Structural Reliability of Aerospace Components

Safe-life and Damage Tolerance Analysis of the TP400 Propeller Shaft

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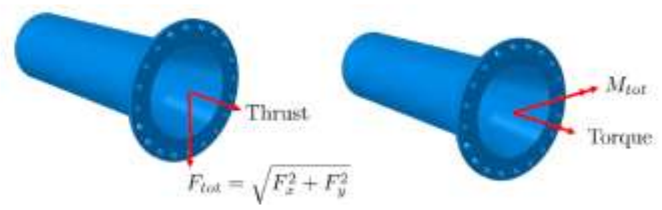
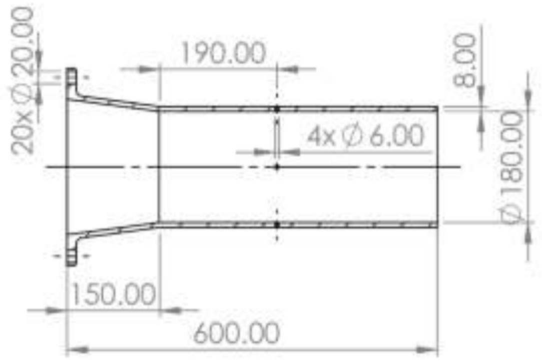
Contents

1 Introduction
1.1 Problem Definition

The **goal** of the project is to do a **safe-life and damage tolerance analysis** of the TP400 propeller shaft, which is a part of the A400M military aircraft.



Assumption: The radial forces generate a moment that is in phase with the bending moment.



During operation:
- Torque and thrust generate a **mean stress** in the section
- Bending moment and radial force generate an **alternate stress** in the section

Stress history data

The stress history (alternate and bending stress) of three mission profiles is given. The stresses are given as local values.

Material data

AISI430 steel	
Elastic modulus [MPa]	210000
Poisson’s ratio [-]	0.3
Yielding stress [MPa]	972.2
UTS [MPa]	1103
KIc [MPa $m^{1/2}$]	112.1

Fatigue data (1)

S [MPa]	N [cycles]	Run-out?
750	20924	0
750	19515	0
750	20012	0
750	31092	0
750	26412	0
650	83565	0
650	65788	0
650	67308	0
650	58303	0
650	110779	0
550	391092	0
550	356807	0
550	308519	0
550	276038	0
550	370768	0
450	2000000	1
450	1582332	0
450	1696395	0
450	1855152	0
450	1412137	0

Part 1
Safe-life assessment

1.1) Fit the SN diagram with a 3 parameters model, considering also run-outs.

1.2) Calculate the damage associated to a mission composed only by profile 1. Assume constant scatter for the SN diagram, use the μ-3σ curve. Lump the stress history in blocks of approximately 15 MPa. Assume a CV of 5% for the loads and use the μ+3σ percentile.

1.3) Calculate the damage associated to a standard mission (profile 1 + profile 2)

1.4) Which is the maximum number of hours of flight for a standard mission?

1.5) Probabilistic question: Plot a curve of failure probability P_f against life.

Part 2
Deterministic damage-tolerance assessment

2.1) Determine the critical crack size using the EPFM theory.

2.2) Find the maximum number of missions for profile 1. Lump the signal and randomize the sequence.

2.3) Find the maximum number of standard missions (profile 1 + profile 2). Lump the signal and randomize the sequence.

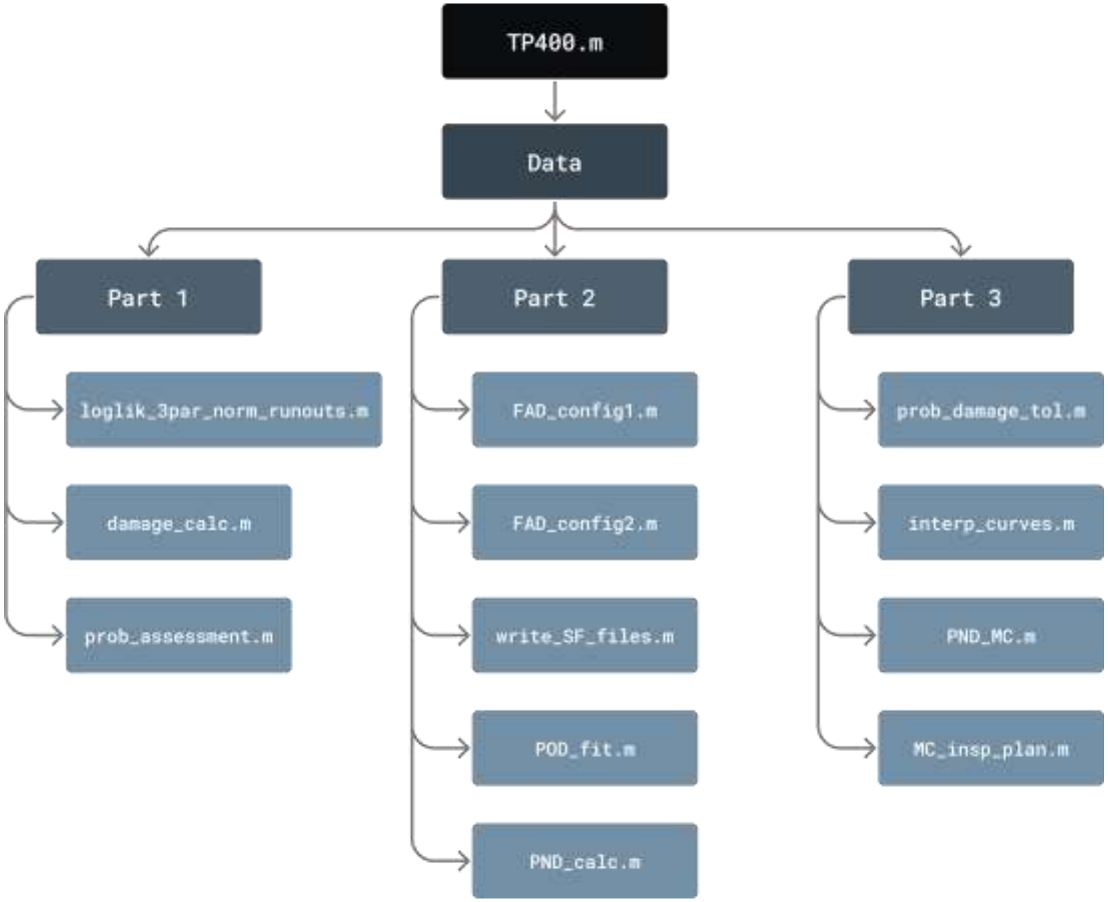
2.4) If the target life of 3000h is not met, find the number of inspections ensuring a failure probability compliant with regulations (2e-5). Use the POD curves in the Advisory Circular AC-70.2.

Part 3
Probabilistic damage tolerance assessment

3.1) Starting from the anomalies distribution for circular holes in the AC 33-70.2, perform a probabilistic damage tolerance assessment, considering that we have 8 holes on each of the 4 shafts (weakest link). Which is the life with a probability of being exceeded of 2e-5 considering only Profile 1? And which for a standard mission?

3.2) If the target failure probability is not ensured, apply an inspection plan in a probabilistic framework. Use a Monte Carlo simulation according to the scheme proposed in the AC 33-70.2.

1.4 Code Structure



Part 1
Safe-life Assessment

2 Results: Part 1
2.1 Task 1.1

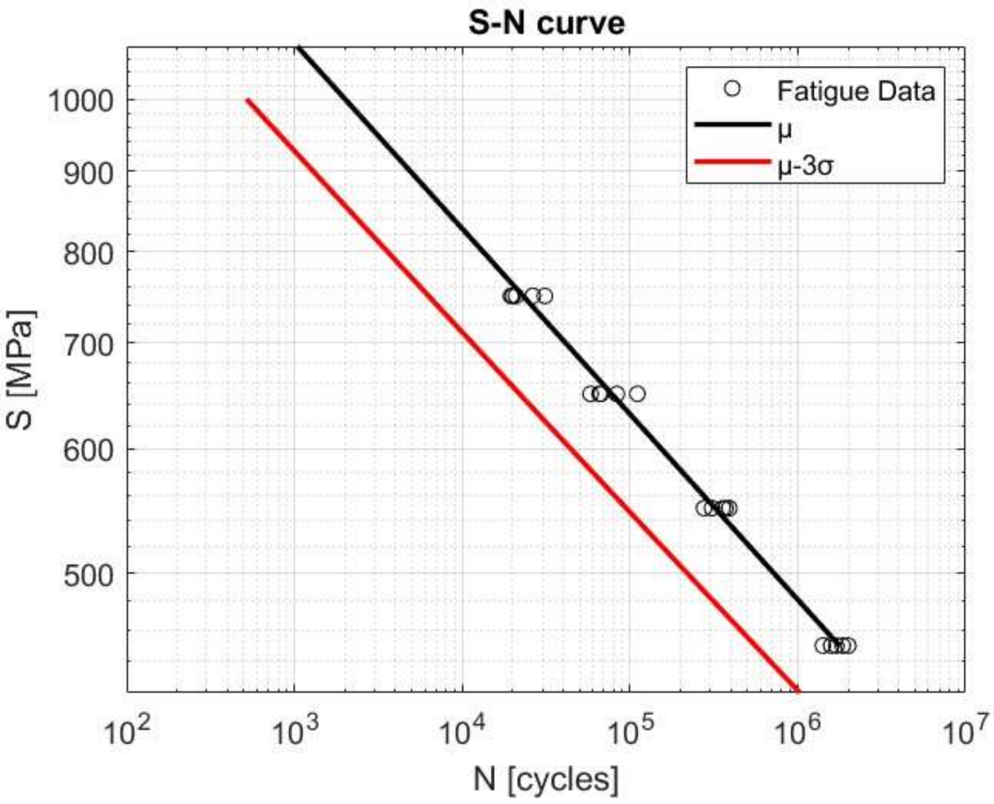
Procedure:

1. Make an initial assumption for x_0. In this case $x_0 = [5 \ -2\ std(\log N)]$.
2. Use the functions fminsearch and fminunc to find the minimum of the log-likelihood function. The log-likelihood function is calculated by the Matlab function shown below:

xx → loglik_3par_norm_runouts.m → loglik

Where the loglik function values are calculated according to:

Failure	Runout
$-0.5 \cdot \ln 2\pi - \ln \sigma_i - 0.5 \cdot z_i^2$	$\ln[1 - \Phi(z_i)]$



Parameters of the model:

$$a_0 = 5.2522$$
$$a_1 = -8.4923$$
$$a_2 = 0.0783$$

Therefore:

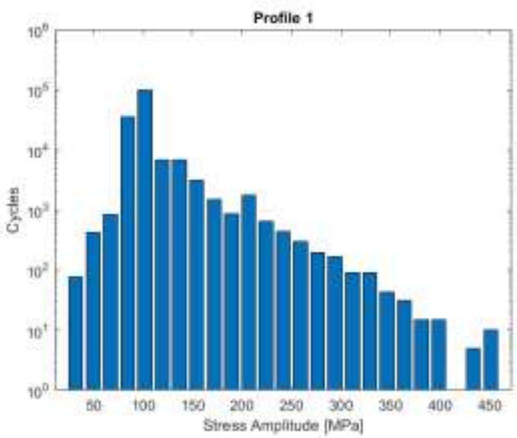
$$\log N_i \sim N(a_0 + a_1 \cdot (\log s_i - \log s_{avg}\, , a_2^2)$$

Procedure:

1. Calculate Sa_{eq} from the equation: $Sa_{eq} = {}^{Sa}/_{1-\frac{Sm}{UTS}}$.
2. Calculate the bins needed for lumping the stress history in blocks of 15 MPa.
3. Lump the stresses using hist ([Nb, Sc] = hist(Sa_{eq}, bins)). This is valid for one cycle of Sa_{eq}, therefore Nb must be multiplied with the real number of cycles and then rounded up, as an integer is needed.
4. Use the **Top of Scatter approach**, where the μ+3σ curve is used for the stresses, while μ-3σ curve is used for the SN diagram. Therefore: $Sc_{3\sigma} = (1 + 3CV) \cdot Sc$, where CV is assumed to be 5% and $Nf_{-3\sigma} = 10^{(a_0+a_1(\log Sc_{3\sigma} - \log Sc_{avg}) - 3a_2)}$. The is $LR = 0.5382$, therefore the approach is valid.
5. The damage of one mission is calculated with the equation: $D = \frac{Nb \cdot number\ of\ cycles}{Nf_{-3\sigma}}$. Calculate the damage for Profile 1 (Task 1.2). Then calculate the damage associated to Profile 2 and add to the damage of Profile 1 in order to obtain the damage for a standard mission (Task 1.3). Ultimately, multiply the inverse of the damage of the standard mission (this is equal with the total admissible missions until failure) with the number of flight hours.

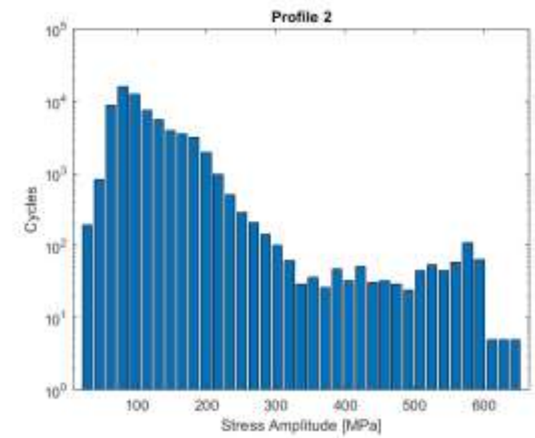
profile, mat, a0, a, a2, logsp_avg, blocks, CV → damage_calc.m → D, MISSIONS, TH Nb, Sc

Profile 1



$D = 5.9983 \cdot 10^{-5}$ (1 mission)
Approx. 16771 missions

Profile 2



$D = 0.0029$ (1 mission)
Approx. 342 missions

$D = 0.003$ (1 mission)
Approx. 335 missions (or 1341 hours)

$$D = \sum_i \frac{n_i}{N_{Fi}}$$

and

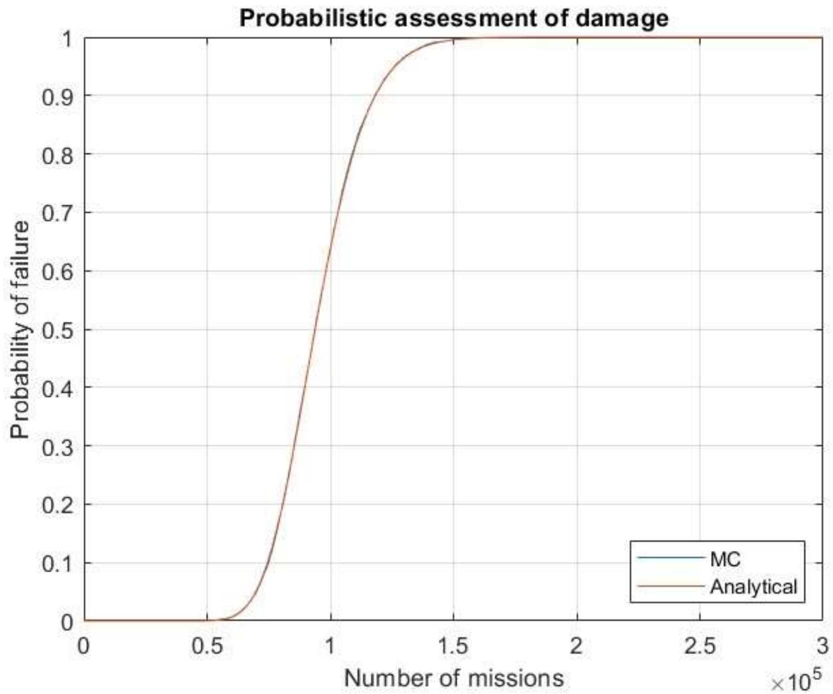
$$\log N = a_0 + a_1\left(\log S - \log S_{avg}\right) = \log 10^{a_0} + a_1\left(\log S/S_{avg}\right)$$
$$= \log 10^{a_0} - \log\left(\frac{S}{S_{avg}}\right)^{-a_1} = \log\left(\frac{C}{\left(\frac{S}{S_{avg}}\right)^m}\right)$$

The parameter C has a log-normal distribution with $\mathrm{N}(\alpha_0, a_2^2)$. The Probabilistic Damage calculation can be computed analytically, with the mean depending on the number of missions and the standard deviation being equal with a_2, or by applying a MC procedure.

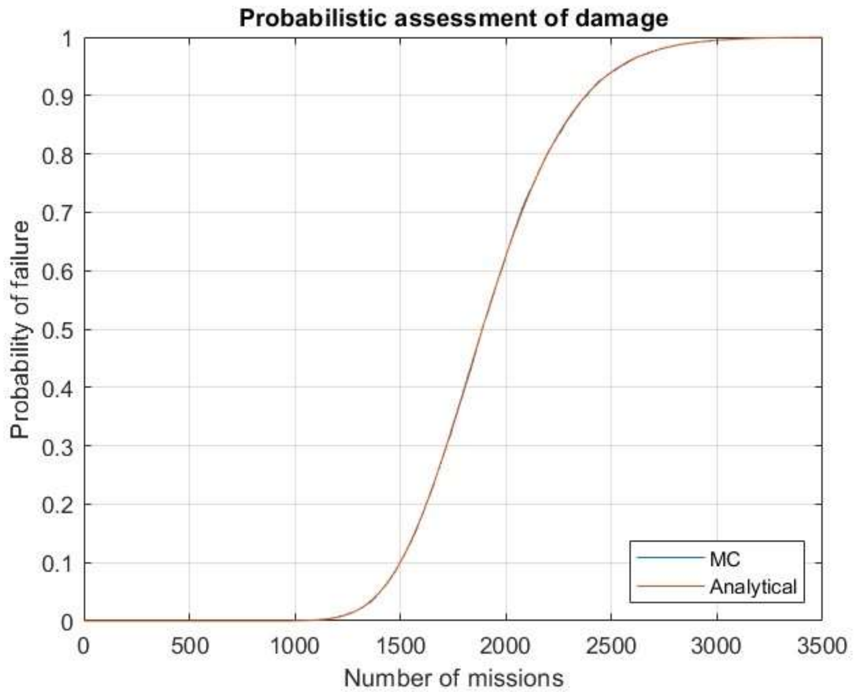
$$D = \sum_i \frac{n_i}{\frac{C}{\left(\frac{S_i}{S_{avg}}\right)^m}}$$

1. N simulations of C
2. Calculate N fracture cycles N_f, for each stress level
3. Calculate N mission damages (D)
4. Loop for $N_{missions}$ and calculate the accumulated damage for each simulation
5. Find the values of $D \geq 1$ and calculate $P_f = \frac{N_{D\geq 1}}{N}$

Profile 1

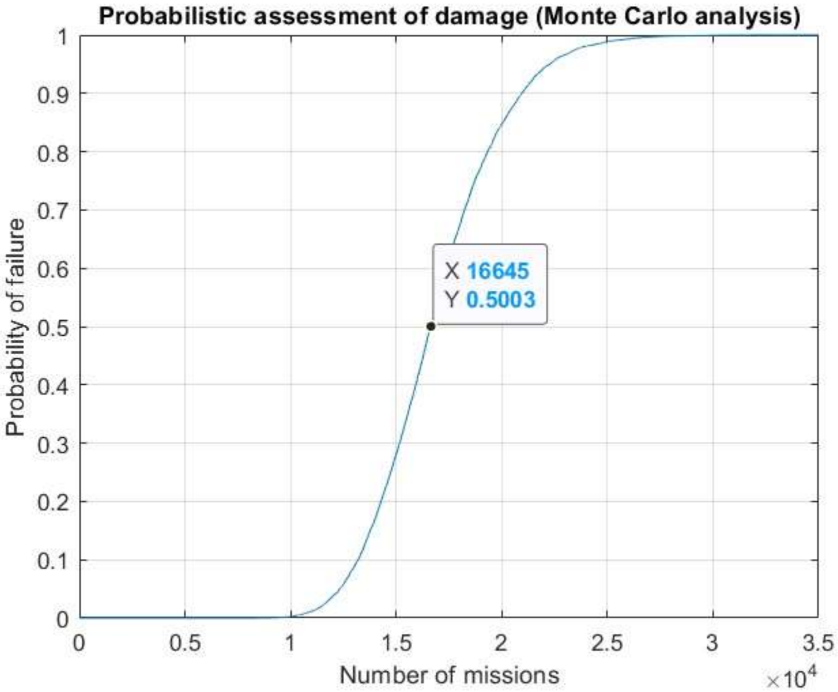


Standard Mission

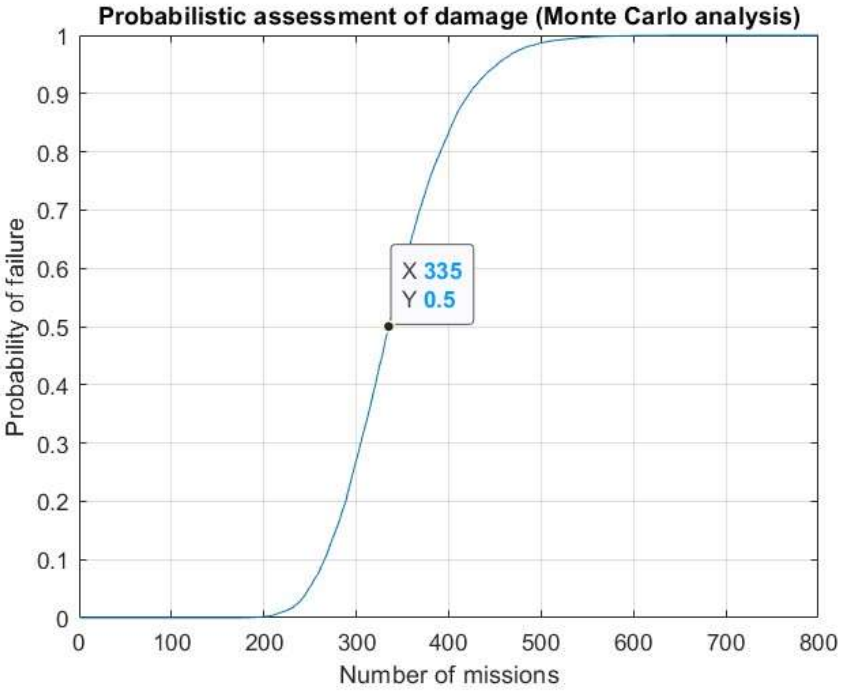


Validation

Profile 1

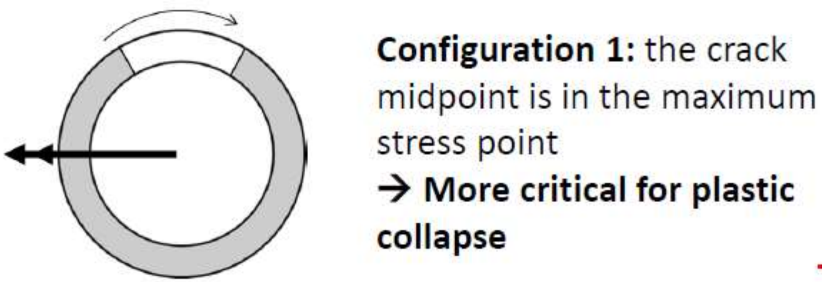


Standard Mission

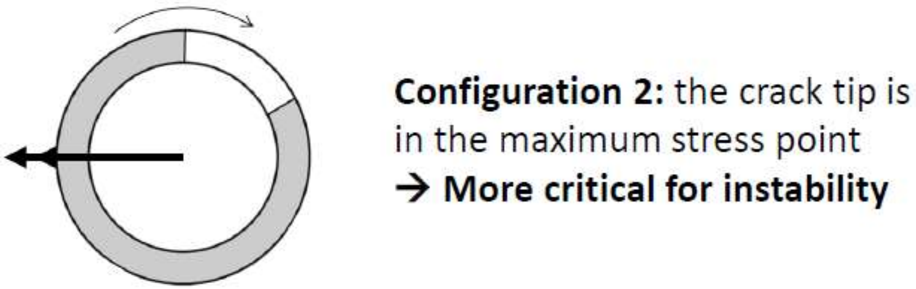


Part 2
Deterministic Damage Tolerance Assessment

Determine the critical crack size for the following configurations:



Approximated as a circumferential through-wall crack

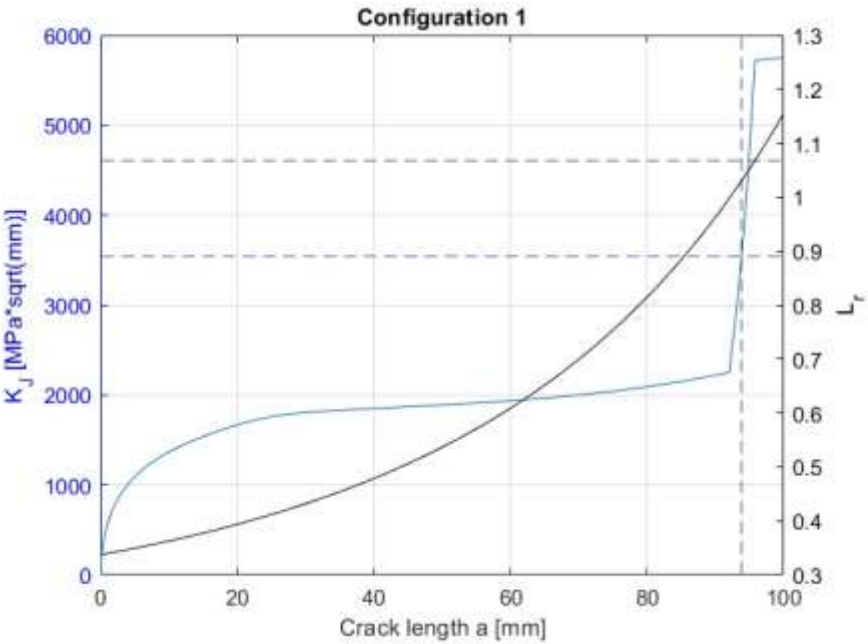


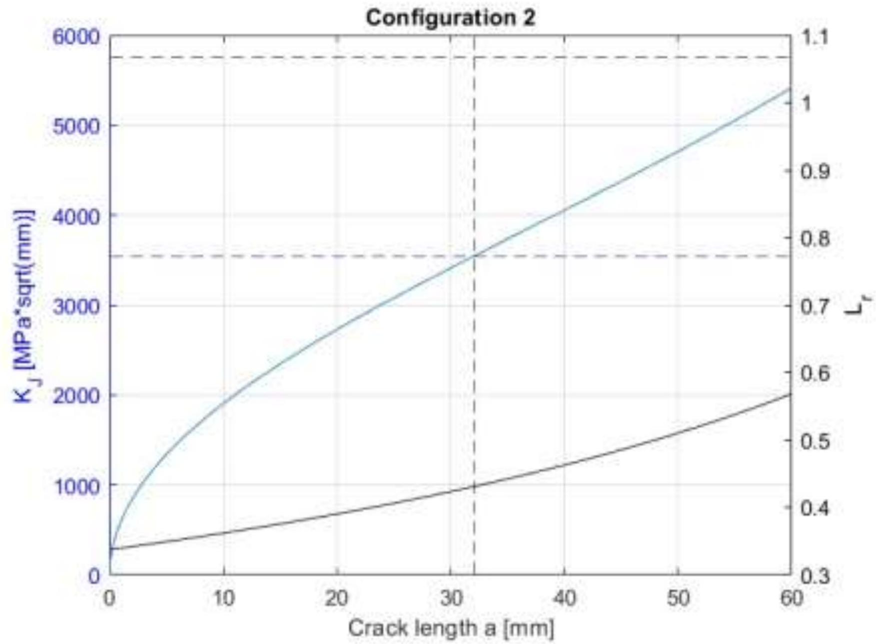
Approximated as a plate with a middle crack

The maximum stress of Profile 3 is used for the determination of the critical crack size, which corresponds to a bending moment of 70 kNm.

Procedure:

1. Assume a maximum crack size and create an equispaced array.
2. Define K_{IC} and Lr_{max} (from material data).
3. Calculate the ligament yielding factor L_r and the stress intensity factor K_I for each crack size. For Configuration 2, the procedure is straightforward, while for Configuration 1, for each crack size the parameter λ must be calculated and then the parameters M_3 and M_4 are interpolated to calculate K_I.
4. Calculate the correction factor $f(L_r)$ for ligament yielding for each crack size using the continuous yielding option (the code gives the option to calculate the plateau option also).
5. The critical crack size for each configuration is the smallest value between: Instability (when K_J reached K_{IC}) and plastic collapse (when L_r reaches $L_{r_{max}}$). The critical crack size of the part is the smallest value between the two configurations.





$a_{crit_1} = 93.994\ mm$

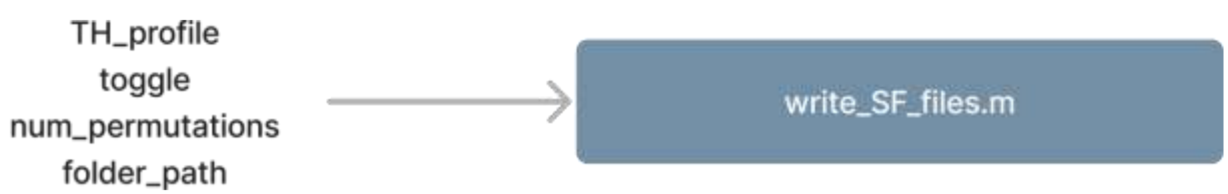
$a_{crit_2} = 32.012\ mm$

$a_{crit} = 32.012\ mm$

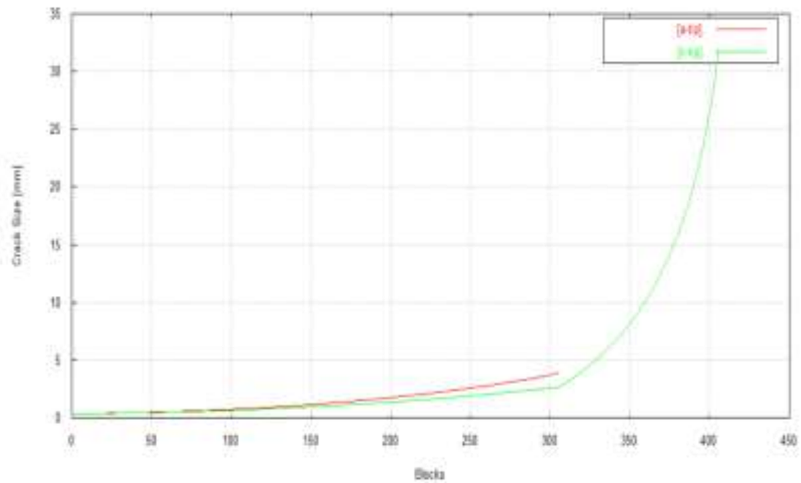
3 Results: Part 2
3.2 Tasks 2.2, 2.3

Procedure:

1. From the damage_calc.m function, the lumped stresses are available (note: The stresses used are the mean ones, not the ones used for the top of scatter approach).
2. The lumped signal is randomized 10 times and 10 SF files are created.
3. The lumped signal is inserted in NASGRO, considering an initial semi-circular defect of a=0.38mm and the maximum admissible crack from the previous task is used for the termination of the crack growth analysis. This is done for all SF files.
4. The load sequence with the least number of admissible missions is used for the following analysis (note: the load sequence didn't affect the number of missions – but number of cycles).



Profile 1

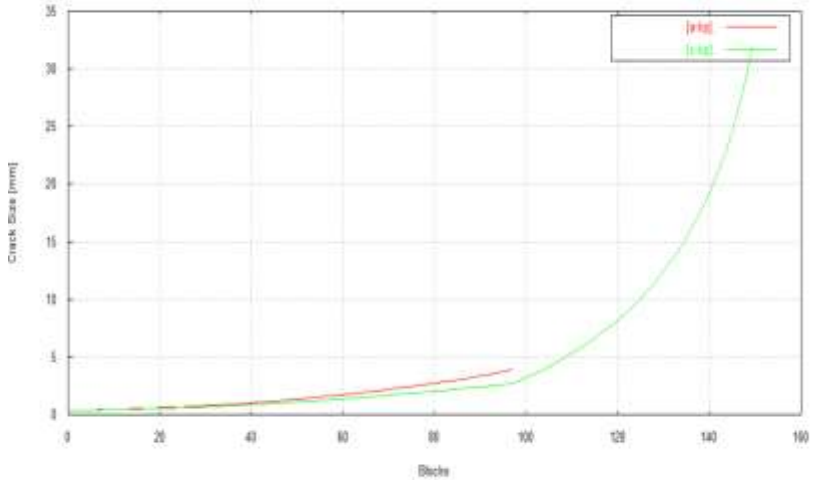



```
FINAL RESULTS:
Crack size reached user-specified crack size limit:
# Crack size of current crack model (TC03)    =  32.0153
# Converted limit after transition to TC03    =  32.0120
# Original limit specified at a-tip of SC11 =*********

# at Cycle No.                5043
# of Load Step No.      15
# of Block No.           6
# of Schedule No.        5
# Crack Size  c =    32.0073
# Total Cycles      =      65353680
```

Number of admissible missions:
406 (approx. 1218 hours)

Standard Mission




```
FINAL RESULTS:
Crack size reached user-specified crack size limit:
# Crack size of current crack model (TC03)    =  32.0212
# Converted limit after transition to TC03    =  32.0120
# Original limit specified at a-tip of SC11 =*********

# at Cycle No.                39781
# of Load Step No.      28
# of Block No.          50
# of Schedule No.        2
# Crack Size  c =    32.0132
# Total Cycles      =      33983059
```

Number of admissible missions:
150 (approx. 600 hours)

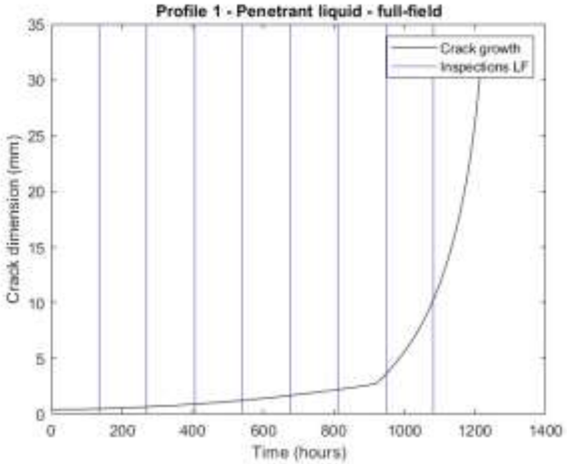
Since **the target life of 3000h is not met**, inspections intervals should be defined to ensure a P_f of 2e-5.

Procedure:

1. Firstly, the POD data given must be fitted through a normal distribution. This is done by the POD_fit.m function, which finds the 0.5 probability of detection value of the data (mu) and the standard variation. Then the CDF is calculated with mu and sigma.
2. From the previous calculations in NASGRO, the crack growth profile (avsN) is loaded.
3. Using a while loop, the avsN curve is divided into equispaced inspection intervals.
4. The avsN curve is interpolated to find the crack size at the inspection intervals.
5. The POD curve fit is interpolated to find the probability of detection at the crack size.
6. The probability of failure is calculated as: $PND = \prod(1 - \mathrm{POD})$.
7. Steps 2-6 are repeated until the probability of failure is less that the target probability.

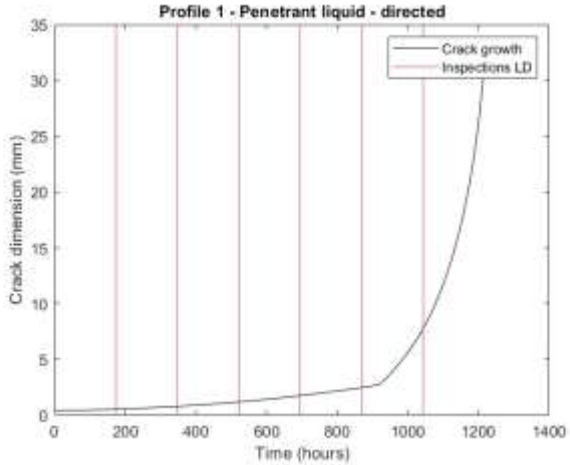
Profile 1

Penetrant liquid: full field



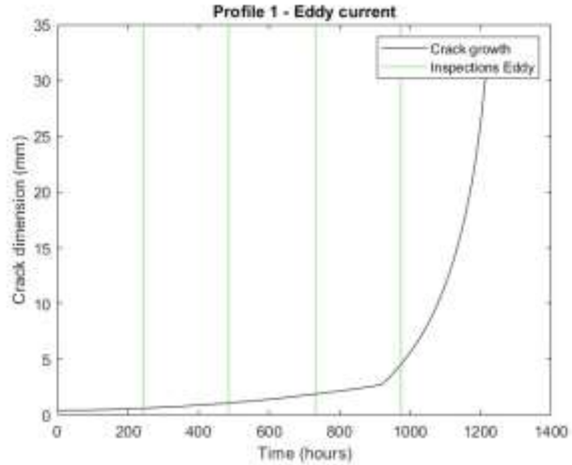
Inspection interval: 135.25 hours

Penetrant liquid: directed



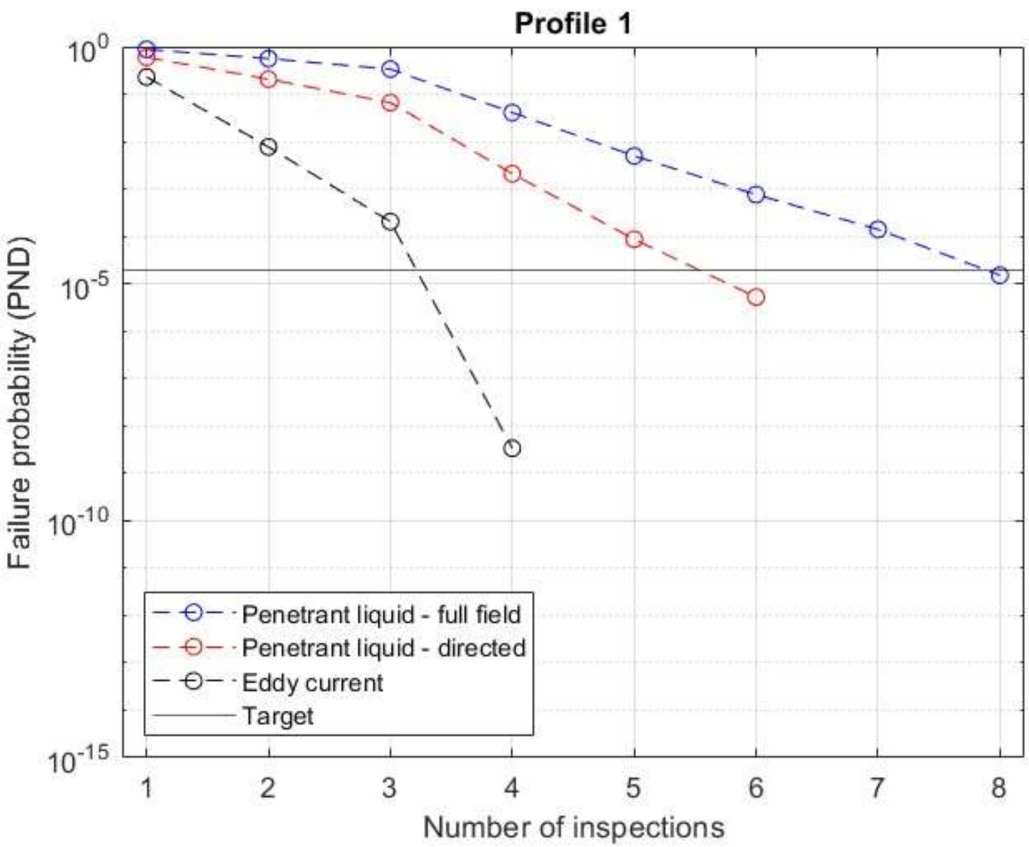
Inspection interval: 173.90 hours

Eddy current

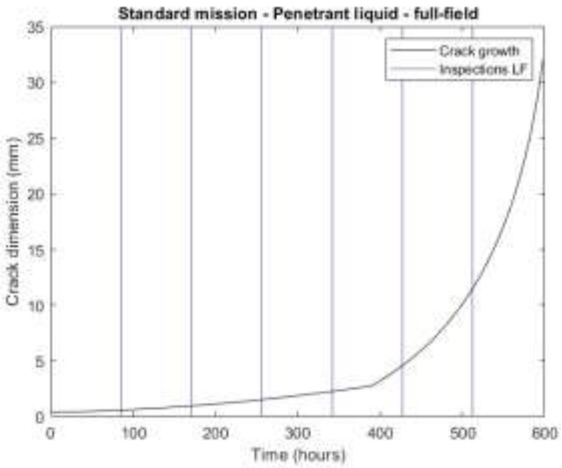
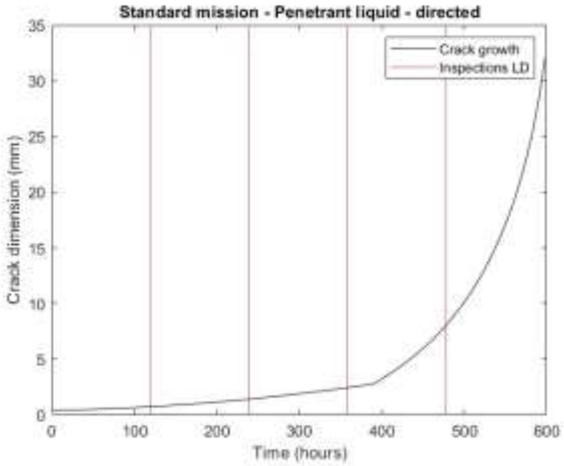
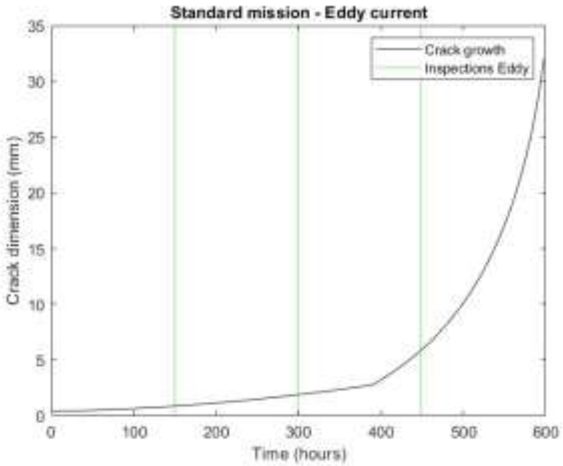


Inspection interval: 243.46 hours

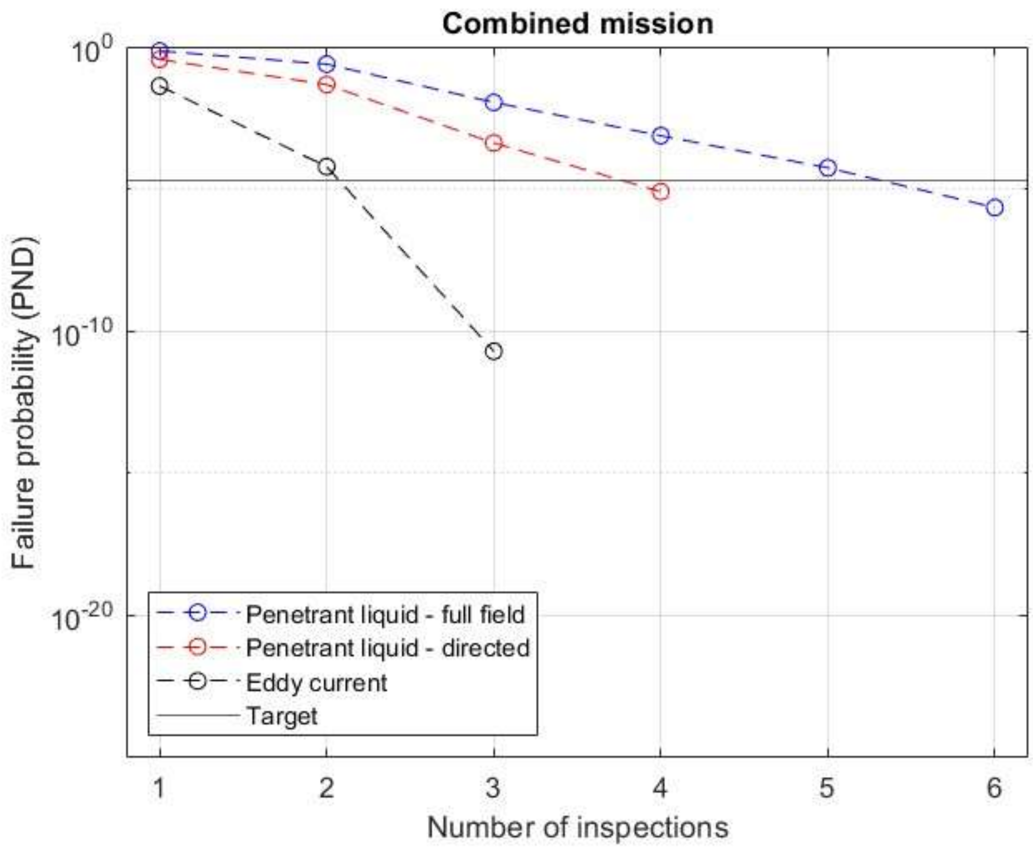
Profile 1


Profile 1
Failure probability (PND)
10^0
10^-5
10^-10
10^-15
1
2
3
4
5
6
7
8
Number of inspections
Penetrant liquid - full field
Penetrant liquid - directed
Eddy current
Target

Standard Mission

Penetrant liquid: full field	Penetrant liquid: directed	Eddy current
		
Inspection interval: 85.36 hours	Inspection interval: 119.50 hours	Inspection interval: 149.38 hours

Standard Mission



Part 3
Probabilistic Damage Tolerance Assessment

4 Results: Part 3

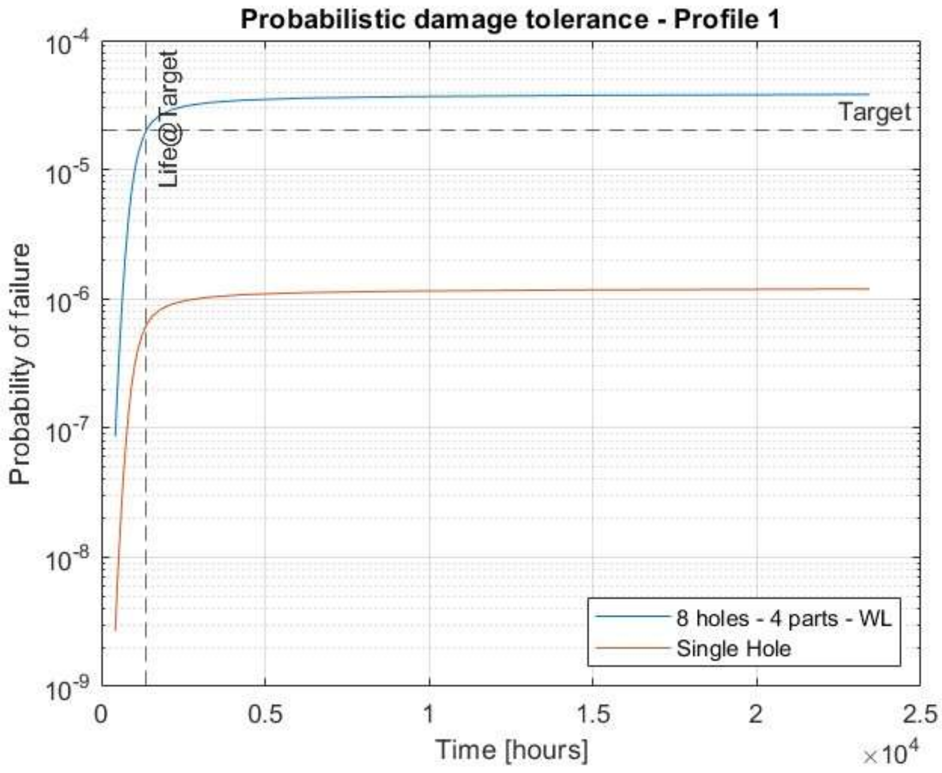
4.1 Task 3.1

Procedure:

1. Create an anomaly crack size array from 0.001 to 0.100 inches. For this task 20 crack sizes were calculated, distributed unevenly due to the large variance of life in the smaller region of cracks.
2. Use NASGRO's Fatigue Crack Growth to calculate the missions (or hours) needed for each crack size to reach the critical crack size calculated previously.
3. Calculate the ***Probability of Exceedance (POE)*** for a single hole, using the formula $F(x) = v * 5.42 \cdot 10^{-6} * e^{-61.456x}$. The parameter v is equal to 1, as L/D is larger than 1.3.
4. Calculate the ***Probability of Failure (hpf)***, by calculating the POE with the area of the hole (area = circumference * depth).
5. Calculate the ***Probability of Failure (POF)***, by using the weakest link approach. The POF is given by the equation: $POF(n) = 1 - (1 - hpf)^n$, where n is the number of holes. For 8 holes in each of the 4 shafts, n is equal to 32.
6. Find the hours needed for a POF of 2e5.

n_missions, crack_geom → prob_damage_tol.m → Pf_series, life_profile

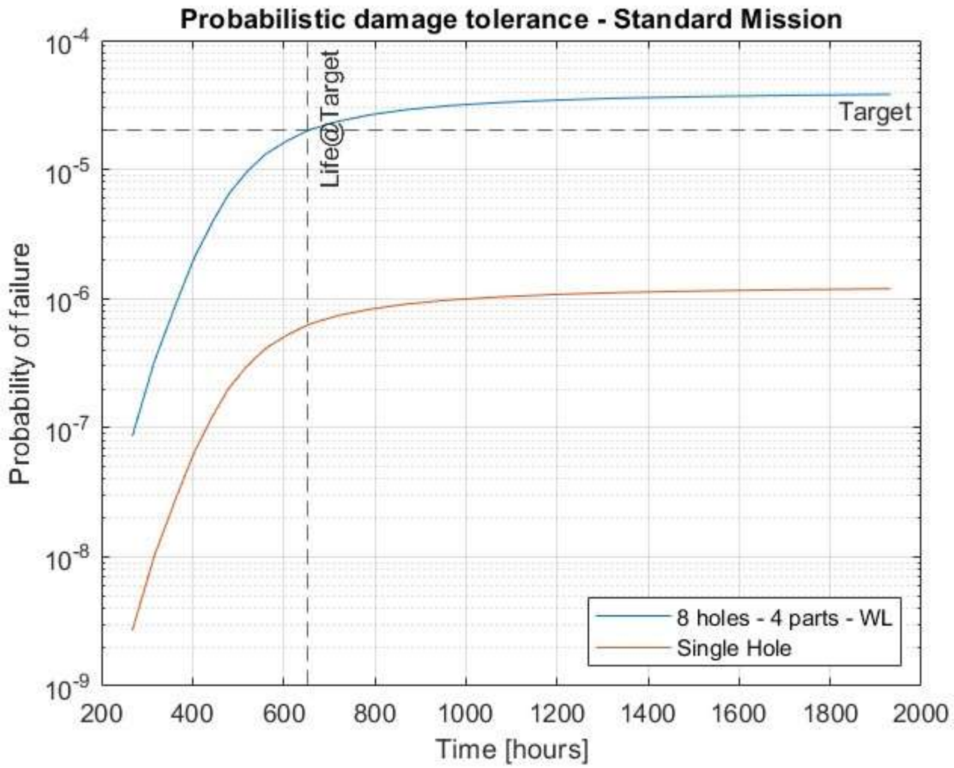
Profile 1



The life with a probability of being exceeded of 2e-5 is:

1367.9 hours

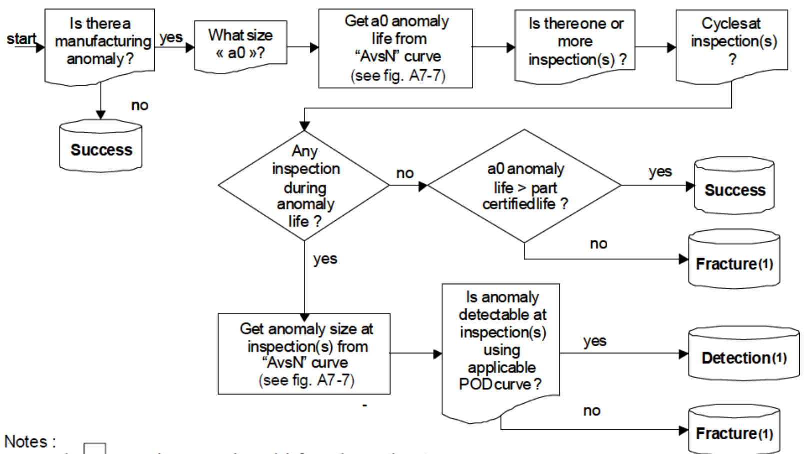
Standard Mission



The life with a probability of being exceeded of 2e-5 is:

652.7 hours

AC 33-70.2



Notes :
- each requires a random pick from the pertinent random variable distribution
-(1) record the number of cycles at fracture or detection for each hole

$$\text{Conditional prob. of fracture before N cycles} = \frac{\text{number of fractured holes before N cycles}}{\text{total sample size}}$$

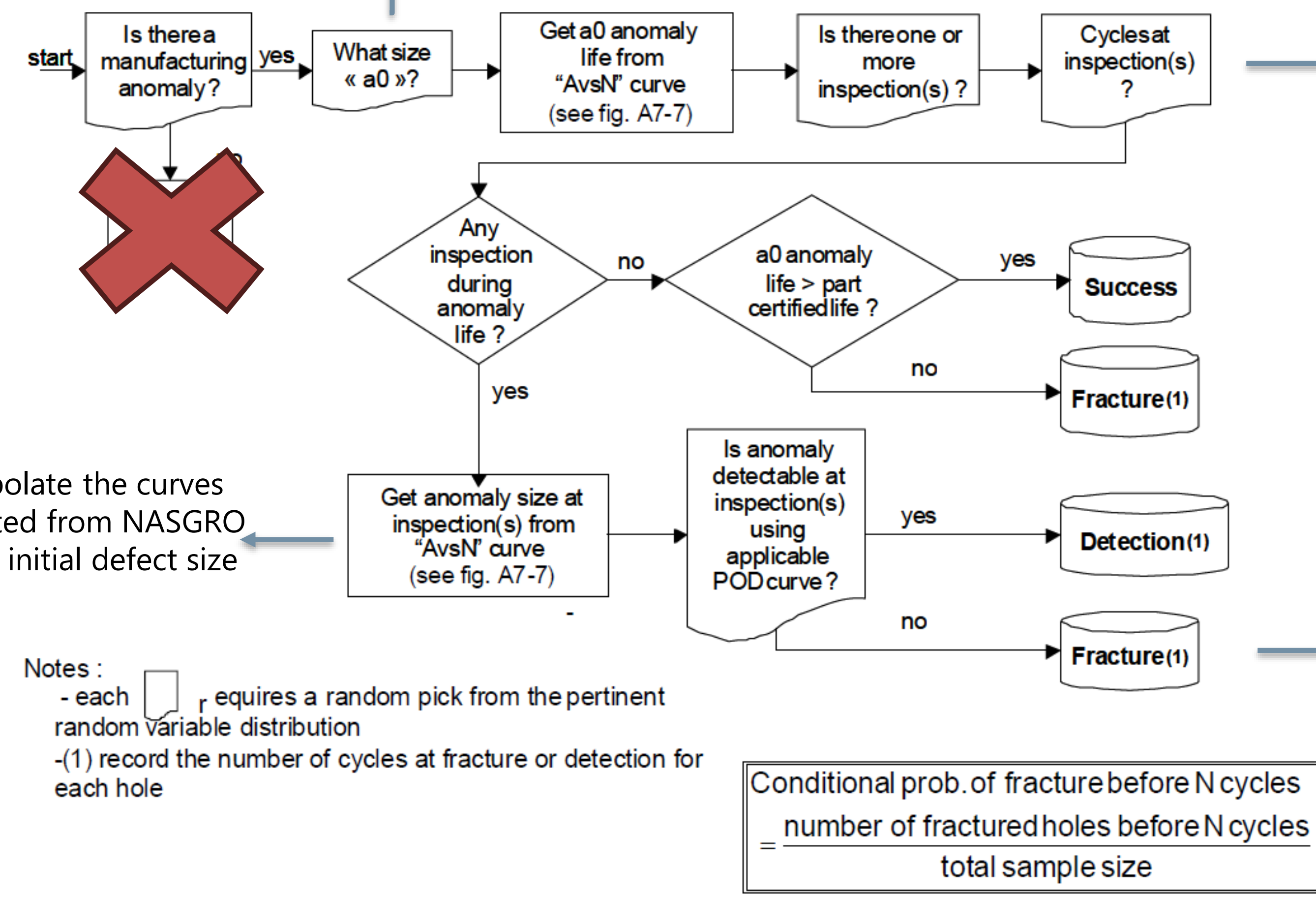
e. Relative risk calculation.

(1) The POF for each location is calculated by integrating the total surface area of all the holes in the feature, anomaly distribution, residual life, and inspection POD (if applicable) from the previous steps (see figure A1-3). The POF of each hole can be calculated by either an integrated probabilistic method or a "Monte Carlo" method. In the Monte Carlo method, the number of simulations required is related to the computed risk. The general rule is that the number of simulations should be at least 2 orders of magnitude higher than the computed risk. For example, if risk is 1 failure in 10^4 parts, the number of samples required is 10^6. This ensures that about 100 "failed" parts are involved in the assessment. The results for the limiting location are multiplied by the number of holes in the feature to determine the total feature POF.

4 Results: Part 3

4.2 Task 3.2

Exponential distribution

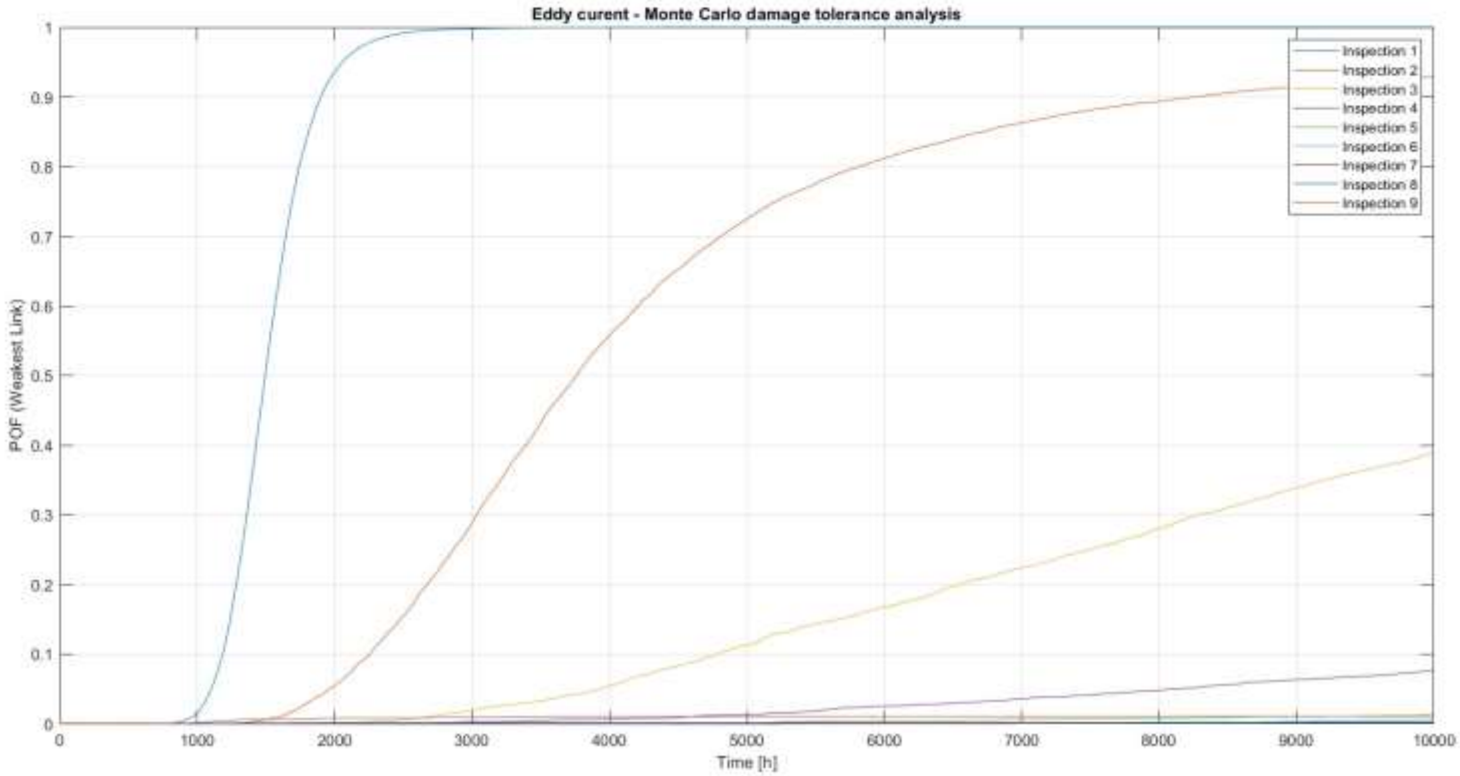


$N(t_{insp}, t_{insp} 0.033^2)$

Interpolate the curves calculated from NASGRO for the initial defect size

Failure if there is no detection at any inspection interval

Profile 1 (1e5 simulations)



5 Conclusions
5.1 Comparison

Profile	Part 1 [hours]	Part 2 [hours]	Part 3 [hours]
Profile 1	50313	1218	1367.9
Standard Mission	1341	600	652.7

- Part 1, which considers no anomalies in the parts, has the least conservative approach, with a big deviation in the calculated life relative to Part 2, Part 3.
- Part 2 is a deterministic damage tolerance analysis, which considers an initial defect presence in the part. In essence, the initial defect is the average defect size given in Part 3. The approach seems much more reliable than Part 1, but ultimately is a little more conservative than Part 3. This is because in Part 3 all the entire initial defect domain is considered, which has more cracks distributed to the lower end (exponential distribution).