



POLITECNICO
MILANO 1863

Application 1
Safe-life and damage tolerance analysis of TP400 propeller shaft

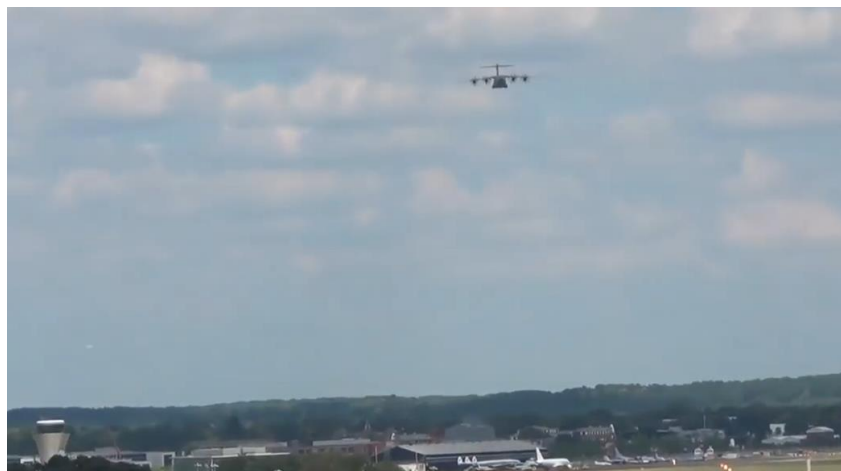
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Avio Aero



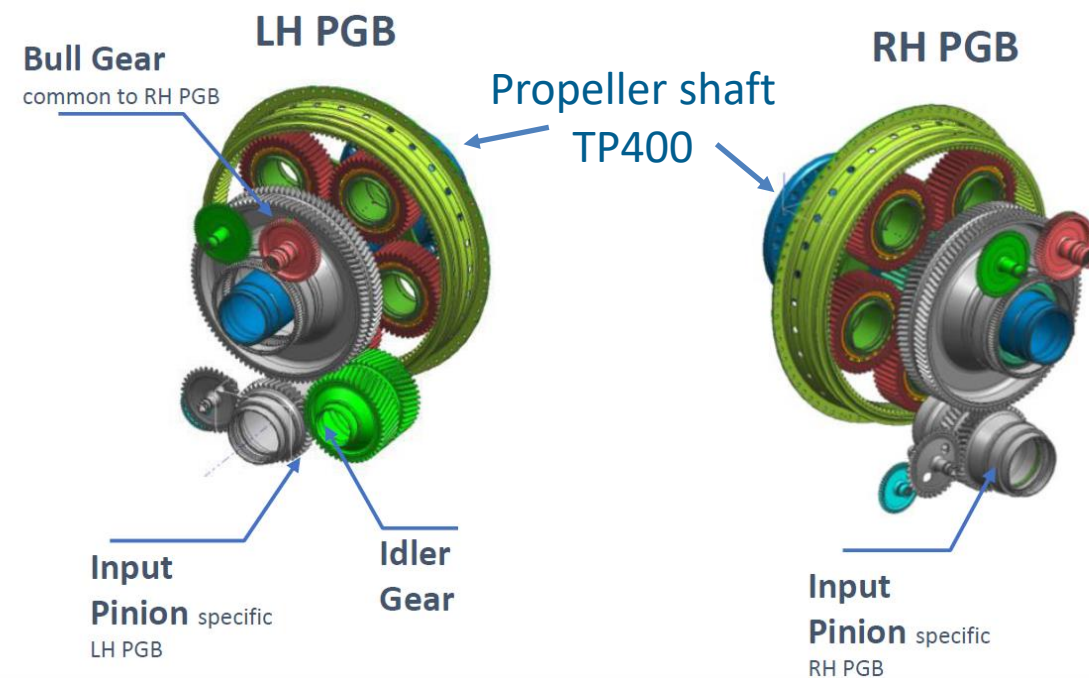
Introduction (1)



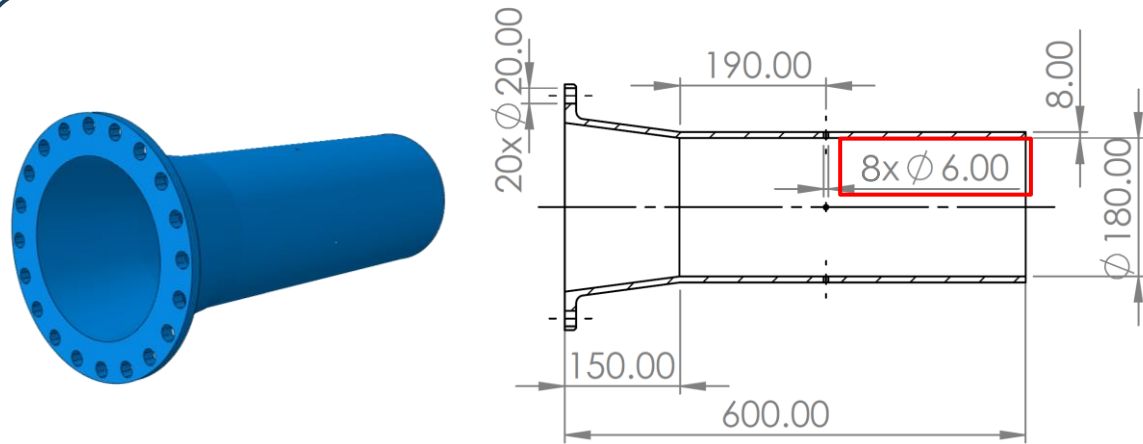
Airbus A400M
Military airlifter



Gear transmission

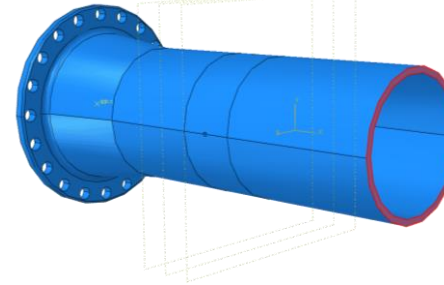


Introduction (2)

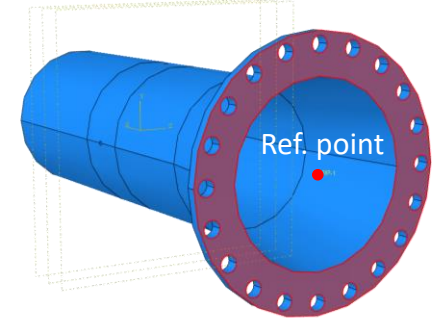


Shaft geometry

Numerical model for local stress identification

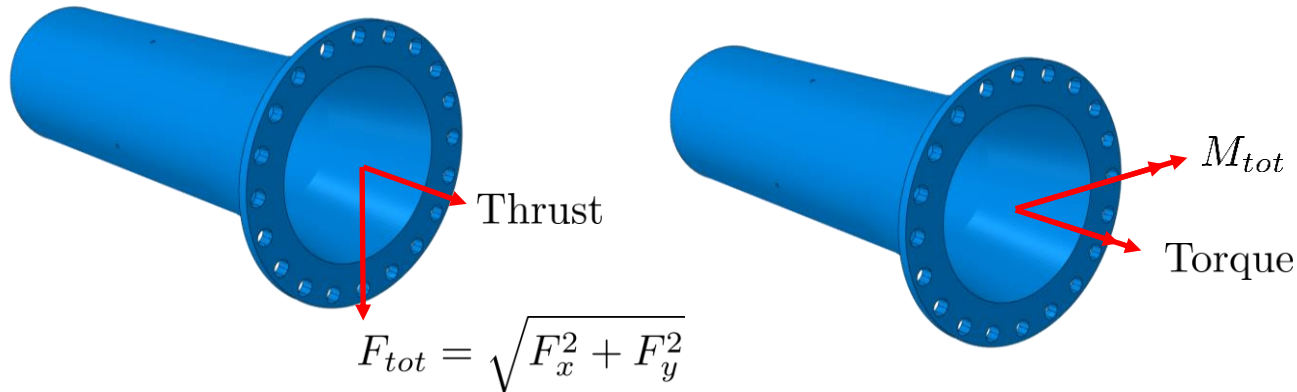


One end is encastered



The other is coupled with a ref. point for load application

Loads acting on the shaft



For simplicity, we assume that radial forces generate a moment in phase with the bending moment (conservative assumption)

During operation, the shaft rotates:

→ Torque and thrust generate a constant state of stress (**mean stress**) in the section

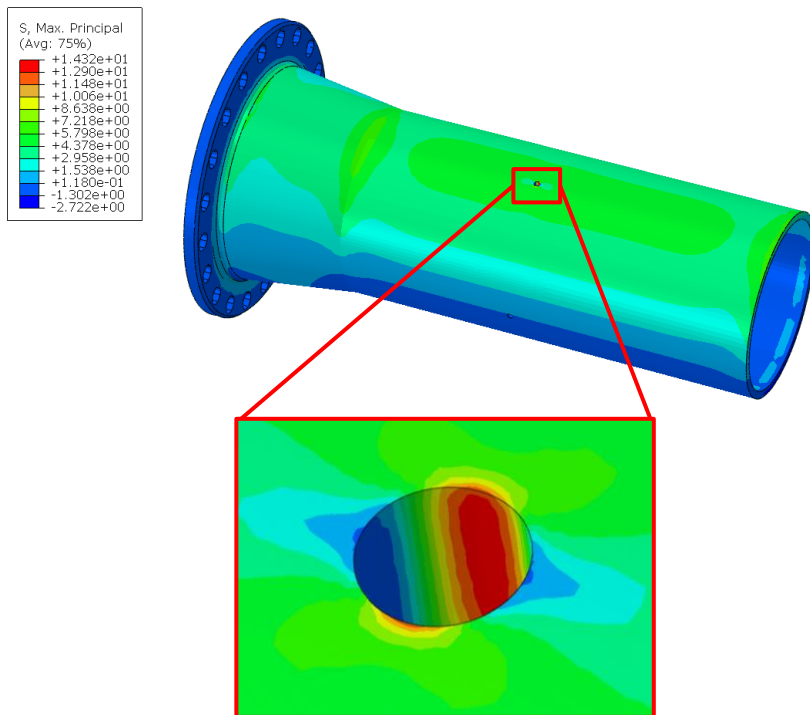
→ Bending moment and radial force generate an **alternate stress** state in the section

Stress history data (1)

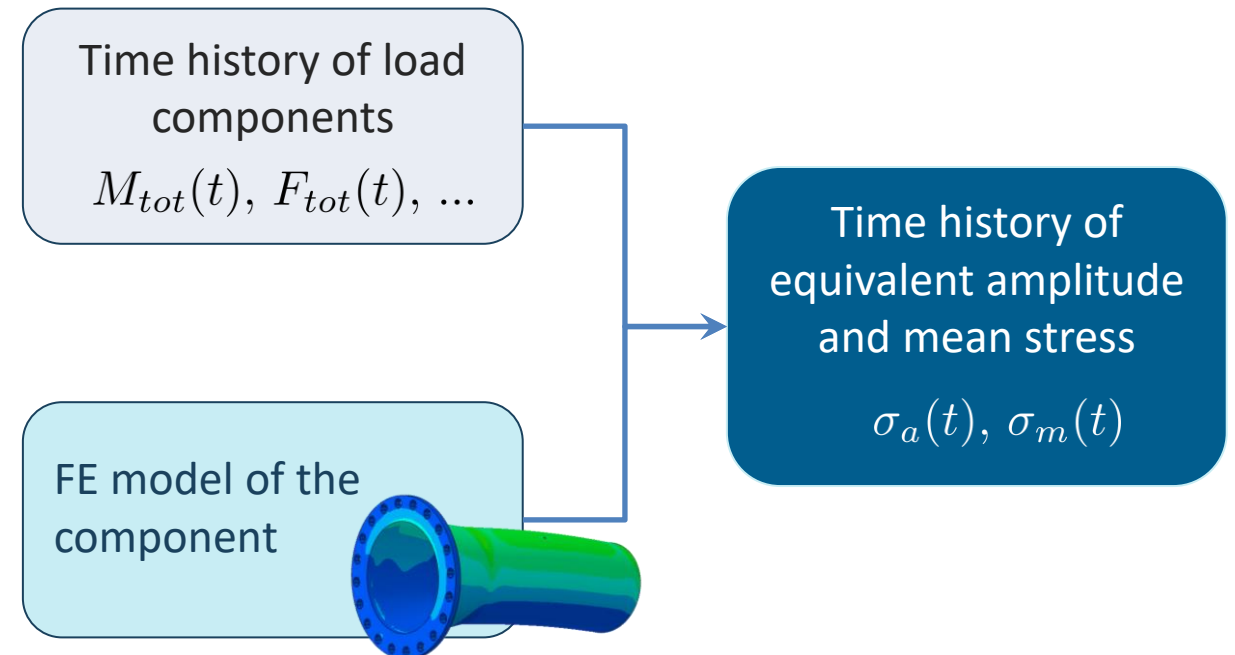
We have the measurements of the loads acting on the shaft for 3 different mission profiles:

- Profile 1: approximately 3 hours of flight
- Profile 2: approximately 1 hours of flight
- Profile 3: extreme flight mission (to be used for maximum admissible crack)

→ A standard mission is composed by profile 1 + profile 2 (4 hours of flight)



Most critical point is the drain hole



Stress history data (2)

- For each profile, you have an excel file with three columns: time, alternate stress and mean stress (maximum value during the sampling time)
- The shaft rotates with a constant rotational speed ω of 900 rpm
- The stresses are given as local values, i.e. they already embed the concentration factor given by the hole ($K_t = 3$)

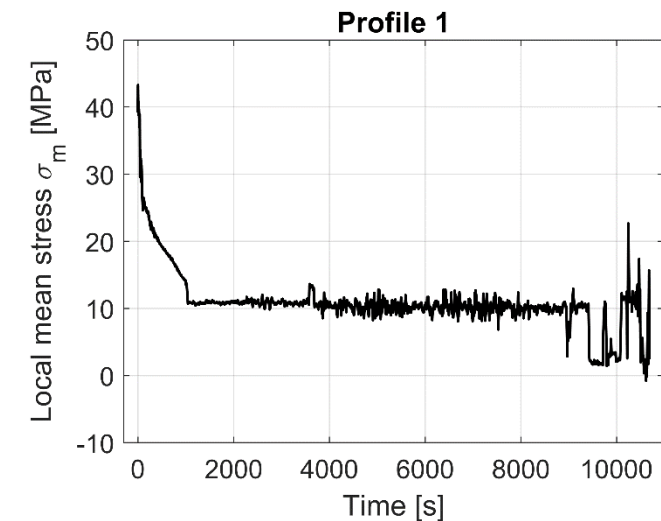
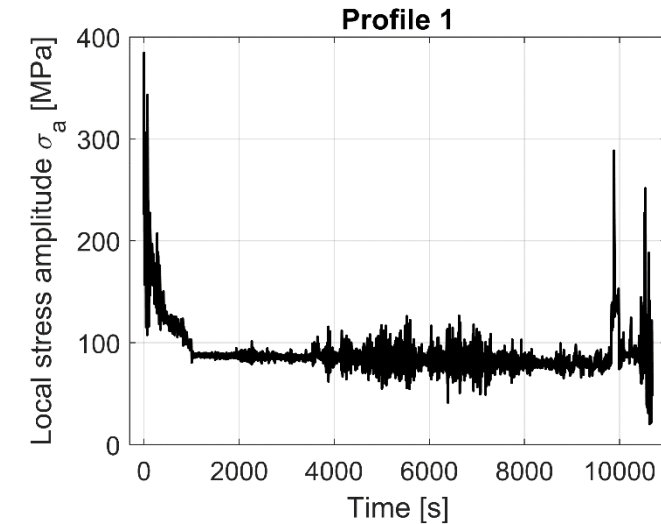
t [s]	σ_a [MPa]	σ_m [MPa]
0	225.707	39.179
0.156	259.589	40.100
0.313	272.900	40.566
0.469	295.062	40.900
0.625	340.689	41.814
0.781	364.744	42.269
0.938	360.365	42.503
1.094	370.745	42.809
1.25	384.434	43.201
1.406	380.566	43.282
1.563	376.857	43.148
...

Each line correspond to a number or cycles:

$$n_i = \omega(t_{i+1} - t_i)$$

ω in revolutions per second!

Round n up to the next integer value.



Part 1 – Material data

Material: AISI430 steel

Elastic modulus [MPa]	210000
Poisson's ratio	0.3
Yielding stress [MPa]	972.2
UTS [MPa]	1103
KIc [MPa m ^{1/2}]	112.1

Fatigue data from the excel files **fatigue_dataX.xlsx**,
where **X is your data code**

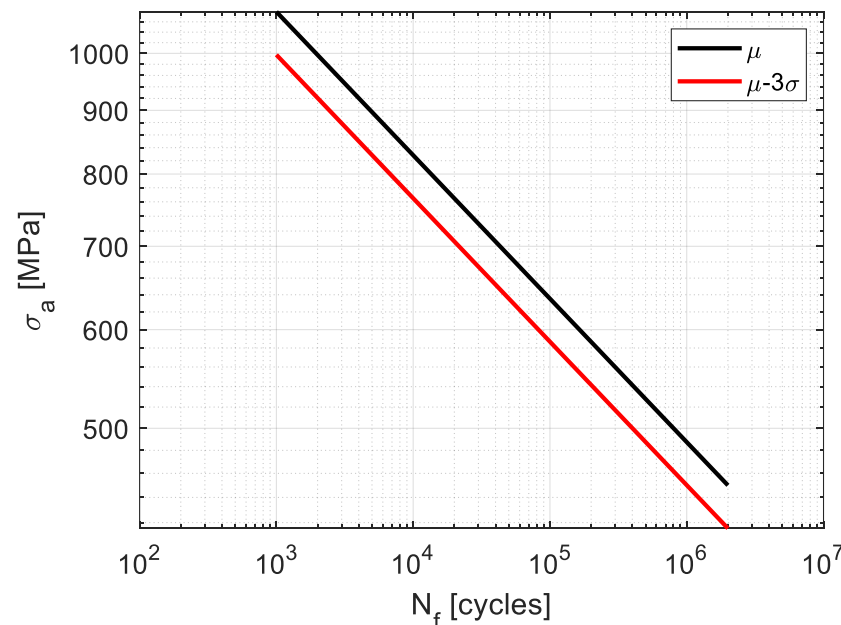
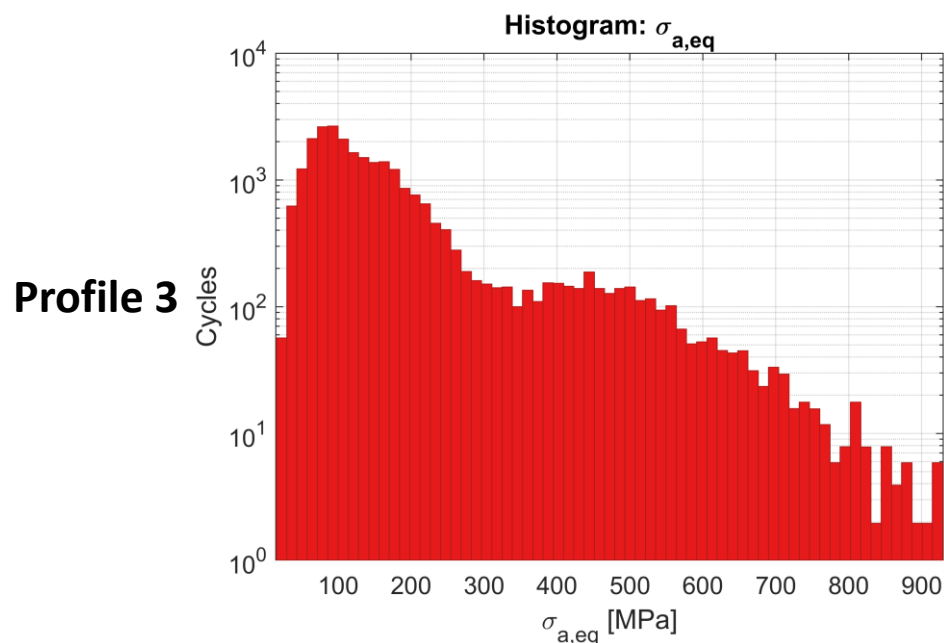
S [MPa]	N [cycles]	Run-out?
750	20924	0
750	19515	0
750	20012	0
750	31092	0
750	26412	0
650	83565	0
650	65788	0
650	67308	0
650	58303	0
650	110779	0
550	391092	0
550	356807	0
550	308519	0
550	276038	0
550	370768	0
450	2000000	1
450	1582332	0
450	1696395	0
450	1855152	0
450	1412137	0

Data for SN fitting

Part 1 – Task 1

Safe-life assessment

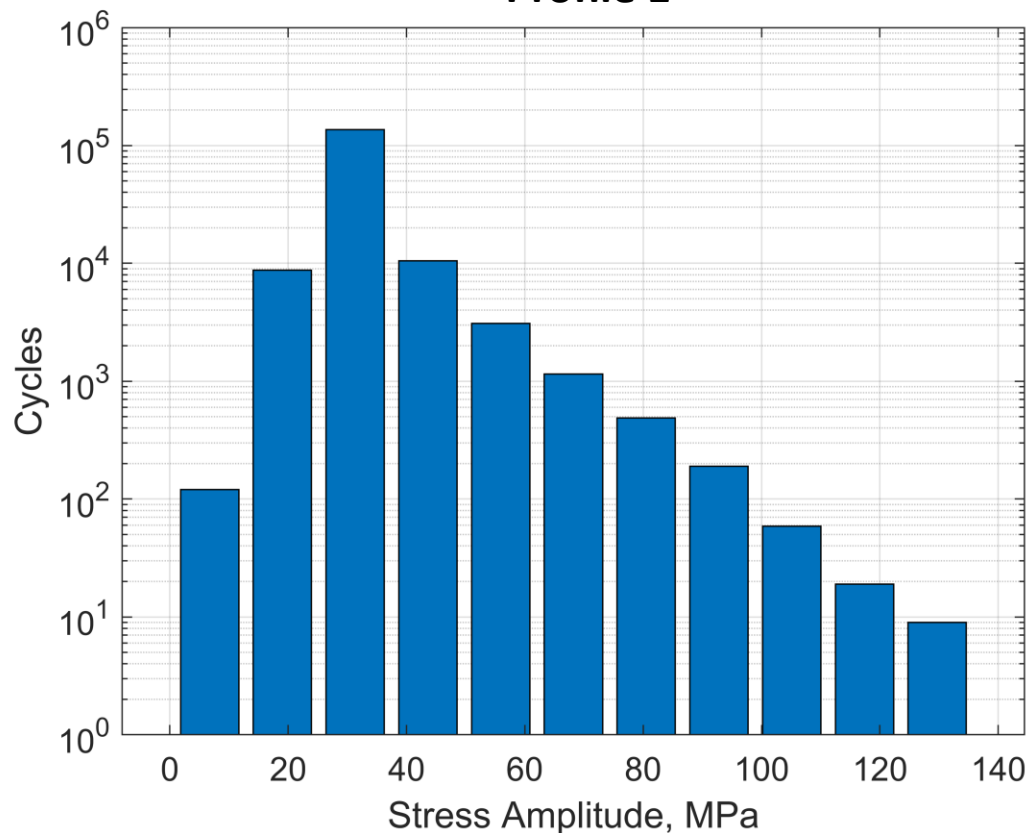
- 1.1) Fit the SN diagram with a 3 parameters model, considering also run-outs.
- 1.2) Calculate the damage associated to a mission composed only by profile 1. Assume constant scatter for the SN diagram, use the $\mu-3\sigma$ curve. Lump the stress history in blocks of approx. 15 MPa. Assume a CV of 5% for the loads and use the $\mu+3\sigma$ load percentile.
- 1.3) Calculate the damage associated to a standard mission (profile 1 + profile 2)
- 1.4) Which is the maximum number of hours of flight for standard missions?
- 1.5) Probabilistic question: Plot a curve of failure probability Pf against life.



SN diagram

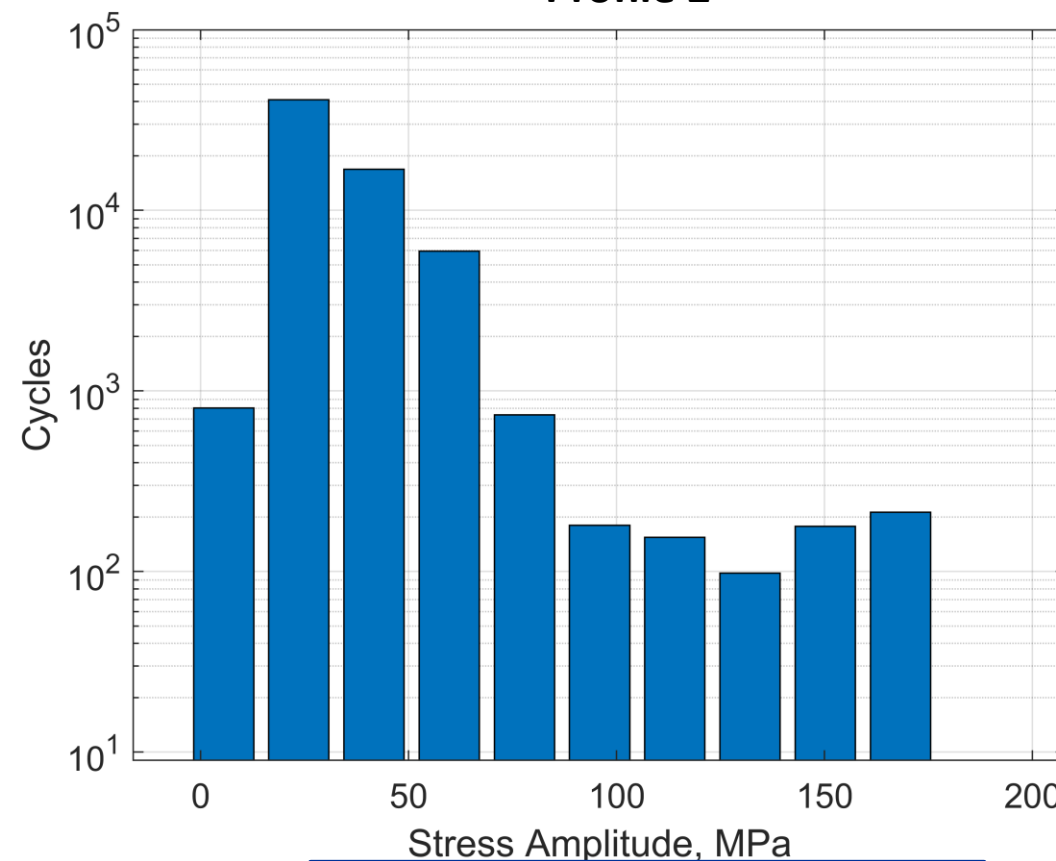
Part 1 – Task 1 Results

Profile 1



$D = 2.06 \times 10^{-6}$ (1 mission)
 → approx. 500 000 missions

Profile 2



$D = 2.27 \times 10^{-4}$ (1 mission)
 → approx. 5 000 missions

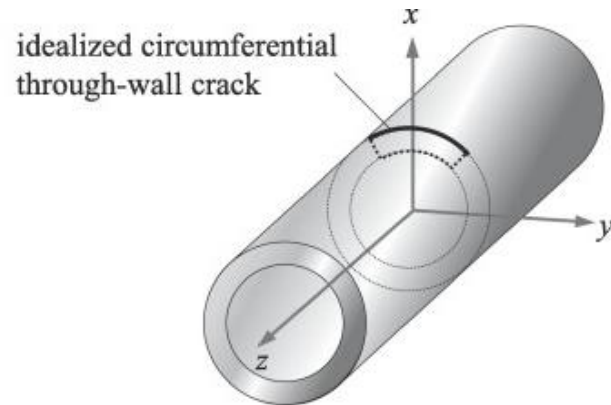
Profile 1+2

$D = 2.29 \times 10^{-4}$ (1 mission)
 → approx. 4 500 missions

Part 2 – Task 1

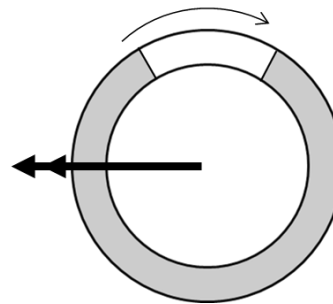
2.1) Determine the critical crack size using the EPFM theory

- Use FAD with option 1
- Use the maximum stress from profile 3 (tip: it corresponds approximately to a bending moment of 70 kN m applied in the critical section of the shaft)

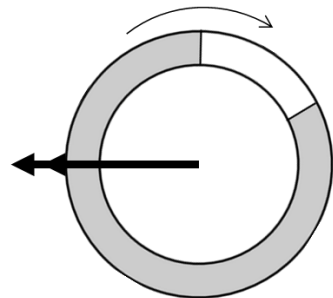


Once the crack grows, we can idealize the component as a pipe with a circumferentially oriented crack

- The crack is fixed in the section
 - The bending moment is rotating
- **Consider the two following cases**



Configuration 1: the crack midpoint is in the maximum stress point
→ **More critical for plastic collapse**

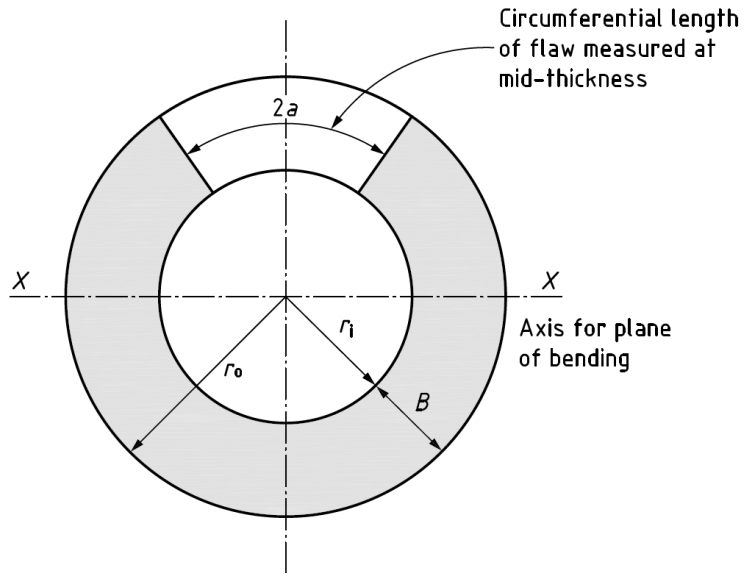


Configuration 2: the crack tip is in the maximum stress point
→ **More critical for instability**

The critical crack size is the smallest value among the two

Part 2 – Task 1

Determine the critical crack size using the EPFM theory – Configuration 1



Stress intensity factor solutions

$$K_I = M_b P_b \sqrt{\pi a}$$

$$\begin{cases} M_b = M_3 + M_4 \\ P_B = \frac{64M}{\pi(D^4 - d^4)} D/2 \\ \lambda = [12(1 - \nu^2)]^{0.25} \frac{a}{\sqrt{r_m B}} \end{cases}$$

λ	M_3	M_4
0	0	1
0.251	0.021	0.828
0.502	0.028	0.733
1.505	0.054	0.544
2.257	0.063	0.45
3.261	0.069	0.364
4.515	0.074	0.299
5.518	0.079	0.264
6.772	0.088	0.23
7.776	0.1	0.205
9.032	0.119	0.179

Ligament yielding factor

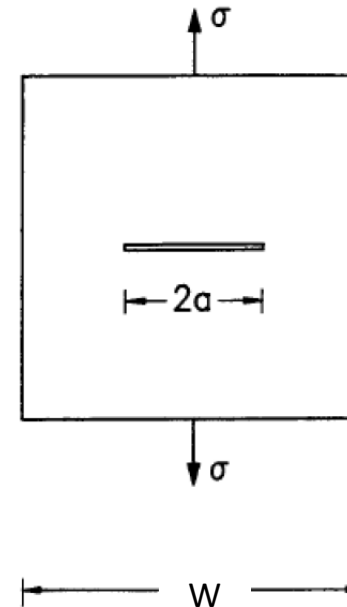
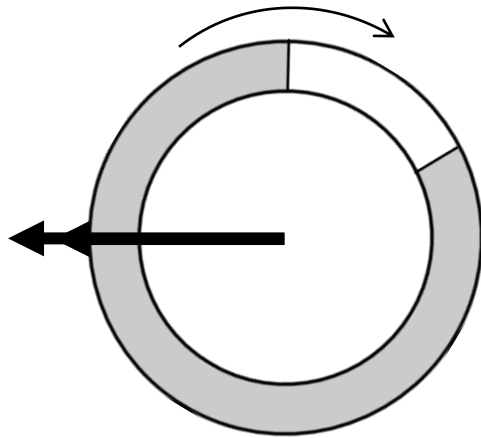
$$L_r = \frac{\sigma_{ref}}{\sigma_Y} \quad \alpha = \frac{a}{r_m}$$

$$\sigma_{ref} = \frac{\pi P_B}{\pi - \alpha - 2 \frac{\sin^2 \alpha}{\pi - \alpha} - \frac{\sin(2\alpha)}{2}}$$

Part 2 – Task 1

Determine the critical crack size using the EPFM theory – Configuration 2

→ The cylinder is approximated with a plate subjected to tension, with width equal to half of the circumference. The stress is the maximum bending stress.



$$\sigma = P_B$$

$$W = \pi r_m$$

Stress intensity factor

$$K_I = Y \sigma \sqrt{\pi a}$$

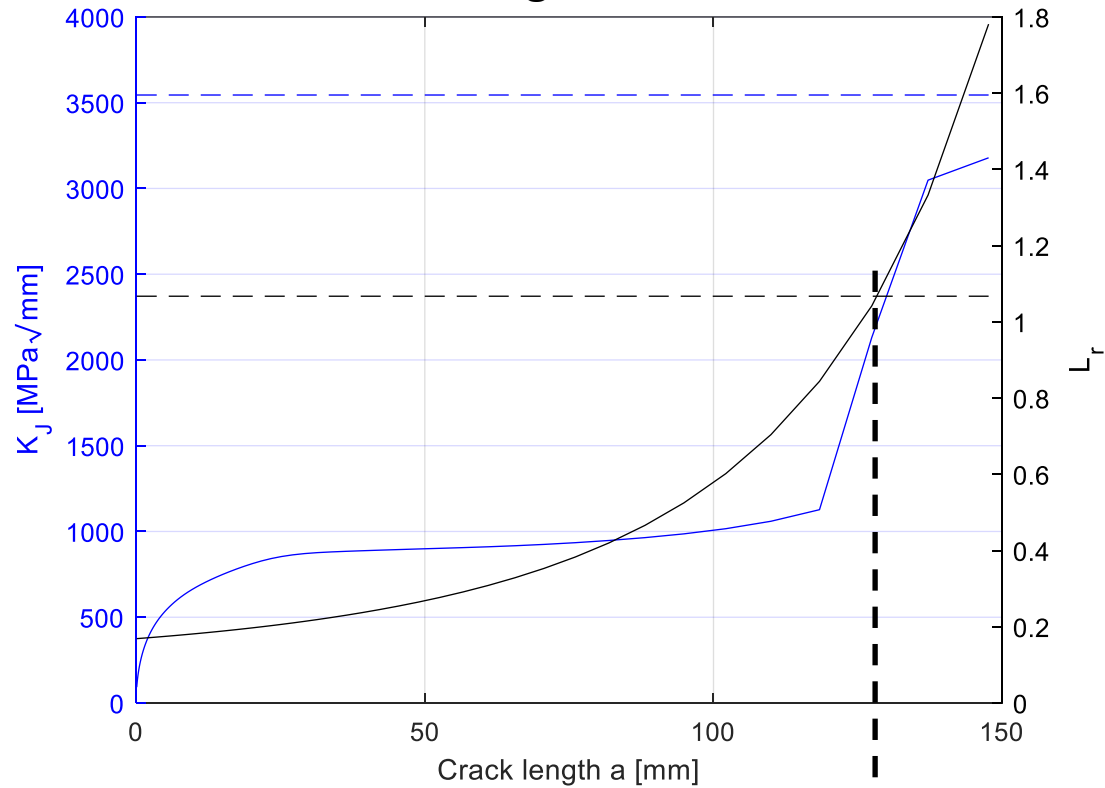
$$Y = [\sec(\pi a/W)]^{0.5}$$

Ligament yielding factor

$$L_r = \frac{\sigma / (1 - 2a/W)}{\sigma_Y}$$

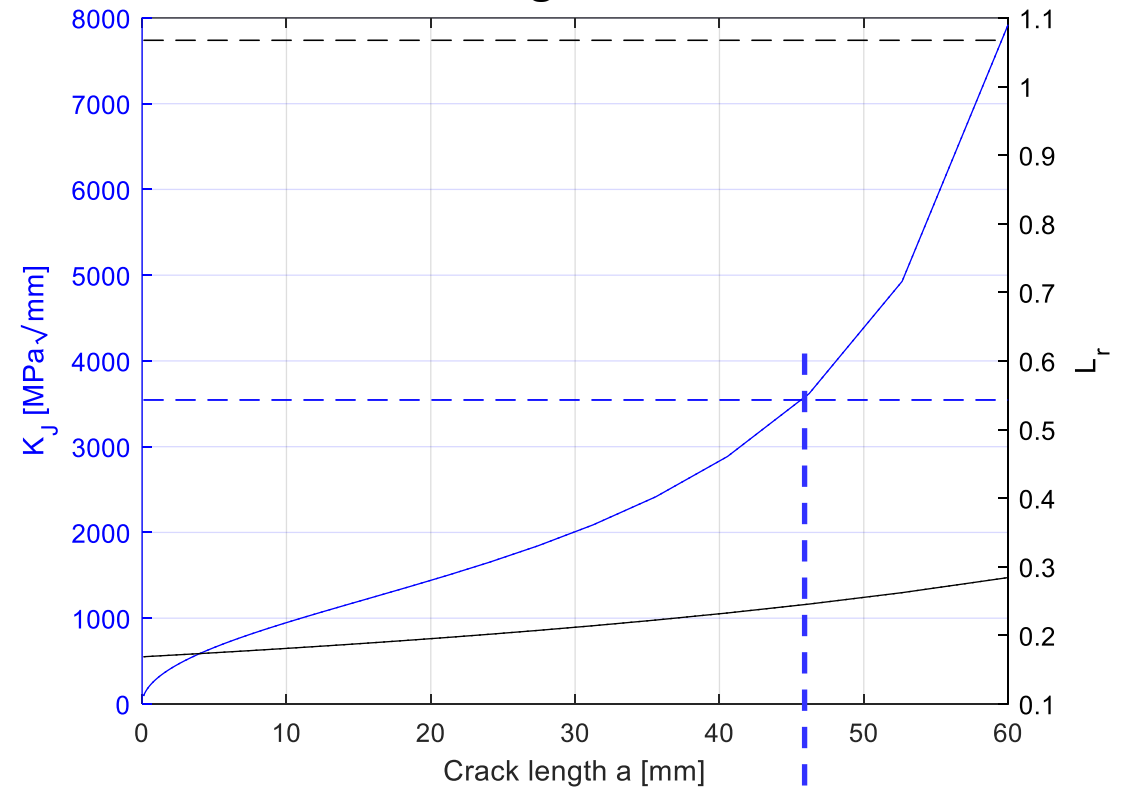
Part 2 – Task 1 Results

Configuration 1



$a_{crit,1} = 128.2$ mm

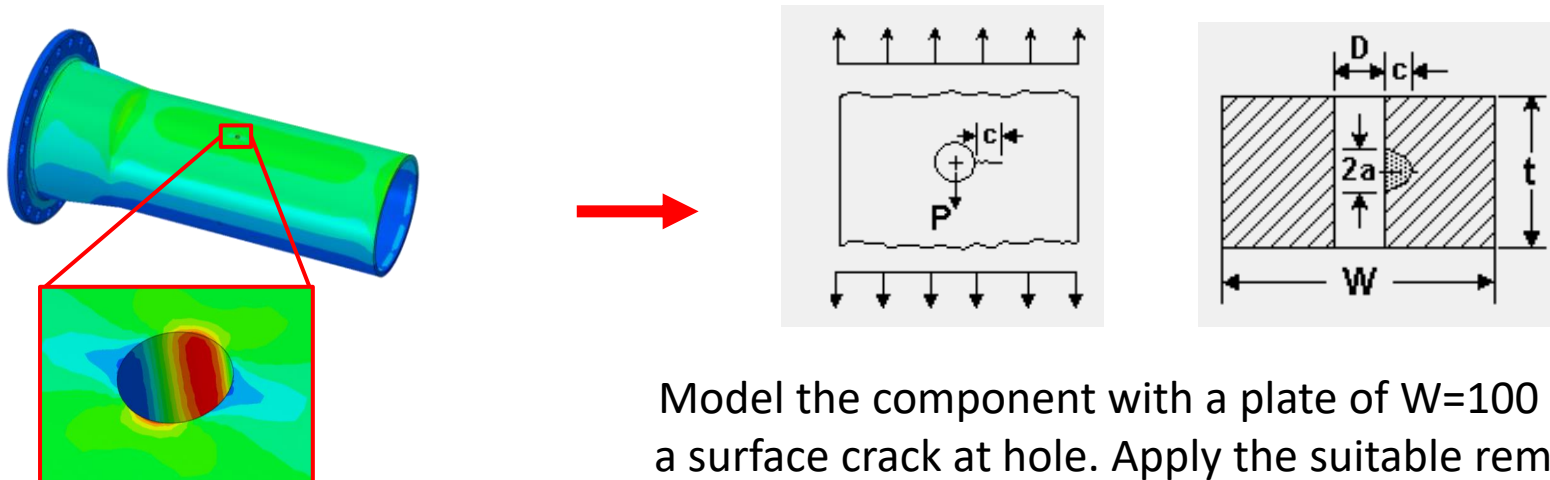
Configuration 2



$a_{crit,2} = 45.6$ mm

Part 2 – Task 2

Perform a damage tolerance analysis considering an initial semi-elliptical surface crack at hole. Consider an initial semi-circular defect with $a = 0.38$ mm. Use the maximum admissible crack computed for Task 2.



Model the component with a plate of $W=100$ mm, with a surface crack at hole. Apply the suitable remote stress ($K_t = 3$, \neq local stress)!

Material: AISI430 steel, different grades according to your project code

Project Code	Material name	NASGRO Material code
1	4340, 1103-1241 UTS	C-4-DE-13AB1
2	4340, 965-1103 UTS	C-4-DD-11AB1
3	4340, 1241-1380 UTS	C-4-DF-13AB1
4	4340, 1379-1571 UTS	C-4-DG-13AB1

Part 2 – Task 2

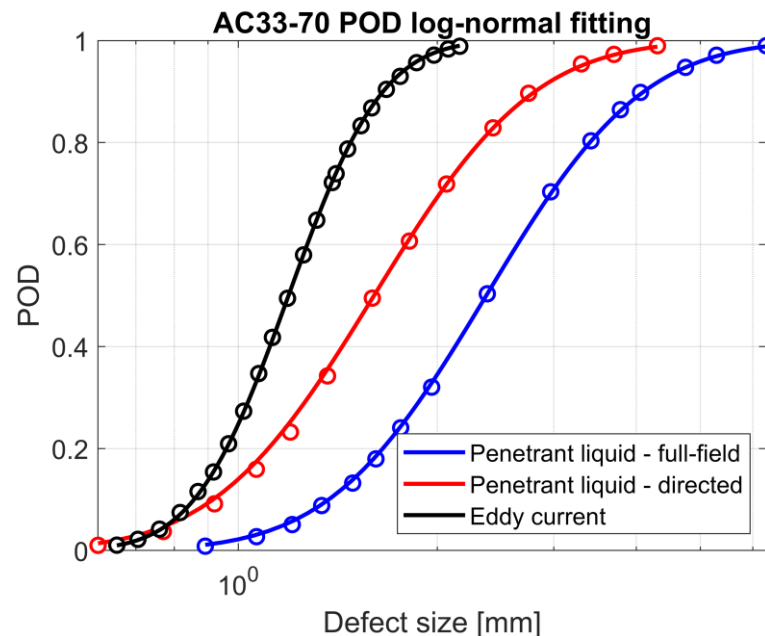
Perform a damage tolerance analysis considering an initial semi-elliptical surface crack at hole. Consider an initial semi-circular defect with $a = 0.38$ mm. Use the maximum admissible crack computed for task 2 (correct the material properties if needed).

Deterministic Damage tolerance assessment:

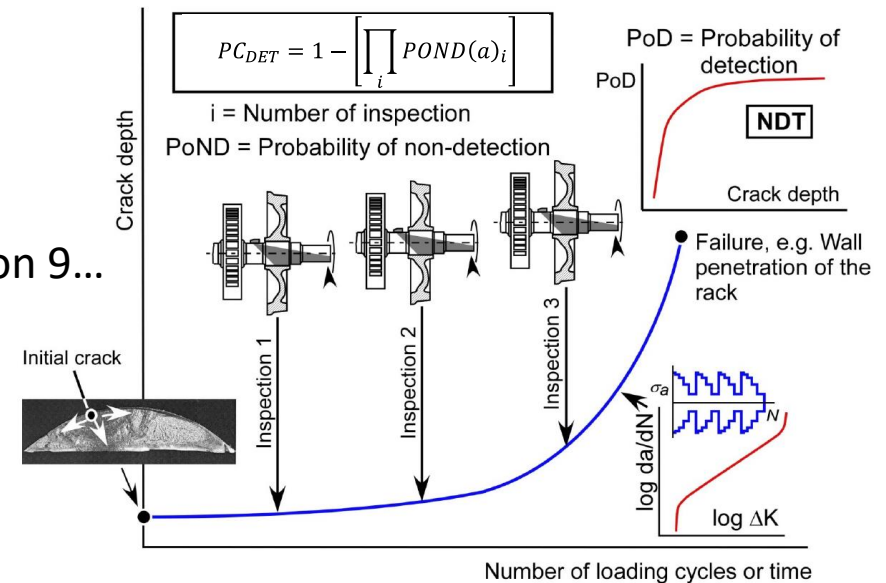
2.2) Find the maximum number of missions for Profile 1. Lump the signal and randomize the sequence.

2.3) Find the maximum number of standard mission (Profile 1 + Profile 2). Lump the signal and randomize the sequence.

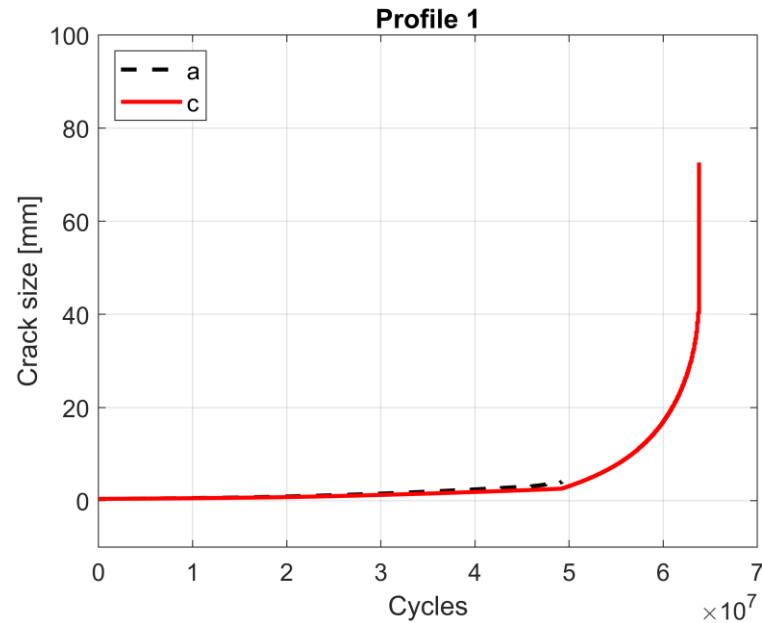
2.4) If the target life of 3000h is not met, find the number of inspections ensuring a failure probability compliant with regulations ($2e-5$). Use the POD curves in the Advisory Circular AC 33-70.2 (Appendix 4)



From Lesson 9...



Part 2 – Task 2 Results



ADVISORY: Crack depth + Yield zone > thickness.

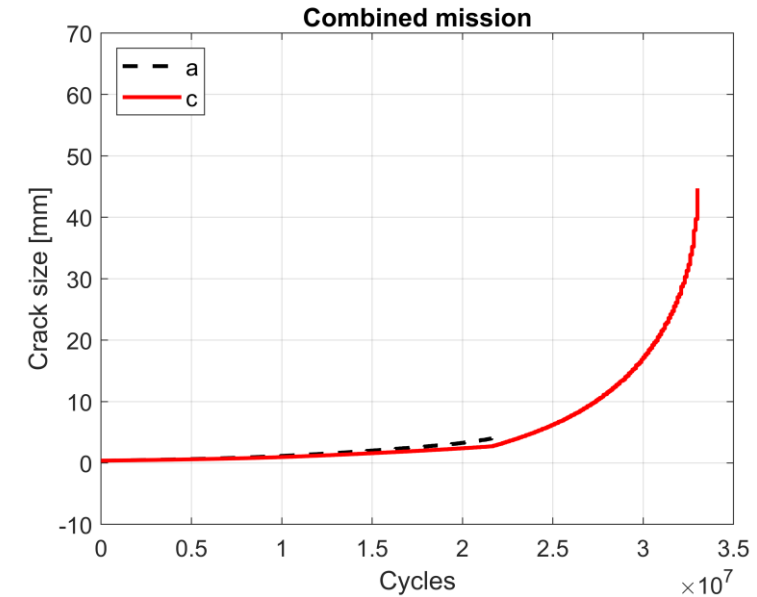
FINAL RESULTS:

Unstable crack growth, max stress intensity exceeds critical value:

K max = 109.9 K(limit) = N.A. K cr = 109.9

at Cycle No. 53590
 # of Load Step No. 1
 # of Block No. 399
 # of Schedule No. 1
 # Crack Size c = 0.725941E-01
 # Total Cycles = 63843836

Numer of admissible missions: 399
 (approx. 1200 h flight)



ADVISORY: Crack depth + Yield zone > thickness.

FINAL RESULTS:

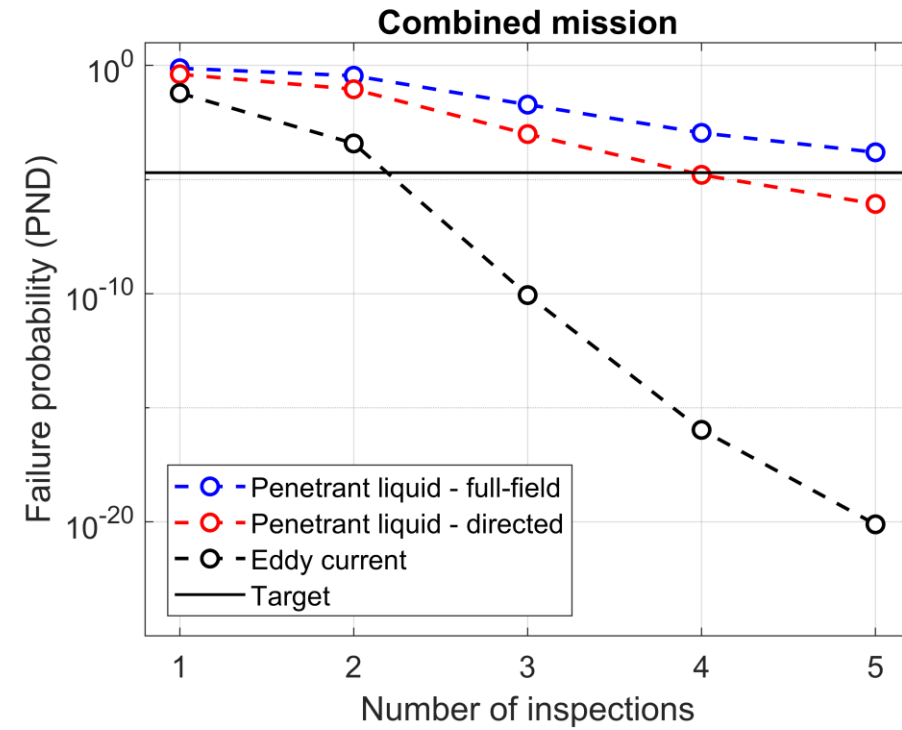
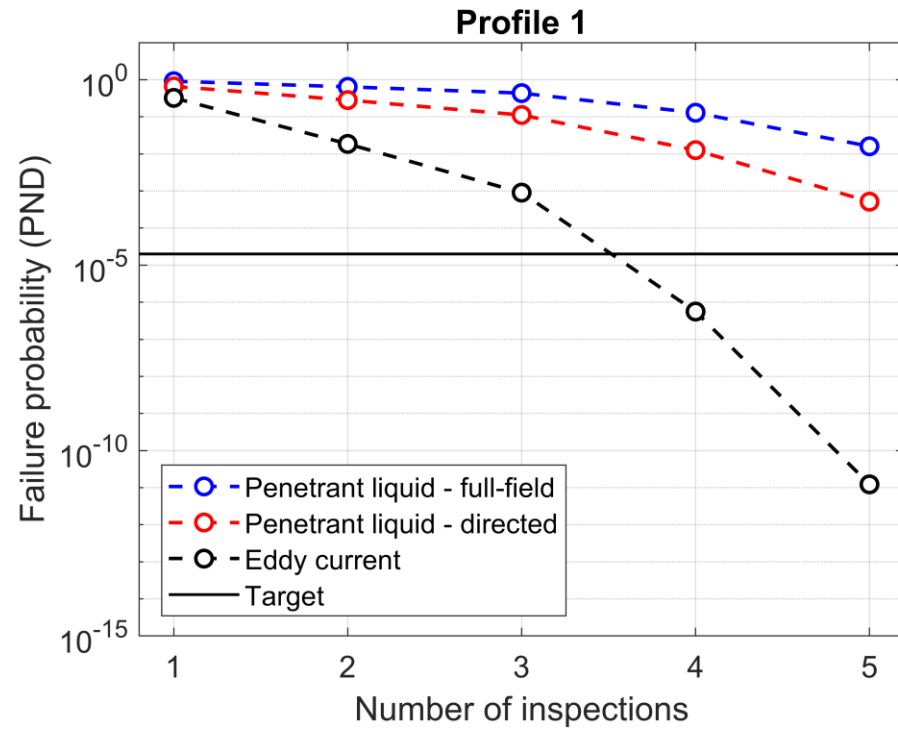
Unstable crack growth, max stress intensity exceeds critical value:

K max = 126.4 K(limit) = N.A. K cr = 109.9

at the very beginning
 # of Load Step No. 17
 # of Block No. 146
 # of Schedule No. 1
 # Crack Size c = 0.447020E-01
 # Total Cycles = 33026301

Numer of admissible missions: 146
 (approx. 600 h flight)

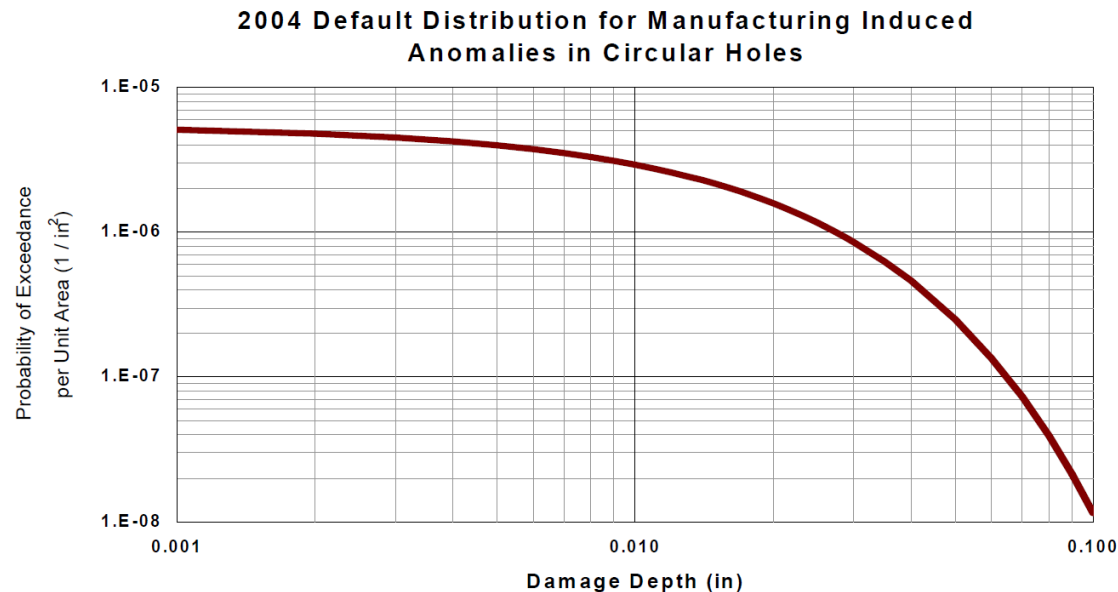
Part 2 – Task 2 Results



Part 3 – Task 1

Probabilistic Damage Tolerance assessment:

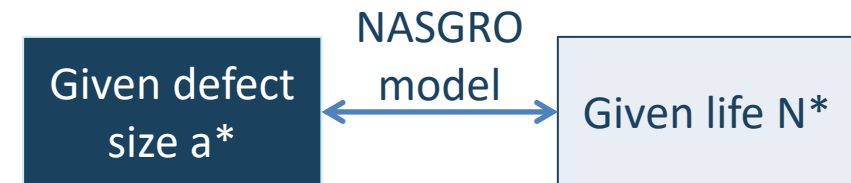
3.1) Starting from the anomalies distribution for circular holes in the AC 33-70.2, perform a probabilistic damage tolerance assessment, considering that we have 8 holes on each of the 4 shafts (weakest link). Which is the life with a probability of being exceeded of $2e-5$ considering only Profile 1? And for a standard mission?



Equation A2-1

$$F(x) = v * 5.42E-06 * \text{EXP}(-61.546*(x))$$

$a \rightarrow$ defect size
 $N \rightarrow$ life



$$\text{Prob}(a \leq a^*) = \text{Prob}(N > N^*)$$

Part 3 – Task 2

Probabilistic Damage Tolerance assessment (optional):

3.2) If the target failure probability is not ensured, apply an inspection plan in a probabilistic framework. Use a Monte Carlo simulation according to the scheme proposed in the AC 33-70.2 (Figure A7-6).

