Heterogeneous Deleveraging

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Abstract

I empirically show that different types of houses experience substantially different rates of return over nation-wide housing cycles, even within narrow geographical areas. In order to understand the implications of this rate of return heterogeneity for the propagation of macroeconomic shocks, I present a dynamic assignment model in which housing is segmented by various quality tiers. I show that the model's unique equilibrium features unidirectional propagation of shocks within the housing market – changes in households' valuation of low-end homes have spill-over effects to house prices across the entire quality spectrum. I characterize the strength of these spill-over effects analytically, examine how they shape the cross-sectional response of house prices to changes in the economic environment, and show that they can lead to an amplification channel for the response of aggregate consumption expenditures to changes in house prices. In the data, consistent with the theory, shocks that affect high-income households only induce changes in the prices of high-end homes, whereas shocks that affect low-income households affect house prices in the entire market.

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1 Introduction

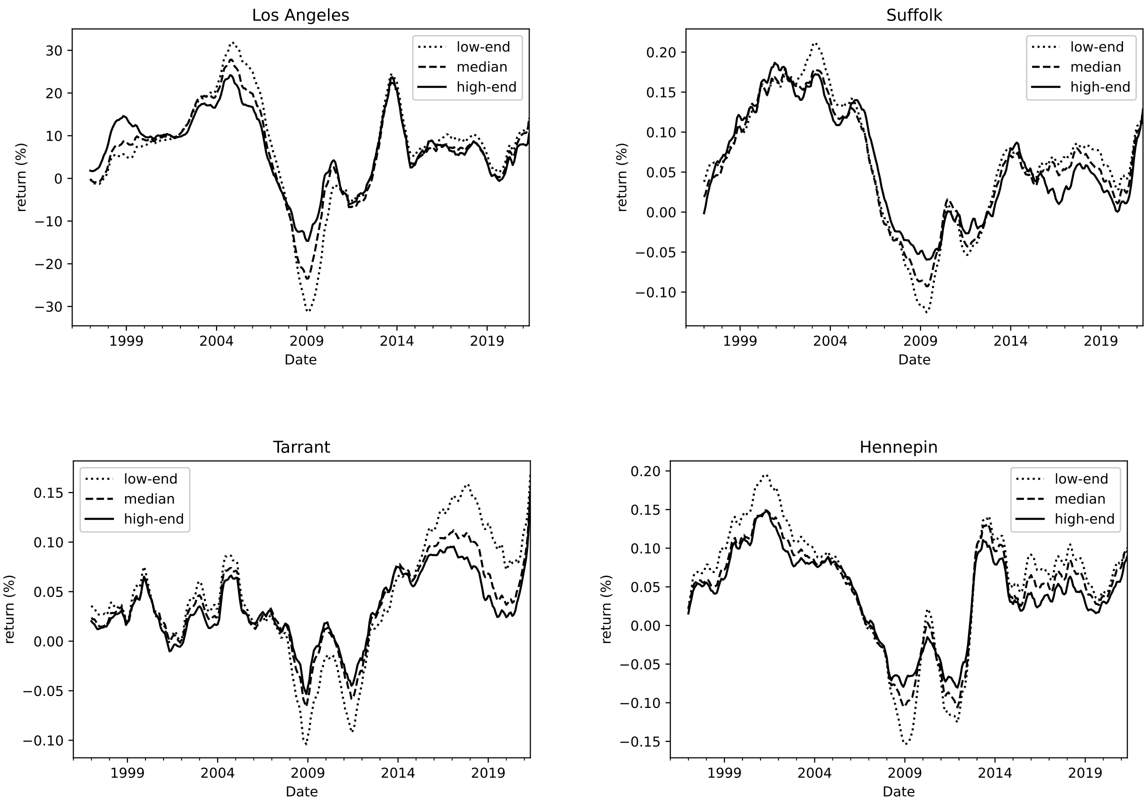
Housing markets are segmented: even within a narrow geographical region, there is substantial heterogeneity in relative capital gains for different types of houses. Figure 1 illustrates this fact. It plots the annualized capital gains of houses for counties in various metro areas within the United States from 1990-2021, disaggregated by quality tiers, alongside the county median. In Los Angeles, for example, low-end houses depreciated approximately three times as much as high-end homes (in percentage terms) in the year 2009. As I will show later, this disparity in relative rates of return is a robust feature for local housing markets across the entire United States.

This rate of return heterogeneity can have crucial macroeconomic consequences. Shocks that affect house prices can lead to large distributional effects on wealth through capital gain variation. Moreover, the unequal incidence of housing wealth effects can in turn be an important driver for other macroeconomic aggregates, such as consumption expenditures. Yet, in macroeconomic models with a housing sector, housing is typically modelled as a commodity that can be bought and sold at a common per unit price from a homogeneous stock. Consequently, capital gains in these models are constant across all households, even if these households are widely heterogeneous with respect to their income, age, and wealth.

In order to understand the implications of this rate of return heterogeneity for the propagation of macroeconomic shocks, I present an analytically tractable, dynamic assignment model in which housing is segmented by various quality tiers. I show that the presence of housing market segmentation substantially alters the response of house prices to shocks relative to the homogeneous housing benchmark. In the unique equilibrium of the model, shocks propagate within the housing market *unidirectionally* – changes in households' valuations of low-end homes have spill-over effects to house prices across the entire quality spectrum. I characterize the strength of these spill-over effects analytically, and examine how they shape the response of the entire *cross-section* of house prices to changes in economic conditions. The model also gives rise to an amplification channel for the response of house price changes to aggregate consumption expenditure through the heterogeneous incidence of capital gains.

I operationalize this model empirically to measure the importance of housing market segmentation in the data. In the data, shocks that affect high-income households only induce price changes in high-end homes, consistent with the unidirectional propagation of shocks. In contrast, shocks that affect low-income households affect house price in the entire market through pricing spill-overs. These tests empirically reject the predictions of the homogeneous housing

Figure 1



Notes: Annualized rates of return for counties in various metro areas (Greater Los Angeles, New York-Newark-Jersey City, Dallas-Fort Worth-Arlington, Minneapolis-Saint Paul). House price data from Zillow Research (single family homes index). The low-end (high-end) series plots average annualized returns for zip codes in the bottom (top) quartile of the zip code price distribution within the given county in the year 1999. The median series represents the median house price value within a given a county.

benchmark, and suggest that the presence of segmentation is an important feature in order to understand how macroeconomic shocks affect the housing market.

Model. My work builds on the seminal work by Landvoigt, Piazzesi and Schneider (2015), and Määttänen and Terviö (2013). In the model, there exist a continuum of indivisible houses that provide different flows of housing services to households. In equilibrium, households match to houses of varying qualities according to the net present value of their income. Because houses are priced competitively, this gives rise to a continuum of pricing equations, in which each household is a marginal investor for the house that they purchase. These pricing equations relate households' willingness to pay for inframarginal quality units to their marginal rate of substitution between consumption and housing services. Households' marginal rate of substitution, in turn, depends on aggregate economic conditions, such as the level of interest rates or the availability of credit. Heterogeneity in housing capital gains is then a consequence of heterogeneous variations in marginal rates of substitution in response to different economic shocks.

Theoretical Results. The existence of housing market segmentation gives rise to distinct asset pricing implications relative to the homogeneous housing benchmark. First, I show that certain "localized" shocks that affect only a subset of households can spill-over into neighboring quality tiers by affecting the willingness to pay of neighboring households along the income distribution. This gives rise to a mechanism in which localized shocks that affect the prices of low-end homes "trickle up" to the valuations of high-end homes in general equilibrium. However, localized shocks that directly affect the prices of high-end homes do not "trickle down" the quality ladder to low-end house valuations. This is a consequence of the positive assortative matching between housing qualities and income in the equilibrium assignment. Intuitively, the price of a given quality tier is determined by the willingness to pay for additional housing services by households that reside strictly *below* that tier. For this reason, the preferences of wealthy households – that reside in high-end homes – will not affect the prices of lower quality segments.

Second, I conduct comparative statics of the cross-section of capital gains in response to *aggregate* shocks. I show that changes in credit conditions mostly affect the returns to low-end homes. Interest rate changes, as well as changes in the perceptions of future house values, mostly affect the returns of high-end homes (both in absolute and percentage terms). This is because credit shocks mostly affect the willingness to pay of constrained, low-income households. In contrast, future house values are mostly pay-off relevant to unconstrained, high-income households through their effect on intertemporal consumption smoothing. As such, the cross-sectional distribution of capital gains is sensitive to the type of shock under consideration. Nevertheless, the presence of pricing spill-overs implies that credit shocks (and other "low-end" shocks) can have a large impact on prices over the whole quality ladder. Even if few households are credit constrained in equilibrium, the average house price index is sensitive to changes in credit conditions.

Third, I demonstrate that market segmentation gives rise to an amplification channel for various aggregate shocks relative to the homogeneous housing benchmark. Intuitively, this is because market segmentation creates a positive covariance between capital gain incidence (which are wealth effects in the model) and marginal propensities to consume (MPCs). In contrast, in the absence of market segmentation, the housing market gives rise to a form of dampening for *all* aggregate fluctuations. This is because capital gains contrivedly become a decreasing function of liquid assets, which establishes a strictly negative covariance between wealth effects and MPCs. Moreover, I show that market segmentation introduces large redistributive effects, which would otherwise be absent in the homogeneous housing benchmark.

Empirical Analysis. Finally, I turn to empirically testing the key predictions of the theory. To do so, I exploit cross-sectional variation in the incidence of various shocks across local housing markets in the United States. In the first test, I isolate plausibly exogenous variation in mortgage lending that occurred over 2002-2005 to create a proxy for a localized shock to *low-end* households. I show that this credit shock leads to monotonicity in housing capital gains across the quality ladder: low-end homes appreciated more than high-end homes, as predicted by the theory. However, I also provide evidence for the existence of "trickle up" forces, which cause an economically significant increase in the prices of homes at the top end of the quality distribution.

In the second test, I confirm the absence of "trickle down" effects by isolating plausibly exogenous variation in stock market wealth changes. I show that stock market wealth is primarily held in "high-end" segments of the house quality distribution. These high-end shocks cause a statistically and economically significant increase on the prices of high-end homes, but have no effect on lower quality tiers. Moreover, the results are robust to local general equilibrium spillovers as in Chodorow-Reich et al. (2021) or Guren et al. (2020). Consequently, the data suggests that there is strong *directionality* in the propagation of shocks within the housing market.

Relation to Existing Literature. This paper lies at the intersection of several strands of literature. First, it contributes to the literature on within-city house price dynamics. Ortalo-Magné and Rady (2006) construct a life-cycle model of the housing market in which households live for four periods and show that the prices of flats display more volatility than houses. In contrast, my model features a simpler life-cycle specification (in which households only live for two periods), but permits a more transparent characterization of the cross-section of capital gains in response to different shocks. I primarily focus on how *demand-side* changes affect house prices. A related work by Guerrieri et al. (2013) demonstrates how *supply-side* effects due to gentrification can cause the prices of low-end homes tend to appreciate more in MSAs during nation-wide housing booms.

Määttänen and Terviö (2013) also consider an assignment model of heterogeneous househoulds to heterogeneous house quality types in a static framework. They show that shocks propagate upwards – but not downwards – in the quality ladder in no-trade equilibria. I generalize this result to cases in which housing transactions are realized in equilibrium and transparently characterize the strength of pricing spill-overs. Moreover, their framework (being static) cannot answer how credit frictions or perceptions regarding the values of *future* homes affect capital gains and propagate onto the rest of the economy.

Of particular relevance is the work of Landvoigt et al. (2015). They construct a model with a continuum of housing quality tiers and quantify it using micro-data on San Diego County during the 2000-2005 boom. They show that the increase in the availability of cheap credit, along with a change in the housing quality distribution over this period can jointly generate the cross-sectional capital gains observed in the data. Their work is primarily quantitative in nature. In contrast, I provide sharp theoretical results on the cross-sectional distribution of capital gains to a wide range of economic shocks, which I test empirically in the data.

Second, this paper relates to a large empirical and theoretical literature on the effect of macroeconomic shocks on house price movements (Kiyotaki, Michaelides, and Nikolov, 2011; Favilukis, Ludvigson, and Nieuwerburgh, 2017; Greenwald, 2018; Cox and Ludvigson, 2019; Guren et al., 2020; Kaplan, Mitman, and Violante, 2020; Gao, Sockin, and Xiong, 2020; Garriga and Hedlund, 2021; Greenwald and Guren, 2021; Mian and Sufi, 2022). This literature primarily characterizes how shocks affect average house prices *across* different housing markets and is often quantitative in nature. Instead, my work differs in its analytical approach and empirically tests the heterogeneous response of different house prices *within* a given market.

Finally, this work relates to papers by Mian and Sufi (2011), Mian, Rao, and Sufi (2013), Mian and Sufi (2014), Ganong and Noel (2017), Berger et al. (2018), and Guren et al. (2021), which analyze the effect of house price changes on consumption expenditures. My work offers a complementary mechanism for the propagation of house price changes to the rest of the macroeconomy through heterogeneity in capital gain incidence.[1] Moreover, most theoretical work in this literature examines the consumption response to house price changes in partial equilibrium. In contrast, I emphasize the importance of general equilibrium forces in shaping the transmission mechanism of housing wealth effects through the cross-sectional incidence of capital gains. Hence, the *type* of underlying shock is crucial in determining the response of aggregate consumption expenditures through the housing market.

Outline. The rest of the paper proceeds as follows. Section 2 documents additional motivating facts regarding housing market segmentation. Section 3 sets up the model. Section 4 presents the main theoretical results. Section 5 tests the key predictions of the theory. Section 6 concludes.

[1] The redistributive effects of house price changes and its role in shaping aggregate consumption expenditures is also emphasized in Kiyotaki, Michaelides, and Nikolov (2011).

2 Facts About Housing Market Segmentation

I first provide quantitative evidence for the premise of this paper – that housing markets are segmented. To this end, I analyze the rates of return of different types of houses within various local housing markets across the United States. My definition of a local housing market is that of a county: this is consistent with prior work on the within-market variation in housing prices (Guerrieri et al. 2013, Landvoigt et al. 2015), as well as empirical work that quantifies the effect of net worth changes on local employment (Mian and Sufi, 2014).[2] In doing so, I implicitly assume that the set of reasonable housing choices for households lie within their given county.[3] Moreover, I assume that the ranking of various houses within a county is common across *all* given households that reside there. As such, the relative granularity in which the analysis is conducted is important – this assumption is likely to be satisfied for regions where internal commuting and migration frictions are limited, but not for bigger areas, such as states.

2.1 Data

My measure of house prices comes from the Zillow zip-code level price indices. The Zillow index is a hedonically adjusted price index. It uses detailed information on properties, collected from public records, to create a seasonally adjusted measure of the typical home value in a given region. I restrict the analysis to the years 2000-2020. Many zip codes are missing prior to the year 2000, which makes it difficult to characterize the cross-sectional evolution of house prices preceding this time period. In order to construct weighted averages of house price growth, I compute total number of houses by county and zip code from the American Community Survey (ACS). I use the ACS five-year survey whose corresponding year is closest to the year in which I disaggregate houses into different quality tiers.

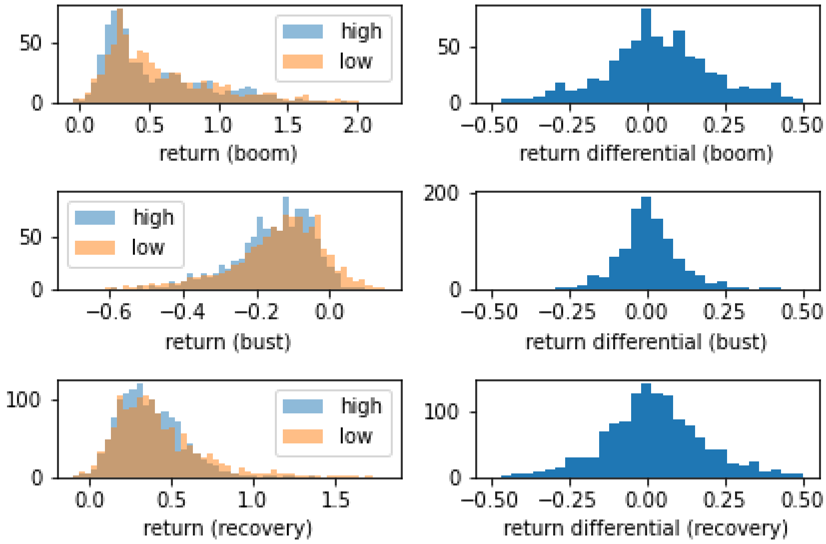
2.2 Methodology

I treat each county as a common housing market that is segmented in various quality tiers. I follow Landvoigt et al. (2015) in assuming that market prices approximately reflect a one-dimensional index of housing quality. The reasoning for this is that any changes in the pay-off relevant characteristics of the house, such as land, location, and structure, should be incorporated into house valuations in competitive equilibrium. Moreover, in the theoretical model

[2] The results are quantitatively similar when using MSAs as the relevant definition of a market with no change in the main results.

[3] There are, of course, more sophisticated methods to identify housing markets using the search behaviour of households as in Piazzesi et al. (2020) or a variant of the stochastic block model used in Nimczik (2018). Doing so, however, requires data on the search behaviour of households for houses, which is difficult to obtain and beyond the scope of this paper.

Figure 2



Notes: Relative rates of return for all counties during the boom (first row), bust (middle row), and recovery (third row). The first column plots the histograms of cumulative returns for the bottom and top quartile of house qualities for the corresponding time period. The second column plots the return difference of the bottom quartile relative to the top quartile. Counties whose average house price growth was less than 2% in the boom and recovery and −2% in the bust have been dropped from the sample.

presented in the next section, I show that price is a sufficient statistic for a given housing type's quality ranking.

Consequently, I sort zip codes in a particular county into different quality quantiles according to their initial price in a given year. When looking at changes in prices experienced by houses of a given quality over a particular time frame, the year in which houses are sorted into quality tiers is somewhat arbitrary. However, the relative ranking of median house prices across zip codes within counties is very persistent. As such, all the empirical results are insensitive to the year in which zip codes are sorted into different quality quartiles.

2.3 Cross-Sectional Patterns

The national house price cycle can be trisected into the 2000-2006 boom, the 2006-2012 bust, and its subsequent recovery. For this reason, it is natural to analyze cross-sectional distribution of house prices within each time frame separately.

The left column of Figure 2 plots the average growth rates of housing prices within the top and bottom quality tier quartiles for all counties and each time period under consideration. The right column of Figure 2 plots the differences between the bottom and top quality quartiles *within* each county. Two observations immediately stand out. First, there is significant variation in the across county growth rate of house prices, even in the same relative quality

Table 1

	return differential > 10%	
	percentage of counties	average return differential
boom (2000-06)	47%	25%
bust (2006-12)	23%	19%
recovery (2012-19)	43%	24%

Notes: This table depicts summary statistics for return differences across counties in the United States. Return differential is defined as the average return between the lowest and highest quality quartiles within a given county over the corresponding period.

tier. Second, there is significant variation in the *intra*-county relative growth rates of various quality tiers.

Moreover, in the first and third rows of Figure 2, where the national housing market was trending upwards (boom and recovery), the distribution of low quality growth rates places more mass on its right tail relative to its high quality counterpart. Hence, in periods of national house price appreciation, low quality housing tends to appreciate more on average relative to high-end housing. This observation is consistent with the results in Guerrieri et al. (2013), who demonstrate that houses with lower initial prices at the city and MSA level experience larger capital gains during nationwide housing increases. Indeed, across counties that experienced a county-wide average house price growth of more than 10% during the boom (recovery) low-end housing appreciated by 4.6% (4.9%) more (in absolute terms) than high-end housing, on average. However, the reverse observation is true during the bust: high-end homes *depreciated* on average by 1.4% more relative to low-end homes.

Although these numbers might appear small, a closer inspection of the data reveals substantial housing market segmentation. For example, across the 862 counties in the sample during the boom, 47% experienced an absolute difference in the returns of the low-end relative to high-end homes of more than 10%. Of these counties, the vast majority (66%) saw low-end homes appreciate more, with an *average* difference across quality tiers of 25%. These numbers are clearly non-negligible.[4] During the bust, of the 1195 counties in the sample, about a quarter (23%) experienced a similar disconnect in relative rates of return. Of these, the majority (59%) saw high-end homes depreciate more than low-end homes, with an average absolute difference of 20% (for the other 41%, low-end homes depreciated more on average by 16%). These observations are also summarized in Table 1.

What drives relative differences in returns within the housing market? The large heterogeneity present in relative rates of return would suggest that different housing markets were

[4] Similar numbers hold for the recovery phase. 42% of countries witnessed absolute differences in rates of return between quality tiers of more than 10%, 63% of which saw low-end homes appreciate more (by 27% on average).

exposed heterogeneously to changes in economic conditions during the boom, bust, and recovery. Hence, the types of shocks that different markets are exposed to are likely to matter in mediating which segments of the housing market will appreciate more during a boom, or depreciate more during a bust. This analysis merits a theoretical basis.

3 A Model of Segmented Markets

To study the key drivers of heterogeneity in capital gains, as well as its role in amplifying macroeconomic shocks, I present a simple macroeconomic model with a housing sector. The model embeds two key features of housing markets that will be crucial in generating heterogeneous capital gains across different types of houses, (i) *segmentation* across houses of varying quality and (ii) *indivisibility* of housing units.

Agents and Preferences. Time is discrete and indexed by $t \in \mathbb{N}$. The economy is populated by an overlapping generations of households. Each period, a mass $m > 1$ of households is born and all households live for two periods. Household age is indexed by $j \in \{0, 1\}$ and will be referred to interchangedly as “young" and "old".

Households have perfect foresight and there is no aggregate uncertainty. They derive utility over consumption streams of a numeraire good and housing. I assume that households may only choose to purchase housing when they are born, and sell on to the next generation's young when they are old. Payoffs are specified as follows:

$$\mathcal{U}\left(\{c_{t+j,j}\}_{j=0}^{1}, h_{t,1}\right) = \log(c_{t,0}) + \beta \log(c_{t+1,1}) + \chi \log(h_{t,1}) \tag{1}$$

where $c_{t,j}$ and $h_{t,j}$ denote the consumption of the numeraire good and housing quality purchased at time t and age j, respectively. The parameter $\chi > 0$ is a utility weight on housing services and $0 < \beta < 1$ is the household's discount factor. Finally, I assume that households incur a proportional transaction cost $\delta \in (0, 1)$ from selling a house.

Housing Market. The housing market consists of a collection of indivisible housing units. Houses are heterogeneous and are indexed in terms of their quality $h \in \mathbb{R}^+$. This one-dimensional index can be thought of as an appropriately weighted aggregator of the various amenities households care about in a house (including location, number of bathrooms, quality of neighboring schools, etc.). House quality in any given period is distributed according to a strictly increasing cumulative distribution function $H(\cdot)$ and where the support of $H(\cdot)$ is $[\underline{h}, \bar{h}] \subset \mathbb{R}^+$. Each house can accommodate at most one household, who must be an owneroccupier.

Households can purchase at most one house of a given quality. A house of quality h at time t trades at a price $q_t(h)$ in a competitive market. Note that this gives rise to a price *function* – a price for each house quality tier. Households have the option to not become home-owners, in which case they receive an exogenous outside flow of housing services of quality $0 < b < \underline{h}$ at zero cost. This assumption ensures that household utility is well-defined if households choose not to enter into the housing market. I normalize the total quantity of housing units available for purchase to unity. Since $m > 1$, at least some households will opt for this outside option in equilibrium.

Credit Frictions. Households can participate in credit markets in their youth by either lending or borrowing at a gross interest rate of $R > 0$. However, the presence of credit frictions implies that households are subject to a loan-to-value constraint

$$d_{t,1} \leq (1-\theta)q_{t+1}(h_{t,1}) \tag{2}$$

where $d_{t,1}$ is the household's choice of debt at time t.[5] Note that this constraint implies that non-home owning households cannot borrow at all with respect to their second-period income.

In order to isolate the direct effect of various changes in macroeconomic conditions on the cross-section of house prices, I assume a small open economy and take R to be exogenous. However, in the following analysis, I also characterize how changes in rates affect house prices in order to inform the full general equilibrium effect.

Endowments. Households are born without any initial wealth, but are heterogeneous in their first period income. In particular, each household receives income y at age 0, which is distributed according to a strictly increasing cumulative distribution function $F(\cdot)$ with support $[\underline{y}, \bar{y}] \subset \mathbb{R}^+$. In the second period, households receive a common income stream $y_1 > 0$. This single margin of heterogeneity is a tractable way to generate sorting between households and house quality type, but is not necessary to obtain the main results. In Appendix B.1, I demonstrate how the model can be extended to incorporate household heterogeneity in old income as well.

[5] This can be microfounded as in Kiyotaki and Moore (1997) or Iacoviello (2005). Borrowers can repudiate on their debt obligations while lenders can only repossess the borrower's assets by paying a proportional transaction $\theta q_{t+1}(h_{t,j})$. Lenders will therefore never lend more than $(1-\theta)q_{t+1}(h_{t,j})$ in a subgame perfect equilibrium.

3.1 Equilibrium

We are now in a position to state the household problem. A household with income y that is born at time t takes the pricing functions $\{q_t(h)\}_t^{t+1}$ as given, and solves

$$\sup_{\{c_{t+j,j}\}_{j=0}^{1}, h_{t,1}, d_{t,1}} \mathcal{U}\left(\{c_{t+j,j}\}_{j=0}^{1}, h_{t,1}\right) \tag{3}$$

$$\text{s.t. } c_{t,0} + q_t(h_{t,1}) = y + d_{t,1} \tag{4}$$

$$c_{t+1,1} = y_1 - Rd_{t,1} + (1-\delta)q_{t+1}(h_{t,1}) \tag{5}$$

$$d_{t,1} \leq (1-\theta)q_t(h_{t,1}) \tag{6}$$

where (4)-(5) are the household's first and second-period budget constraint, respectively, and (6) is the loan-to-value constraint described above.

In the following analysis, I focus on the response of the economy to unanticipated changes in various parameters from a stationary equilibrium. A stationary competitive equilibrium in this economy is defined as follows:

Definition 1. *Given a distribution of income earnings $F(\cdot)$, a distribution of house qualities $G(\cdot)$, and a gross interest rate R, a stationary equilibrium is a set of time-invariant individual decision functions $\{c_j^*\}_{j=0}^{1}$: $[\underline{y}, \bar{y}] \to \mathbb{R}^+$, d_1^*: $[\underline{y}, \bar{y}] \to \mathbb{R}$ and h_1^*: $[\underline{y}, \bar{y}] \to \{b\} \times [\underline{h}, \bar{h}]$, as well as a pricing function q: $[\underline{h}, \bar{h}] \to \mathbb{R}^+$, such that*

1. *The individual decision functions solve the household problem* (3) - (6)
2. *The market for each housing quality h clears*

$$H(h) = m\int_{\underline{y}}^{\bar{y}} \mathbb{1}[b < h_1^*(y) \leq h]dF(y), \quad \forall\, h \in [\underline{h}, \bar{h}] \tag{7}$$

3. *The housing pricing function is constant $q_t(\cdot) = q(\cdot)$, $\forall\, t \in \mathbb{N}$*

Equation (7) ensures that the number of household that choose a housing quality less than h is equal to the total number of available houses below that quality. The key difference in this definition from the standard case with homogeneous housing is that there exist a *continuum* of market clearing conditions, one for each house quality tier. This is the natural extension of the more standard market clearing condition in which the per unit price of housing clears through the *total* amount of housing demanded.

3.2 Comments

Housing Supply In analyzing the response of house price changes to various aggregate shocks, I purposefully abstract from the determinants of housing *supply* movements to isolate *demand*-side factors. This can be viewed as a useful approximation over short-term horizons, when housing supply is inelastic. However, declining housing markets are generally associated with prices that are below the minimum profitable production cost, which makes the supply schedule across all housing qualities inelastic (Glaeser and Gyourko, 2018). In Appendix B.2, I present an extension of this model with free entry into a construction sector in which house prices are entirely demand-determined in response to contractionary shocks.

Rental Markets The absence of rental markets is inessential to the main analysis. The model can be extended to allow for a fixed rental rate of housing, in which case the assumption $b < \underline{h}$ would reflect the observed segmentation between rental and owner-occupied markets (Garriga and Hedlund, 2020). Appendix B.3 presents an extension of the model in which non-home owners can purchase rental units from a homogeneous rental stock.

Trading-Up Note that houses are priced by *young* households because they enjoy housing services later in life. Hence, the model abstracts from the "trading-up" of houses over the life-cycle, but naturally incorporates the well-documented importance of first-buyers in shaping aggregate housing fluctuations (Ortalo-Magné and Rady, 2006). Moreover, although this is a simplified model of housing that abstracts from long-term mortgages (given that it is only a two-period model), it captures various realistic features of the housing market. First, housing is a good that can be used to transfer resources across states. Second, credit frictions impede the household from borrowing up to the full value of the house. Third, house price changes during the second period of life constitute pure *wealth effects*. This is consistent with a large empirical and theoretical literature documenting that the effect of house price changes on consumption is closely approximated by marginal propensities to consume (Mian et al., 2013 ; Berger et al., 2017).

4 Theoretical Results

I now present the main theoretical results in four parts. First, I characterize the main features of the stationary equilibrium. I show that it admits a unique, differentiable price function which can be characterized through two implicit ordinary differential equations. Moreover, the equilibrium exhibits *positive assortative matching* (PAM) between household income and house quality. Second, I consider how the price function responds to certain kinds of "localized" shocks which affect certain quintiles of the income distribution, but leave other quintiles unchanged. I demonstrate that there is a sense in which shocks to house prices can only propagate upwards across house quality tiers (a "trickle-up" effect), but never downwards.

Third, I turn my attention to how house prices respond to aggregate shocks – unanticipated changes to the loan-to-value constraint, interest rates, transaction costs, and future income. I show that the resulting housing capital gains (or losses) are strongly nonlinear in house quality and that the shape crucially depends on the type of shock under consideration. Finally, I show that the non-linear incidence of capital gains across the wealth distribution can be an important driver of other macroeconomic aggregates, such as aggregate consumption.

4.1 Equilibrium Characterization

The first lemma characterizes certain properties the endogenous pricing function $q^*(\cdot)$ must satisfy in any stationary equilibrium.

Lemma 1. *$q^*(\cdot)$ is strictly increasing and differentiable for all $h \in (\underline{h}, \bar{h})$.*

Proof. See Appendix A.1. □

This lemma permits further characterization of the pricing function through the household's first-order condition.

Proposition 1. *The derivative of the pricing function is characterized by the following two equations. For an unconstrained household with income y:*

$$q'(h_1^*(y))\left(1 - \frac{1-\delta}{R}\right) = \underbrace{\chi \frac{c_0^*(y)}{h_1^*(y)}}_{MRS} \tag{8}$$

where $c_0^*(y) = \frac{1}{1+\beta}\left(y + \frac{y_1}{R} - \frac{R+\delta-1}{R} q(h_1^*(y))\right)$. For a constrained household:

$$q'(h_1^*(y))\left(1 - \frac{1-\delta}{R}\right) = \chi \frac{1}{h}\left(\frac{x}{\tilde{c}_0^*(y)} + \frac{(1-x)\beta R}{\tilde{c}_1^*(y)}\right)^{-1} \tag{9}$$

where $x \equiv \frac{\theta R}{R+\delta-1}$, $\tilde{c}_0^*(y) = y - \theta q(h_1^*(y))$ and $\tilde{c}_1^*(y) = y_1 - [R(1-\theta) - (1-\delta)]\, q(h_0^*(y))$.

Proof. See Appendix A.2. □

Equation (8) demonstrates that the slope of the pricing function at a given house quality tier is dictated by the *marginal rate of substitution* (MRS) between consumption and housing services of the household that purchases that housing type in equilibrium. The willingness of households to sacrifice consumption goods for additional housing services therefore determines the *rate of change* of prices across the quality spectrum. This is contrast to the homogeneous housing stock case, in which the MRS of all households prices a common *per-unit* price of housing.

Equation (9) illustrates the impact of credit constraints on the pricing function. Relative to the unconstrained case, we see that what determines the rate of change of house prices is a *distorted* marginal rate of substitution – consumption enters the MRS through a convex combination of marginal utilities across young and old age. Clearly, if $\tilde{c}_1^*(y) = \beta R \tilde{c}_0^*(y)$, the household lies on its Euler equation and the two equations are identical. Since the marginal utility of consumption is convex, the presence of credit frictions implies *flatter* house prices across quality tiers.

Notably, the presence of market segmentation and indivisibilities imply that each household becomes a "marginal" investor for the house quality that it optimally chooses to purchase in equilibrium. In other words, changes in the MRS for a particular household will affect the rate of change of prices only at that households' neighboring quality tiers. Shocks that heterogeneously affect the MRS of households along the income distribution will therefore naturally give rise to heterogenous capital gains along the house quality distribution.

However, there are two difficulties in obtaining a clean characterization of capital gain heterogeneity in response to different kinds of shocks. First, it is difficult to establish how exactly a shock will affect a household's MRS. For example, a higher weight on housing utility χ will uniformly increase the MRS for all households, which will raise house prices by making them steeper. In general equilibrium, however, this increase in house prices will reduce the amount of resources available for consumption, having a counter-acting effect on the MRS through the budget constraint. The challenge is to disentangle how these general equilibrium effects shape the pricing function.

Second, it is difficult to establish how changes in a household's MRS will affect house prices in the absence of additional information on the housing choice function $h^*(y)$. This is needed in order to relate changes in the willingness to pay for housing services along the *income* distribution to changes in prices along the *quality* distribution. The next lemma demonstrates that every equilibrium features an assignment of households to houses in which house quality is *strictly* increasing in income.

Proposition 2. *The following two statements are true in any stationary equilibrium.*

1. *There exists a $y_p \in (\underline{y}, \bar{y})$ such that households with $y > y_p$ are homeowners.*

2. *The policy function $h_1^*(y)$ is strictly increasing in y for all $y > y_p$.*

Proof. See Appendix A.3. □

This result, which follows from the weak supermodularity inherent in the households' payoffs, allows us to characterize the allocation of houses to income types. To see this, note that monotonicity in the decision rules implies that the market clearing condition (7) must now equate the mass of all home-owning households below a particular quality with the corresponding supply:

$$m\left(F(y(h)) - F(y_p)\right) = H(h), \quad \forall\, h > \underline{h} \tag{10}$$

or, equivalently,

$$y(h) = F^{-1}\left(\frac{1}{m}\left(H(h) + F(y_p)\right)\right), \quad \forall\, h > \underline{h} \tag{11}$$

which gives rise to the *inverse* matching function $y(\cdot) \equiv \left(h_1^*\right)^{-1}(\cdot)$ in closed form. As such, we can index household's policy functions through their *equilibrium* choice of housing.

This observation will suffice in allowing us to characterize how the pricing function will respond to certain kinds of *localized* shocks. Before doing so, I settle the question of equilibrium existence and uniqueness.

Proposition 3. *A stationary equilibrium exists and is unique.*

Proof. See Appendix A.4. □

4.2 Price Response to Localized Shocks

This section introduces an irrelevance result with respect to the effect of certain kinds of shocks on the pricing function. I consider an unanticipated, permanent change to the income distribution $F(\cdot)$ to $\tilde{F}(\cdot)$. I denote all equilibrium variables following the shock with a tilde.

Proposition 4. *Suppose $F(\cdot)$ and $\tilde{F}(\cdot)$ satisfy $F(y) = \tilde{F}(y)\ \forall\, y < \tilde{y}$, where $\tilde{y} \in [\underline{y}, \bar{y}]$. Then, $\tilde{q}(h) = q(h)\ \forall\, h \leq h(\tilde{y})$.*

Proof. See Appendix A.5. □

This proposition states that shocks that affect richer households will have no effect on house prices below that income group. Intuitively, this is because price changes only flow in the same direction in which *incentives* bind. Increasing the income of a household will increase the demand for housing services *above* its original housing tier, but will leave the prices below that tier unaffected. Thus, there is a distinct lack of a "trickle down" effect. House prices at low quality tiers are invariant to distributional changes in the income of households that match to *better* houses.

However, it is crucial to note that this reasoning does *not* generalize to the reverse case. Changing the income of poorer households – while leaving the income of richer households unaffected – will generally change the *entire* distribution of house prices. The following corollary illustrates this statement for a particular kind of shock: one which causes the income distribution of poor households to first-order stochastically dominate the initial distribution.

Corollary 1. *Suppose $\tilde{F}(y) \leq F(y)$ for all $y \in [\underline{y}, y_p]$ where $y_p \in [\underline{y}, \bar{y}]$ and where the inequality is strict for some open interval. Suppose further $\tilde{F}(y) = F(y)$ for all $y_p \in (y_p, \bar{y}]$. Then, $q(h) \leq \tilde{q}(h)$ for all $h \in [\underline{h}, \bar{h}]$. Moreover, $q(h) < \tilde{q}(h)$ for all $h \in (h_1^*(y_p), \bar{h}]$.*

Proof. See Appendix A.6. □

The above corollary demonstrates that increasing the income of households below a particular income quantile will increase the prices of *all* homes. At its core, this is a non-trivial implication of the positive assortative matching inherent within the competitive equilibrium. Intuitively, incentives for housing choice bind *upwards*. Absent any price changes, increasing the income of a particular household will induce it to demand better quality homes. However – due to positive assortative matching – richer households must still reside in the homes that now a less wealthy household is demanding. The only way these richer households can continue to reside in their original quality tiers is if they themselves bid more for housing. One therefore gets pricing spill-overs that occur from the bottom of the house quality distribution to the top.

Hence, the presence of market segmentation implies that changes in marginal rates of substitution can "trickle up" along the quality ladder, but shocks can never "trickle down". In this sense, shocks to poorer households are most important in shaping the cross-section of housing returns because they affect the house prices of all qualities *above* them through spill-overs in quality tiers.[6]

A variant of this "trickle up" effect is also present in Määttänen and Terviö (2013), who consider the effect of income inequality on house prices under no-trade equilibria, and in Terviö (2008), who considers the determinants of CEO pay.[7] These results stand in stark contrast to the predictions from models with a homogeneous housing stock, in which changes in the marginal rate of substitution of any given household will affect the house prices of all households in the economy. Moreover, this result is robust to any assumptions on the functional

[6]The preceding results generalize to the case in which we consider changes in the housing distribution $G(h)$. As long as as the housing distribution remains unchanged for all h below some threshold $\tilde{h}$, prices are guaranteed to remain constant for all $h < \tilde{h}$.

[7]In both these works, the magnitude of these spill-over effects is constant across the quality spectrum. In contrast, I demonstrate in the next section that spillovers in this model *dissipate* across quality spectrum and map the rate at which they dissipate to model primitives for a wide class of matching functions.

forms of the income and housing distribution, or the value of credit constraints and interest rates. The generalizability of this result begs for a data-driven test and motivates my empirical analysis in section 5.

4.3 Price Response to Aggregate Shocks

In this section, I consider how changes to certain *aggregate* shocks affect the cross-sectional distribution of capital gains. Because these changes typically affect the MRS of all households in the economy, we cannot use the results in the previous section to characterize the distribution of capital gains. However, with segmented housing markets, some shocks invariably affect some marginal rates of substitution more than others. Consequently, price changes will be more pronounced for quality tiers bought by households whose MRS change more. The distribution of capital gains thus crucially depends on the *type* of shock under consideration.

However, characterizing the distribution of capital gains for arbitrary functional forms is made challenging because the pricing function can be highly non-linear across quality tiers. This may in principle induce non-monotonicities in constraint status, where very wealthy household are credit constrained because the slope of the pricing function increases at a faster rate than their income.

The below proposition characterizes the pricing function in closed form for unconstrained households under the assumption of credit constraint monotonicity. Later in this section, I provide exact parametric restrictions under which credit constraint monotinicity holds. Nevertheless, this characterization permits us to see how the prices respond to various shocks above the credit constraint threshold.

Proposition 5. *Assume there exists a threshold $y^* \in (\underline{y}, \bar{y})$ such that households with $y > y^*$ are not credit constrained. Then, $q(h)$ satisfies the following equation for all $h > h(y^*)$:*

$$q(h) = \underbrace{\frac{\chi}{\omega(1+\beta)} \int_{h^*}^{h} \left(\frac{\tilde{h}^{\frac{\chi}{1+\beta}-1}}{h^{\frac{\chi}{1+\beta}}} \right) y(\tilde{h}) d\tilde{h}}_{(1)} + \underbrace{\left(\frac{y_1}{R}\right)\left(\frac{1}{\omega}\right)\left(1 - \left(\frac{h^*}{h}\right)^{\frac{\chi}{1+\beta}}\right)}_{(2)} + \underbrace{\left(\frac{h^*}{h}\right)^{\frac{\chi}{1+\beta}} q(h^*)}_{(3)} \quad (12)$$

where $\omega \equiv \left(1 - \frac{1-\delta}{R}\right)$ and $h^* \equiv h(y^*)$.

Proof. See Appendix A.7. □

This result establishes three robust features of the effect of changes in economic fundamentals on the cross-sectional distribution of capital gains. First, as seen through term (2), changes in *future* income y_1 has a relatively higher impact on the prices of *high*-end homes. This is because changes in future income is mostly payoff relevant to wealthier households that are not on their credit constraint. This affects the MRS of non-credit constrained households, and these spillovers accumulate upwards across the quality spectrum. Consequently, capital gains in respone to changes in future income are *increasing* in house quality.

Second, changess in credit conditions will mostly affect the prices of *low*-end homes. Note that θ does not directly appear in the above pricing equation because credit is pay-off irrelevant to unconstrained households. However, credit will still affect the prices of high-end homes through changes in $q(h^*)$ within term (3). Hence, the effects of credit will be more pronounced for low house qualities, but dissipate as one moves upwards the quality ladder. Moreover, these spill-overs dissipate at a rate that is proportional to $h^{-\frac{\chi}{1+\beta}}$. Intuitively, this is linked to the slackness in the housing market – when χ is high (or β is low), small changes in consumption have a large effect on the household's marginal rate of substitution. This makes it more difficult for households to accommodate pricing pressures from below.

Third, high-end homes are particularly sensitive to changes in interest rates or transaction costs. This is embodied through ω, which captures the proportional change in the *net present value of liabilities* of a household when purchasing a more expensive home. Whenever term (1) is increasing in house quality, changes in interest rates or transaction costs will have a *proportionately* larger increase for high-end homes.

In order to characterize these effects formally, I make the following three parametric assumptions:

Assumption 1. *$R\theta(1+\beta) > R + \delta - 1$, and y_1 and the ratio $\sqrt{\frac{b}{\underline{h}}}$ is large enough.*

Assumption 2. *The inverse matching function is linear. That is, $y(h) = kh$ for some $k > 0$.*

Assumption 3. *χ is not too large.*[8]

Assumption 1 ensures that there are a strictly positive mass of credit-constrained households in equilibrium by making housing sufficiently desirable relative to the outside option. Assumption 2 restricts the inverse matching function to be within a *linear class*. In other words, the ratio of income to house quality is constant for all households. This will permit to characterize the effect of various shocks in closed form both for capital gains (absolute price differences) and returns (percentage changes). Assumption 3 ensures that credit-constraint status is monotone in income. I now state the main result.

Proposition 6 (Comparative Statics)**.** *Suppose Assumptions 1 - 3 are satisfied. Then, there exists an $h^* \in (\underline{h}, \bar{h})$ such that households with $y > y(h^*)$ are not credit constrained (and all other home-*

[8] Exact parametric restrictions in terms of the model primitives can be found in the Appendix.

purchasing households are credit constrained). Moreover, the pricing function responds as follows to various permanent, unanticipated change to various parameters:

1. *Loan-to-value constraint* (θ) — $\frac{\partial q(h)}{\partial\theta\partial h} > 0$ *and* $\frac{\partial \log q(h)}{\partial\theta\partial h} > 0$ *for* $h > h(y^*)$*. Moreover,* $\lim_{h\to\infty} \frac{\partial \log q(h)}{\partial\theta} = 0$

2. *Interest rate* (R) — *There exists a threshold* y' *such that* $\frac{\partial q(h)}{\partial R\partial h} < 0$ *for* $h > h(y')$*. Moreover,* $\lim_{h\to\infty} \frac{\partial \log q(h)}{\partial R} = -\frac{1-\delta}{R(R+\delta-1)}$

3. *Transaction cost* (δ)— *There exists a threshold* y' *such that* $\frac{\partial q(h)}{\partial\delta\partial h} < 0$ *and* $\frac{\partial \log q(h)}{\partial\delta\partial h} < 0$ *for* $h > h(y')$*. Moreover,* $\lim_{h\to\infty} \frac{\partial \log q(h)}{\partial\delta} = -\frac{1}{R+\delta-1}$

4. *Future income* (y_1) — *There exists a threshold* y' *such that* $\frac{\partial q(h)}{\partial y_1\partial h} > 0$, $\frac{\partial \log q(h)}{\partial y_1\partial h} < 0$ *for* $h \geq h(y')$*. Moreover,* $\lim_{h\to\infty} \frac{\partial \log q(h)}{\partial y_1} = 0$

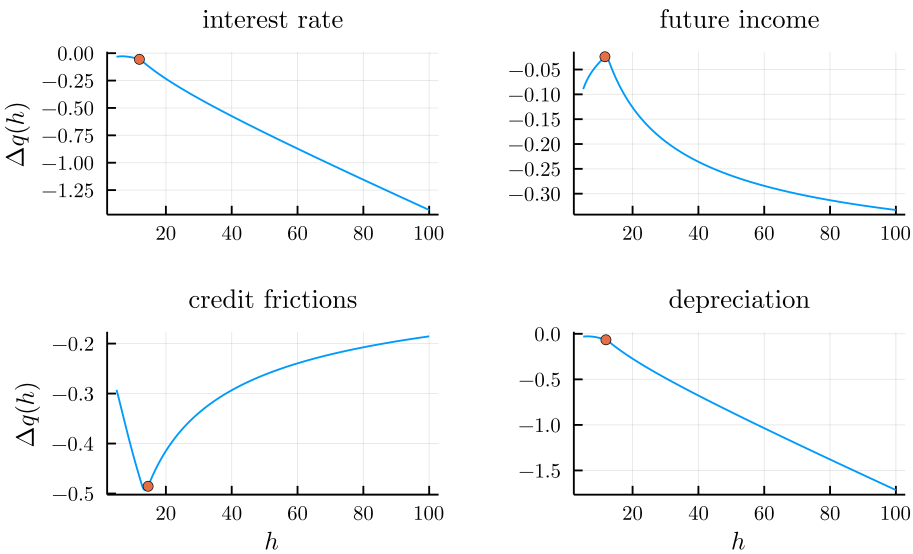
Proof. See Appendix A.8. □

This result formalizes the discussion above and establishes that shocks can have a highly non-linear effect on the pricing function. Moreover, the incidence of capital gains along the income distribution will crucially depend on the type of shock under consideration. Note that the result characterizes both *absolute* capital gains as well as *returns* (percentage changes) in response to different kinds of shocks. To make the proposition as transparent as possible, Figure 3 and 4 plot changes in the pricing function for a simple parameterization of the model.

The top-left panel in Figure 3 and Figure 4 plot the change in the pricing function (in absolute and percentage terms, respectively) due to a 10% increase in the net interest rate. An interest rate change has little effect on the prices of low-end homes that constrained households match to. This is because changes in the interest rate affects the net present value of liabilities, which is something that is mostly pay-off relevant to *unconstrained* households. These losses accumulate as one moves up the quality ladder due to spillovers. As such, capital losses are monotonic in quality – both in absolute and percentage terms. A similar reasoning applies for changes in transaction costs and future income changes. However, because changes in future income do not *proportionately* affect the valuation of a house, percentage returns dissipate as one moves up the quality ladder.

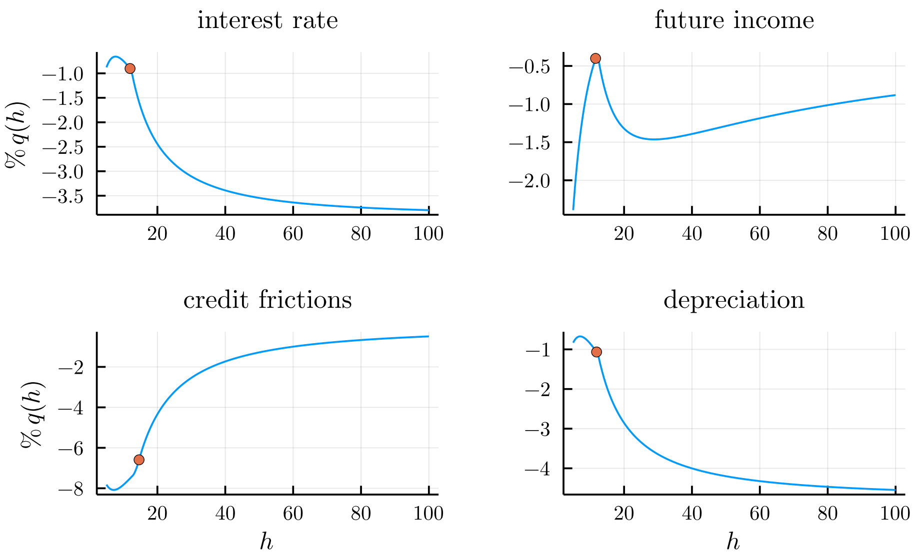
In contrast, a ten percent permanent increase in the loan-to-value ratio (bottom left) results in larger capital losses amongst low-end homes. Moreover, these capital losses are monotonic and decreasing for $h > h(y^*)$, as stated in the proposition. This is because constrained households are more sensitive to changes in the loan-to-value constraint relative to their wealthier counterparts. An increase in credit frictions magnifies the discrepancy between the marginal

Figure 3



Notes: absolute price differences in response to various shocks. Each subplot depicts the capital gains of various quality tiers in response to a 10% contractionary shock. Parameters are $(\beta, k, R, \theta, y_1, \underline{h}, b, \underline{y}, \delta, m) = (1, 1, 1.1, 0.2, 1.0, 5, 1, 0.65, 0.1, 2)$. The distribution for housing and income are Pareto with a common shape parameter of two.

Figure 4



Notes: capital gains (in percent) in response to various shocks. Each subplot depicts the capital gains of various quality tiers in response to a 10% contractionary shock. Parameters are $(k, R, \theta, y_2, \underline{h}, b, \underline{y}, \delta, m) = (1, 1.1, 0.2, 1.0, 5, 1, 0.65, 0.1, 2)$. The distribution for housing and income are Pareto with a common shape parameter of two.

valuation of consumption across periods for constrained households. As a result, low-end homes experience greater capital losses, both in absolute and relative terms.

Observe that returns converge to zero as quality increases, but prices still drop substantially for mid-range homes through spill-overs in quality tiers. Hence, market segmentation gives rise to a mechanism in which changes in credit can have a large affect on *average* house prices, even if relatively few households are credit constrained. This is contrast to the homogeneous housing stock case, in which case changes in credit can have a large effect on the per unit price of housing only if a large number of households are credit constrained (Greenwald and Guren, 2022). In this sense, market segmentation can amplify the effect of credit conditions on house price fluctuations.

The above analysis shows that different shocks can have widely different ramifications for the cross-sectional distribution of house prices, a result that is obscured when one only looks at returns in the aggregate. It also sheds light on exactly what kind of shocks are most important in driving capital gains for constrained households. These results might also be promising in identifying the key shocks that drive housing market returns within a given period by analyzing the distributional changes at the micro level. For example, there is a recent literature that scrutinizes whether the boom-bust cycle of the Great Recession was driven by credit conditions or beliefs (Cox and Ludvigson, 2019; Kaplan et al., 2020). If changes in credit conditions were the dominant force in the boom-bust cycle, we should expect low-end homes to have more volatile returns relative to mid-tier and high-end homes. In contrast, if changes in beliefs were the primary driver (as captured by changes in transaction costs), high-end homes should have experienced the greatest capital losses.

Example. I conclude this section by providing a simple, illustrative example which transparently illustrates the effects of various shocks on the entire pricing function. I assume the income distribution and the house quality distribution is Pareto with a common shape parameter $\alpha > 0$:

$$F(y) = 1 - \left(\frac{\underline{y}}{y}\right)^{\alpha}, \quad G(h) = 1 - \left(\frac{\underline{h}}{h}\right)^{\alpha} \tag{13}$$

and cutoffs $\underline{y}$ and $\underline{h}$, respectively. Assuming $m = 1$ (so that all households are home-owners) and using (10), we obtain the following inverse matching function in terms of house qualities:

$$y(h) = \frac{\underline{y}}{\underline{h}} h \tag{14}$$

for all $h \geq \underline{h}$, which is linear in housing quality. In order to obtain the constrained pricing function in closed form, I also assume that borrowing constraints are maximally tight $\theta \approx 1$, that housing depreciates fully $\delta = 1$ and that households are perfectly patient $\beta = 1$. Finally, I take $\underline{h} \to 0$ and $\underline{y} \to 0$ such that $\frac{\underline{y}}{\underline{h}} \to 1$,[9] This yields the following pricing ODE for constrained households:

$$q'(h) = \frac{\chi}{\theta} \frac{h - \theta q(h)}{h} \tag{15}$$

This is a separable ordinary differential equation, the solution of which is

$$q(h) = \frac{\chi}{(1+\chi)\theta} h \tag{16}$$

since the price of the house of lowest quality must be zero given that the poorest household has zero income and cannot borrow.

Hence, we can readily observe the impact of credit constraints along all households that will be credit constrained. In particular, in response to a temporary tightening in the borrowing constraint, the price change will be

$$\frac{d \ln q(h)}{d\theta} = -\frac{1}{\theta} < 0 \tag{17}$$

which is *independent* of the house quality h. Therefore, the model predicts that as long as households are credit constrained (and credit constraints are sufficiently tight), house price appreciation or depreciation in response to changes in credit conditions will be uniform across all such house qualities. What can be said about house qualities for which the matched households are not credit constrained? Letting h^*, y^* denote the lowest housing and income level above which households are not constrained,[10] one can show that the pricing ODE for an unconstrained household satisfies

$$q'(h) = \chi \frac{h + \frac{y_1}{R} - q(h)}{2h} \tag{18}$$

the uniqe solution of which is

$$q(h) = \frac{\chi}{2+\chi} h - \phi \left(\frac{1}{h}\right)^{\frac{\chi}{2}} \left(\frac{\theta y_1}{R}\right)^{\frac{\chi}{2}+1} + \frac{y_1}{R} \qquad \text{for} \qquad h > h^* \tag{19}$$

[9]Formally, we can consider sequence of economies in which cut-offs of the income and housing distributions converge to zero at the same rate. Taking the limits of the cut-offs to zero is necessary in order to prevent indeterminacy in equilibrium. Otherwise, multiple equilibria would arise due to the presence of discrete choice. This is common in assignment models with indivisible assets due to the generic non-uniqueness of the core. (Shapley and Shubik, 1972)

[10]Under the preceding assumptions, this is given by $y^* = \theta \frac{1+\chi}{R} y_1$.

where $\phi \equiv \frac{\chi(1+\chi)^{\frac{\chi}{2}+1}}{2+\chi}$

In the limit as $h \to \infty$, pricing for high quality houses becomes linear, but is non-linear for lower-tier houses. Moreover, we can transparently see how different shocks affect the cross-sectional distribution of capital gains. A tightening in the borrowing constraint will induce constant capital gains for all housing qualities up to h^*, and decreasing capital gains thereafter, exactly as predicted by Proposition 12.

Changes in y_1 will only affect house prices for households that can afford to tap into their future earnings. Indeed, an increase in y_1 leaves house prices for constrained households entirely unaffected, but raises house prices for higher quality tiers. Similarly, a decrease in the interest rate increases the present value of income, which boosts house prices for all qualities greather than h^*. However, it does not change the prices of low-end homes because the owners of these homes are credit constrained: marginal changes in the future income of these households does not affect their willingness to pay in their first period.

4.4 The Consumption Effect of Capital Gains

The analysis above illustrated that the response of house prices to certain shocks can be highly non-linear across housing quality tiers, and that the shape of capital gains across the house quality distribution crucially depends on the *type* of shock in question. This section illustrates that this heterogeneous incidence can also be an important driver of other macroeconomic aggregates, such as aggregate consumption expenditures. This is because the incidence of these capital gains does not fall randomly upon the population. Rather, larger capital losses can be experienced precisely by those households most important in driving aggregate consumption expenditures. In particular, the effect of house price changes on consumption will depend on how capital gains covary with a “distorted" marginal propensity to consume (MPC): a weighted difference in the MPC of young households, keeping their housing choice fixed, and the MPC of old households.

Concretely, let $\widetilde{MPC}_0(h)$ denote the additional consumption of a young household due to an incremental income increase, holding his housing policy choice fixed at h. Moreover, let $MPC_1(h)$ denote the marginal propensity to consume of an old household that has purchased a house quality of h. The next proposition characterizes the effect of house price changes on aggregate consumption expenditures.

Proposition 7. *Suppose the economy is in a stationary equilibrium at time t and consider an unanticipated change to $x \in \{\theta, R, \delta, y_1\}$. Then, the effects of house price changes on aggregate consumption*

dC_t is (to first-order):

$$dC_t = \underbrace{\mathbb{E}[\overline{MPC}(h)]\mathbb{E}[dq_t(h)]}_{\textit{homogeneous response}} + \underbrace{cov(\overline{MPC}(h), dq_t(h))}_{\textit{matching "multiplier"}} \tag{20}$$

where $\overline{MPC}(h) = (1-\delta)MPC_1(h) - \widetilde{MPC_0}(h)$ and the expectation is over house qualities.

Proof. See Appendix A.9. □

The key insight in proving this result is to notice that changes in house prices constitute a pure *negative* income effect to young, home-purchasing households, and a *positive* wealth effect to old, home-owning households. Hence, the aggregate effect on consumption expenditures depends on the relative MPCs of home-owners (adjusted for depreciation) to young purchasers, holding their housing policy function fixed. Intuitively, the presence of housing indivisibility and positive assortative matching implies that any given change in house prices will not affect the relative quality tier that a household will target in equilibrium. Hence, we can abstract from *substitution* effects that arise due to house price changes. This is in contrast to models with a homogeneous housing stock, in which changes in house prices create substitution, income, and collateral effects on household policy functions, thereby making such a simple characterization intractable (Berger et al., 2018).

This proposition also illustrates that – conditional on the *average* fall in house prices in an economy – the *covariance* of MPCs with capital gains can be an important force in shaping aggregate consumption expenditures. As we have seen so far, the incidence of these capital gains can be highly non-linear and crucially depend on the underlying shock. Modelling house prices in general equilibrium is therefore important in order to discipline the incidence of capital gains. This is once again in contrast to models with a homogeneous housing stock, in which *any* kind of shock generally creates a negative covariance of capital gains with MPCs.[11]

However, the stylized model presented above implies that the MPCs for old households – for whom house price movements constitute pure wealth effects – are constant along the income (and wealth) distribution. In order to introduce MPC heterogeneity in a tractable way, I extend the model to allow for a *warm-glow* bequest motive as in De Nardi (2004). For simplicity,

[11] To see this, consider a similar model in which households can purchase housing at a common per unit price q. Any changes in q will generate constant returns to the housing stock values across the entire wealth distribution. However, poorer households, which have higher marginal propensities to consume, will also purchase a lower stock of housing. Consequently, the incidence of capital gains will fall mainly upon wealthier households with low MPCs. In models with homogeneous housing, these mechanics must artificially generate a form of *dampening*:

$$cov(MPC(h), dq(h)) = cov(MPC(h), hdq) < 0, \quad \forall h \tag{21}$$

I assume that bequests leave the economy.[12] I assume that payoffs take the form:

$$\mathcal{U}\left(\{c_{t+j,j}\}_{j=0}^{1}, v_{t+1,1}, h_{t,1}\right) = \log(c_{t,1}) + \beta\left(\log(c_{t+1,2}) + \log(v_{t+1,1} + \phi)\right) + \chi\log(h_{t,1}) \tag{22}$$

where $v_{t+1,1}$ denotes the choice of bequests at old age $\phi > 0$ parameterizes the extent to which bequests are a *luxury* good. The appendix demonstrates that all previous results regarding the incidence of capital gains are valid for this extended utility function as well. To see how this creates heterogeneity in the marginal propensity to consume out of house price changes, consider a sudden, proportional decrease in house prices at time t by a factor $d\lambda$. The consumption response of old households to this wealth effect is

$$dc_{t,1}(h) = \begin{cases} -(1-\delta)q_t(h)d\lambda & \text{if} \quad v_{t,1}(h) = 0 \\ -\frac{1}{2}(1-\delta)q_t(h)d\lambda & \text{if} \quad v_{t,1}(h) > 0 \end{cases} \tag{23}$$

Note that $v_{t,1} > 0$ if and only if a household's income is greater than some threshold y^{**}. This capture the empirically relevant feature that MPCs are declining in wealth (Kaplan and Violante, 2014).[13]

The next corollary illustrates that the homogeneous response and matching multiplier term in (20) work in the same direction in response to a credit supply shock, thereby giving rise to an amplification channel for the response of aggregate consumption expenditures to house price changes.

Corollary 2. *Suppose* $1 - \delta > \frac{2}{1+2\beta}$[14] *and Assumptions 1-3 hold. Consider a permanent change to* θ *and suppose* ϕ *is large enough. Then,*

$$cov(MPC(h), dq(h)) > 0, \quad for \quad h > h(y^*) \tag{24}$$

Proof. See Appendix A.10. □

Credit shocks therefore result in larger capital losses for precisely those households that are most important in driving consumption expenditures. In contrast, other shocks, such as changes in interest rates or depreciation, induce capital losses that is mostly incident upon low MPC

[12] Allowing bequests to be passed on the next generation's young is inessential to the main results.

[13] The effect of house price changes on the consumption expenditures of non-home-owning households is zero. Home-owning households with low MPCs can therefore be viewed as wealthy hand-to-mouth (Kaplan and Violante, 2014) due to their low *liquid* wealth.

[14] This assumption is needed to ensure the marginal propensity to consume out of housing wealth is larger for old households than for young households, so that unanticipated house price drops result in an initial reduction in consumption. Christelis et al. (2021) provide evidence that this indeed the case.

households. Hence, *conditional* on the same *average* decline in house prices, credit shocks induce a larger aggregate consumption response through changes in house prices. This suggests that the underlying shock affecting house prices can give rise to a “matching multiplier", an amplification channel that is a product of the model’s equilibrium sorting.[15]

5 Testing the Theory

The presence of market segmentation implies that shocks propagate within the housing market *unidirectionally*. Changes in the marginal rates of substitution of relatively wealthy households will affect the prices of the high-end homes these households match to, with no effect on the prices of lower quality tiers – a lack of a “trickle-down" effect. However, changes in the marginal rates of substitution of relatively poor households will have price effects on *all* homes through spill-overs in quality tiers – a “trickle up" effect.

This section provides empirical support for both of these predictions. I first begin by directly testing for the first mechanism (the lack of a *trickle-down* effect) through plausibly exogenous variation to stock market wealth changes. Low-income households tend to hold little to no stock market wealth (Kaplan and Violante, 2014), while stock market wealth varies widely both across and within counties. I follow Chodorow-Reich et al. (2020) in constructing a shift-share design in which identification is derived from the differential effects of stock-market movements across regions. The theory suggests that the identified stock-market movements should have a larger effect on high-end homes, since the buyers of these homes are precisely the ones most exposed to wealth effects from stock market fluctuations. Moreover, we should expect an economically insignificant effect on low-end homes, as the corresponding buyers for these homes have little stock-market wealth and the theory predicts that shocks cannot trickle down the quality ladder.

I then provide evidence for the second mechanism (the *trickle-up* effect) through an episode that provides for a plausibly exogenous localized shock to *low-income* households: the historic increase in private label securitization (PLS) in the mortgage market from 2002 to 2005 (Justiniano et al., 2017; Mian and Sufi, 2022). I show that the mortgage growth that originated from increased private label securitization mostly mostly targeted subprime, low-income households. As such, we should expect low-end homes to appreciate relatively more than high-end homes. However, the response of prices due to an increase in subprime credit spills over to adjacent housing tiers, even if the households that reside in these tiers are not the ultimate

[15] The notion of a “matching multiplier" has also been explored in the context of income changes to the aggregate economy, as in Guvenen et al. (2017), or Patterson (2020).

beneficiaries of sub-prime credit. Hence, mid-end and high-end homes should appreciate as well.

5.1 The stock-market wealth shock

The test for the absence of a trickle down effect exploits the regional heterogeneity in stock market wealth using a Bartik-style shift-share instrument, as in Chodorow-Reich et al. (2020). Intuitively, we may obtain exogenous variation in stock market wealth by taking advantage of the fact that changes in aggregate stock prices (the shifter) should affect housing markets with different levels of stock market wealth (the shares) differentially. I begin by describing the data and methodology in more detail, provide evidence that the shock is sufficiently “targetted" to high-income households residing in high-end zip codes, and then present the main results.

Data. My aim is to explore the effect of changes in stock market wealth in a given housing market on the cross-sectional distribution of house prices across quality tiers. In order to do so, I need data on house prices, stock market wealth, and stock market returns. Furthermore, the cross-sectional data needs to be at a sufficiently granular level so I can provide evidence that the shock is primarily localized within high-end homes. I continue using Zillow zip-code level price indices for single family homes as my measure of house prices, and define each housing market as a county as in section 2.2.[16] In the model, the price ranking of a house is a sufficient statistic for its relative quality in a given housing market. For this reason, I continue to sort zip codes into quality tiers according to their relative price ranking within a county.

Next, I describe my construction of local stock market wealth, which closely follows that of Chodorow-Reich, Nenov, and Simsek (2020). In particular, I use publicly available IRS Statistics of Income (SOI) data, which contains zip code aggregates of annual dividend income reported on individual tax returns over the period 2005-2019. I then convert dividend income to stock market wealth by multiplying dividends by the price to earnings ratio of the S&P 500 in the relevant year. I am then able to compute variations in stock market wealth across ZIP codes using this measure in conjunction with the monthly total return on the S&P 500 (which I use as the shifter). I interpolate total stock market wealth at the monthly level in order to conduct my analysis at the monthly frequency. In practice, this introduces little measurement error because the time series for total stock market wealth is very persistent. IRS SOI data also reports the number of tax returns by ZIP code, which I use in order to construct stock market wealth measures on a per capita basis.

[16] The results are robust to defining a housing market as an MSA.

Methodology. I denote my main regressor as $Shock_{c,t} \equiv W_{c,t-1} \times R_{c,t-1}$, where $W_{c,t-1}$ is stock market wealth per capita in period $t-1$ in county c and $R_{c,t-1}$ is the return on the S&P 500 between $t-1$ and t. Motivated by the theoretical results presented above, I assume the following specification:

$$\triangle_h p_{z,c,t} = \sum_{q=1}^{4} \left[\beta_{q,h} \times \mathbb{1}\{z \in q\} \times Shock_{c,t}\right] + \Gamma_h' X_{c,z,t} + \varepsilon_{z,c,t,h} \tag{25}$$

where $\triangle_h p_{z,c,t}$ is the absolute change in prices for a particular zip code z in county c between $t+h$ and $t-1$, $\mathbb{1}\{\cdot\}$ is an indicator for whether a particular zip code belongs to a housing quartile q, $X_{c,z,t}$ is a vector of zip-county-time specific controls, and $\varepsilon_{z,c,t,h}$ is an idiosyncratic error component.

The coefficients $\{\beta_q\}_{q=1}^4$ capture the heterogeneous response of house prices in various quality quarties to changes in total stock market wealth per capita in a given housing market. Below, I provide evidence that stock-market wealth is mostly held by households in high-end homes ($q = 4$). Consequently, the theory predicts that house prices in high-end quartiles should appreciate ($\beta_4 > 0$), but the price response should be muted effect on all lower quality quartiles ($\beta_{q\neq 4} \approx 0$) since prices cannot spill-over downwards.

The key assumption for identification in this framework is that high and low wealth counties are not heterogeneously affected by other aggregate variables that co-move with stock market returns, conditional on the controls. This condition mirrors the parallel trends assumption in a continuous difference-in-difference design with multiple treatments (Goldsmith-Pinkham et al. 2018). Chodorow-Reich et al. (2020) argue extensively that this assumption is satisfied in their specification that focuses on the response of labor market variables to changes in stock market wealth. Below, I provide further evidence for exogeneity of the shock by illustrating the absence of pre-trends prior to the treatment.

Results. I first begin by providing evidence that the shock is sufficiently targeted to high-end zip codes to make it a genuine high-end shock. Table 2 shows the average value of the shock over 2005-2019 for different quality quartiles.

Table 2 demonstrates that most of the treatment falls mostly upon high-end zip codes. In particular, households that reside in zip codes within the highest quality quartile hold approximately seven times as much as stock market wealth, on average, than households residing in zip codes in the bottom quartile. The heterogeneous incidence of the shock on zip codes *within* a county makes this specification particularly well-suited to test for trickle-down effects. If price are to spill-over downwards in general equilibrium, we should see significant effects for

Table 2

quartile	average $\lvert Shock_{z,t} \rvert$ (per capita)
1	106.2
2	166.2
3	261.3
4	716.6

Notes: Average (absolute value of the) stock market wealth shock across quality quartiles over 2005-2019. For example, a number of 106.2 for quartile 1 would imply that low-end ZIP codes experienced an average monthly return of 106.2 dollars per month over the period 2005-2019 on a per capita basis.

houses in the bottom three quality quartiles.

Table 3 shows the results of the regression (25) with the inclusion of various controls. The table reports the coefficients for $\{\beta_q\}_{q=1}^4$ for $h = 12$, but I show that the results are robust to different time horizons by plotting local projections following Jorda (2005). In my preferred specification (final column), I non-parametrically control for time-invariant aggregate shocks that covary with stock market returns *and* the share of wealth within a given county by including county fixed effects. I also control for unobservable variables that might heterogeneously affect different quartiles depending on their initial level of stock market wealth by including quartile fixed effects. Furthermore, I include time fixed effects and clean up any residual correlation in the stock market wealth shock by including 12 lags of $S_{c,t}$. I also exploit only within state variation by including state$\times$time fixed effects. Finally, I double cluster all standard errors at the county and time. Clustering by time soaks up any remaining serial correlation on stock market returns, while clustering by county allows households in the same county to experience common shocks. The final sample consists of 2397225 time-ZIP code observation pairs.

The impact of a one dollar stock wealth increase per capita for a given county increases the prices of high-end homes by 1.27 dollars over a six-month horizon.[17] Moreover, the effect of a stock wealth increase is statistically indistinguishable from zero for the bottom two quartiles, and is small (but in line with the direct effect of the shock) for the third quality quartile. This result is remarkably consistent with the theory and demonstrates that the asset price response to wealth changes of richer households is entirely localized within their own quality quartile.

[17]This does not imply a marginal propensity to consume that is greater than one for non-durable consumption because – as shown in Table 2 – the incidence of the treatment is unequally distributed across quality quartiles.

Table 3

	(1)	(2)	(3)	(4)
	$\triangle_{12} p_{z,c,t}$	$\triangle_{12} p_{z,c,t}$	$\triangle_{12} p_{z,c,t}$	$\triangle_{12} p_{z,c,t}$
β_1	0.453	0.214	0.357	0.0542
	(0.113)	(0.446)	(0.183)	(0.744)
β_2	0.441	0.339	0.401	0.206
	(0.065)	(0.143)	(0.081)	(0.206)
β_3	0.661*	0.591*	0.590*	0.377*
	(0.018)	(0.022)	(0.021)	(0.028)
β_4	1.672**	1.614**	1.456**	1.274**
	(0.001)	(0.001)	(0.003)	(0.004)
lagged returns	✓	✓	✓	✓
time FE	✓	✓	✓	
county FE		✓	✓	✓
quartile FE			✓	✓
state×time FE				✓
N	2397225	2397225	2397225	2397225

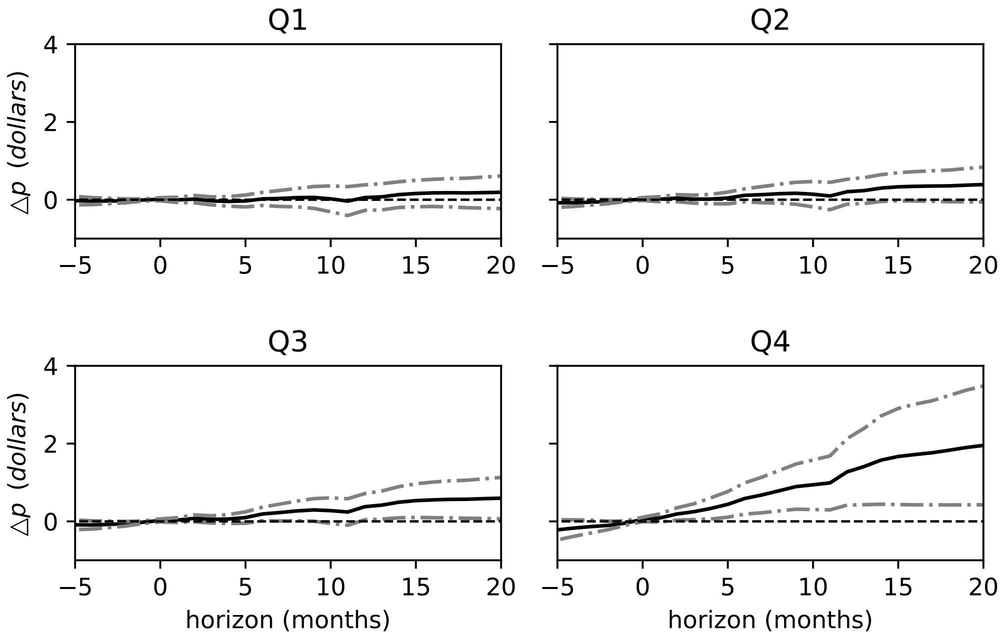
Notes: The table reports alternative specifications for the baseline regression. The dependent variable is the six-month change in the house price level for a given zip code. Robust t-statistics are in parenthesis and are double-clustered by county and time. $*$ denotes significance at the 5% level, and $**$ at the 1% level.

In order to show that the results are robust to different time horizons, Figure 5 plots the local projections of (25) over different time horizons. The shock occurs at period -1 and is equivalent to a one dollar per capita increase in the stock market wealth for a given county. The only pronounced increase in house prices occurs in the highest quality quartile, exactly as predicted by the theory. Note that the increase appears persistent – this is consistent with the interpretation that the stock market is perceived to be close to a random walk and mirrors the results of Chodorow-Reich et al. (2020). The price effects for lower quality quartiles are small and statistically indistinguishable from zero at long time horizons. This suggests that any additional spill-overs likely to occur through the general equilibrium increase in high-end homes (e.g. through standard Keynesian reasoning) is not sufficient to create a meaningful impact on lower tier homes. Consequently, the results point to the far-reaching robustness of the model's mechanism.

5.2 Trickle up: The sub-prime credit shock

I next turn my attention to the second test, the trickle-up effect from lower to higher quality tiers. In order to test for the trickle-up mechanism, I isolate a subprime credit shock that was

Figure 5



Notes: Local projections to column 4 of (25). The grey dotted lines depict 95% confidence intervals, based on standard errors clustered two-way by county and time. The shock occurs at $h = -1$.

mostly incident on low-income households residing in low-end ZIP codes. In particular, I exploit the differential city-level exposure to the 2003 expansion in private label securitization (PLS). Existing research has documented that there was a sudden, pronounced increase in the PLS market in the summer of 2003 (Justiniano et al. 2017). I follow Mian and Sufi (2022) in hypothesizing that banks that relied more heavily on non-core-deposit financing were able to expand their mortgage lending more aggressively during this time period. Mian and Sufi (2022) argue that exposure to high non-core-liability (NCL) lenders provides a plausibly exogenous source of variation in mortgage credit supply across housing markects, and show that there were no differential pre-trends in credit in more exposed markets.

I document that counties that were more exposed to high NCL lenders indeed experienced greater mortgage growth, but that the increase was mostly concentrated in *low-end* ZIP codes. As a result, this natural experiment provides an ideal setting to test the extent to which there are spill-overs across quality tiers. As before, I present the data, methodology and then the main results.

Data. Testing the trickle-up mechanism requires data on house prices, mortgage origination growth, and the liabilities of issuing institutions. For house price data, I continue using the publicly available ZIP code level Zillow house price data as before. I collect data on the flow of new mortgage loans originated in the years 2000-2010 through the "Home Mortgage Disclosure Act" (HMDA) data set. HMDA records various characteristics of each mortgage and applicant at the loan application level. I collect information on the applicant's total loan amount, purpose of borrowing (refinancing/home purchase/home improvement), income, and whether or not (and to which kind of agency) the loan was sold to in the secondary market within that given year. Each institution in the HMDA dataset is given a unique HMDA identifier. I obtain the non-core liabilities of each lender by linking these financial institutions to Call Report data using a key provided by the Federal Reserve Board.

I continue to define low quality tiers as ZIP codes that fall within the lowest price quartile within a county for a given base year. I use 2002 as the base year as it precedes the surge in the PLS market, but the choice of a base year is immaterial given that the ranking of ZIP codes across years is persistent. I drop counties for which we have fewer than four ZIP code-observations, leading to a final sample of 9135 ZIP codes and 136 counties. Finally, I only consider mortgages originated for the purpose of purchasing a house.

Methodology. I first need to obtain a measure of the county-level exposure to high-NCL lenders. To do so, I follow Mian and Sufi (2022) in constructing an average of the 2002 NCL ratios of mortgage lenders in the area, where the average is weighted by a lender's amount of mortgage originations in 2002. Concretely,

$$NCLShare_{c,2002} = \sum_b \omega_{c,b,2002} \times NCL_{b,2002} \tag{26}$$

and where

$$\omega_{c,b,2002} = \frac{Originations_{c,b,2002}}{\sum_b Originations_{c,b,2002}} \tag{27}$$

where $NCL_{b,2002}$ denotes the non-core liabilities of bank b in 2002, and $Originations_{g,b,2002}$ denotes the amount of mortgage originations from bank b to county c in 2002. Mian and Sufi (2022) argue that this geographic measure of NCL exposure is a valid instrument for credit.

In order to analyze the impact of this subprime loan expansion on house prices, I follow Mian and Sufi (2022) and Greenwald and Guren (2022) in regressing the percentage change in house prices directly on the 2002 NCL ratio. This is sufficient to provide evidence for the trickle-up mechanism, although it implies that we cannot interpret the coefficients in terms of units of credit. Appendix C shows that the trickle-up mechanisms is robust to a classical 2SLS procedure, although the coefficients are estimated with greater noise. I consider the following specification, which closely mirrors the one used for the stock-market wealth shock:

Table 4

	Growth 02-05
γ_1	7.303**
	(0.016)
γ_2	5.868***
	(0.003)
γ_3	2.391
	(0.108)
γ_4	0.378
	(0.822)
Quartile FE	✓
N	9453

Notes: The table reports the coefficients from Equation (29) for the percentage growth in originations over 2002-2005 (column 1) and 2000-2002 (column 2). All standard errors are clustered at the county-level and robust p-values are in parentheses. $*$ denotes significance at the 5% level, $**$ at the 1% level, and $***$ at the 0.1% level.

$$\triangle_{2005,2002} Price_{z,c} = \sum_{q=1}^{4} \left[\beta_q \times \mathbb{1}\{z \in q\} \times NCLShare_{c,2002} \right] + \Gamma' X_{z,c} + \varepsilon_{z,c} \tag{28}$$

where $\triangle_{2005,2002} Price_{z,c}$ is the log-change in house prices in zip code z and county c over 2002-2005 and $\varepsilon_{z,c}$ reflects unmodelled determinants of house price growth. The identifying assumption is that counties did not experience shocks post-treatment that were correlated with their level of the NCL share as of 2002. Mian and Sufi (2022) argue extensively that this orthogonality assumption is satisfied, and I provide further evidence for this below by showing the lack of pre-trends prior to treatment.

Results. I first show that the additional supply in credit that is induced by being in a high NCL county is mostly incident on low-end ZIP codes within a county. This is necessary in order to ensure that price variation that is attributed to the "trickle-up" effect that occurs in general equilibrium is not confounded with the direct effect of providing more credit to high-end ZIP codes. To this end, I run the following regression:

$$\triangle_{2005,2002} Originations_{z,c} = \sum_{q=1}^{4} \left[\gamma_q \times \mathbb{1}\{z \in q\} \times NCLShare_{c,2002} \right] + \Gamma' X_{z,c} + \varepsilon_{z,c} \tag{29}$$

The coefficients γ_q, depicted in Table 4, capture the expected increase in mortgage origination that occurs in a quality tier q when the NCL exposure of a given housing market increases from

Table 5

	(1)
	$\triangle_{2005,2002} Price$
β_1	2.911^{**}
	(0.003)
β_2	3.349^{***}
	(0.000)
β_3	3.023^{***}
	(0.001)
β_4	2.298^{**}
	(0.04)
Quartile FE	✓
N	9135

Notes: The table reports the coefficients from Equation (28). The dependent variable is the percentage change in the house price in a given zip code. Robust p-values are in parentheses. $*$ denotes signifiance at the 5% level, $**$ at the 1% level, and $***$ at the 0.1% level.

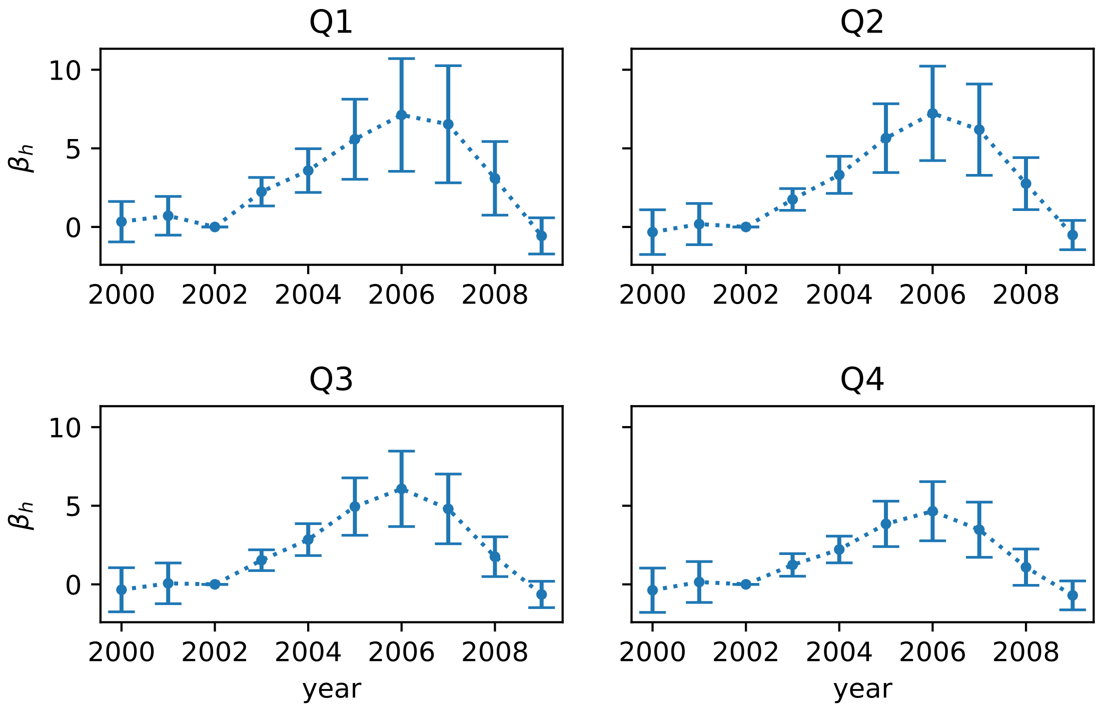
zero to one.

Table 4 shows that the treatment due to changes in the NCL share is almost entirely localized in low-end ZIP codes. An increase in the NCL share from zero to one results in an sevenfold increase in the amount of mortgages that are originated in the lowest quality quartile, but an imprecisely estimated 0.3-fold increase for the highest quality quartile. This is consistent with the notion that the vast majority of privately securitized loans were originated to buyers in low-end, and therefore low-income and low-credit score neighborhoods.[18]

Having shown that the treatment is sufficiently targetted to low-end zip codes and provided evidence for its exogeneity, I next turn my attention to its effect on house prices across the quality spectrum. Table 5 depicts the coefficients β_q from this regression. Changing the county-wide NCL ratio from zero to one increases the prices of homes in the bottom two quartiles of zip codes by an additional 290% and 335%, respectively. Moreover, there are significant trickle-up effects: a unit increase in a county's NCL ratio is associated with a 230% increase in house prices for the highest quality quartile. This is in spite of the relatively few (albeit imprecisely estimated) originations to these high-end homes. Moreover, the results are consistent with the theory in that $\beta_1 > \beta_4 > 0$. Low quality homes do appreciate more in response to

[18]Consistent with this analysis, Mian and Sufi (2022) provide evidence that high NCL expsoure is correlated with lower income, low credit scores, and a younger populations using data from TransUnion.

Figure 6



Notes: The figure reports the coeffients β_h for each quality tier from Equation (30). The figure shows 95% confidence bands, based on standard errors clustered at the county-level.

a credit expansion than high-end homes, but high-end homes appreciate through spill-overs that dissipate across the quality ladder.

The main threat to a causal interpretation of these findings is that counties with a high NCL ratio are also impacted by other covariates that affect house prices and co-move with local mortgage growth. In order to provide further evidence for the exogeneity of the NCL-based credit expansion, I also follow an event-study approach and run the following regression separately for each quality quartile:

$$\triangle_{t,2002}Prices_{z,c,t} = \xi_c + \psi_t + \sum_{h \neq 2002} \left[\beta_{q,h} \times \mathbb{1}_{h=2002} \times NCLShare_{2002}\right] + \varepsilon_{z,c,t} \tag{30}$$

where $\triangle_{t,2002}Prices_{z,c,t}$ denotes the percentage change in house prices that occured between t and 2002 in zip code z and county c and ξ_c and ψ_t denote county and year fixed effects, respectively. The coefficients $\{\beta_h\}$ give the relative growth in prices that is attributed to being in a high NCL county in 2002 (which is the omitted year).

Figure 6 plots the estimated coefficients $\{\beta_{q,h}\}$ for each quality quartile q. The estimated coefficients show that there was no pre-trend and a sharp rise in house prices for high NCL areas starting in 2003. The rise is most pronounced for houses in the lowest quality quartile,

Table 6

	(1)
	$\triangle_{2005,2002} Income$
β_1	-0.073**
	(0.008)
β_2	-0.066
	(0.076)
β_3	-0.093*
	(0.032)
β_4	0.035
	(0.120)
Quartile FE	✓
N	9135

Notes: The table reports the coefficients from Equation (28). The dependent variable is the percentage change in annual gross adjusted income in a given zip code from 2002-2005. Robust p-values are in parentheses. $*$ denotes signifiance at the 5% level, $**$ at the 1% level, and $***$ at the 0.1% level.

but we also see an economically significant increase in the prices of high-end homes. At its peak, the coefficient for zip codes in the highest quality quartile was around sixty percent (4.6) the magnitude of the corresponding coefficient for low-end homes (7.1). This analysis suggests that trickle-up forces that occur in general equilibrium can have a meaningful effect on house prices across the quality distribution and can amplify the response of asset prices to shocks that affect low-income segments of the population.

It is important to note that these regressions do not control for other general equilibrium forces that might further increase the price of high-end homes. For example, high NCL counties may experience greater income growth due to their increased credit exposure through changes in aggregate demand. This would increase house prices amongst all quality quartiles, thereby biasing the coefficients in (28) upwards. This "contamination" of the partial equilibrium effect with local general equilibrium effects is a general feature of empirical designs that use variation across regions for identification (Guren et al. 2020).

In order to address these concerns, I use IRS zip code level SOI data to see if high NCL areas also experienced higher income growth relative to NCL areas. Table 6 reports the coefficients from regression (28) using income as the dependent variable. The effect of NCL exposure on income growth is low for all quality quartiles and statistically indistinguishable from zero for the highest quality quartile. In all cases, the standard errors are small. These results suggest that it is the direct, partial equilibrium, effect of increased credit that is responsible for house price growth as opposed to indirect effects that arise through Keynesian channels. Appendix C shows that these results are also robust to longer time horizons by employing a similar event-study regression as in (30).

6 Concluding Remarks

This paper has developed an assignment model in which housing is segmented by various quality tiers. In doing so, I have characterized how the entire cross-section of house prices responds to various changes in the economic environment and demonstrated the consequences of capital gain heterogeneity for shock amplification and wealth inequality. The key to understanding the asymmetric effects of macroeconomic fluctuations on house prices lies in the heterogeneous impact of these shocks on the marginal rates of substitution of different households. Moreover, the positive assortative matching between housing quality and income in the equilibrium assignment implies that price changes flow in the same direction that incentives bind: trickle up effects can substantially affect average house prices in the economy, but price changes cannot trickle down.

I have highlighted, both theoretically and empirically, that the cross-sectional response of house prices is enormously heterogeneous and depends on the nature of the shock that hits the economy. As such, the effect of house prices on consumption cannot be reduced to a single sufficient statistic, as is often done in the literature. Market segmentation gives rise to an amplification channel for various aggregate shocks to operate through the housing market. In contrast, the homogeneous housing stock benchmark always generates dampening.

The data lends support to the theoretical predictions of the model. Trickle-up effects due to low-end shocks (proxied by subprime mortgage growth) have substantial, monotonic effects on house prices in higher quality tiers. High-end shocks (proxied by stock market wealth changes) only affect high-end tiers – a one dollar per capita increase in stock market wealth in a given housing market increases the prices of high-end homes by 1.3 dollars over a twelve-month horizon. In contrast, trickle-down effects are both statistically and economically insignificant.

Although the homogeneous housing stock framework has dominated macroeconomic modelling of the housing market, this paper argues that treating housing as a heterogeneous, private-value asset is indispensable in obtaining a complete view of housing within the macroeconomy. A more serious treatment of the price formation – and evolution – of housing in the presence of market segmentation is bound to be a fruitful avenue for future research.

References

Beraja, M., Fuster, A., Hurst, E., and Vavra, J. (2019). Regional heterogeneity and the refinancing channel of monetary policy. *The Quarterly Journal of Economics*, 134(1):109–183.

Berger, D., Guerrieri, V., Lorenzoni, G., and Vavra, J. (2018). House prices and consumer spending. *The Review of Economic Studies*, 85(3):1502–1542.

Chodorow-Reich, G., Nenov, P. T., and Simsek, A. (2021). Stock market wealth and the real economy: A local labor market approach. *American Economic Review*, 111(5):1613–57.

Christelis, D., Georgarakos, D., Jappelli, T., Pistaferri, L., and Van Rooij, M. (2021). Heterogeneous wealth effects. *European Economic Review*, 137:103805.

Cox, J. and Ludvigson, S. C. (2021). Drivers of the great housing boom-bust: Credit conditions, beliefs, or both? *Real Estate Economics*, 49(3):843–875.

De Nardi, M. (2004). Wealth inequality and intergenerational links. *The Review of Economic Studies*, 71(3):743–768.

Demyanyk, Y. and Van Hemert, O. (2011). Understanding the subprime mortgage crisis. *The review of financial studies*, 24(6):1848–1880.

Favilukis, J., Ludvigson, S. C., and Van Nieuwerburgh, S. (2017). The macroeconomic effects of housing wealth, housing finance, and limited risk sharing in general equilibrium. *Journal of Political Economy*, 125(1):140–223.

Fuster, A. and Zafar, B. (2021). The sensitivity of housing demand to financing conditions: evidence from a survey. *American Economic Journal: Economic Policy*, 13(1):231–65.

Glaeser, E. and Gyourko, J. (2018). The economic implications of housing supply. *Journal of economic perspectives*, 32(1):3–30.

Greenwald, D., Guren, A., et al. (2019). Do credit conditions move house prices? *Unpublished manuscript, MIT*, page 3.

Guerrieri, V., Hartley, D., and Hurst, E. (2013). Endogenous gentrification and housing price dynamics. *Journal of Public Economics*, 100:45–60.

Guren, A. M., McKay, A., Nakamura, E., and Steinsson, J. (2021). Housing wealth effects: The long view. *The Review of Economic Studies*, 88(2):669–707.

Guvenen, F., Schulhofer-Wohl, S., Song, J., and Yogo, M. (2017). Worker betas: Five facts about systematic earnings risk. *American Economic Review*, 107(5):398–403.

Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American economic review*, 95(3):739–764.

Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2019). Credit supply and the housing boom. *Journal of Political Economy*, 127(3):1317–1350.

Kaplan, G., Mitman, K., and Violante, G. L. (2020). The housing boom and bust: Model meets evidence. *Journal of Political Economy*, 128(9):3285–3345.

Kaplan, G. and Violante, G. L. (2014). A model of the consumption response to fiscal stimulus payments. *Econometrica*, 82(4):1199–1239.

Kiyotaki, N., Michaelides, A., and Nikolov, K. (2011). Winners and losers in housing markets. *Journal of Money, Credit and Banking*, 43(2-3):255–296.

Kiyotaki, N. and Moore, J. (1997). Credit cycles. *Journal of political economy*, 105(2):211–248.

Landvoigt, T., Piazzesi, M., and Schneider, M. (2015). The housing market (s) of san diego. *American Economic Review*, 105(4):1371–1407.

Lee, D., Mayer, C., and Tracy, J. (2013). *5. A New Look at Second Liens*. University of Chicago Press.

Määttänen, N. and Terviö, M. (2014). Income distribution and housing prices: An assignment model approach. *Journal of Economic Theory*, 151:381–410.

Mian, A., Rao, K., and Sufi, A. (2013). Household balance sheets, consumption, and the economic slump. *The Quarterly Journal of Economics*, 128(4):1687–1726.

Mian, A. and Sufi, A. (2009). The consequences of mortgage credit expansion: Evidence from the us mortgage default crisis. *The Quarterly journal of economics*, 124(4):1449–1496.

Mian, A. and Sufi, A. (2014). What explains the 2007–2009 drop in employment? *Econometrica*, 82(6):2197–2223.

Mian, A. and Sufi, A. (2022). Credit supply and housing speculation. *The Review of Financial Studies*, 35(2):680–719.

Nimczik, J. S. (2017). Job mobility networks and endogenous labor markets.

Ortalo-Magne, F. and Rady, S. (2006). Housing market dynamics: On the contribution of income shocks and credit constraints. *The Review of Economic Studies*, 73(2):459–485.

Patterson, C. et al. (2019). The matching multiplier and the amplification of recessions. *Unpublished Manuscript, Northwestern University*.

Piazzesi, M., Schneider, M., and Stroebel, J. (2020). Segmented housing search. *American Economic Review*, 110(3):720–59.

Shapley, L. and Shubik, M. (1977). Trade using one commodity as a means of payment. *Journal of political economy*, 85(5):937–968.

Shiller, R. J. (2015). *Irrational exuberance*. Princeton university press.

A Mathematical Appendix

A.1 Proof of Lemma 1

Proof. The fact that $q(\cdot)$ is strictly increasing follows from market clearing and the fact that payoffs are strictly increasing in housing quality. By Lebesgue's Differentiation Theorem, monotonicity of $q(\cdot)$ then implies that $q(h)$ is differentiable almost everywhere on $(\underline{h}, \bar{h})$. Where the derivative exists, the first-order condition satisfies

$$q'(h_0^*(y)) = f(q(h_0^*(y)), y) \tag{31}$$

for some continuous function $f(\cdot, \cdot)$ and $y \in [\underline{y}, \bar{y}]$.[19]

Now, in order to prove differentiability everywhere, I first argue that $q(\cdot)$ must be continuous. Suppose it is not and consider a point of discontinuity $h' \in [\underline{h}, \bar{h}]$. Because the utility derived from housing is continuous in h, any household would prefer adjusting their housing choice in either direction as opposed to a discrete jump to $q(h)$. Moreover, this adjustment is feasible because $H(\cdot)$ is strictly increasing and the support of $H(\cdot)$ is a connected set. This proves continuity.

Next, consider the decision functions $c_j^*(\cdot)$ for $j \in \{0, 1\}$ and $h_1^*(\cdot)$. Since $q(\cdot)$ is continuous, the conditions of the maximum theorem are satisfied and the decision functions are also continuous with respect to y. Hence, $f(q(h_0^*(y)), y)$ is a continuous function, being the composition of continuous functions. Moreover, the fact that $F(\cdot)$ is strictly increasing implies that the function is well-defined on the interval $[\underline{y}, \bar{y}]$. The continuity of $f(q(h_0^*(\cdot)), \cdot)$ on the closed interval $(\underline{y}, \bar{y})$ then implies that $q'(\cdot)$ is differentiable on $(\underline{h}, \bar{h})$ by the Fundamental Theorem of Calculus.

□

A.2 Proof of Proposition 1

Proof. I suppress the dependence of the decision functions on y for notational simplicity. Using the Karush-Kuhn-Tucker conditions for the household problem, we obtain the following equation for constrained households.

$$q'(h_1) \left(\frac{(1-\theta)\beta R - (1-\delta)\beta}{c_1} + \frac{\theta}{c_0} \right) = \chi \frac{1}{h_1} \tag{32}$$

[19] See the proof of Proposition 1 for proof of continuity and characterization of $f(\cdot, \cdot)$.

Multiplying and dividing by a factor of $\frac{R+\delta-1}{R}$, and letting $x \equiv \frac{\theta R}{R+\delta-1}$ we obtain

$$q'(h_1)\left(1-\frac{1-\delta}{R}\right) = \chi\frac{1}{h_1}\left(\frac{x}{c_0}+\frac{(1-x)\beta R}{c_1}\right)^{-1} \tag{33}$$

Finally, we substitute for c_0 and c_1 using (4) - (6). This yields (9) directly.

Households that are not constrained lie on their Euler equation

$$c_1 = \beta R c_0 \tag{34}$$

The pricing equation for unconstrained households is therefore

$$q'(h_1)\frac{1-\frac{1-\delta}{R}}{c_0} = \chi\frac{1}{h_1} \tag{35}$$

Since (6) is not binding, we substitute d_1 out of (4) (5) and use (34) to solve for c_0. This yields

$$c_0 = \frac{1}{1+\beta}\left(y+\frac{y_1}{R}-\frac{R+\delta-1}{R}q(h_1)\right) \tag{36}$$

□

A.3 Proof of Proposition 2

Proof. I set up the problem to make use of Topkis' Theorem. The challenge is to express the household's payoff only in terms of y and h and characterize its second derivative. To this end, we substitute for optimal consumption in terms of y and h. For an unconstrained household, this is given by

$$c_0(y,h) = \frac{1}{1+\beta}\left(y+\frac{y_1}{R}-\frac{R+\delta-1}{R}q(h)\right) \tag{37}$$

$$c_1(y,h) = \frac{\beta R}{1+\beta}\left(y+\frac{y_1}{R}-\frac{R+\delta-1}{R}q(h)\right) \tag{38}$$

Similarly, we have for a constrained household

$$\tilde{c}_0(y,h) = y - \theta q(h) \tag{39}$$

$$\tilde{c}_1(y,h) = y_1 - (R(1-\theta)-(1-\delta))q(h) \tag{40}$$

The consumption functions will therefore depend on the level of debt d (because this dictates whether households will be constrained or not), the housing quality choice h, and the level of first-period income y. Let $V_c(h,y)$ and $V_u(h,y)$ be the payoff of a household with first-period income y that chooses a house of quality h for a constrained and unconstrained household, respectively. We must show that these functions are supermodular in h, y.

Step 1: $V_c(h,y)$ and $V_u(h,y)$ are twice differentiable in h, y

We have

$$V_c(h,y) = \log(c_0(y,h)) + \beta \log(c_1(y,h)) + \chi \log(h) \tag{41}$$

and similarly for a constrained household. By Lemma 1, we know that $q'(h)$ is everywhere differentiable on $(\underline{h}, \bar{h})$. Twice differentiability of $V_c(\cdot,\cdot)$ and $V_u(\cdot,\cdot)$ on $(\underline{h}, \bar{h}) \times (\underline{y}, \bar{y})$ then follows.

Step 2: The cross-derivatives are strictly positive

The twice differentiability of $V_c(h,y)$ and $V_u(h,y)$ implies that we can characterize supermodularity through the second-partial derivative. For the constrained household:

$$\frac{\partial^2 V_c(h,y)}{\partial h \partial y} = q'(h)\left(\frac{\theta}{c_0^2}\right) > 0 \tag{42}$$

Similarly, it is straightforward to show for an unconstrained household:

$$\frac{\partial^2 V_u(h,y)}{\partial h \partial y} = q'(h)\frac{R+\delta-1}{(1+\beta)Rc_0^2} > 0 \tag{43}$$

Hence, the value function is supermodular in h, y. By Topkis's theorem $h_1^*(y)$ must be strictly increasing for both constrained and unconstrained households. Finally, a household that switches from being constrained to unconstrained (or vice versa) would not want to decrease their housing quality due to the continuity of the house quality decision function. Finally, at the boundary $\underline{h}$, $q(\underline{h})$ must be such so that the marginal household is indifferent to purchasing the lowest quality home or obtaining the outside option. By the supermodularity of the payoff function, it follows that there must exist a $y_p \in [\underline{h}, \bar{h}]$ such that all households with $y > y_p$ purchase housing.

□

A.4 Proof of Proposition 3

Proof. The *allocation* of households to house qualities is unique by Proposition 2 and the argument outlined in the main text. Equilibrium uniqueness and existence then follows by showing that there is a unique pricing function that satisfies (8) and (9).

First, note that the initial condition $q(\underline{h})$ is well-defined and unique. It is the price that makes the marginal household with income y_p indifferent to owning or obtaining his outside option. There are four cases to consider: (i) the household is credit-constrained when purchasing the house, and unconstrained otherwise, (ii) unconstrained when purchasing the house and constrained otherwise, (iii) constrained in both cases, or (iv) unconstrained in both cases. Because the down-payment for the house is strictly positive, we need only consider (i), (iii), and (iv). For case (i), the initial condition is given by the smallest positive root of $q(\underline{h})$ to the following equation:

$$\frac{\beta R}{(1+\beta)^2}\left(y_p + \frac{y_2}{R}\right)^{1+\beta} b^{\chi} = \left(y_p - \theta q(\underline{h})\right)\left(y_2 - ((1-\theta)R - (1-\delta)q(\underline{h})\right)^{\beta} \underline{h}^{\chi} \tag{44}$$

For case (iii), it is the smallest positive solution to:

$$(y_p)(y_2)^{\beta} b^{\chi} = \left(y_p - \theta q(\underline{h})\right)\left(y_2 - ((1-\theta)R - (1-\delta)q(\underline{h})\right)^{\beta} \underline{h}^{\chi} \tag{45}$$

For case (iv), it is the smallest positive solution to:

$$\frac{\beta R}{(1+\beta)^2}\left(y_p + \frac{y_2}{R}\right)^{1+\beta} b^{\chi} = \frac{\beta R}{(1+\beta)^2}\left(y_p + \frac{y_1}{R} - \frac{R+\delta-1}{R} q(\underline{h})\right)^{1+\beta} \underline{h}^{\chi} \tag{46}$$

Note that in all cases, y_p is the unique solution to

$$m(1 - F(y_p)) = 1 \tag{47}$$

As $\underline{h} > b$, t is straightforward to see that a solution in all cases exists. We may substitute for the inverse matching function $y(h)$ into (9) and (8) to obtain an equation in both cases of the form

$$q'(h) = f(q(h), h, y(h)) \tag{48}$$

for some continuously differentiable function $f(\cdot,\cdot,\cdot)$. We next verify the conditions for the Picard-Lindelöf theorem. Continuity in h follows from the continuity of the inverse matching function. Lipschitz continuity follows from the fact that $f(q(h), y(h), h)$ is continuously differentiable on the compact interval $h \in [\underline{h}, \bar{h}]$. Therefore, the derivative is bounded and $f(q(h), y(h), h)$ is Lipschitz continuous in h.[20]

Finally, we must show that the constraint status of households is unique. Note that the marginal household that does not purchase housing is not credit constrained if and only if

$$y_1 < R\beta y_p \tag{49}$$

[20]This follows from the mean value theorem: $f(q_2,\cdot,\cdot) - f(q_1,\cdot,\cdot) = f'(q_3,\cdot,\cdot)(q_2 - q_1)$ for some $q_3 \in (q_2, q_1)$. But $f'(q,\cdot,\cdot)$ is continuous on a compact set, and therefore bounded. Lipschitz continuity follows.

Suppose (49) is satisfied, and (46) yields a price that satisfies the home-owning household's credit constraint. By the continuity of the decision functions, households are unconstrained for some half-open interval $[\underline{h}, \underline{h} + \varepsilon)$, where $\varepsilon > 0$. Otherwise, households are constrained for some half-open interval. It is impossible to find prices in which households are both unconstrained and constrained because the left-hand side of (46) dominates the left-hand side of (44) by household optimality. If (49) does not hold, then the home-owning household is credit constrained.

We construct the pricing function iteratively, using either (9) or (8) depending on the constraint status of the household. Since the initial constraint status of the household is is well defined (by the argument above) and the construction on each interval is unique, the pricing function is unique on the interval $[\underline{h}, \bar{h}]$. □

A.5 Proof of Proposition 4

Proof. Note that the matching function is identical in both economies for $y < y_p$. Since the price function is unique on $[\underline{y}, y_p]$, the two price functions must be identical on that interval. □

A.6 Proof of Corollary 1

Proof. The inverse matching function in the two economies is:

$$y(h) = F^{-1}\left(\frac{1}{m}\left(H(h) + F(y_p)\right)\right), \quad \forall\, h > \underline{h} \tag{50}$$

$$\tilde{y}(h) = \tilde{F}^{-1}\left(\frac{1}{m}\left(H(h) + \tilde{F}(y_p)\right)\right), \quad \forall\, h > \underline{h} \tag{51}$$

Note that $\tilde{F}(y_p) = F(y_p) = \frac{m-1}{m}$ by market clearing. Since $\tilde{F}(\cdot) \leq F(\cdot)$, we have $\tilde{F}^{-1}(y) > F^{-1}(y)$ and $\tilde{y}(h) > y(h)$. By (9) and (8), we observe that both pricing functions satisfy the following ODEs:

$$\tilde{q}(h) = \tilde{f}(h, q(h)) \tag{52}$$

$$q(h) = f(h, q(h)) \tag{53}$$

where $\tilde{f}(h, q(h)) > f(h, q(h))$. For the same initial condition $q(\underline{h})$, we have $\tilde{q}(h) \geq q(h)$ by Petrovitsch's Theorem. However, the initial conditions must satisfy $\tilde{q}(\underline{h}) \geq q(h)$ (see Proposition 3). But this can only increase $\tilde{q}(h)$ further at every $h \in [\underline{h}, \bar{h}]$. Towards a contradiction, suppose that $\tilde{q}(h^*, q(\underline{h})) > \tilde{q}(h^*, \tilde{q}(\underline{h}))$ for some $h^* \in [\underline{h}, \bar{h}]$, where $q(h, a)$ is the unique solution to the ODE

$$\tilde{q}'(h) = \tilde{f}(h, \tilde{q}(h)), \quad q(\underline{h}) = a \tag{54}$$

Since $\tilde{q}(h, q(\underline{h}))$ and $\tilde{q}(h, \tilde{q}(\underline{h}))$ are continuous functions of h, and $\tilde{q}(\underline{h}, q(\underline{h})) < \tilde{q}(\underline{h}, \tilde{q}(\underline{h}))$, and $\tilde{q}(h^*, q(\underline{h})) > \tilde{q}(h^*, \tilde{q}(\underline{h}))$, there must exist an $h^{**} \in (\underline{h}, h^*)$ such that the two functions are equal by the Intermediate Value Theorem. But this contradicts equilibrium uniqueness of the pricing function for the initial condition $q(h^{**})$. □

A.7 Proof of Proposition 5

Proof. Directly follows by integrating (8). □

A.8 Proof of Proposition 6

I first state the exact version of the assumptions stated in the text:

Assumption 4. *The ratio* $\left(\frac{b}{\underline{h}}\right)$ *and* y_1 *are sufficiently large so that:*

$$\frac{R}{R+\delta-1}\left(y_p + \frac{y_1}{R} - \left(y_p + \frac{y_1}{R}\right)\left(\frac{b}{\underline{h}}\right)^{\frac{\chi}{1+\beta}}\right) > \frac{\beta R y_p - y_1}{R\theta(1+\beta) - (R-(1-\delta))} \tag{55}$$

Assumption 5. *χ is not too large and satisfies:*

$$\left((1-\delta-R+(1+\beta)R\theta)\phi - \beta R k\right)\underline{h} > \left(\frac{(1-\delta-R+(1+\beta)R\theta)}{\omega R} - 1\right)\bigg) y_1 \tag{56}$$

where $\phi = \frac{k\chi}{\omega(\chi+1+\beta)}$

The proof of this proposition is divided into two parts. In the first part, I prove that the constraint status is interior and unique. In the second part, I prove the properties of the pricing function stated in the proposition.

Part 1: Uniqueness and interiority of h^*

Proof. Step 1. First, I prove that a positive mass of households are credit constrained in equilibrium. Note that $y_p > \frac{y_1}{\beta R}$ implies that the marginal household that does not purchase a house is not credit constrained (if this condition is not satisfied then the marginal household that purchases a house is clearly also credit constrained and the proof is trivial). Next, suppose that the home-purchasing household is not credit-constrained. This implies that we must have

$$c_0 + q(\underline{h}) - y_p \leq (1-\theta)q(\underline{h}) \tag{57}$$

Substituting for the unconstrained consumption function and solving for $q(\underline{h})$, we see that it must satisfy

$$q(\underline{h}) \leq \frac{\beta R y_p - y_1}{R\theta(1+\beta) - (R - (1-\delta))} \tag{58}$$

But (46) implies that the price of $\underline{h}$ is

$$q(\underline{h}) = \frac{R}{R+\delta-1}\left(y_p + \frac{y_1}{R} - \left(y_p + \frac{y_1}{R}\right)\left(\frac{b}{\underline{h}}\right)^{\frac{\chi}{1+\beta}}\right) \tag{59}$$

which contradicts 4. Hence, the marginal household must be credit-constrained.

Step 2. Next, I prove that there exists an interior threshold h^*. Households stop becoming credit-constrained at the point at which they equalize their marginal utility. Substituting for the constrained household's consumption function into (34), we obtain:

$$\beta R(kh^* - \theta q(h^*)) = y_1 + (1 - \delta - R(1-\theta)q(h^*)) \tag{60}$$

where I have substituted for the inverse matching function using A2. Solving for $q(h^*)$, we obtain the following function in terms of h

$$t(h) = \frac{\beta Rkh - y_1}{1 - \delta - R + (1+\beta)R\theta} \tag{61}$$

The intersection of $q(h)$ with $t(h)$ yield the points at which households switch constraint status. We must show that $q(h)$ intersects $t(h)$ exactly once. To this end, note that $t(\underline{h}) < q(\underline{h})$ by 4. The proof proceeds by constructing bounds for the ODE that $q(h)$ must satisfy and then using Petrovitsch's Theorem.

In particular, using (9), we construct the following bounds for the ODE of credit-constrained households:

$$q'(h) = \frac{R}{R+\delta-1}\frac{1}{h}\left(\frac{x}{c_0} + \frac{(1-x)\beta R}{c_1}\right)^{-1} \tag{62}$$

$$q'(h) \leq \frac{R}{R+\delta-1}\frac{1}{h}\left(\frac{x\beta R}{c_0} + \frac{(1-x)\beta R}{c_1}\right)^{-1} \tag{63}$$

$$q'(h) \leq \frac{1}{\beta(R+\delta-1)}\frac{y_1 - ((1-\theta)R - (1-\delta))}{h} \tag{64}$$

where I have used the fact that $c_1 > \beta R c_0$ for credit constrained households. By Petrovitsch's Theorem, the pricing function for constrained households must be bounded above by $\bar{q}(\cdot)$, where $\bar{q}(\cdot)$ solves

$$\bar{q}'(h) \leq \frac{1}{\beta(R+\delta-1)} \frac{y_1 - ((1-\theta)R - (1-\delta))}{h} \tag{65}$$

The solution to this ODE is given by

$$\bar{q}(h) = \frac{y_1}{\gamma_2}\left(1 - \left(\frac{\underline{h}}{h}\right)^{\gamma_1\gamma_2}\right) + \left(\frac{\underline{h}}{h}\right)^{\gamma_1\gamma_2} q(\underline{h}) \tag{66}$$

where

$$\gamma_1 \equiv \frac{1}{\beta(R+\delta-1)}, \qquad \gamma_2 \equiv (1-\theta)R - (1-\delta) \tag{67}$$

which intersects $t(h)$ at a finite h. Since $q(h)$ is increasing and $t(\underline{h}) < q(\underline{h})$, this proves that $q(h)$ also intersects $t(h)$ at a finite h.

Step 3. I now prove uniqueness of the constraint threshold. Note that since both $t(h)$ and $q(h)$ are increasing, and $q(h) > t(h)$ for all $\underline{h} \leq h < h^*$, we must have $t'(h^*) > q'(h^*)$ at the intersection point. Since $t'(h)$ is constant for all h, concavity of $q(h)$ implies the two curves will never intersect above h^*. This will prove uniqueness.

Using (12), substituting for the inverse matching function using A2, and taking the derivative twice, we note that $q(h)$ is concave for $h > h^*$ if and only if

$$\left(\phi h^* + \frac{y_1}{\omega R}\right) > q(h^*) \tag{68}$$

where $\phi = \frac{k\chi}{\omega(\chi+1+\beta)}$

By definition, $q(h^*) = t(h^*)$. Substituting and solving for h^* yields

$$\Big((1-\delta-R+(1+\beta)R\theta)\phi - \beta Rk\Big)h^* > \left(\frac{(1-\delta-R+(1+\beta)R\theta)}{\omega R} - 1\right)\Big) y_1 \tag{69}$$

which yields the sufficient condition in terms of model primitives:

$$\Big((1-\delta-R+(1+\beta)R\theta)\phi - \beta Rk\Big)\underline{h} > \left(\frac{(1-\delta-R+(1+\beta)R\theta)}{\omega R} - 1\right)\Big) y_1 \tag{70}$$

which is satisfied if χ is not too large. This proves that h^* is unique and interior.

□

Part 2 The pricing function for the linear class is given by

$$q(h) = \frac{\chi}{\omega(\chi+1+\beta)}\left(h - \left(\frac{(h^*)^{\frac{\chi+1+\beta}{1+\beta}}}{h^{\frac{\chi}{1+\beta}}}\right)\right) + \left(\frac{y_1}{R}\right)\left(\frac{1}{\omega}\right)\left(1-\left(\frac{h^*}{h}\right)^{\frac{\chi}{1+\beta}}\right) + \left(\frac{h^*}{h}\right)^{\frac{\chi}{1+\beta}} q(h^*) \tag{71}$$

where the equation holds for $h > h^*$. The proof for the second part relies on the following two simple lemmas.

Lemma 2. *Consider a twice differentiable function $u : \mathbb{R}^2 \to \mathbb{R}$*

$$u(x,y) = f(x) + g(x,y) \tag{72}$$

where $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R}^2 \to \mathbb{R}$. Suppose $u(x,y) > 0$, $u'_x(x,y) > 0$, and $g''_{xy}(x,y) > 0$. Then,

$$u''_{xy}(x,y) > 0 \quad \textit{and} \quad \frac{\partial^2 \ln u}{\partial x \partial y}(x,y) > 0 \tag{73}$$

Proof. Suppressing dependence on x, y, we have $\frac{\partial^2 \ln u}{\partial x \partial y} = \frac{g_{xy}(f+g)-g_y(f_x+g_x)}{(f+g)^2}$, which is strictly positive by the assumptions above. □

Lemma 3. *Consider a twice differentiable function $u : \mathbb{R}^2 \to \mathbb{R}$*

$$u(x,y) = f(y)x + c + \eta(y,x) \tag{74}$$

where $f : \mathbb{R} \to \mathbb{R}$ and $\eta : \mathbb{R}^2 \to \mathbb{R}$. Suppose $u(x,y) > 0$, $f'(y) < 0$, $c > 0$, and $\lim_{x\to\infty} \eta(\cdot,x) = 0$, $\lim_{x\to\infty} x\eta'_x(\cdot,x) = 0$, $\lim_{x\to\infty} \eta'_y(\cdot,x) = 0$, and $\lim_{x\to\infty} x\eta''_{xy}(\cdot,x) = 0$. Then,

$$u''_{xy}(x,y) < 0 \quad \textit{and} \quad \frac{\partial^2 \ln u}{\partial x \partial y}(x,y) < 0 \tag{75}$$

for all $x > x'$, where $x' \in \mathbb{R}$.

Proof. The assumptions above imply that $\lim_{x\to\infty} = \frac{\partial^2 \ln u}{\partial x \partial y} = \frac{cf'(y))}{u(x,y)^2}$. Hence, there exists an $x' \in \mathbb{R}$ such that $\frac{\partial^2 \ln u}{\partial x \partial y} < 0$ for all $x > x'$. □

Proof. (**Proposition 6**) *Credit frictions*: It is straightforward to see that equaiton (71) satisfy the conditions of the Lemma 1, where x can be substituted for h and y can be substituted for θ. Differentiation of the ODEs and a simple limit yields the result.

Interest rates: Follows from simple differentiation of (71) and taking the limit.

Transaction costs: It is straightforward to see that (71) and satisfy the conditions of the Lemma 2, where x can be substituted for h and y can be substituted for δ. A simple limit then yields the remainder of the results.

Future income: I claim that $\frac{\partial q(h)}{\partial y_1} < \frac{1}{R+\delta-1}$ for all $h < h^*(y)$. Suppose otherwise. Let h_a be the first occurrence of h for which $\frac{\partial q(h)}{\partial y_1} \geq \frac{1}{R+\delta-1}$. Simple differentiation of the initial condition reveals that $\frac{\partial q(\underline{h})}{\partial y_1} < \frac{1}{R+\delta-1}$. By the continuity of this derivative, the mean value theorem thus implies that $h_a > \underline{h}$. Differentiating (9) with respect to y_1, we obtain an equation of the form

$$\frac{\partial q'(h)}{\partial y_1} = f(h, q(h))\left(\frac{x}{c_1^2}(-\theta\frac{\partial q(h)}{\partial y}) + \frac{(1-x)\beta R}{c_2^2}(1-((1-\theta)R-(1-\delta))\frac{\partial q(h)}{\partial y_2})\right) \tag{76}$$

where $f(h, q(h))$ is strictly positive. Using the fact that $c_2 > \beta R c_1$ (since agents are credit constrained), we obtain

$$\frac{\partial q'(h)}{\partial y_1} < f(h, q(h))\left(\frac{1}{c_1^2}(\frac{1}{R} - \frac{R+\delta-1}{R}\frac{\partial q(h)}{\partial y_1})\right) \tag{77}$$

Hence, a sufficient condition for $\frac{\partial q'(h)}{\partial y_2} < 0$ is that $\frac{\partial q(h)}{\partial y} \geq \frac{1}{R+\delta-1}$. But since $\frac{\partial q(\underline{h})}{\partial y_2} < \frac{1}{R+\delta-1}$, this implies that no such h_a can exist, which is a contradiction.

Hence, $\frac{\partial q(h)}{\partial y_1} < \frac{1}{R-1+\delta}$ for all $h < h^*(y)$. Simple differentiation then yields the result. □

A.9 Proof of Proposition 7

Proof. House price changes for old households are pure wealth effects. Hence, their effect on consumption is $(1-\delta)MPC(h)$. Assuming the absence of substitution effects, the effect of house price changes on consumption for young households is $-MP\tilde{C}(h)$.

It remains to show that there are no substitution effects for young households. To this end, note that house price changes do not change relative income rankings. Hence, by Proposition 2, the matching function before and after the credit shock is unchanged for young households. But this implies that there can be no substitution effect. The result then follows by integrating over house quality distribution and decomposing the expectation function. □

A.10 Proof of Corollary 2

Proof. The $\widetilde{MPC}$ of a young household that is not credit constrained and does not leave bequests is given by $\frac{1}{1+\beta}$. The MPC of an old household that does leaves bequests is unity, and any house price changes are attenuated in the budget constraint by a factor of $(1-\delta)$. Similarly, the $\widetilde{MPC}$ for young households that leaves bequests is $\frac{1}{1+2\beta}$ and is $\frac{1}{2}$ for old households that leave bequests. The condition

$$1 - \delta > \frac{2}{1+2\beta} \tag{78}$$

ensures that $\overline{MPC}(h) > 0$ for all $h \in (h^*, \bar{h}]$.

The condition ϕ is large enough ensures that households start leaving bequests above the credit constraint threshold, which results in non-trivial variation in marginal propensities to consume above y^*. By Proposition 6, capital losses are a strictly decreasing function of housing quality for $h > h(y^*)$. Moreover, MPCs and $\widetilde{MPC}$s are a non-decreasing function of housing quality. Hence, the conditions of Theorem 1 and Theorem 2 in Behboodian (2006) are satisfied and we must have $cov(\overline{MPC}_t(h), dq_t(h)) > 0$ for $h > h(y^*)$. $\square$

A.11 Extension to Bequests

Clearly, the introduction of bequests do not change the positive assortative matching of the equilibrium, which leaves the inverse matching function unchanged. For ϕ large enough, it also does not change the initial conditions of the pricing function. Hence, it remains to show that credit constraint status monotonicity holds under a linear matching function with bequests. We may obtain the following pricing ODE for households that leave bequests:

$$\begin{aligned} q(h) = \frac{\chi}{\omega(\chi+1+2\beta)} \left(h - \left(\frac{(h^*)^{\frac{\chi+1+2\beta}{1+2\beta}}}{h^{\frac{\chi}{1+2\beta}}} \right) \right) \\ + \left(\frac{y_1 + \phi}{R} \right) \left(\frac{1}{\omega} \right) \left(1 - \left(\frac{h^*}{h} \right)^{\frac{\chi}{1+2\beta}} \right) + \left(\frac{h^*}{h} \right)^{\frac{\chi}{1+2\beta}} q(h^*) \end{aligned} \tag{79}$$

for all $h > h^{**} > h^*$, where h^{**} is the threshold at which households start leaving bequests

$$h^{**} = \inf\{h : c_1(h) > \phi\} \tag{80}$$

This function continues to be concave if the parametric restrictions underlying Proposition 12 are satisfied. Hence, credit constraint monotonicity is also satisfied. The response of the pricing function to various shocks is then entirely identical to the arguments outlined in Proposition 12.

B Model Extensions

B.1 Income Heterogeneity

This subsection extends the model to the case in which income at old age is heterogeneous. I show that the trickle-up result also holds in this more general environment (Proposition 4), and the effects of different shocks on the cross-sectional distribution of house prices characterized in the main text (Proposition 12) continues to hold under the assumption of credit constraint monotonicity and linear inverse matching functions. I consider the following form on old income heterogeneity:

Assumption 6. *$y_1 = g(y) + \varepsilon_y$, where*

1. *$g(\cdot)$ is a weakly increasing function with positive range*
2. *ε_y is a random variable with positive and bounded support*

The conditions within Assumption 6 ensure that greater income in youth is associated with weakly higher income, but the distribution of idiosyncratic risk can be quite arbitrary. This is sufficient to ensure that utility is strictly increasing in first-period income.

Proposition 8. *Suppose Assumption 6 is satisfied and the two following parametric conditions hold: (i) $(1-\delta) \leq (1-\theta)R$, and (ii) $(R-1)(1-\delta+\beta R) < \delta R(1+\beta)$. Then,*

1. *There exists a $y_p \in (\underline{y}, \bar{y})$ such that households with $y > y_p$ are homewoners.*
2. *The policy function $h_1^*(y)$ is strictly increasing in y for all $y > y_p$.*

Proof. The proof is an application of Topkis' Theorem. The twice-differentiability of the value function follows (with only minor modifications) from the proof of Proposition 4. It remains to show that the cross-derivative of the value function is strictly positive in h and y. For a constrained household, we have

$$\frac{\partial V}{\partial y} = \frac{1}{c_0} + \beta g'(y)\mathbb{E}_\varepsilon \frac{1}{c_1} \tag{81}$$

Differentiating with respect to h, we have:

$$\frac{\partial^2 V}{\partial y \partial h} = \frac{\theta}{c_0^2} q'(h) + \beta g'(y)\left((1-\delta) - (1-\theta)R\right)\mathbb{E}_\varepsilon \frac{q'(h)}{-c_1^2} \tag{82}$$

Under the assumptions of the proposition, each term is strictly positive. I next turn to unconstrained households. The difficulty here is that we do not have a closed form expression for consumption. However, we may use the Euler equality:

$$\frac{1}{c_0} = \beta R \mathbb{E}\left[\frac{1}{c_1}\right] \tag{83}$$

We may differentiate with respect to h and y to obtain the following two equations

$$\frac{1}{c_0^2}\left(1+\frac{\partial d}{\partial y}\right) = \beta R \mathbb{E}\left(\frac{1}{c_1^2}\right)\left(g'(y) - R\frac{\partial d}{\partial y}\right) \tag{84}$$

$$\frac{1}{c_0^2}\left(-q'(h)+\frac{\partial d}{\partial h}\right) = \beta R \mathbb{E}\left(\frac{1}{c_1^2}\right)\left((1-\delta)q'(h) - R\frac{\partial d}{\partial h}\right) \tag{85}$$

where d is the agent's choice of debt. Differentiating the value function twice, we obtain:

$$\begin{aligned}\frac{\partial^2 V}{\partial y \partial h} = -\frac{1}{c_0^2}\left(-q'(h)+\frac{\partial d}{\partial h}\right)\left(1+\frac{\partial d}{\partial y}\right) + \frac{1}{c_0}\frac{\partial^2 d}{\partial y \partial h} + \beta \mathbb{E}\left(\frac{1}{c_1}\right)\left(-R\frac{\partial^2 d}{\partial y \partial h}\right) \\ -\beta R \mathbb{E}\left(\frac{1}{c_1^2}\right)\left(g'(y) - R\frac{\partial d}{\partial y}\right)\left((1-\delta)q'(h) - R\frac{\partial d}{\partial h}\right)\end{aligned} \tag{86}$$

Using (83) and (84), we obtain

$$\frac{\partial^2 V}{\partial y \partial h} = -\frac{1}{c_0^2}\left(1+\frac{\partial d}{\partial y}\right)\left(-\delta q'(h) + \frac{\partial d}{\partial h}(R-1)\right) \tag{87}$$

I now provide bounds for $\frac{\partial d}{\partial y}$ and $\frac{\partial d}{\partial h}$ such that the cross-derivative is strictly positive. Using Jensen's inequality, (83), and (84), we obtain

$$1+\frac{\partial d}{\partial y} > g'(y) - R\frac{\partial d}{\partial y} \tag{88}$$

Note that this implies $\frac{\partial d}{\partial y} > -1$ if $g'(y) > 0$. Moreover, using (83) and using Jensen's inequality, we obtain

$$1 < (\beta R)^2 \mathbb{E}\left(\left(\frac{c_0}{c_1}\right)^2\right) \tag{89}$$

Combining this expression with (85), we obtain

$$\beta R\left(-q'(h)+\frac{\partial d}{\partial h}\right) < (1-\delta)q'(h) - R\frac{\partial d}{\partial h} \tag{90}$$

It is then easily verified that the cross-derivative is strictly positive if the conditions on R outlined in the proposition are satisfied. Global supermodularity then follows from continuity of the policy function. Hence, $h_1^*(y)$ cannot decrease as one crosses the constraint threshold.

□

The presence of positive assortative matching implies that a variant of Proposition 4 continues to hold. In particular consider two economies with different incomes CDFs $F(\cdot)$ and $\tilde{F}(\cdot)$ that are otherwise identical. Furthermore, let $q(\cdot)$ and $\tilde{q}(\cdot)$ be the respective price functions in these two economies. We have:

Proposition 9. *Suppose $F(\cdot)$ and $\tilde{F}(\cdot)$ satisfy $F(y) = \tilde{F}(y)\ \forall\, y < \tilde{y}$, where $\tilde{y} \in [\underline{y}, \bar{y}]$. Suppose further that $\tilde{q}(\underline{h}) = q(\underline{h})$ (i.e. the prices of the lowest home qualities are identical in these two economies). Then, $\tilde{q}(h) = q(h)\ \forall\, h \leq h(\tilde{y})$.*

Proof. The Lipschitz continuity of marginal utility implies uniqueness given an initial condition. The result then follows from the proof of Proposition 4. □

We may also characterize the effects of aggregate shocks on the distribution of house prices under the assumptions of (i) credit constraint monotonicity, (ii) linear inverse matching functions, (iii) $g(y) = ky$ for some $k \in \mathbb{R}^+$, and (iv)$\varepsilon = 0$ (no idiosyncratic risk). In this case, the results of Proposition 12 continue to hold (where the comparative statics to future income are now meaningless), which follows with minor modifications from the proof of the original proposition.

B.2 Supply-Side Movements

This section extends the main model to incorporate a construction sector. I show that all the results in the main text continue to hold in the presence of *contractionary* shocks (defined as shocks that reduce the average value of housing), as the price of housing falls below the cost of producing a new housing unit.

Developers. I assume that there exist a large number of potential developers that can choose to enter the housing market at a cost of c. Once a developer enters the housing market, they draw a random house quality from the distribution of quality quartiles $G(\cdot)$, which they may then sell to the household at its competitive price. I take $G(\cdot)$ to be exogenous, and interpret the random draw of housing as idiosyncratic risk within the development process. As such, the presence of developers can change the *mass* of the total housing stock, but will not change the relative densities between different quality tiers. Developers are risk-neutral. Free entry for developers therefore implies

$$\mathbb{E}\left[q_t(h)\right] \geq c \tag{91}$$

I assume that c is sufficiently low so that a positive stock of housing exists in equilibrium.

Let s be the total mass of housing in equilibrium. In order to maintain market clearing for all quality quartiles, I further assume that c is sufficiently high so that $s < m$.[21] This implies that the home-ownership rate is strictly less than one.

Analysis. I first present a simple result that relates developer costs to house prices.

Proposition 10. *A reduction in c strictly decreases house prices along all quality tiers.*

Proof. The inverse matching function in this economy is given by

$$y(h) = F^{-1}\left(\frac{s}{m}G(h) + F(y(\underline{h}))\right) \tag{92}$$

where $y(\underline{h}) = F^{-1}\left(1 - \frac{s}{m}\right)$. Differentiating with respect to s, we obtain

$$\frac{\partial y(h)}{\partial s} = \frac{\frac{1}{m}(G(h) - 1)}{f\left(F^{-1}\left(\frac{s}{m}G(h) + F(y(\underline{h}))\right)\right)} < 0 \tag{93}$$

It is also straightforward to show that the price which makes a household indifferent between the outside option b and the lowest house quality $\underline{h}$ is strictly decreasing in y. The initial price function was given by an equation of the form

$$q'(h) = r(q, y(h)) \tag{94}$$

whereas the new pricing function is given by

$$\tilde{q}'(h) = \tilde{r}(q, y(h)) \tag{95}$$

with $\tilde{r}(q, \cdot) < r(q, \cdot)$ and with a lower initial condition. Suppose momentarily that the initial condition had not changed. Then, the fact that the new price function is lower follows from a direct application of Petrovitsch's Theorem. However, a decrease in the initial condition lowers the pricing function further still through the argument contained in the proof of Corollary 1. □

The above result also sheds light on the effect of migration on house prices. An increase in m raises house prices for all house quality tiers. Consider now a shock that decreases average house prices permanently.[22] Because the housing stock does not depreciate over time, the total supply of housing is such so that the new average price is below the minimum profitable cost. As such, contractionary shocks in this setting are isomorphic to an economy with inelas-

[21] Having $s > m$ would lead to vacant houses, which would necessitate a new equilibrium definition.

[22] Note that an increase in θ, or a decrease in R, δ, or y_1 lower house prices for all quality tiers in the main text.

tic housing supply, a point also illustrated in Glaeser and Gyourko (2018) in a model with a homogeneous housing stock.

B.3 Rental Markets

This section endogenizes the household's outside option by incorporating a homogeneous rental market into the main model. I derive conditions under which positive assortative matching holds in this new environment, and show that the main results are robust to this richer framework.

I assume that renting households can now purchase rental units from a homogeneous housing stock at a per unit price of p_r. The pay-off of a renting household that consumes c_0, c_1, and purchases r rental units is given by

$$\log(c_0) + \log(c_1) + \chi\gamma \log(r) \tag{96}$$

where $0 < \gamma < 1$ is a utility penalty of renting, as opposed to owning, a home. Moreover, I assume that the total quantity of the rental stock is fixed at $\bar{R} > 0$. In the remainder of the analysis, I assume that $\underline{y}$ is sufficiently large relative to y_1 so that renting households are not credit constrained. This considerably simplifies some of the algebra in the remainder of the analysis, but is inessential to the main results. I now derive a condition under which positive assortative matching holds in this environment. To this end, define the average income of renting households

$$\bar{Y} \equiv \int_{\underline{y}}^{y_p} y dF(y)$$

where y_p is the income for the marginal home-owning household (I will show that the average income takes this form momentarily). We have the following lemma:

Proposition 11. *Suppose the following condition is satisfied:*

$$\underline{h} > \left(\frac{y_p + \frac{y_1}{R}}{m\bar{Y} + \frac{y_1}{R}} \bar{R} \right)^{\gamma} \tag{97}$$

where $y_p = F^{-1}\left(1 - \frac{1}{m}\right)$. *Then, there exists an equilibrium such that (i) all households with* $y > y_p$ *become homeowners, and (ii) the housing policy function* $h_1^*(y)$ *is strictly increasing in* y *for all* $y > y_p$.

Proof. The proof proceeds by conjecturing that all households with $y \leq y_p$ are renters, and then verifying the conditions for positive assortative matching. The household's first-order condition gives rise to the policy function:

$$h = \frac{\chi\gamma\left(\frac{1}{1+\beta}\left(y + \frac{y_1}{R} - p_r r\right)\right)}{p_r} \tag{98}$$

where it is here that we have used the fact that renters are not credit constrained. Aggregating and imposing market clearing for the rental market, we can obtain the price for rents in closed form

$$p_r = \frac{\frac{\chi\gamma}{1+\beta}\left(m\bar{Y} + \frac{y_1}{R}\right)}{\left(1 + \frac{\chi\gamma}{1+\beta}\right)\bar{R}} \tag{99}$$

We may then substitute the price back into the household's rental policy function (98) to obtain

$$r = \frac{y + \frac{y_1}{R}}{m\bar{Y} + \frac{y_1}{R}}\bar{R} \tag{100}$$

Clearly, rental choice is increasing in income. Under the assumption of the proposition, the utility from the lowest house quality tier exceeds the utility derived from optimally choosing one's rental housing in the rental market. The price of the lowest quality tier is such so as to make the household with income y_p indifferent to renting and owning. Since the pay-off function is supermodular in house quality and income, all households with income $y < y_p$ strictly prefer renting to paying y_p's indifference price. This proves (i). The fact that the housing policy function is strictly increasing in income follows from Proposition 4. □

Note that the average price of rents is determined *independently* of house prices – it is the price of the lowest house quality quartile that adjusts to make the marginal household indifferent to renting and home-owning. This observation, in conjunction with positive assortative matching, implies that the main results are robust to the inclusion of a rental sector. The only additional consideration here is that certain aggregate shocks will now induce "variation" in the outside option by changing the price of rents. In particular, we obtain the following alternative to Proposition 12.

Proposition 12 (Comparative Statics)**.** *Suppose Assumptions 1 - 3 are satisfied. Then, there exists an $h^* \in (\underline{h}, \bar{h})$ such that households with $y > y(h^*)$ are not credit constrained (and all other home-purchasing households are credit constrained). Moreover, the pricing function responds as follows to various permanent, unanticipated change to various parameters:*

1. *Loan-to-value constraint (θ) —* $\frac{\partial q(h)}{\partial\theta\partial h} > 0$ *and* $\frac{\partial \log q(h)}{\partial\theta\partial h} > 0$ *for $h > h(y^*)$. Moreover,* $\lim_{h\to\infty} \frac{\partial \log q(h)}{\partial\theta} = 0$

2. *Interest rate (R) — There exists a threshold y' such that $\frac{\partial q(h)}{\partial R \partial h} < 0$ for $h > h(y')$. Moreover,* $\lim_{h\to\infty} \frac{\partial \log q(h)}{\partial R} = -\frac{1-\delta}{R(R+\delta-1)}$

3. *Transaction cost (δ)— There exists a threshold y' such that $\frac{\partial q(h)}{\partial \delta \partial h} < 0$ and $\frac{\partial \log q(h)}{\partial \delta \partial h} < 0$ for $h > h(y')$. Moreover,* $\lim_{h\to\infty} \frac{\partial \log q(h)}{\partial \delta} = -\frac{1}{R+\delta-1}$

Proof. Follows directly from the proof of Proposition 12. □

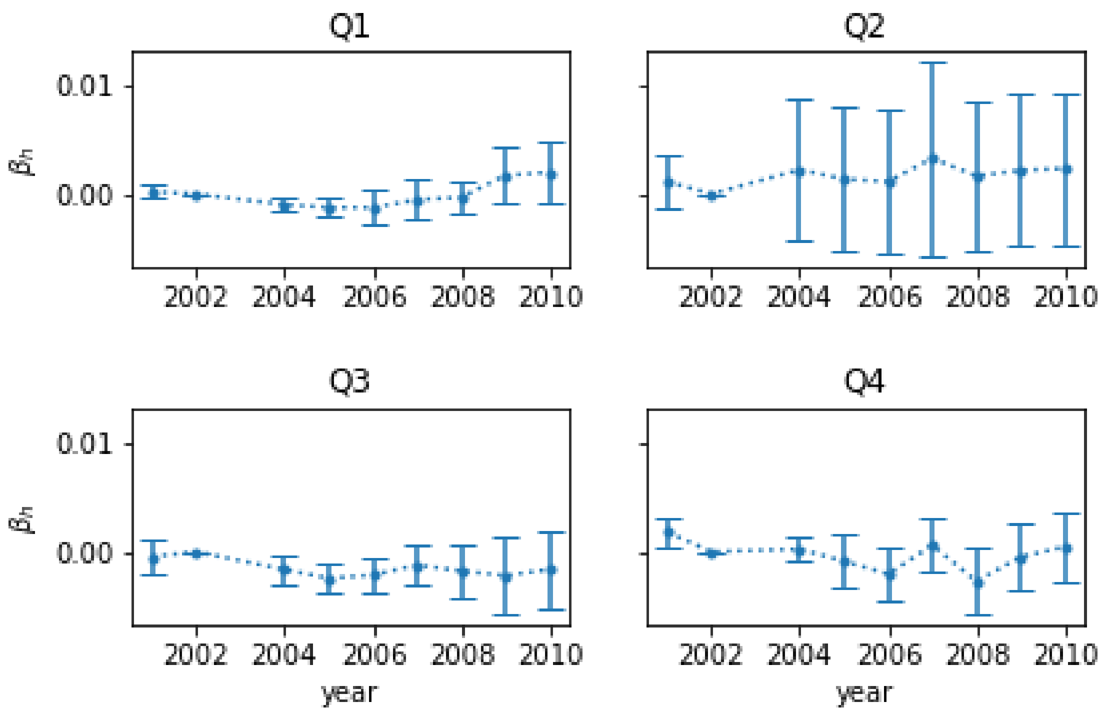
C Additional Tables and Figures

Table 7

	(1)
	$\triangle_{2005,2002} Price$
β_1	1.800
	(1.260)
β_2	0.760^{***}
	(0.182)
β_3	1.380^{*}
	(0.650)
β_4	1.672^{*}
	(0.844)
Quartile FE	✓
N	9135

Notes: The table reports the coefficients from Equation (28), using the NCL share as an instrument for the total change in credit growth in the corresponding county from 2002-2005. The dependent variable is the percentage change in the house price in a given zip code. Robust standard errors are in parentheses. $*$ denotes signifiance at the 5% level, $**$ at the 1% level, and $***$ at the 0.1% level.

Figure 7



Notes: The figure reports the coefficients β_h for each quality tier from Equation (30) using the percentage change in income as the dependent level. 2002 is the base year and observations are at the zipcode level. The figure shows 95% confidence bands, based on standard errors clustered at the county-level. The years 2000 and 2003 have been omitted due to the lack of zipcode level income data.