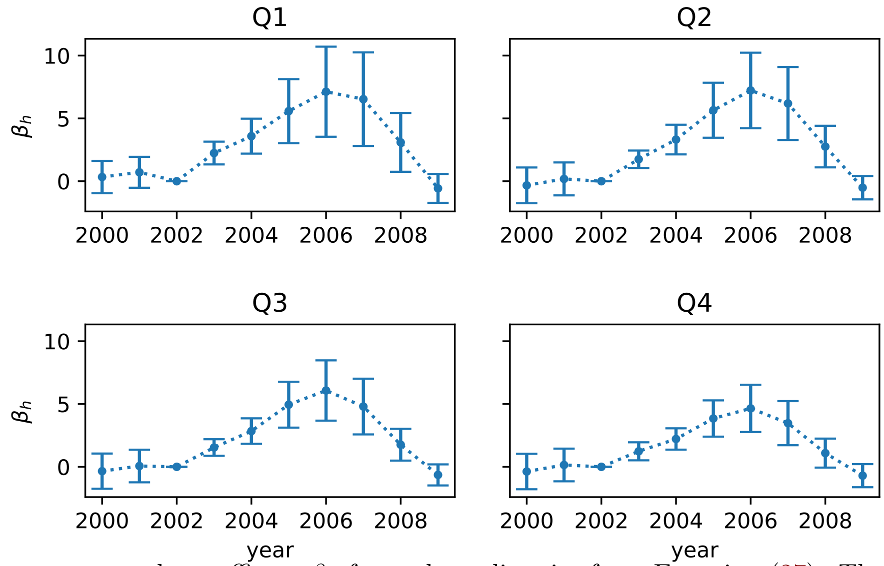
$$\triangle_{t,2002} Prices_{z,c,t} = \xi_c + \psi_t + \sum_{h\neq 2002} \left[\beta_{q,h} \times NCLShare_{2002}\right] + \varepsilon_{z,c,t} \tag{27}$$

where $\triangle_{t,2002} Prices_{z,c,t}$ denotes the percentage change in house prices that occurred between t and 2002 in zip code z and county c and ξ_c and ψ_t denote county and year fixed effects, respectively. The coefficients $\{\beta_{q,h}\}$ give the relative growth in prices that is attributed to being in a high NCL county in 2002 (which is the omitted year) for quartile q.

Figure 6 plots the estimated coefficients $\{\beta_{q,h}\}$ for each quality. The estimated coefficients

[19] Mian and Sufi (2022) provide direct evidence that high NCL exposure is correlated with lower income, low credit scores, and a younger populations using data from TransUnion.

Figure 6: Price Responses From a Credit Supply Shock



Note: The figure reports the coeffients β_h for each quality tier from Equation (27). The figure shows 95% confidence bands, based on standard errors clustered at the county-level.

show that there was no pre-trend and a sharp rise in house prices for high NCL areas starting in 2003. The rise is most pronounced for houses in the lowest quality quartile, but we also see an economically significant increase in the prices of high-end homes. At its peak, the coefficient for zip codes in the highest quality quartile was around sixty percent (4.6) the magnitude of the corresponding coefficient for low-end homes (7.1). This analysis suggests that trickle-up forces that occur in general equilibrium can have a meaningful effect on house prices across the quality distribution. As predicted by the theory, shocks that affect low-income households are crucial in shaping the response of house prices across the *entire* market. However, shocks that affect high-income households do not trickle down to lower quality tiers. These two results in tandem highlight the role of segmentation in shaping house price dynamics.

Additional General Equilibrium Spill-Overs. It is important to note that these regressions do not control for other general equilibrium forces that might further increase the price of high-end homes. For example, high NCL counties may experience greater income growth due to their increased credit exposure through changes in aggregate demand. This would increase house prices amongst all quality quartiles, thereby biasing the coefficients in (25) upwards. This "contamination" of the partial equilibrium effect with local general equilibrium effects is a general feature of empirical designs that use variation across regions for identification Guren et al. (2021).

In order to address these concerns, I use IRS zip code level SOI data to see if high NCL areas also experienced higher income growth relative to NCL areas. Table 4 reports the coefficients from regression (25) using income as the dependent variable. The effect of NCL exposure on income growth is low for all quality quartiles and statistically indistinguishable from zero for the highest quality quartile. In all cases, the standard errors are small. These results suggest that it is the direct, partial equilibrium, effect of increased credit that is responsible for house price growth as opposed to indirect effects that arise through Keynesian channels. Appendix E shows that these results are also robust to longer time horizons by employing a similar event-study regression as in Equation (27).

7 Concluding Remarks

In this paper, I develop a dynamic assignment model in which housing is segmented by various quality tiers. I show that the presence of housing market segmentation substantially alters how macroeconomic shocks affect the housing market relative to the homogeneous housing benchmark: shocks "trickle up" but do not "trickle down" across quality tiers. I characterize how the entire cross-section of house prices responds to aggregate shocks and show that two important forces in the housing market operate on different ends of the wealth and housing quality distribution: changes in credit conditions mostly affect low-end homes, while changes in interest rates and perceptions about future house prices mostly affect high-end homes. Moreover, I show that the unequal incidence of capital gains that arises from market segmentation can amplify the response of aggregate consumption expenditures to house prices changes.

Finally, I test the model's key prediction of unidirectional shock propagation in the data. Trickle-up effects due to low-end shocks (as measured by subprime mortgage growth) have large, but dissipating effects on house prices in higher quality tiers. High-end shocks (as measured by stock market wealth changes) only affect high-end tiers: trickle down effects are both statistically and economically insignificant. These results are jointly consistent with the theory and highlight the role of housing market segmentation in shaping house price dynamics in the data.

Although the homogeneous housing stock framework has dominated macroeconomic modelling of the housing market, this paper argues that treating housing as a heterogeneous, private-value asset is indispensable in obtaining a complete view of housing within the macroeconomy. A more quantitative treatment of the evolution of house prices in the presence of market segmentation and its macroeconomic consequences may prove a fruitful avenue for future research.

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A Mathematical Appendix

A.1 Proof of Proposition 1

We first prove the first part of the proposition.

Lemma 2. $h^*(\cdot)$ *is strictly increasing and differentiable for all* $h \in (\underline{h}, \bar{h})$

Proof. The fact that $q(\cdot)$ is strictly increasing follows from market clearing and the fact that payoffs are strictly increasing in housing quality. By Lebesgue's Differentiation Theorem, monotonicity of $q(\cdot)$ then implies that $q(h)$ is differentiable almost everywhere on $(\underline{h}, \bar{h})$. Where the derivative exists, the first-order condition satisfies

$$q'(h_0^*(y)) = f(q(h_0^*(y)), y) \tag{28}$$

for some continuous function $f(\cdot, \cdot)$ and $y \in [\underline{y}, \bar{y}]$.[20]

Now, in order to prove differentiability everywhere, I first argue that $q(\cdot)$ must be continuous. Suppose it is not and consider a point of discontinuity $h' \in [\underline{h}, \bar{h}]$. Because the utility derived from housing is continuous in h, any household would prefer adjusting their housing choice in either direction as opposed to a discrete jump to $q(h)$. Moreover, this adjustment is feasible because $H(\cdot)$ is strictly increasing and the support of $H(\cdot)$ is a connected set. This proves continuity.

Next, consider the decision functions $c_j^*(\cdot)$ for $j \in \{0, 1\}$ and $h_1^*(\cdot)$. Since $q(\cdot)$ is continuous, the conditions of the maximum theorem are satisfied and the decision functions are also continuous with respect to y. Hence, $f(q(h_0^*(y)), y)$ is a continuous function, being the composition of continuous functions. Moreover, the fact that $F(\cdot)$ is strictly increasing implies that the function is well-defined on the interval $[\underline{y}, \bar{y}]$. The continuity of $f(q(h_0^*(\cdot)), \cdot)$ on the closed interval $(\underline{y}, \bar{y})$ then implies that $q'(\cdot)$ is differentiable on $(\underline{h}, \bar{h})$ by the Fundamental Theorem of Calculus.

□

We now prove the second part of the proposition.

Proof. I suppress the dependence of the decision functions on y for notational simplicity. Using the Karush-Kuhn-Tucker conditions for the household problem, we obtain the following equation for constrained households.

$$q'(h_1)\left(\frac{(1-\theta)\beta R - (1-\delta)\beta}{c_1} + \frac{\theta}{c_0}\right) = \chi \frac{1}{h_1} \tag{29}$$

Multiplying and dividing by a factor of $\frac{R+\delta-1}{R}$, and letting $x \equiv \frac{\theta R}{R+\delta-1}$ we obtain

$$q'(h_1)\left(1 - \frac{1-\delta}{R}\right) = \chi \frac{1}{h_1}\left(\frac{x}{c_0} + \frac{(1-x)\beta R}{c_1}\right)^{-1} \tag{30}$$

Finally, we substitute for c_0 and c_1 using (4) - (6). This yields (9) directly.

[20]See the proof of Proposition 1 for proof of continuity and characterization of $f(\cdot, \cdot)$.

Households that are not constrained lie on their Euler equation

$$c_1 = \beta R c_0 \tag{31}$$

The pricing equation for unconstrained households is therefore

$$q'(h_1)\frac{1-\frac{1-\delta}{R}}{c_0} = \chi\frac{1}{h_1} \tag{32}$$

Since (6) is not binding, we substitute d_1 out of (4) (5) and use (31) to solve for c_0. This yields

$$c_0 = \frac{1}{1+\beta}\left(y + \frac{y_1}{R} - \frac{R+\delta-1}{R}q(h_1)\right) \tag{33}$$

□

A.2 Proof of Proposition 2

Proof. I set up the problem to make use of Topkis' Theorem. The challenge is to express the household's payoff only in terms of y and h and characterize its second derivative. To this end, we substitute for optimal consumption in terms of y and h. For an unconstrained household, this is given by

$$c_0(y,h) = \frac{1}{1+\beta}\left(y + \frac{y_1}{R} - \frac{R+\delta-1}{R}q(h)\right) \tag{34}$$

$$c_1(y,h) = \frac{\beta R}{1+\beta}\left(y + \frac{y_1}{R} - \frac{R+\delta-1}{R}q(h)\right) \tag{35}$$

Similarly, we have for a constrained household

$$\tilde{c}_0(y,h) = y - \theta q(h) \tag{36}$$

$$\tilde{c}_1(y,h) = y_1 - (R(1-\theta) - (1-\delta))q(h) \tag{37}$$

The consumption functions will therefore depend on the level of debt d (because this dictates whether households will be constrained or not), the housing quality choice h, and the level of first-period income y. Let $V_c(h,y)$ and $V_u(h,y)$ be the payoff of a household with first-period income y that chooses a house of quality h for a constrained and unconstrained household, respectively. We must show that these functions are supermodular in h, y.

Step 1: $V_c(h,y)$ and $V_u(h,y)$ are twice differentiable in h, y

We have

$$V_c(h,y) = \log(c_0(y,h)) + \beta\log(c_1(y,h)) + \chi\log(h) \tag{38}$$

and similarly for a constrained household. By Proposition 1, we know that $q'(h)$ is everywhere differentiable on $(\underline{h}, \bar{h})$. Twice differentiability of $V_c(\cdot,\cdot)$ and $V_u(\cdot,\cdot)$ on $(\underline{h}, \bar{h}) \times (\underline{y}, \bar{y})$ then follows.

Step 2: The cross-derivatives are strictly positive

The twice differentiability of $V_c(h, y)$ and $V_u(h, y)$ implies that we can characterize supermodularity through the second-partial derivative. For the constrained household:

$$\frac{\partial^2 V_c(h, y)}{\partial h \partial y} = q'(h) \left(\frac{\theta}{c_0^2} \right) > 0 \tag{39}$$

Similarly, it is straightforward to show for an unconstrained household:

$$\frac{\partial^2 V_u(h, y)}{\partial h \partial y} = q'(h) \frac{R + \delta - 1}{(1+\beta) R c_0^2} > 0 \tag{40}$$

Hence, the value function is supermodular in h, y. By Topkis's theorem $h_1^*(y)$ must be strictly increasing for both constrained and unconstrained households. Finally, a household that switches from being constrained to unconstrained (or vice versa) would not want to decrease their housing quality due to the continuity of the house quality decision function. Finally, at the boundary $\underline{h}$, $q(\underline{h})$ must be such so that the marginal household is indifferent to purchasing the lowest quality home or obtaining the outside option. By the supermodularity of the payoff function, it follows that there must exist a $y_p \in [\underline{h}, \bar{h}]$ such that all households with $y > y_p$ purchase housing.

□

A.3 Proof of Proposition 3

Proof. The *allocation* of households to house qualities is unique by Proposition 2 and the argument outlined in the main text. Equilibrium uniqueness and existence then follows by showing that there is a unique pricing function that satisfies (8) and (9).

First, I argue that the initial condition $q(\underline{h})$ is well-defined and unique. It is the price that makes the marginal household with income y_p indifferent to owning or obtaining their outside option. There are four cases to consider: (i) the household is credit-constrained when purchasing the house, and unconstrained otherwise, (ii) unconstrained when purchasing the house and constrained otherwise, (iii) constrained in both cases, or (iv) unconstrained in both cases. For case (i), $q(\underline{h})$ must solve the following equation:

$$\frac{\beta R}{(1+\beta)^2} \left(y_p + \frac{y_1}{R} \right)^{1+\beta} b^{\chi} = (y_p - \theta q(\underline{h})) \left(y_1 - ((1-\theta)R - (1-\delta)) q(\underline{h}) \right)^{\beta} \underline{h}^{\chi} \tag{41}$$

Note that the left hand-side is the payoff to an unconstrained household that opts for the outside option (after exponentiating), whereas the right-hand side is the payoff to a constrained household that purchases the lowest quality house (after exponentiating). Both terms on the RHS are positive due to the non-negativity of consumption in both periods. Similarly, for case (ii), $q(\underline{h})$ must solve:

$$(y_p)(y_1)^{\beta} b^{\chi} = \frac{\beta R}{(1+\beta)^2} \left(y_p + \frac{y_1}{R} - \frac{R + \delta - 1}{R} q(\underline{h}) \right)^{1+\beta} \underline{h}^{\chi} \tag{42}$$

For case (iii), $q(\underline{h})$ must solve

$$(y_p)(y_1)^\beta b^\chi = (y_p - \theta q(\underline{h}))\left(y_1 - ((1-\theta)R - (1-\delta))q(\underline{h})\right)^\beta \underline{h}^\chi \tag{43}$$

For case (iv), $q(\underline{h})$ must solve

$$\frac{\beta R}{(1+\beta)^2}\left(y_p + \frac{y_1}{R}\right)^{1+\beta} b^\chi = \frac{\beta R}{(1+\beta)^2}\left(y_p + \frac{y_1}{R} - \frac{R+\delta-1}{R}q(\underline{h})\right)^{1+\beta} \underline{h}^\chi \tag{44}$$

Furthermore, the income of the marginal home-owning household, y_p, is the unique solution to

$$m(1 - F(y_p)) = 1 \tag{45}$$

Note that the marginal household that does not purchase a house is unconstrained if and only if

$$y_p > \frac{y_1}{\beta R} \tag{46}$$

Suppose this condition is satisfied, so that the relevant cases are (i) and (iv). The left-hand side of Equations (41) and (44) are independent of $q(\underline{h})$. Further, the assumption that $x \equiv \theta R/(R+\delta-1) \in (0,1)$ implies that $(1-\theta)R - (1-\delta) > 0$. Hence, the RHS of both equations is strictly decreasing in $q(\underline{h})$.

I now argue that a unique solution exists. Note that the RHS of (41) is equal to the RHS of (44) at the constraint threshold. When $d_1^*(y(\underline{h})) \leq (1-\theta)q(\underline{h})$, the home-owning household is unconstrained and $q(\underline{h})$ given by Equation (44). When $d_1^*(y(\underline{h})) > (1-\theta)q(\underline{h})$, the solution is given by Equation (41). We may therefore construct a continuous function that is strictly decreasing and is given by the RHS of either (41) or the RHS of (44) depending on the households' constraint status. This function is greater than the LHS of either (41) or (44) for $q(\underline{h}) = 0$ (in which the household is unconstrained by assumption) and tends to zero as $q(\underline{h})$ increases. Hence, a unique, positive solution exists by the intermediate value theorem. A similar argument shows the initial condition is unique and well-defined when $y_p \leq y_1/(\beta R)$.

I now show that the solution to the ODEs (9) and (8) is unique. We may substitute the inverse matching function to obtain the following equation for an unconstrained household:

$$\left(1 - \frac{1-\delta}{R}\right)\frac{\mathrm{d}}{\mathrm{d}h}q^*(h) = \chi\frac{c_0^*(h)}{h} \tag{47}$$

where $c_0^*(y) = \frac{1}{1+\beta}\left(y(h) + \frac{y_1}{R} - \frac{R+\delta-1}{R}q^*(h)\right)$. For a constrained household, we obtain:

$$\left(1 - \frac{1-\delta}{R}\right)\frac{\mathrm{d}}{\mathrm{d}h}q^*(h) = \chi\frac{1}{h}\left(\frac{x}{\tilde{c}_0^*(h)} + \frac{(1-x)\beta R}{\tilde{c}_1^*(h)}\right)^{-1} \tag{48}$$

where $\tilde{c}_0^*(h) = y(h) - \theta q^*(h)$ and $\tilde{c}_1^*(h) = y_1 - [R(1-\theta) - (1-\delta)]\,q^*(h)$.

We now verify the conditions for the Picard-Lindelöf theorem. Continuity in h follows from the continuity of the inverse matching function. Lipschitz continuity follows from the fact that the RHS of the ODEs are continuously differentiable on the compact interval $h \in [\underline{h}, \bar{h}]$. Therefore, the derivative is bounded and the RHS is Lipschitz continuous in h.[21]

We construct the pricing function iteratively, using either (9) or (8) depending on the constraint status of the household. Since the initial constraint status of the household is well defined and the construction on each interval is unique, the pricing function is unique on the interval $[\underline{h}, \bar{h}]$. □

A.4 Proof of Theorem 1

Proof. Note that the matching function is identical in both economies for $y < y_p$. Since the price function is unique on $[y, y_p]$, the two price functions must be identical on that interval. □

A.5 Proof of Lemma 1

Proof. The inverse matching function in the two economies is:

$$y(h) = F^{-1}\left(\frac{1}{m}\left(H(h) + F(y_p)\right)\right), \quad \forall\, h > \underline{h} \tag{49}$$

$$\tilde{y}(h) = \tilde{F}^{-1}\left(\frac{1}{m}\left(H(h) + \tilde{F}(y_p)\right)\right), \quad \forall\, h > \underline{h} \tag{50}$$

Note that $\tilde{F}(y_p) = F(y_p) = \frac{m-1}{m}$ by market clearing. Since $\tilde{F}(\cdot) \leq F(\cdot)$, we have $\tilde{F}^{-1}(y) > F^{-1}(y)$ and $\tilde{y}(h) > y(h)$. By (9) and (8), we observe that both pricing functions satisfy the following ODEs:

$$\tilde{q}(h) = \tilde{f}(h, q(h)) \tag{51}$$

$$q(h) = f(h, q(h)) \tag{52}$$

where $\tilde{f}(h, q(h)) > f(h, q(h))$. For the same initial condition $q(\underline{h})$, we have $\tilde{q}(h) \geq q(h)$ by Petrovitsch's Theorem. However, the initial conditions must satisfy $\tilde{q}(\underline{h}) \geq q(h)$. But this can only increase $\tilde{q}(h)$ further at every $h \in [\underline{h}, \bar{h}]$. Towards a contradiction, suppose that $\tilde{q}(h^*, q(\underline{h})) > \tilde{q}(h^*, \tilde{q}(\underline{h}))$ for some $h^* \in [\underline{h}, \bar{h}]$, where $q(h, a)$ is the unique solution to the ODE

$$\tilde{q}'(h) = \tilde{f}(h, \tilde{q}(h)), \quad q(\underline{h}) = a \tag{53}$$

Since $\tilde{q}(h, q(\underline{h}))$ and $\tilde{q}(h, \tilde{q}(\underline{h}))$ are continuous functions of h, and $\tilde{q}(\underline{h}, q(\underline{h})) < \tilde{q}(\underline{h}, \tilde{q}(\underline{h}))$, and $\tilde{q}(h^*, q(\underline{h})) > \tilde{q}(h^*, \tilde{q}(\underline{h}))$, there must exist an $h^{**} \in (\underline{h}, h^*)$ such that the two functions are equal by the Intermediate Value Theorem. But this contradicts equilibrium uniqueness of the pricing function for the initial condition $q(h^{**})$. □

[21] This follows from the mean value theorem: $f(q_2) - f(q_1) = f'(q_3)(q_2 - q_1)$ for some $q_3 \in (q_2, q_1)$ and a function $f(\cdot)$. But $f'(q)$ is continuous on a compact set, and therefore bounded. Lipschitz continuity follows.

A.6 Proof of Proposition 4

Proof. First, I derive Equation (12). From Equation (47), we have:

$$\left(1-\frac{1-\delta}{R}\right)\frac{\mathrm{d}}{\mathrm{d}h}q^*(h)=\frac{\frac{\chi}{1+\beta}\left(y(h)+\frac{y_1}{R}-\frac{R+\delta-1}{R}q^*(h)\right)}{h} \tag{54}$$

Recall that $\omega\equiv 1-(1-\delta)/R$. Rearranging and multiplying by $h^{\frac{\chi}{1+\beta}-1}$ yields:

$$\omega\left(\frac{\chi}{1+\beta}h^{\frac{\chi}{1+\beta}-1}q^*(h)+h^{\frac{\chi}{1+\beta}}\frac{\mathrm{d}}{\mathrm{d}h}q^*(h)\right)=h^{\frac{\chi}{1+\beta}-1}\frac{\chi}{1+\beta}\left(y(h)+\frac{y_1}{R}\right) \tag{55}$$

Note that we may factor the left-hand side as follows:

$$\omega\frac{\mathrm{d}}{\mathrm{d}h}\left(h^{\frac{\chi}{1+\beta}}q^*(h)\right)=\frac{\chi}{1+\beta}\left(h^{\frac{\chi}{1+\beta}-1}y(h)+h^{\frac{\chi}{1+\beta}-1}\frac{y_1}{R}\right) \tag{56}$$

Integrating the above from h_c to h then yields Equation (12).

Next, I show that constraint status is monotone. To this end, I characterize when households are constrained as a function of house prices. Note that the first-period consumption for an unconstrained household is

$$c_0^*(h)=\frac{1}{1+\beta}\left(y(h)+\frac{y_1}{R}-\frac{R+\delta-1}{R}q^*(h)\right) \tag{57}$$

Moreover, debt satisfies

$$d_1^*=c_0^*(h)+q^*(h)-y(h) \tag{58}$$

A household is therefore unconstrained if and only if

$$c_0^*(h)+q^*(h)-y(h)\leq(1-\theta)q^*(h) \tag{59}$$

Which is equivalent to:

$$\left(\theta-\frac{R+\delta-1}{(1+\beta)R}\right)q^*(h)\leq\frac{\beta}{(1+\beta)}\left(y(h)-\frac{y_1}{\beta R}\right) \tag{60}$$

Under Assumption 1, the RHS is strictly positive, since $y(h)$ is strictly increasing and $y_p>y_1/(\beta R)$. If $\left(\theta-\frac{R+\delta-1}{(1+\beta)R}\right)\leq 0$, then all households are not credit constrained, irrespective of the price of housing.[22] If $\left(\theta-\frac{R+\delta-1}{(1+\beta)R}\right)>0$, then households are not credit constrained if and only if:

$$q^*(h)\leq\frac{\beta}{R(1+\beta)\theta-R+\delta-1}y(h)-\frac{y_1}{R(1+\beta)\theta-R+\delta-1} \tag{61}$$

A sufficient condition for constraint status monotonicity is that $q^*(h)$ does not grow as fast as the

[22] Intuitively, this is because θ is relatively small (the credit constraint is slack) and first period consumption decreases as house prices increase.

RHS of Equation (61) for $h > h_c$. Using the Leibniz formula, the derivative of (12) is:

$$\begin{aligned} q^{*'}(h) =& \frac{\chi}{\omega(1+\beta)}\frac{y(h)}{h} - \int_{h_c}^{h} \frac{\chi^2}{\omega(1+\beta)^2}\frac{\tilde{h}^{\frac{\chi}{1+\beta}-1}}{h^{\frac{\chi}{1+\beta}+1}} y(\tilde{h})\mathrm{d}\tilde{h} \\ &+ \frac{\chi y_1}{(1+\beta)R\omega} h_c^{\frac{\chi}{1+\beta}} h^{-\frac{\chi}{1+\beta}-1} - \frac{\chi}{1+\beta} h_c^{\frac{\chi}{1+\beta}} h^{-\frac{\chi}{1+\beta}-1} q^*(h_c) \end{aligned} \tag{62}$$

A sufficient condition is that the RHS of Equation (62) is smaller than the RHS of (61) for all $h > h_c$, which is satisfied if χ is small enough. This proves that constraint status is monotone. □

A.7 Proof of Theorem 2

The proof requires the following lemma.

Lemma 3. *Consider a twice differentiable function* $u : \mathbb{R}^2 \to \mathbb{R}$

$$u(x, y) = f(x) + g(x, y) \tag{63}$$

where $f : \mathbb{R} \to \mathbb{R}$ *and* $g : \mathbb{R}^2 \to \mathbb{R}$. *Suppose* $u(x, y) > 0$, $u'_x(x, y) > 0$, $u'_y(x, y) < 0$ *and* $g''_{xy}(x, y) > 0$. *Then,*

$$u''_{xy}(x, y) > 0 \quad \textit{and} \quad \frac{\partial^2 \log u}{\partial x \partial y}(x, y) > 0 \tag{64}$$

Proof. Suppressing dependence on x and y, we have:

$$\frac{\partial^2 \log u}{\partial x \partial y} = \frac{g_{xy}(f+g) - g_y(f_x + g_x)}{(f+g)^2}$$

which is strictly positive by the assumptions above. □

Proof. (*Theorem 2*) The pricing function for linear matching functions $y = kh$ is given by

$$\begin{aligned} q(h) = & \frac{k\chi}{\omega(\chi+1+\beta)} \left(h - \left(\frac{(h_c)^{\frac{\chi+1+\beta}{1+\beta}}}{h^{\frac{\chi}{1+\beta}}} \right) \right) \\ & + \left(\frac{y_1}{R}\right)\left(\frac{1}{\omega}\right)\left(1 - \left(\frac{h_c}{h}\right)^{\frac{\chi}{1+\beta}}\right) + \left(\frac{h_c}{h}\right)^{\frac{\chi}{1+\beta}} q(h_c) \end{aligned} \tag{65}$$

where the equation holds for $h > h_c$ and recall that $\omega \equiv \frac{R+\delta-1}{R}$

Credit Frictions. First, I prove that $\frac{\partial q(h_c)}{\partial \theta} < 0$. To this end, I show that $q(\underline{h})$ is decreasing in θ. θ only affects $q(\underline{h})$ if the household is constrained. Hence, we must show that the RHS of Equation (43) is decreasing. Note that this is equivalent to:

$$-\frac{\theta}{c_0^*(y_p)} + \frac{\theta\beta R}{c_1^*(y_p)} < 0 \tag{66}$$

For a constrained household, $c_0^* > \beta R c_1^*(y_p)$. Hence, the initial condition is decreasing.

Next, I show that the ODE (9) is decreasing in θ. Differentiating the RHS of (9) yields:

$$
\begin{aligned}
&-\chi\frac{1}{h}\left(\frac{x}{c_0^*(h)}+\frac{(1-x)\beta R}{c_1^*(h)}\right)^{-2} \\
&\times\left(\frac{x}{c_0^{*2}}-\frac{(1-x)\beta R^2}{c_1^{*2}(h)}+\frac{R}{(R+\delta-1)c_0^*(h)}-\frac{\beta R^2}{(R+\delta-1)c_1^*(h)}\right)
\end{aligned}
\tag{67}
$$

The last two terms in the second parentheses are positive by the Euler inequality. We can bound the sum of the first two terms as follows:

$$
\frac{x}{c_0^{*2}}-\frac{(1-x)\beta R^2}{c_1^{*2}(h)}>\frac{(\beta R)^2x-(1-x)\beta R^2}{c_1^{*2}}
\tag{68}
$$

where we have used the Euler inequality. The RHS is positive if and only if $x \geq \frac{1}{1+\beta}$, which is a necessary condition for a household to be credit constrained by Equation (61), which characterizes the constraint threshold.

By Petrovitsch's Theorem, it follows that $\frac{\partial q(h)}{\partial \theta} \leq 0$ for all $h > \underline{h}$, with strict inequality if some households are credit constrained. Hence, Equation (65) satisfies the conditions of Lemma 3, where x can be substituted for h and y can be substituted for θ. The result then follows.

Interest Rates. The claim that there exists a threshold y' such that $\frac{\partial^2 q(h)}{\partial R \partial h} < 0$ for $h > h(y')$ follows from differentiating Equation (65) and taking the limit. To prove that $\lim_{h\to\infty}\frac{\partial \log q(h)}{\partial R} = -\frac{1-\delta}{R(R+\delta-1)}$, note that

$$
\lim_{h\to\infty}\frac{\partial \log q(h)}{\partial R}=\frac{\partial}{\partial R}\log\left(\frac{k\chi}{\omega(\chi+1+\beta}\right)=-\frac{(1-\delta)}{R(R+\delta-1)}
\tag{69}
$$

Transaction Costs. The claim that there exists a threshold y' such that $\frac{\partial^2 q(h)}{\partial \delta \partial h} < 0$ for $h > h(y')$ follows from differentiating Equation (65) and taking the limit. Moreover,

$$
\lim_{h\to\infty}\frac{\partial \log q(h)}{\partial \delta}=\frac{\partial}{\partial \delta}\log\left(\frac{k\chi}{\omega(\chi+1+\beta}\right)=-\frac{1}{R+\delta-1}
\tag{70}
$$

Future Income. From Equation (65), we have:

$$
\frac{\partial^2 q(h)}{\partial h \partial y_1}=\left(\frac{h_c}{h}\right)^{\frac{\chi}{1+\beta}}\frac{1}{h}\left(\frac{1}{R+\delta-1}-\frac{\partial q(h_c)}{\partial y_1}\right)
\tag{71}
$$

I claim that $\frac{\partial q(h)}{\partial y_1} < \frac{1}{R+\delta-1}$ for all $h < h_c$. Suppose otherwise. Let h_a be the first occurrence of h for which $\frac{\partial q(h)}{\partial y_1} \geq \frac{1}{R+\delta-1}$. If the marginal household is not credit constrained, then it is straightforward to see that $\frac{\partial q(h)}{\partial y_1} < \frac{1}{R+\delta-1}$ from Equation (44). If the marginal household is constrained, we can implicitly differentiate Equation (41) to obtain:

$$\left(-\frac{1+\frac{\beta}{R}}{y_p+\frac{y_1}{R}}+\frac{\beta}{c_1^*}\right)\mathrm{d}y_1=\left(\frac{\theta}{c_0^*}+\frac{(1-\theta)R-1+\delta}{c_1^*}\right)\mathrm{d}q(\underline{h}) \tag{72}$$

If the LHS is negative, then it trivially follows that $\frac{\partial q(\underline{h})}{\partial y_1}<\frac{1}{R+\delta-1}$. Otherwise, we can construct the following bound on the response of the $\mathrm{d}q^*(\underline{h})$:

$$\mathrm{d}q(\underline{h}) \leq \frac{\frac{\beta}{c_1^*}}{\frac{\theta}{c_0^*}+\frac{(1-\theta)R-1+\delta}{c_1^*}}\mathrm{d}y_1 \tag{73}$$

$$\leq \frac{\beta}{\theta\beta R+(1-\theta)R-1+\delta} \tag{74}$$

where the second line uses the Euler inequality. Hence, a sufficient condition is:

$$\frac{\beta}{R\theta(\beta-1)+R-1+\delta}\leq\frac{1}{R+\delta-1} \tag{75}$$

$$\implies (\beta-1)(R+\delta-1)\leq R\theta(\beta-1) \tag{76}$$

Which is always true given our assumption that

$$x\equiv\frac{R\theta}{R+\delta-1}\leq 1 \tag{77}$$

Hence, $\frac{\partial q(\underline{h})}{\partial y_1}<\frac{1}{R+\delta-1}$. By the continuity of this derivative, the mean value theorem thus implies that $h_a>\underline{h}$. Differentiating (9) with respect to y_1, we obtain the following equation

$$\begin{aligned}\left(1-\frac{1-\delta}{R}\right)\frac{\partial q'(h)}{\partial y_1}=&\chi\frac{1}{h}\left(\frac{x}{c_0^*(h)}+\frac{(1-x)\beta R}{c_1^*(h)}\right)^{-2}\left(\frac{x}{c_0^{*2}}\left(-\theta\frac{\partial q(h)}{\partial y_1}\right)\right.\\&\left.+\frac{(1-x)\beta R}{c_1^{*2}}(1-((1-\theta)R-(1-\delta))\frac{\partial q(h)}{\partial y_1}\right)\end{aligned}$$

Using the fact that $c_1^*>\beta R c_0^*$ (since agents are credit constrained), we obtain

$$\begin{aligned}\left(1-\frac{1-\delta}{R}\right)\frac{\partial q'(h)}{\partial y_1}<&\frac{\chi\beta R}{hc_1^{*2}}\left(\frac{x}{c_0^*(h)}+\frac{(1-x)\beta R}{c_1^*(h)}\right)^{-2}\left(\beta Rx\left(-\theta\frac{\partial q(h)}{\partial y_1}\right)\right.\\&\left.+(1-x)(1-((1-\theta)R-(1-\delta))\frac{\partial q(h)}{\partial y_1}\right)\end{aligned}$$

We can simplify the terms in the second parentheses to obtain:

$$x(R(1-\theta)-(1-\delta)-\beta R\theta)\frac{\partial q(h)}{\partial y_1}+(1-\delta-R(1-\theta))\frac{\partial q(h)}{\partial y_1}+1-x \tag{78}$$

Note that the first term is negative under the assumption that households are credit constrained from Equation (61). Moreover, $\frac{\partial q(h)}{\partial y_1}$ is positive. Hence, we can construct an upper bound to the term given by:

$$(1-\delta-R(1-\theta))\frac{\partial q(h)}{\partial y_1}+1-x \tag{79}$$

But this implies that a sufficient condition for $\frac{\partial q'(h)}{\partial y_1}<0$ is that $\frac{\partial q(h)}{\partial y_1}\geq\frac{1}{R+\delta-1}$. But since $\frac{\partial q(\underline{h})}{\partial y_2}<\frac{1}{R+\delta-1}$, continuity implies implies that no such presumed h_a can exist, which is a contradiction.

Hence, $\frac{\partial q(h)}{\partial y_1}<\frac{1}{R-1+\delta}$ for all $h<h_c$. This proves that $\frac{\partial^2 q(h)}{\partial h \partial y_1}>0$.

To prove that there exists a y' such that such that $\frac{\partial \log q(h)}{\partial y_1}<0$ for all $y>y'$, note that:

$$\frac{\partial^2 \log q(h)}{\partial y_1 \partial h}=\frac{qq_{yh}-q_h q_y}{q^2} \tag{80}$$

The numerator of this ratio contains only one constant equal to

$$-\frac{k\chi}{\omega^2(\chi+1+\beta)R} \tag{81}$$

while all other terms in the numerator are at least of order $O\left(h^{-\frac{-\chi}{1+\beta}}\right)$. Hence, $\frac{\partial \log q(h)}{\partial y_1}<0$ for for all $y>y'$. □

A.8 Proof of Proposition 5

Proof. House price changes for old households are pure wealth effects. Hence, their effect on consumption is $(1-\delta)MPC(h)$. Assuming the absence of substitution effects, the effect of house price changes on consumption for young households is $-\widetilde{MPC}(h)$.

It remains to show that there are no substitution effects for young households. To this end, note that house price changes do not change relative income rankings. Hence, by Lemma 1, the matching function before and after the credit shock is unchanged for young households. But this implies that there can be no substitution effects. The result then follows by integrating over house quality distribution and decomposing the expectation function. □

B Model Extensions

B.1 Income Heterogeneity

This subsection extends the model to the case in which income at old age is heterogeneous. I show that the trickle-up result also holds in this more general environment (Theorem 1), and the effects of different shocks on the cross-sectional distribution of house prices characterized in the main text (Theorem 2) continues to hold under the assumption of credit constraint monotonicity and linear inverse matching functions. I consider the following form on old income heterogeneity:

Assumption 2. $y_1=g(y)+\varepsilon_y$, *where*

1. *$g(\cdot)$ is a weakly increasing function with positive range*
2. *ε_y is a random variable with positive and bounded support*

Assumption 2 ensures that greater income in youth is associated with weakly higher income, but the distribution of idiosyncratic risk can be quite arbitrary. This is sufficient to ensure that utility is strictly increasing in first-period income.

Proposition 6. *Suppose Assumption 2 is satisfied and the two following parametric conditions hold: (i) $(1-\delta) \leq (1-\theta)R$, and (ii) $(R-1)(1-\delta+\beta R) < \delta R(1+\beta)$. Then,*

1. *There exists a $y_p \in (\underline{y}, \bar{y})$ such that households with $y > y_p$ are homewoners.*
2. *The policy function $h_1^*(y)$ is strictly increasing in y for all $y > y_p$.*

Proof. The proof is an application of Topkis' Theorem. The twice differentiability of the value function follows (with only minor modifications) from the proof of Proposition 2. It remains to show that the cross-derivative of the value function is strictly positive in h and y. For a constrained household, we have

$$\frac{\partial V}{\partial y} = \frac{1}{c_0} + \beta g'(y)\mathbb{E}\frac{1}{c_1} \tag{82}$$

Differentiating with respect to h, we have:

$$\frac{\partial^2 V}{\partial y \partial h} = \frac{\theta}{c_0^2} q'(h) + \beta g'(y)\left((1-\delta) - (1-\theta)R\right)\mathbb{E}\frac{q'(h)}{-c_1^2} \tag{83}$$

Under the assumptions of the proposition, each term is strictly positive. I next turn to unconstrained households. The difficulty here is that we do not have a closed form expression for consumption. However, we may use the Euler equality:

$$\frac{1}{c_0} = \beta R \mathbb{E}\left[\frac{1}{c_1}\right] \tag{84}$$

We may differentiate with respect to h and y to obtain the following two equations

$$\frac{1}{c_0^2}\left(1 + \frac{\partial d}{\partial y}\right) = \beta R \mathbb{E}\left(\frac{1}{c_1^2}\right)\left(g'(y) - R\frac{\partial d}{\partial y}\right) \tag{85}$$

$$\frac{1}{c_0^2}\left(-q'(h) + \frac{\partial d}{\partial h}\right) = \beta R \mathbb{E}\left(\frac{1}{c_1^2}\right)\left((1-\delta)q'(h) - R\frac{\partial d}{\partial h}\right) \tag{86}$$

where d is the agent's choice of debt. Differentiating the value function twice, we obtain:

$$\begin{aligned}\frac{\partial^2 V}{\partial y \partial h} = &-\frac{1}{c_0^2}\left(-q'(h) + \frac{\partial d}{\partial h}\right)\left(1 + \frac{\partial d}{\partial y}\right) + \frac{1}{c_0}\frac{\partial^2 d}{\partial y \partial h} + \beta \mathbb{E}\left(\frac{1}{c_1}\right)\left(-R\frac{\partial^2 d}{\partial y \partial h}\right) \\ &- \beta R \mathbb{E}\left(\frac{1}{c_1^2}\right)\left(g'(y) - R\frac{\partial d}{\partial y}\right)\left((1-\delta)q'(h) - R\frac{\partial d}{\partial h}\right)\end{aligned} \tag{87}$$

Using (84) and (85), we obtain

$$\frac{\partial^2 V}{\partial y \partial h} = -\frac{1}{c_0^2}\left(1 + \frac{\partial d}{\partial y}\right)\left(-\delta q'(h) + \frac{\partial d}{\partial h}(R-1)\right) \tag{88}$$

I now provide bounds for $\frac{\partial d}{\partial y}$ and $\frac{\partial d}{\partial h}$ such that the cross-derivative is strictly positive. Using Jensen's inequality, (84), and (85), we obtain

$$1 + \frac{\partial d}{\partial y} > g'(y) - R\frac{\partial d}{\partial y} \tag{89}$$

Note that this implies $\frac{\partial d}{\partial y} > -1$ if $g'(y) > 0$. Hence, the first parentheses of the RHS of Equation (88) is positive. Moreover, using (84) and using Jensen's inequality, we obtain

$$1 < (\beta R)^2 \mathbb{E}\left(\left(\frac{c_0}{c_1}\right)^2\right) \tag{90}$$

Combining this expression with (86), we obtain

$$\beta R\left(-q'(h) + \frac{\partial d}{\partial h}\right) < (1-\delta)q'(h) - R\frac{\partial d}{\partial h} \tag{91}$$

It is then easily verified that the cross-derivative is strictly positive if the conditions on R outlined in the proposition are satisfied. Global supermodularity then follows from continuity of the policy function. Hence, $h_1^*(y)$ cannot decrease as one crosses the constraint threshold.

□

The presence of positive assortative matching implies that a variant of Theorem 1 continues to hold. In particular consider two economies with different incomes CDFs $F(\cdot)$ and $\tilde{F}(\cdot)$ that are otherwise identical. Furthermore, let $q(\cdot)$ and $\tilde{q}(\cdot)$ be the respective price functions in these two economies. We have:

Proposition 7. *Suppose $F(\cdot)$ and $\tilde{F}(\cdot)$ satisfy $F(y) = \tilde{F}(y)\ \forall\, y < \tilde{y}$, where $\tilde{y} \in [\underline{y}, \bar{y}]$. Suppose further that $\tilde{q}(\underline{h}) = q(\underline{h})$ (i.e. the prices of the lowest home qualities are identical in these two economies). Then, $\tilde{q}(h) = q(h)\ \forall\, h \leq h(\tilde{y})$.*

Proof. The Lipschitz continuity of marginal utility implies uniqueness given an initial condition. The result then follows from the proof of Theorem 1. □

We may also characterize the effects of aggregate shocks on the distribution of house prices under the assumptions of (i) credit constraint monotonicity, (ii) linear inverse matching functions, (iii) $g(y) = ky$ for some $k \in \mathbb{R}^+$, and (iv) $\varepsilon_y = 0$ (no idiosyncratic risk). In this case, the results of Theorem 2 continue to hold (where the comparative statics to future income are now meaningless), which follows with minor modifications from the proof of the original proposition.

B.2 Supply-Side Movements

This section extends the main model to incorporate a construction sector. I show that all the results in the main text continue to hold in the presence of *contractionary* shocks (defined as shocks that reduce the average value of housing), as the price of housing falls below the cost of producing a new housing unit.

Developers. I assume that there exist a large number of potential developers that can choose to enter the housing market at a cost of c. Once a developer enters the housing market, they draw a random house quality from the distribution of quality quartiles $G(\cdot)$, which they may then sell to the household at its competitive price. I take $G(\cdot)$ to be exogenous, and interpret the random draw of housing as idiosyncratic risk within the development process. As such, the presence of developers can change the *mass* of the total housing stock, but will not change the relative densities between different quality tiers. Developers are risk-neutral. Free entry for developers therefore implies

$$\mathbb{E}\left[q_t(h)\right] \geq c \tag{92}$$

I assume that c is sufficiently low so that a positive stock of housing exists in equilibrium. Let s be the total mass of housing in equilibrium. In order to maintain market clearing for all quality quartiles, I further assume that c is sufficiently high so that $s < m$.[23] This implies that the home-ownership rate is strictly less than one.

Analysis. I first present a simple result that relates developer costs to house prices.

Proposition 8. *A reduction in c strictly decreases house prices along all quality tiers.*

Proof. The inverse matching function in this economy is given by

$$y(h) = F^{-1}\left(\frac{s}{m}G(h) + F(y(\underline{h}))\right) \tag{93}$$

where $y(\underline{h}) = F^{-1}\left(1 - \frac{s}{m}\right)$. Differentiating with respect to s, we obtain

$$\frac{\partial y(h)}{\partial s} = \frac{\frac{1}{m}(G(h) - 1)}{f\left(F^{-1}\left(\frac{s}{m}G(h) + F(y(\underline{h}))\right)\right)} < 0 \tag{94}$$

It is also straightforward to show that the price which makes a household indifferent between the outside option b and the lowest house quality $\underline{h}$ is strictly decreasing in y. The initial price function was given by an equation of the form

$$q'(h) = r(q, y(h)) \tag{95}$$

whereas the new pricing function is given by

$$\tilde{q}'(h) = \tilde{r}(q, y(h)) \tag{96}$$

[23] Having $s > m$ would lead to vacant houses, which would necessitate a new equilibrium definition.

with $\tilde{r}(q,\cdot) < r(q,\cdot)$ and with a lower initial condition. Hence, the new price function must be uniformly lower follows from a direct application of Petrovitsch's Theorem. $\square$

The above result also sheds light on the effect of migration on house prices. An increase in m raises house prices for all house quality tiers. Consider now a shock that decreases average house prices permanently.[24] Because the housing stock does not depreciate over time, the total supply of housing is such so that the new average price is below the minimum profitable cost. As such, contractionary shocks in this setting are isomorphic to an economy with inelastic housing supply, a point also illustrated in Glaeser and Gyourko (2018) in a model with a homogeneous housing stock.

B.3 Rental Markets

This section endogenizes the household's outside option by incorporating a homogeneous rental market into the main model. I derive conditions under which positive assortative matching holds in this new environment, and show that the main results are robust to this richer framework.

I assume that renting households can now purchase rental units from a homogeneous housing stock at a per unit price of p_r. The pay-off of a renting household that consumes c_0, c_1, and purchases r rental units is given by

$$\log(c_0) + \log(c_1) + \chi\gamma \log(r) \tag{97}$$

where $0 < \gamma < 1$ is a utility penalty of renting, as opposed to owning, a home. Moreover, I assume that the total quantity of the rental stock is fixed at $\bar{R} > 0$. In the remainder of the analysis, I assume that $\underline{y}$ is sufficiently large relative to y_1 so that renting households are not credit constrained. This considerably simplifies some of the algebra in the remainder of the analysis, but is inessential to the main results. I now derive a condition under which positive assortative matching holds in this environment. To this end, define the average income of renting households

$$\bar{Y} \equiv \int_{\underline{y}}^{y_p} y dF(y)$$

where y_p is the income for the marginal home-owning household (I will show that the average income takes this form momentarily). We have the following lemma:

Proposition 9. *Suppose the following condition is satisfied:*

$$\underline{h} > \left(\frac{y_p + \frac{y_1}{R}}{m\bar{Y} + \frac{y_1}{R}} \bar{R} \right)^{\gamma} \tag{98}$$

where $y_p = F^{-1}\left(1 - \frac{1}{m}\right)$. Then, there exists an equilibrium such that (i) all households with $y > y_p$ become homeowners, and (ii) the housing policy function $h_1^*(y)$ is strictly increasing in y for all $y > y_p$.

[24] Note that an increase in θ, or a decrease in R, δ, or y_1 lower house prices for all quality tiers in the main text.

Proof. The proof proceeds by conjecturing that all households with $y \leq y_p$ are renters, and then verifying the conditions for positive assortative matching. The household's first-order condition gives rise to the policy function:

$$h = \frac{\chi\gamma\left(\frac{1}{1+\beta}\left(y + \frac{y_1}{R} - p_r r\right)\right)}{p_r} \tag{99}$$

where it is here that we have used the fact that renters are not credit constrained. Aggregating and imposing market clearing for the rental market, we can obtain the price for rents in closed form

$$p_r = \frac{\frac{\chi\gamma}{1+\beta}\left(m\bar{Y} + \frac{y_1}{R}\right)}{\left(1 + \frac{\chi\gamma}{1+\beta}\right)\bar{R}} \tag{100}$$

We may then substitute the price back into the household's rental policy function (99) to obtain

$$r = \frac{y + \frac{y_1}{R}}{m\bar{Y} + \frac{y_1}{R}}\bar{R} \tag{101}$$

Clearly, rental choice is increasing in income. Under the assumption of the proposition, the utility from the lowest house quality tier exceeds the utility derived from optimally choosing one's rental housing in the rental market. The price of the lowest quality tier is such so as to make the household with income y_p indifferent to renting and owning. Since the pay-off function is supermodular in house quality and income, all households with income $y < y_p$ strictly prefer renting to paying y_p's indifference price. This proves (i). The fact that the housing policy function is strictly increasing in income then follows from Proposition 2. □

Note that the average price of rents is determined *independently* of house prices – it is the price of the lowest house quality quartile that adjusts to make the marginal household indifferent to renting and home-owning. This observation, in conjunction with positive assortative matching, implies that the main results are robust to the inclusion of a rental sector. The only additional consideration here is that certain aggregate shocks will now induce "variation" in the outside option by changing the price of rents. In particular, we obtain the following alternative to Theorem 2.

Theorem 3 (Comparative Statics)**.** *Suppose Assumptions 1 is satisfied. Then, there exists an $h_c \in (\underline{h}, \bar{h})$ such that households with $y > y(h_c)$ are not credit constrained (and all other home-purchasing households are credit constrained). Moreover, the pricing function responds as follows to various permanent, unanticipated change to various parameters:*

1. *Loan-to-value constraint (θ) — $\frac{\partial q(h)}{\partial\theta\partial h} > 0$ and $\frac{\partial \log q(h)}{\partial\theta\partial h} > 0$ for $h > h(y_c)$. Moreover,* $\lim_{h\to\infty} \frac{\partial \log q(h)}{\partial\theta} = 0$
2. *Interest rate (R) — There exists a threshold y' such that $\frac{\partial q(h)}{\partial R\partial h} < 0$ for $h > h(y')$. Moreover,* $\lim_{h\to\infty} \frac{\partial \log q(h)}{\partial R} = -\frac{1-\delta}{R(R+\delta-1)}$

3. *Transaction cost (δ)— There exists a threshold y' such that* $\frac{\partial q(h)}{\partial\delta\partial h} < 0$. *Moreover,* $\lim_{h\to\infty}\frac{\partial \log q(h)}{\partial\delta} = -\frac{1}{R+\delta-1}$

Proof. Follows directly from the proof of Proposition 1 and Theorem 2. □

B.4 Extension to Bequests

In order to introduce MPC heterogeneity in a tractable way, I extend the model to allow for a *warm-glow* bequest motive as in De Nardi (2004). For simplicity, I assume that bequests leave the economy.[25] I assume that payoffs take the form:

$$\begin{aligned}\mathcal{U}\left(\{c_{t+j,j}\}_{j=0}^{1}, v_{t+1,1}, h_{t,1}\right) =& \log(c_{t,0}) + \beta\log(c_{t+1,1}) \\ &+ \log(v_{t+1,1}+\phi) + \chi\log(h_{t,1})\end{aligned} \tag{102}$$

where $v_{t+1,1}$ denotes the choice of bequests at old age and $\phi > 0$ parameterizes the extent to which bequests are a *luxury* good. To see how this creates heterogeneity in the marginal propensity to consume out of house price changes, consider a sudden, proportional decrease in house prices at time t by a factor $d\lambda$. The consumption response of old households to this wealth effect is

$$dc_{t,1}(h) = \begin{cases} -(1-\delta)q_t(h)d\lambda & \text{if} \quad v_{t,1}(h) = 0 \\ -\frac{1}{2}(1-\delta)q_t(h)d\lambda & \text{if} \quad v_{t,1}(h) > 0 \end{cases} \tag{103}$$

Note that $v_{t,1} > 0$ if and only if a household's income is greater than some threshold y_b. This capture the empirically relevant feature that MPCs are declining in wealth (Kaplan and Violante, 2014).[26]

The next corollary illustrates that the homogeneous response and matching multiplier term in Proposition 5 work in the same direction in response to a credit supply shock, thereby giving rise to an amplification channel for the response of aggregate consumption expenditures to house price changes.

Proposition 10. *Suppose $1-\delta > \frac{2}{1+2\beta}$ and Assumption 1 holds. Consider a permanent change to θ and suppose ϕ is large enough, so that $h(y_b) > h(y_c)$. Then,*

$$cov(\overline{MPC}(h), dq(h)) > 0, \quad for \quad h > h_c \tag{104}$$

Proof. See Appendix C.1. □

The assumption $1-\delta > \frac{2}{1+2\beta}$ is needed to ensure the marginal propensity to consume out of housing wealth is larger for old households than for young households, so that unanticipated house

[25] Allowing bequests to be passed on the next generation's young is inessential to the main results.

[26] The effect of house price changes on the consumption expenditures of non-home-owning households is zero. Home-owning households with low MPCs can therefore be viewed as wealthy hand-to-mouth (Kaplan and Violante, 2014) due to their low *liquid* wealth.

price drops result in an initial reduction in consumption.[27]

C Additional Theoretical Results

C.1 Proof of Proposition 10

Proof. The introduction of bequests do not change the positive assortative matching of the equilibrium, which leaves the inverse matching function unchanged. For ϕ large enough so that $h_b > h_c$, it also does not change the initial conditions of the pricing function. Hence, it remains to show that credit constraint status monotonicity holds under a linear matching function with bequests. Following the arguments of Proposition 1, we can show that the bequests change the pricing function as follows in a stationary equilibrium:

$$\begin{aligned} q(h) = \frac{\chi}{\omega(\chi+1+2\beta)} \left(h - \left(\frac{(h_b)^{\frac{\chi+1+2\beta}{1+2\beta}}}{h^{\frac{\chi}{1+2\beta}}} \right) \right) \\ + \left(\frac{y_1+\phi}{R} \right) \left(\frac{1}{\omega} \right) \left(1 - \left(\frac{h_b}{h} \right)^{\frac{\chi}{1+2\beta}} \right) + \left(\frac{h_b}{h} \right)^{\frac{\chi}{1+2\beta}} q(h_b) \end{aligned} \tag{105}$$

for all $h > h_b$, where h_b is the threshold at which households start leaving bequests

$$h_b = \inf\{h : v_1^*(h) > \phi\} \tag{106}$$

and $v_1^*(h)$ is the stationary bequest function.

It is straightforward to see that the response of the pricing function to various shocks are the same as those outlined in Theorem 2 by following the arguments in Appendix A.7.

Next, note that the $\widetilde{MPC}$ of a young household that is not credit constrained and does not leave bequests is given by $\frac{1}{1+\beta}$. The MPC of an old household that does leaves bequests is unity, and any house price changes are attenuated in the budget constraint by a factor of $(1-\delta)$. Similarly, the $\widetilde{MPC}$ for young households that leaves bequests is $\frac{1}{1+2\beta}$ and is $\frac{1}{2}$ for old households that leave bequests. The condition

$$1 - \delta > \frac{2}{1+2\beta} \tag{107}$$

therefore ensures that $\overline{MPC}(h) > 0$ for all $h \in (h_c, \bar{h}]$.

The condition that ϕ is large enough ensures that households start leaving bequests above the credit constraint threshold, which results in non-trivial variation in marginal propensities to consume above y^*. By Theorem 2, capital losses are a strictly decreasing function of housing quality for $h > h(y^*)$. Moreover, MPCs and $\widetilde{MPC}$s are a non-decreasing function of housing quality. Hence, the conditions of Theorem 1 and Theorem 2 in Behboodian (2006) are satisfied and we must have $cov(\overline{MPC}_t(h), dq_t(h)) > 0$ for $h > h(y^*)$. ∎

[27] Christelis et al. (2021) provide some evidence that housing wealth effects are larger for older households.

D Data Appendix

D.1 Additional Details on Stock Market Wealth Shock Construction

Comprehensive stock market wealth data at the zip code level does not exist. I therefore follow the approach of Chodorow-Reich et al. (2021) in imputing stock market wealth at the zip code level. In particular, I use publicly available IRS Statistics of Income (SOI) data, which contains zip code aggregates of annual dividend income reported on individual tax returns over the period 2005-2019. I then convert dividend income to stock market wealth by multiplying dividends by the price to earnings ratio of the S&P 500 in the relevant year. I am then able to compute variations in stock market wealth across ZIP codes using this measure in conjunction with the monthly total return on the S&P 500 (which I use as the shifter). SOI data is available at a yearly frequency. Consequently, I interpolate total stock market wealth at the monthly level in order to conduct my analysis at the monthly frequency. In practice, this introduces little measurement error because the time series for total stock market wealth is persistent. IRS SOI data also reports the number of tax returns by ZIP code, which I use in order to construct stock market wealth measures on a per capita basis.

D.2 Additional Details on Credit Shock Construction

I collect data on the flow of new mortgage loans originated in the years 2000-2010 through the "Home Mortgage Disclosure Act" (HMDA) data set. HMDA records various characteristics of each mortgage and applicant at the loan application level. I collect information on the applicant's total loan amount, purpose of borrowing (refinancing/home purchase/home improvement), income, and whether or not (and to which kind of agency) the loan was sold to in the secondary market within that given year. Each institution in the HMDA dataset is given a unique HMDA identifier. I obtain the non-core liabilities of each lender by linking these financial institutions to Call Report data using a key provided by the Federal Reserve Board.[28]

I continue to define low quality tiers as zip codes that fall within the lowest price quartile within a county for a given base year. I use 2002 as the base year as it precedes the expansion in the PLS market, but the choice of a base year is immaterial given that the ranking of zip codes across years is persistent. I drop counties for which we have fewer than four zip code-observations, leading to a final sample of 9135 zip codes and 136 counties. Finally, I only consider mortgages originated for the purpose of purchasing a house.

E Additional Tables and Figures

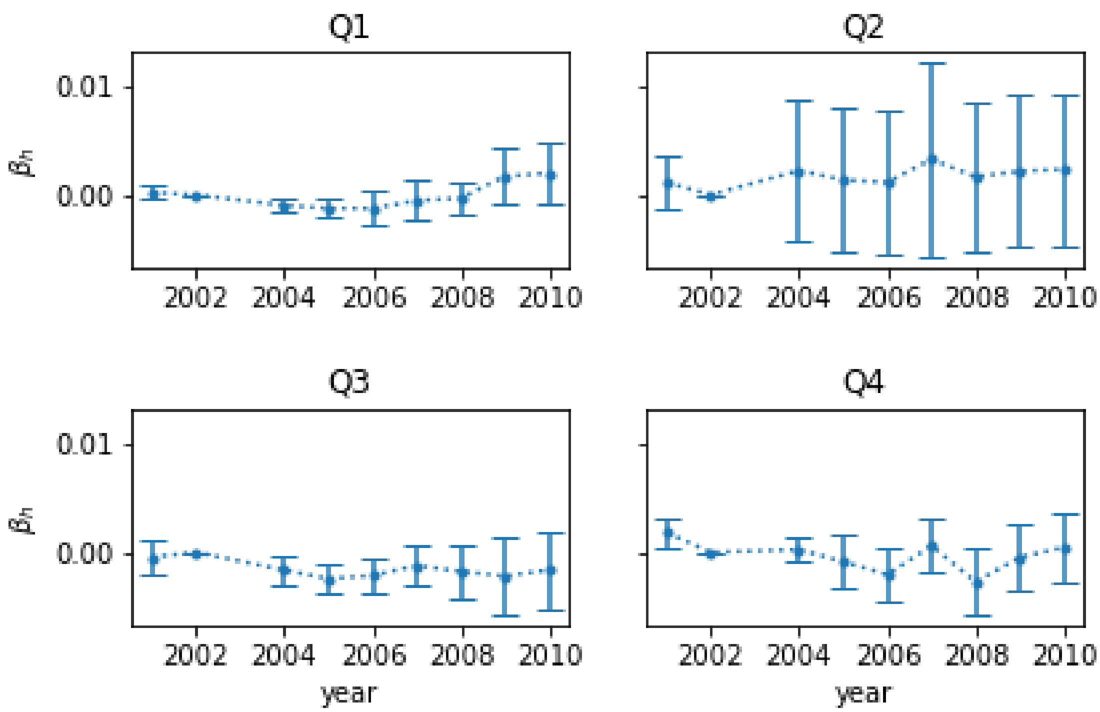
[28] The key is available on Neil Bhutta's website: https://sites.google.com/site/neilbhutta/data?authuser=0

Table 5

	(1)
	$\triangle_{2005,2002} Price$
β_1	1.800
	(1.260)
β_2	0.760***
	(0.182)
β_3	1.380*
	(0.650)
β_4	1.672*
	(0.844)
Quartile FE	✓
N	9135

Note: The table reports the coefficients from Equation (25), using the NCL share as an instrument for the total change in credit growth in the corresponding county from 2002-2005. The dependent variable is the percentage change in the house price in a given zip code. Robust standard errors are in parentheses. $*$ denotes signifiance at the 5% level, $**$ at the 1% level, and $***$ at the 0.1% level.

Figure 7



Note: The figure reports the coefficients β_h for each quality tier from Equation (27) using the percentage change in income as the dependent level. 2002 is the base year and observations are at the zipcode level. The figure shows 95% confidence bands, based on standard errors clustered at the county-level. The years 2000 and 2003 have been omitted due to the lack of zip code level income data.