Draft for Public Comment

Form 36

DPC: 17/30347641 DC

BSI Group Headquarters

389 Chiswick High Road London W4 4AL

Tel: +44 (0)20 8996 9000
Fax: +44 (0)20 8996 7400
www.bsigroup.com

Date: 10 May 2017
Origin: International

Latest date for receipt of comments: 01 July 2017

Project No. 2016/02504

Responsible committee: ISE/101/6 Fatigue testing of metals and metal matrix composites

Interested committees:

Title: Draft BS ISO 12107 Metallic materials - Fatigue testing - Statistical planning and analysis of data.

Please notify the secretary if you are aware of any keywords that might assist in classifying or identifying the standard or if the content of this standard

i) has any issues related to 3rd party IPR, patent or copyright
ii) affects other national standard(s)
iii) requires additional national guidance or information

WARNING: THIS IS A DRAFT AND MUST NOT BE REGARDED OR USED AS A BRITISH STANDARD. THIS DRAFT IS NOT CURRENT BEYOND 01 July 2017

This draft is issued to allow comments from interested parties; all comments will be given consideration prior to publication. No acknowledgement will normally be sent. **See overleaf for information on the submission of comments.**

Further copies of this draft may be purchased from BSI Shop http://shop.bsigroup.com or from BSI Customer Services, Tel: +44(0) 20 8996 9001 or email cservices@bsigroup.com. British, International and foreign standards are also available from BSI Customer Services.

Information on the co-operating organizations represented on the committees referenced above may be obtained from http://standardsdevelopment.bsigroup.com

Responsible Committee Secretary: Ms Takiyah Williams (BSI)
Direct tel: 02089967442
E-mail: takiyah.williams@bsigroup.com

Introduction

This draft standard is based on international discussions in which the UK has taken an active part. Your comments on this draft are invited and will assist in the preparation of the consequent standard. Comments submitted will be reviewed by the relevant BSI committee before sending the consensus UK vote and comments to the international secretariat, which will then decide appropriate action on the draft and the comments received.

If the international standard is approved, it is possible the text will be published as an identical British Standard.

UK Vote

Please indicate whether you consider the UK should submit a negative (with reasons) or positive vote on this draft.

Submission of Comments

- The guidance given below is intended to ensure that all comments receive efficient and appropriate attention by the responsible BSI committee. **Annotated drafts are not acceptable and will be rejected.**
- All comments must be submitted, preferably electronically, to the Responsible Committee Secretary at the address given on the front cover. Comments should be compatible with version 6.0 or version 97 of Microsoft Word for Windows, if possible; otherwise comments in ASCII text format are acceptable. **Any comments not submitted electronically should still adhere to these format requirements.**
- All comments submitted should be presented as given in the example below. Further information on submitting comments and how to obtain a blank electronic version of a comment form are available from the BSI website at: **http://drafts.bsigroup.com/**

Template for comments and secretariat observations

Date: xx/xx/20xx	Document: ISO/DIS xxxx

1	2	(3)	4	5	(6)	7
MB	Clause No./ Subclause No./Annex **(e.g. 3.1)**	Paragraph/Figure/ Table/Note	**Type of comment**	**Commend (justification for change) by the MB**	**Proposed change by the MB**	**Secretariat observations on each comment submitted**
	3.1	Definition 1	ed	Definition is ambiguous and needs clarifying.	Amend to read '...so that the mains connector to which no connection...'	
	6.4	Paragraph 2	te	The use of the UV photometer as an alternative cannot be supported as serious problems have been encountered in its use in the UK.	Delete reference to UV photometer.	

DRAFT INTERNATIONAL STANDARD
ISO/DIS 12107

ISO/TC **164**/SC **5** Secretariat: **ANSI**

Voting begins on: **2017-05-10** Voting terminates on: **2017-08-01**

Metallic materials — Fatigue testing — Statistical planning and analysis of data

Matériaux métalliques — Essais de fatigue — Plans et analyse statistique de données

ICS: 77.040.10

This document is circulated as received from the committee secretariat.

Reference number
ISO/DIS 12107:2017(E)

ISO copyright office
Ch. de Blandonnet 8 • CP 401
CH-1214 Vernier, Geneva, Switzerland
Tel. +41 22 749 01 11
Fax +41 22 749 09 47
copyright@iso.org
www.iso.org

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: www.iso.org/iso/foreword.html.

The committee responsible for this document is ISO/ TC 164, *Mechanical testing of metals*, Subcommittee SC 5, *Fatigue testing*.

This second edition cancels and replaces the first edition (ISO 12107:2003), which has been technically revised.

Introduction

It is known that the results of fatigue tests display significant variations even when the test is controlled very accurately. In part, these variations are attributable to non-uniformity of test specimens. Examples of such non-uniformity include slight differences in chemical composition, heat treatment, surface finish, etc. The remaining part is related to the stochastic process of fatigue failure itself that is intrinsic to metallic engineering materials.

Adequate quantification of this inherent variation is necessary to evaluate the fatigue property of a material for the design of machines and structures. It is also necessary for test laboratories to compare materials in fatigue behaviour, including its variation. Statistical methods are necessary to perform these tasks. They include both the experimental planning and procedure to develop fatigue data and the analysis of the results.

Metallic materials — Fatigue testing — Statistical planning and analysis of data

1 Scope

1.1 Objectives

This International Standard presents methods for the experimental planning of fatigue testing and the statistical analysis of the resulting data. The purpose is to determine the fatigue properties of metallic materials with both a high degree of confidence and a practical number of specimens.

1.2 Fatigue properties to be analysed

This International Standard provides a method for the analysis of fatigue life properties at a variety of stress levels using a relationship that can linearly approximate the material's response in appropriate coordinates.

Specifically, it addresses

a) the fatigue life for a given stress, and

b) the fatigue strength for a given fatigue life.

The term “stress” in this International Standard can be replaced by “strain”, as the methods described are also valid for the analysis of life properties as a function of strain. Fatigue strength in the case of strain-controlled tests is considered in terms of strain, as it is ordinarily understood in terms of stress in stress-controlled tests.

1.3 Limit of application

This International Standard is limited to the analysis of fatigue data for materials exhibiting homogeneous behaviour due to a single mechanism of fatigue failure. This refers to the statistical properties of test results that are closely related to material behaviour under the test conditions.

In fact, specimens of a given material tested under different conditions may reveal variations in failure mechanisms. For ordinary cases, the statistical property of resulting data represents one failure mechanism and may permit direct analysis. Conversely, situations are encountered where the statistical behaviour is not homogeneous. It is necessary for all such cases to be modelled by two or more individual distributions.

An example of such behaviour is often observed when failure can initiate from either a surface or internal site at the same level of stress. Under these conditions, the data will have mixed statistical characteristics corresponding to the different mechanisms of failure. These types of results are not considered in this International Standard because a much higher complexity of analysis is required.

Finally, for the *S*-*N* case (discussed in Clause 8), this International Standard addresses only complete data. Runouts (censored data) are not addressed.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534 (all parts), *Statistics — Vocabulary and symbols*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in 3534 and the following apply.

3.1 Terms related to statistics

3.1.1
confidence level
value $1 - \alpha$ of the probability associated with an interval of statistical tolerance

3.1.2
degrees of freedom
ν
number calculated by subtracting from the total number of observations the number of parameters estimated from the data

3.1.3
distribution function
function giving, for every value x, the probability that the random variable X is less than or equal to x

3.1.4
estimation
operation made for the purpose of assigning, from the values observed in a sample, numerical values to the parameters of a distribution from which this sample has been taken

3.1.5
population
totality of individual materials or items under consideration

3.1.6
random variable
variable that may take any value of a specified set of values

3.1.7
sample
one or more items taken from a population and intended to provide information on the population

3.1.8
size
n
number of items in a population, lot, sample, etc.

3.1.9
mean
μ
sum of all the data in a population divided by the number of observations

3.1.10
sample mean
$\hat{\mu}$
sum of all the data in a sample divided by the number of observations

3.1.11
standard deviation
σ
positive square root of the mean squared standard deviation from the mean from a population.

3.1.12
estimated standard deviation
$\hat{\sigma}$
positive square root of the mean squared standard deviation from the mean of a sample.

3.1.13
estimated stress or strain
$\hat{S}$
at a given N

3.1.14
one-sided tolerance limit for a normal distribution
k
number dependent on the failure probability, the confidence level and the degrees of freedom.

3.2 Terms related to fatigue

3.2.1
fatigue life
N
number of cycles observed in the test to achieve the intended failure criterion.

Note The dependent variable in a fatigue test conducted under force or strain control.

3.2.2
fatigue strength
value of stress level S at which a specimen would fail at a given fatigue life

Note 1 to entry: This is expressed in megapascals.

3.2.3
specimen
portion or piece of material to be used for a single test determination and normally prepared in a predetermined shape and in predetermined dimensions

3.2.4
stress or strain level
S
Intensity of the applied test stimulus

Note 1 to entry: The independent variable in a fatigue test conducted under stress or strain control. The stress or strain level can be expressed as amplitude, maximum or range

3.2.5
stress step
d
difference between neighbouring stress levels when conducting the test by the staircase method

Note 1 to entry: This is expressed in megapascals.

4 Statistical distributions in fatigue properties

4.1 Concept of distributions in fatigue

The fatigue properties of metallic engineering materials are determined by testing a set of specimens at various stress levels to generate a fatigue life relationship as a function of stress. The results are usually expressed as an *S-N* curve that fits the experimental data plotted in appropriate coordinates. These are generally either log-log or semi-log plots, with the life values always plotted on the abscissa on a logarithmic scale.

Fatigue test results usually display significant scatter even when the tests are carefully conducted to minimize experimental error. A component of this variation is due to inequalities, related to chemical composition or heat treatment, among the specimens, but another component is related to the fatigue process, an example being the initiation and growth of small cracks under test environments.

The variation in fatigue data are expressed in two ways: the distribution of fatigue life at a given stress and the distribution of strength at a given fatigue life (see References [1] to [5]).

4.2 Distribution of fatigue life

Fatigue life, N, at a given test stress, S, is considered as a random variable. It is frequently observed the distribution of fatigue life values at any stress is normal in the logarithmic metric. That is, the logarithms of the life values follow a normal distribution (See 6.4). This relationship is:

$$P(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x} \right)^2 \right] \mathrm{d}x \tag{1}$$

where $x = \log N$ and μ_x and σ_x are, respectively, the mean and the standard deviation of x.

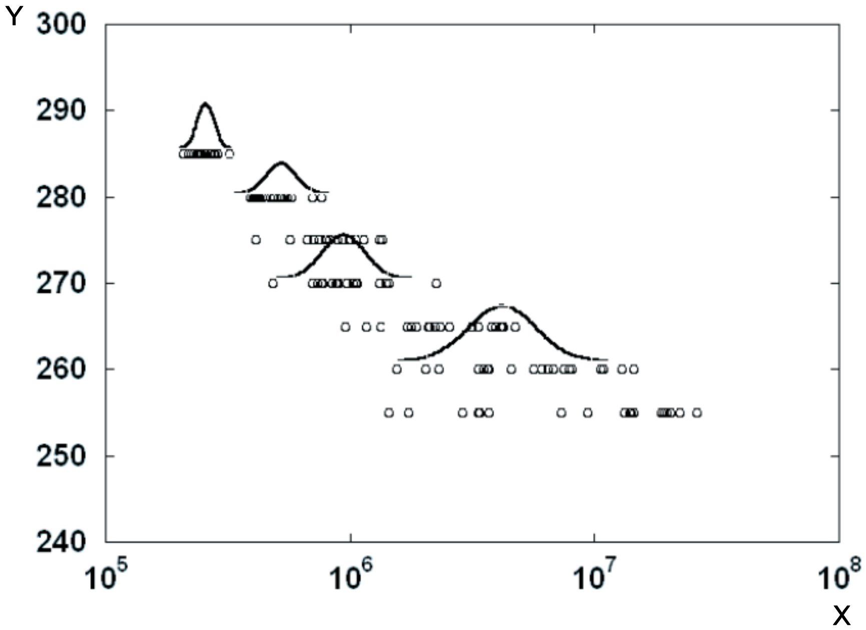
Equation (1) gives the cumulative probability of failure for x. This is the proportion of the population failing at lives less than or equal to x.

Equation (1) does not relate to the probability of failure for specimens at or near the fatigue limit. In this region, some specimens may fail, while others may not. The shape of the distribution is often skewed, displaying even greater scatter on the longer-life side. It also may be truncated to represent the longest failure life observed in the data set.

This International Standard does not address situations in which a certain number of specimens may fail, but the remaining ones do not.

Other statistical distributions can also be used to express variations in fatigue life. The Weibull [4] distribution is one of the statistical models often used to represent skewed distributions. On occasion, this distribution may apply to lives at low stresses, but this special case is not addressed in this International Standard.

Figure 1 shows an example of data from a fatigue test conducted with a statistically based experimental plan using a large number of specimens (see Reference [5]). The shape of the fatigue life distributions is demonstrated for explanatory purposes.



Key

X cycles to failure

Y stress amplitude, in MPa

Figure 1 — Concept of variation in a fatigue property — Distribution of fatigue life at given stresses for a 0,25 % C carbon steel tested in the rotating-bending mode

4.3 Distribution of fatigue strength

Fatigue strength at a given fatigue life, N, is considered as a random variable. It is expressed as the normal distribution:

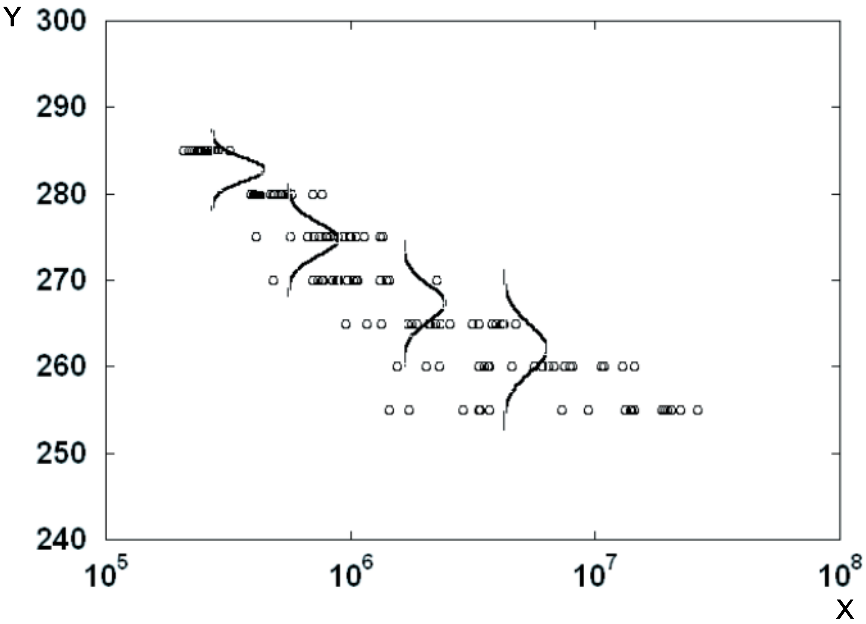
$$P(y) = \frac{1}{\sigma_y \sqrt{2\pi}} \int_{-\infty}^{y} \exp\left[-\frac{1}{2} \left(\frac{y - \mu_y}{\sigma_y} \right)^2 \right] \mathrm{d}y \qquad (2)$$

where $y = S$ (at a given N), and μ_y and σ_y are, respectively, the mean and the standard deviation of y.

Equation (2) gives the cumulative probability of failure for y. It defines the proportion of the population presenting fatigue strengths less than or equal to y.

Other statistical distributions can also be used to express variations in fatigue strength.

Figure 2 is based on the same experimental data as Figure 1. The variation in the fatigue property is expressed here in terms of strength at typical fatigue lives (see Reference [5]).



Key

X cycles to failure

Y stress amplitude, in MPa

Figure 2 — Concept of variation in a fatigue property — Distribution of fatigue strength at typical fatigue lives for a 0,25 % C carbon steel tested in the rotating-bending mode

5 Statistical planning of fatigue tests

5.1 Sampling

It is necessary to define clearly the population of the material for which the statistical distribution of fatigue properties is to be estimated. Specimen selection from the population shall be randomized. It is also important that the specimens be selected so that they accurately represent the population they are intended to describe. A complete plan would include additional considerations.

If the population consists of several lots or batches of material, the test specimens shall be selected randomly from each group in a number proportional to the size of each lot or batch. The total number of specimens taken shall be equal to the required sample size, *n*.

If the population displays any serial nature, e.g. if the properties are related to the date of fabrication, the population shall be divided into groups related to time. Random samples shall be selected from each group in numbers proportional to the group size.

The specimens taken from a particular batch of material will reveal variability specific to the batch. This within-batch variation can sometimes be of the same order of importance as the between-batch variation. When the relative importance of different kinds of variation is known from experience, sampling shall be performed taking this into consideration.

Hardness measurement is recommended for some materials, when possible, to divide the population of the material into distinct groups for sampling. The groups should be of as equal size as possible. Specimens may be extracted randomly in equal numbers from each group to compose a test sample of size n. This procedure will generate samples uniformly representing the population, based upon hardness.

5.2 Allocation of specimens for testing

Specimens taken from the test materials shall be allocated to individual fatigue tests in principle in a random way, in order to minimize unexpected statistical bias. The order of testing of the specimens shall also be randomized in a series of fatigue tests.

When several test machines are used in parallel, specimens shall be tested on each machine in equal or nearly equal numbers and in a random order. The equivalence of the machines in terms of their performance shall be verified prior to testing.

When the test programme includes several independent test series, e.g. tests at different stress levels or on different materials for comparison purposes, each test series should be carried out at equal or nearly equal rates of progress, so that all testing can be completed at approximately the same time.

6 Statistical estimation of fatigue life at a given stress

6.1 Testing to obtain fatigue life data

Conduct the fatigue tests at a given stress, S, on a set of carefully prepared specimens to determine the fatigue life values for each. The number selected will be dependent upon the purpose of the test and the availability of test material. A set of seven specimens is recommended in this International Standard for exploratory tests. For reliability purposes, however, at least 28 specimens are recommended.

6.2 Plotting data on normal probability paper

Plot the fatigue lives on log-normal probability coordinates. The results should plot as a straight line. Should one or two data points (really a very low proportion of the data set) deviate from the curve, this is usually the result of invalid data. Examining test records and failed specimens is useful when there is non-conforming behaviour. The purpose is to identify a cause for such deviant behaviour to learn if these results can be discounted. Other statistical distributions e.g. Weibull may be evaluated. However, since the vast majority of unimodal fatigue results have proven to be distributed log-normally, the standard does not consider Weibull statistics. Subclause 8.3.3 gives some examples of normal probability plots constructed from data used to generate an S-N curve. Refer to these plots to understand how they will appear when the data conform well to the assumption and in other cases when there might be some issues. Please note that for the present case, the y-axis will just be the property in question as opposed to the standardized residuals given the y-axis on the presented plots in 8.3.3.

One other issue is that if the data appear to support two distinct failure distributions, the data should be segregated by the root cause. For example, results for both surface and internal initiation sites should be separated into two groups and evaluated uniquely.

6.3 Estimating distribution parameters

Calculation of the sample mean is performed as follows:

$$\hat{\mu} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{3}$$

where

$\hat{\mu}$ is the sample mean;

x_i is the i th observed value, where $x_i = \log_{10}(N_i)$ and where N_i is the i th observed fatigue value;

n is the number of data points.

Note that the symbol “^” means an estimation based upon a sample.

The sample standard deviation is calculated using the following relationship:

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n}\left(x - \hat{\mu}\right)}{n-1}} \tag{4}$$

6.4 Quantitative evaluation of the assumption of normality

A number of statistical tests have been developed attempting to quantitatively consider the assumption of normality. These tests can sometimes generate conflicting results. However, one that seems quite useful is the Anderson-Darling Test. The details for performing this evaluation as well as others can be found in Reference [9]. Also, there are commercially available statistical software packages that perform quantitative evaluations of normality.

6.5 Estimating the lower limit of the fatigue life

Estimate the logarithm of the lower limit of the fatigue life at a given probability of failure, assuming a log-normal distribution, at the confidence level $1 - \alpha$ from the equation:

$$\hat{x}_{(P,1-\alpha)} = \hat{\mu}_x - k_{(P,1-\alpha,\upsilon)}\,\hat{\sigma}_x \tag{5}$$

The coefficient $k_{(P,\,1-\alpha,\,\nu)}$ is the one-sided tolerance limit for a normal distribution, as given in Table B.1.

P corresponds to the reliability of the prediction (say 99 % probability) and $1 - \alpha$ is the confidence of the reliability statement. These values are generated by integration of the non-central t distribution with non-centrality parameter:

$$\delta = \sqrt{n} \tag{6}$$

The number of degrees of freedom, v, is the same number used in estimating the standard deviation. For the present case, this is $n - 1$.

A worked example is given in A.1.

7 Statistical estimation of fatigue strength at a given fatigue life

7.1 Testing to obtain fatigue strength data

Conduct fatigue tests to generate strength data for a set of specimens in a sequential way using the method known as the staircase method (see Reference [7]).

It is necessary to have rough estimates of the mean and the standard deviation of the fatigue strength for the materials to be tested. Start the test at a first stress level preferably close to the estimated mean strength. Also select a stress step, preferably close to the standard deviation, by which to vary the stress level during the test.

If no information is available about the standard deviation, a step of about 5 % of the estimated mean fatigue strength may be used as the stress step.

Test a first specimen, randomly chosen, at the first stress level to find if it fails before the given number of cycles or not fails. For the next specimen, also randomly chosen, increase the stress level by a step if the preceding specimen did not fail, and decrease the stress by the same amount if it failed. Continue testing until all the specimens have been tested in this way.

Exploratory research requires a minimum of 15 specimens to estimate the mean and the standard deviation of the fatigue strength. Reliability data requires at least 28 specimens.

A worked example of the staircase method is given in A.2.1, together with worked examples of the analyses described in 7.2 and 7.3.

7.2 Statistical analysis of test data

Count the frequencies of failure and non-failure of the specimens tested at different stress levels. Use the analysis for the group with the least number of observations (failure or non-failure).

Denote the stress levels arranged in ascending order by $S_0 \leq S_1 \leq \ldots \leq S_l$, where l is the total number of stress levels of the group with the least number of observations, denote the number of observations by f_i, and denote the stress step by d. Estimate the parameters for the statistical distribution of the fatigue strength, equation (2), from:

$$\hat{\mu} = S_0 + d\left(\frac{A}{C} \pm \frac{1}{2}\right) \tag{7}$$

$$\hat{\sigma} = 1{,}62d\left(D + 0{,}029\right) \tag{8}$$

where:

$$A = \sum_{i=0}^{l} i f_i$$

$$B = \sum_{i=0}^{l} i^2 f_i$$

$$C = \sum_{i=0}^{l} f_i$$

$$D = \frac{BC - A^2}{C^2}$$

In equation (7), take the value of ± 1/2 as:

– 1/2 when the observation analysed is failure;

+ 1/2 when the observation analysed is non-failure.

In Reference [7], it is stated that equation (8) is valid only when $D \geq 0{,}3$. This condition is generally satisfied when $d / \hat{\sigma}_y$ is selected properly within the range 0,5 to 2.

7.3 Estimating the lower limit of the fatigue strength

Estimate the lower limit of the fatigue strength at a probability of failure P for the population at a confidence level of $1 - \alpha$, if the assumption of a normal distribution of the fatigue strength is correct, from the equation:

$$\hat{y}_{(P,1-\alpha)} = \hat{\mu}_y - k_{(P,1-\alpha,\upsilon)}\, \hat{\sigma}_y \qquad (9)$$

where the coefficient $k_{(P,1-\alpha,v)}$ is the one-sided tolerance limit for a normal distribution, as given in Table B.1. Take as the number of degrees of freedom, v, the number that was used in estimating the standard deviation. For the present case, this is $n - 1$, where n is the least number of analysed observations (failure or non-failure).

7.4 Modified method when standard deviation is known

A modified staircase method, with fewer specimens, is possible if the standard deviation is known and only the mean of the fatigue strength needs to be estimated (see Reference [8]).

Conduct tests as in the staircase method described in 7.1, by decreasing or increasing the stress level by a fixed step depending whether the preceding observation was a failure or non-failure, respectively. Choose the initial stress level close to the roughly estimated mean and the stress step approximately equal to the known standard deviation.

A minimum of six specimens is required for exploratory tests and at least 15 for reliability data.

If the test is conducted on n specimens at stress levels S_1, S_2, ... , S_n in a sequential way, then the mean fatigue strength is determined by averaging the test stresses, S_1 to S_{n+1}, without regard to whether each observation was a failure or a non-failure:

$$\hat{\mu}_y = \frac{\sum_{i=1}^{n+1} S_i}{n+1} \qquad (10)$$

The test at S_{n+1} is not carried out, but the stress level itself is determined from the result of n th test.

Estimate the lower limit of the fatigue strength for the population from Formula (9). Take as the number of degrees of freedom that corresponding to the standard deviation used for the test or, if this number is unknown, take it as $n - 1$, where n represents in this context the total number of analysed observations.

In the modified staircase method, it is necessary to know the standard deviation of the fatigue strength. It may be estimated from the *S-N* curve as described in Clause 8.

A worked example is given in A.2.

8 Statistical estimation of the *S-N* curve

8.1 Introduction

Analysis of *S-N* fatigue is performed for the purpose of fitting an appropriate mathematical relationship to test data to generate a curve which yields approximately 50 % probability of failure. Typically, the data exist at a number of stress or strains and represent a continuous single distribution that is log-normally distributed with constant variance as a function of stress or strain.

The basic relationships employed to describe behaviour are used to reflect either linear or curvilinear response. Figures 3 and 4 demonstrate the behaviour in question. Figure 5 presents a case which occurs only on occasion. This more complicated behaviour can be managed by use of the Bastenaire equation. This relationship is useful when the data demonstrates an asymptotic flattening of the curve in the very high life regime while simultaneously displaying a convex downward shape in the high stress or strain region.

Mathematically, the Bastenaire relationship [10, 11] has the following form:

$$N = \frac{A}{S-E}\exp\left[-\left(\frac{S-E}{B}\right)^{C}\right] \tag{11}$$

where

N	is the fatigue life
S	is the stress or strain
A, B, C, E	are curve fit parameters

The Strohmeyer relationship [14] is also useful when there is no high stress downward concavity. It is:

$$\log_{10}(N) = A + B\log_{10}(S-E) \tag{12}$$

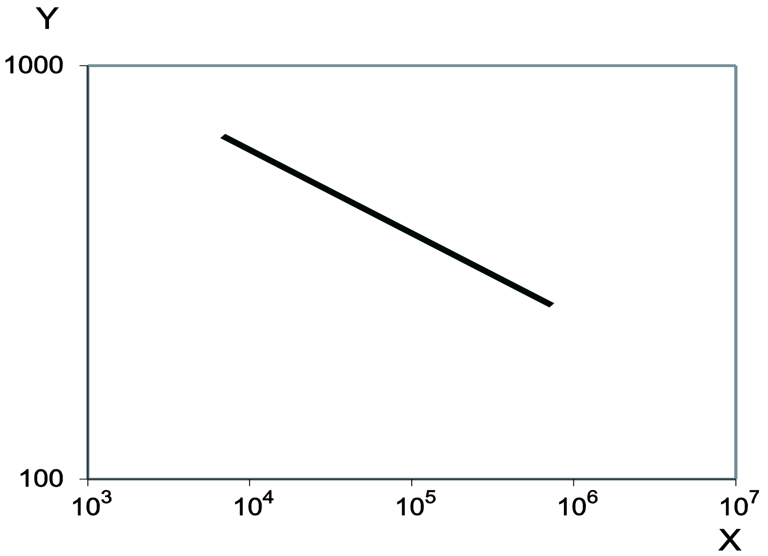
where

N	is the fatigue life

S	is the stress or strain
E	is stress or strain at very long life and must be less than all the stress or strain values in the data
A, B	are curve fit parameters

Application of the Bastenaire or Strohmeyer equations cannot be performed using the method of linear least squares. More advanced concepts are required and are beyond the scope of the present document. They can be found in References [10] and [11]. Additionally, a future standard is planned to address a method suitable for this analysis as well as other advanced topics that remain to be defined.

Note that high stress behaviour can result from performing stress-controlled tests at maximum stress levels exceeding the yield strength. In general, this is an improper test technique because cyclic ratcheting may result. Conversely, this high stress behaviour has been observed in strain-controlled tests as well. Strain-controlled testing is sometimes purposely conducted at strain levels producing stresses exceeding the yield strength. These are usually valid in the absence of specific testing issues, etc., invalidating the results.

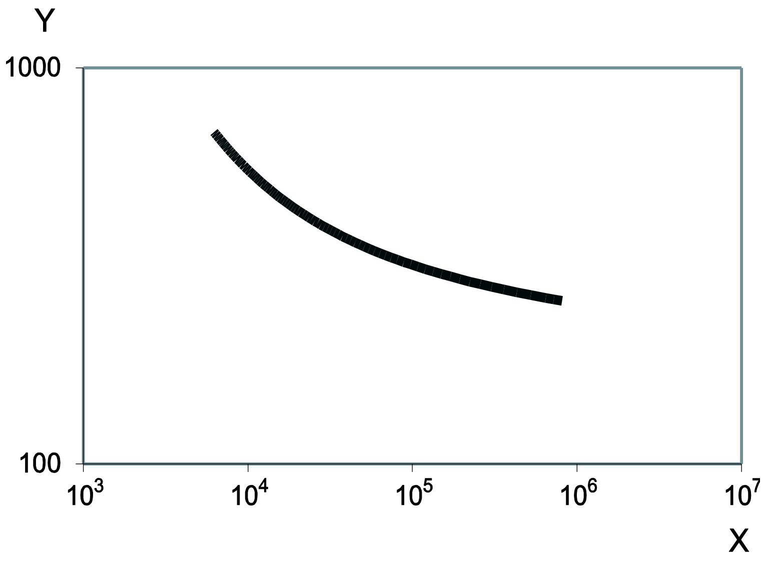


Key

X cycles to failure

Y stress or strain, stress units (MPa) presented

Figure 3 — Typical linear fatigue response

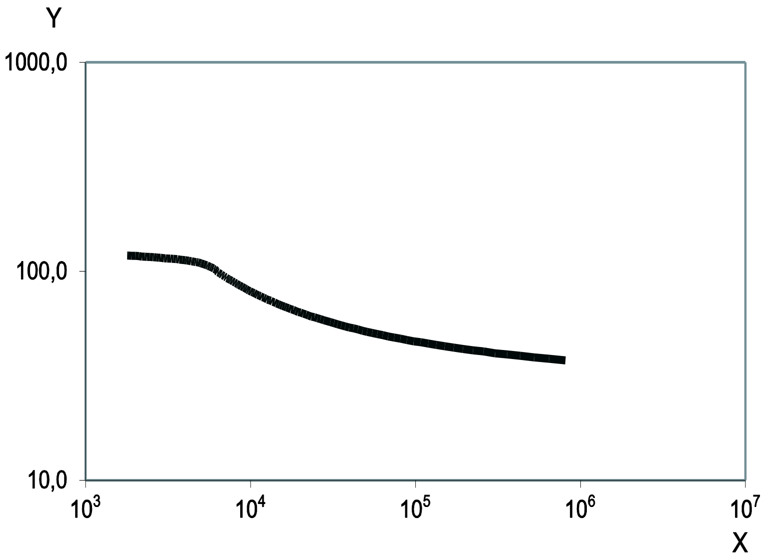


Key

X cycles to failure

Y stress or strain, stress units (MPa) presented

Figure 4 — Typical curvilinear fatigue response



Key

X cycles to failure

Y stress or strain, stress units (MPa) presented

Figure 5 — *S-N* response occasionally observed

The mathematical models appropriate for the majority of the cases the fatigue practitioner will encounter are given below. However, there will be cases which occur, noted above, that our outside the domain of the methodologies presented and require more sophisticated approaches. However, in general these cases occur rather infrequently and the majority of *S-N* curves can be evaluated using the techniques presented for linear or curvilinear response (Figures 3 and 4, respectively). The statistical techniques presented below are based on the method of linear least squares [12], [13]. The more advanced methods require either nonlinear regression methods or maximum likelihood estimation (MLE).

These relationships are:

— Linear fatigue response model

$$\log_{10}\left(N\right) = b_0 + b_1 \log_{10}\left(S\right) \tag{13}$$

where b_0 and b_1 are linear regression coefficients and S can be either stress or strain.

— Curvilinear fatigue response

$$\log_{10}\left(N\right) = b_0 + b_1 \log_{10}\left(S\right) + b_2 \log_{10}^2\left(S\right) \tag{14}$$

where b_0, b_1 and b_2 are linear regression coefficients and S can be either stress or strain.

8.2 Estimation of regression parameters

8.2.1 Estimation of the parameters for the linear model[1]

For all the data, take the logarithms of the stress (or strain) values and the corresponding observed lives. Base 10 logarithms are recommended. Logarithms to another base, for example base e, are acceptable. However, base 10 is recommended, as most practitioners seem to use these values.

Specifically, the model then has the form:

$$\hat{Y}_i = b_0 + b_i X_i \tag{15}$$

where $\hat{Y}_i$ is the predicted value of the dependent variable, $X_i = \log_{10}(S_i)$ and $Y_i = \log_{10}(N_i)$.

$$b_1 = \frac{\sum_{i=1}^{n} X_i Y_i - \frac{\sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n}}{\sum_{i=1}^{n} X_i^2 - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}} \tag{16}$$

$$b_0 = \frac{\left(\sum_{i=1}^{n} Y_i - b_1 \sum_{i=1}^{n} X_i\right)}{n} \tag{17}$$

Note n in this context represents only the number of failures in the data.

Standard deviation calculated from the results of regression analysis is defined as:

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{n-p}} \tag{18}$$

where p is the number of parameters estimated in the model, in this case $p = 2$.

A worked example for estimation of the regression parameters and the standard deviation for the linear model is given in A.3.1.

Correlation coefficient: A useful parameter to assist in the evaluating the quality of the fit is the correlation coefficient, R^2. This parameter presents the proportion of the variation of the data explained

[1] Classically, linear regression refers to any linear combination of explanatory variables. For the purposes of this International Standard, however, a linear model simply means a relationship of the form $y = mx + b$.

by the model to the total variation. The following relationship is normally used for the linear case. It is generalized later in the discussion of the quadratic model.

$$R^2 = \frac{\left(\sum_{i=1}^{n} X_i Y_i - \frac{\sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n} \right)^2}{\left[\left(\sum_{i=1}^{n} X_i^2 - \frac{\left(\sum_{i=1}^{n} X_i \right)^2}{n} \right) \left(\sum_{i=1}^{n} Y_i^2 - \frac{\left(\sum_{i=1}^{n} Y_i \right)^2}{n} \right) \right]} \tag{19}$$

Usually, a value of 0,9 or better is indicative of a good fit.

8.2.2 Estimation of the regression parameters for the quadratic model

Linear regression models that is, all models in which the parameters are linear or can be linearized by a suitable transformation (for example logarithmic) can be calculated using the methods of linear algebra. References [12] and [13] provide the details.

Basically, the solution to the general linear problem is given by:

$$\boldsymbol{b} = \left(\boldsymbol{X'X} \right)^{-1} \boldsymbol{X'Y} \tag{20}$$

where

$\boldsymbol{b}$	is the matrix of the calculated regression parameters
$\boldsymbol{X}'$	is the transpose of the matrix of x values; the independent variables
$\boldsymbol{X}$	is the matrix of X values; the independent variables
$\boldsymbol{Y}$	is the matrix of Y values; the dependent variable

Additional relationships of interest are:

$$R_{\mathrm{SST}} = \sum_{i=1}^{n} \left(Y_i - \bar{Y}_i \right)^2 \tag{21}$$

$$R_{\mathrm{SSR}} = \sum_{i=1}^{n} \left(\hat{Y}_i - \bar{Y}_i \right)^2 \tag{22}$$

$$R_{\mathrm{SSE}} = \sum_{i=1}^{n} \left(Y_i - \hat{Y}_i \right)^2 \tag{23}$$

where

R_{SST}	is the sum of squares total
R_{SSR}	is the sum of squares regression
R_{SSE}	is the sum of squares error
$\bar{Y}$	is the average of all the Y' in the model

The standard deviation for the regression model, given in equation (18) is alternatively expressed as:

$$\hat{\sigma} = \sqrt{\frac{R_{SSE}}{n-p}} \tag{24}$$

where

$\hat{\sigma}$	is the estimated standard deviation
p	is the number of parameters (b's) estimated in the model
	$p = 2$ for a model of the form $y = b_0 + b_1x$ and
	$p = 3$ for a quadratic model, $y = b_0 + b_1x + b_2x^2$
$n - p$	is the degrees of freedom.

The correlation coefficient presented in equation (19) is more generally given by:

$$R^2 = \frac{R_{SSR}}{R_{SST}} \tag{25}$$

8.3 Analysis approach

In general, the simplest model that captures behaviour should be used for the analysis. Usually, a more complex relationship will give a better fit and certainly will increase the R^2 value, but the final model should only be more complex than the linear relationship if it can be shown to significantly reduce the scatter. Analysis of the data using this strategy is encouraged and the following details will help in determining the significance of increasing the complexity.

8.3.1 Plot the curve on the *S-N* diagram

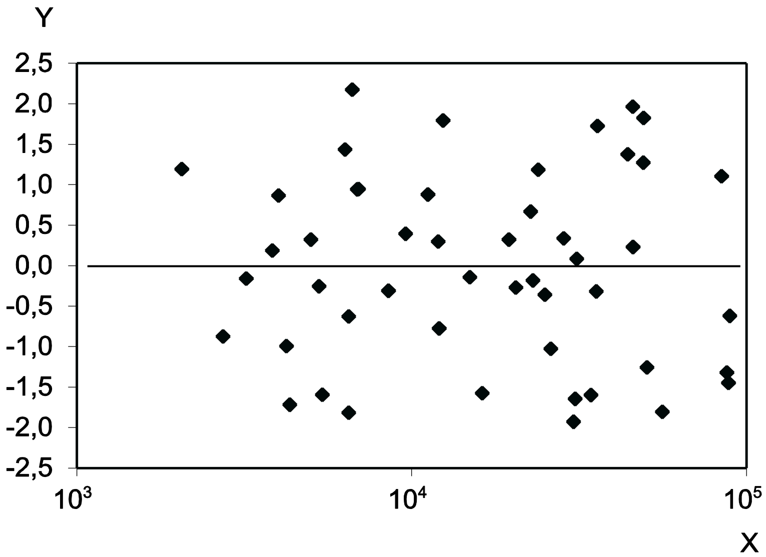
The first analysis, the one assuming linear response in *S-N* coordinates, should be plotted with the data used to generate the curve to obtain an assessment of the overall fit.

8.3.2 Residuals plots

Evaluation of the quality of fit is undertaken by evaluating plots of residuals and plots of residual versus cumulative normal probability. A residual is defined as:

$$e_i = \left(Y_i - \hat{Y}_i \right) \tag{26}$$

One property of the residuals is that they should sum to zero. A plot of the residual versus the corresponding predicted ($\hat{Y}_i$) [$\log_{10}$ (fatigue life)] values should more or less uniformly populate the plot. An example of an acceptable plot and plots with issues that require resolution are given below. Ideally, the residuals should more or less uniformly populate plot curve without biases or trends. As always, the sum of the residuals should be zero, so the residuals should be centred about the zero value on the *y*-axis. Note that the residuals on the *y*-axis have been scaled by the standard deviation. These are referred to as the standardized residuals. Figure 6, as noted, demonstrates the case when a model is adequately capturing response. Figure 7 is the classic case where data was evaluated using a linear response model, but requires a quadratic expression. When the quadratic model is applied, the residuals plot will then appear similar to that shown in Figure 6.

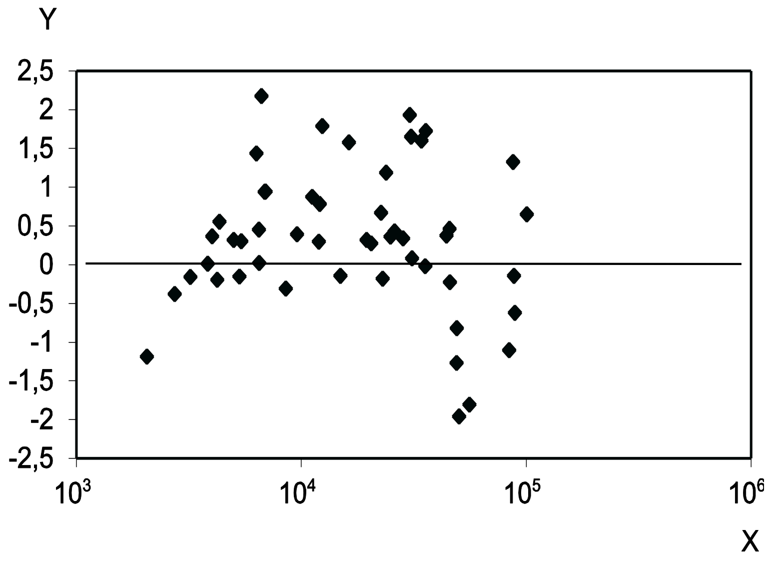


Key

X predicted life

Y standardized residuals

Figure 6 — Residuals plot demonstrating results for a model adequately capturing behaviour

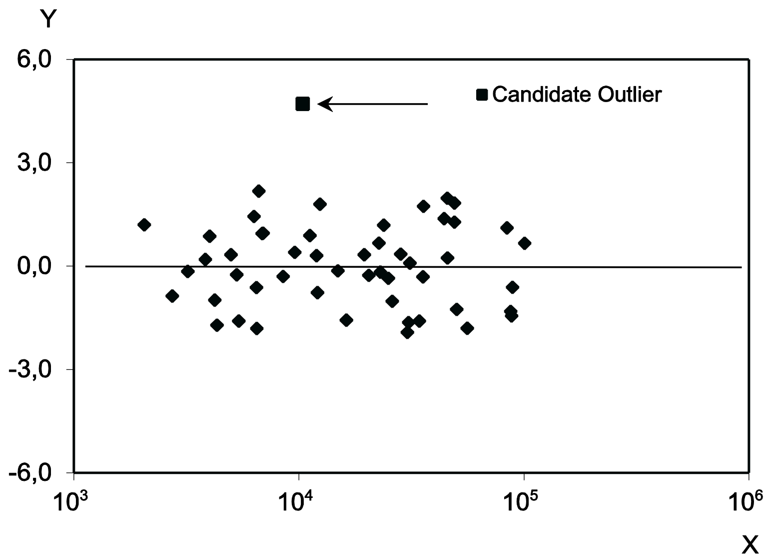


Key

X predicted life

Y standardized residuals

Figure 7 — Residuals plot demonstrating classic behaviour when a linear response model is applied to results requiring a quadratic relationship



Key

X predicted life

Y standardized residuals

Figure 8 — An acceptable residuals plot for the fit, but also demonstrating a candidate outlying result

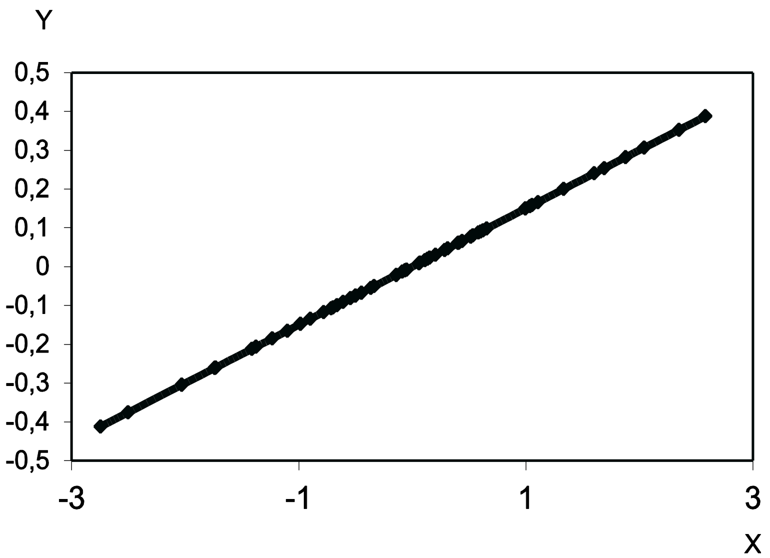
Finally, when a possible outlier(s) is observed, it is appropriate to conduct a careful review of the test records that accompany the results to see if there were any machining and/or testing discrepancies. In addition, it is recommended that a metallurgical and fractographic evaluation of results be made in order to determine if the specimen is a valid observation or if it may have been damaged. Note that the disposition of outlying data can be problematic and subjective. Quantitative statistical tests to evaluate outlying data are available, but these can often generate conflicting results. This is particularly true for cases where the points(s) are suspiciously aberrant, but no so far deviant as to be clearly erroneous.

Unduly high (long-life) results, as well as short-life values, can be problematic; both can inappropriately inflate the scatter and shift the curves. The likely result is higher than necessary estimates of the standard deviation. This can lead to lower tolerance limits than appropriate.

In cases with outliers, the correct perspective is to be judicious, neither automatically rejecting tests that might be problematic, nor retaining such results because no physical reason can be identified. The guidance is to err conservatively in such cases by retaining observations where there is insufficient cause to reject them.

8.3.3 Normal probability plot

Plots of the residuals, or standardized residuals, versus cumulative normal probability are also useful in evaluation of the fit of the model to the data. The residuals are assumed to be normally distributed and this assumption can be evaluated by determining if the residual from the analysis plot reasonably as a straight line.

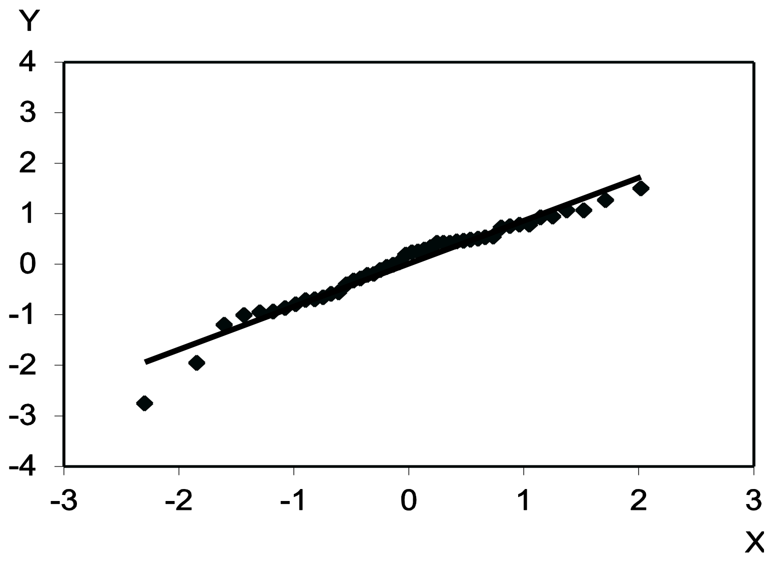


Key

X cumulative normal probability

Y standardized residuals

Figure 9 — Example of a cumulative normal probability plot displaying excellent conformance to normality

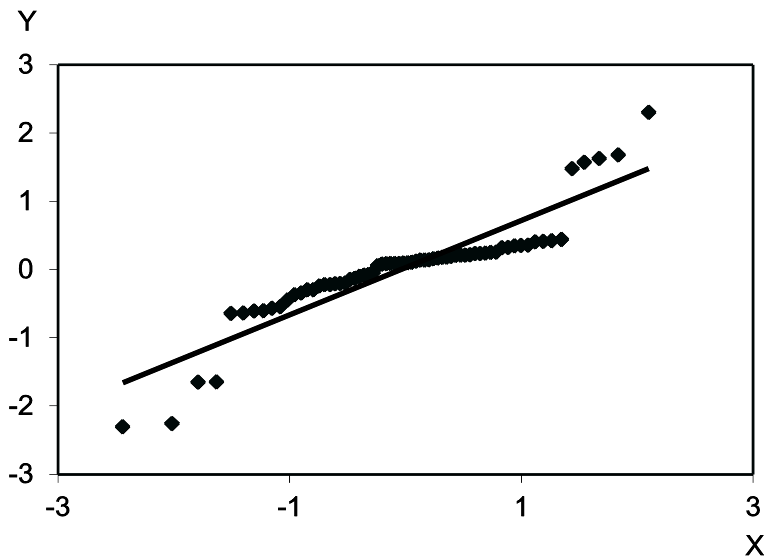


Key

X cumulative normal probability

Y standardized residuals

Figure 10 — Example of a cumulative normal probability plot displaying acceptable conformance to normality



Key

X cumulative normal probability

Y standardized residuals

Figure 11 — Example of a cumulative normal probability plot displaying marginally acceptable conformance to normality

8.3.4 Quantitative measures of normality for the residuals

Should the analyst wish to quantitatively evaluate the assumption of normality, the procedures of Reference [9] (as noted in 6.4) can be applied. For regression analysis, however, it is the residuals that are evaluated as opposed to the actual observations.

8.3.5 Remedial measures

Should the results for the tools evaluating the quality of the fit indicate there might be some issues, the following measures are recommended:

— check for invalid data;

— determine if a more complicated model (quadratic) is required;

— examine the *S-N* curve and the diagnostic plots to see if there are two or more populations within the data requiring segregation into separate data sets. Unique curves for each population may be required.

8.3.6 Determination of the need for a quadratic model

Should the *S-N* plot suggest a lack of fit and the residuals plot have the appearance of Figure 7, then application of the so-called “general linear test” [12] can quantitatively evaluate the significance of using this relationship. To perform this test, first analyse the results with both the linear and quadratic models. The general linear test uses the sum of squares error, R_{SSE}, and the corresponding degrees of freedom for each of the fitted linear and quadratic expressions.

Specifically, this is:

$$F^* = \frac{\left(R_{\text{SSE 1}} - R_{\text{SSE 2}}\right)}{R_{\text{SSE 1}}} \div \frac{\nu_2}{\left(\nu_1 - \nu_2\right)} \tag{27}$$

where

$R_{\text{SSE 1}}$	SSE for the simpler model
$R_{\text{SSE 2}}$	SSE for the candidate (quadratic) model
ν_1	are the degrees of freedom for the simpler model
ν_2	are the degrees of freedom for the candidate model

If $F^* > F_{\alpha,\ \nu_1, \nu_2}$ conclude the quadratic model is significantly improving the fit.

F is the value obtained from the *F* distribution table (or integration of the *F* distribution) at the *p*, number of parameters in the candidate model, the ν_2 degrees of freedom and the choice of the α level.

If the results of this test deem that the quadratic relationship is significant, the curve generated from this model should be plotted against the data to verify the relationship yields reasonable *S-N* character. On occasion, it has been observed that significant quadratic relationships demonstrate inconsistent behaviour with accepted fatigue response. For example, properties can curve downward at low stresses/strains or occasionally curve over at the highest levels of these stimuli. That is, the curve can suggest life improvements as stress or strain increases. Please note, this is within the range of the data and is not referring to any extrapolation. The reason that a significant quadratic model can demonstrate uncharacteristic fatigue behaviour is attributable to several possibilities.

One or more invalid observations exist in the data. Some can become particularly problematic if they are at the lowest or highest levels of stress or strain.

1) The data were not developed according to the recommended procedure presented in 8.4 and biases resulted.

2) More than one failure distribution is contained within the data.

3) The remedial action for the first issue involves suitable identification of invalid results. Then once the outlying data have been removed, the analysis process should be repeated.

For the second issue, either additional data must be appropriately generated or one must accept the linear model and recognize it is not performing an optimal analysis.

For the final issue, the data should be segregated into homogenous populations. If this is a not an option, then other techniques, beyond the scope of this International Standard, are necessary.

Given the *S-N* response is with the quadratic relationship is acceptable, then the diagnostic residuals and normal probability plots should be developed to confirm the model is adequately analysing the data.

8.4 Calculation of the lower tolerance limit

Estimation of the lower limit of the *S-N* curve life at a given probability of failure, assuming a normal distribution, at the confidence level, 1 - α follows that given in 6.5 is given by:

$$\hat{Y}_{TL} = \hat{Y} - k_{(P,1-\alpha,\upsilon)}\hat{\sigma}\left[1 + X_H^{'}(X^{'}X)^{-1}X_H\right]^{\frac{1}{2}} \tag{28}$$

where

$\hat{Y}_{TL}$	is the lower tolerance estimate
$k_{(P,\,1-\alpha,\nu)}$	is the factor for a one-sided tolerance limit at a probability of *P*, a confidence of $1 - \alpha$ and ν degrees of freedom
$\boldsymbol{X}'_{\mathrm{H}}$	is the transpose of the matrix of specific values for which a tolerance estimate is desired; this is also known as the transpose of the "hat matrix"
$\boldsymbol{X}_{\mathrm{H}}$	is the matrix of specific *x* values for which a tolerance estimate is desired; this is the "hat matrix"

For the linear case, this simplifies to

$$\hat{Y}_{TL} = \hat{Y} - k_{(P,1-\alpha,\upsilon)}\hat{\sigma}\left[1 + \frac{1}{n} + \frac{(X - \bar{X})^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}\right]^{\frac{1}{2}} \tag{29}$$

The coefficient k is the one-sided tolerance limit for a normal distribution, as given in Table B.1. These values are generated by integration of the non-central t distribution with non-centrality parameter given in equation (6). The value of k is developed for a given value of the probability of failure, P (one-sided) and the 1 - α confidence value (also one-sided), and v degrees of freedom.

The number of degrees of freedom, v, is the same number used in estimating the standard deviation.

8.5 Experimental plan for the development of *S-N* curves

Data generation to support the development of design curves should be performed by first determining the appropriate sample size. For a regression model, a minimum of 10 observations is necessary for exploratory work, but 28 are necessary for a curve intended for design or reliability pursuits. Regardless of the intention, the stress/strain range over, which the data are to be generated, must be identified. Replicate data are not necessary and are in general discouraged. Then specimen test conditions should be allocated in equal increments of stress or strain. Further, tests should be conducted according to the guidelines of Section 5. Data are generated in this fashion because

— it clearly helps to define if the response is linear or curvilinear, and

— it mitigates against any biases since the data are uniformly distributed.

Fatigue results tend to demonstrate more scatter as the stress or strain diminishes. The uncertainty along the length of the curve increases proportionately. To address this behaviour, it is recommended that specimens be allocated in increments using the double logarithm of strain or stress. This tends to locate more specimens in the higher life regimes.

Worked examples are given in A.3. and A.4.

9 Test report

9.1 Presentation of test results

The test report shall include the following information as appropriate to the type of test.

a) the observed fatigue life at a given stress or strain;

b) the test stress or strain level and the estimated mean fatigue life, plus the estimated standard deviation of the logarithm of the fatigue life. The number of test specimens shall be indicated;

c) a compilation of the experimental fatigue life data obtained for each specimen, the observations such as the mode of failure or non-failure, and indicting the test stress or strain. The R ratio (ratio of minimum to maximum stress or strain), the type of test and the test frequency shall also be reported. If the test was performed in strain control, indicate the method of extensometry and if the test was performed at constant frequency or strain rate. Report the strain rate if test were performed using a constant strain rate. Conversely, if the test frequency was held constant, this value should be reported. Finally, report if the test specimen was smoothed or notched. If notched specimens were used, the corresponding stress concentration factor (K_t) should be reported;

d) a plot of the experimental data on probability coordinates with the line best fitting the results shall be reported. No extrapolation beyond the range of the data shall be presented;

e) the estimated tolerance limit at the selected confidence and reliability level should be reported.

9.2 Fatigue strength at a given life

The test report shall include the following information as appropriate to the type of test:

a) the estimated mean fatigue strength and the estimated standard deviation. Include the number of specimens tested. Report the method used to estimate these parameters, such as the staircase method. In the latter case, the test plan showing the individual results shall be provided as well as the estimated fatigue limit (at the proscribed life, e.g. $1x10^7$ cycles). Tables A.2 and A.3 are examples of test plans followed when performing the staircase method;

b) a list of the experimental at each stress level and the number of cycles to which each specimen was subjected, with observations on failure or non-failure in the order of the test;

c) The estimated lower limit of the fatigue strength at the selected probability, when necessary. No extrapolation of the probability curve is permissible beyond the limits of the data.

9.3 *S-N* curve

The test report shall include the following information as appropriate to the type of test:

a) the estimated mean *S*-*N* curve, showing plots of the experimental data. No extrapolation beyond the limits of the data are permissible;

b) a list of experimental data including the stress or strain level and the number of cycles applied to each specimen. Each should be identified as a failure or non-failure, as appropriate;

c) the estimated lower limit of the of the *S*-*N* curve at the selected probability of failure;

d) plots of residuals and normal probability plots shall be included. Adequate justification for the choice of model, including the standard deviations for the linear or quadratic models, R^2 values, and the results of the general linear test calculation shall be presented. The reviewer should be able to clearly understand the process for final model selection.

Annex A
(informative)

Examples of applications

A.1 Example of statistical estimation of fatigue life

A set of seven data items is given in Table A.1 as an example of fatigue life data at a given fatigue strength. Calculate the average and standard deviation using equations (3) and (4).

$$\hat{\mu} = \frac{\sum_{i=1}^{n} X_i}{n} \tag{A.1}$$

where

$\hat{\mu}$ is the sample mean;

X_i is the i th observed sample value;

n is the number of data points.

In this case, x_i is the logarithm, base 10, of each observation.

Hence,

$$\hat{\mu} = \frac{\sum_{i=1}^{n} \log_{10}(x_i)}{n}$$

$$\hat{\mu} = \frac{\left(\log_{10}\left(6{,}05 \times 10^4\right) + \log_{10}\left(6{,}31 \times 10^4\right) + \ldots\right)}{7} = 4{,}915$$

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \hat{\mu})^2}{n-1}} \tag{A.2}$$

Hence,

$$\hat{\sigma} = \sqrt{\frac{\left(\log_{10}\left(6{,}05 \times 10^4\right) - 4{,}915\right)^2 + \left(\log_{10}\left(6{,}31 \times 10^4\right) - 4{,}915\right)^2 + \ldots}{7-1}} = 0{,}109$$

The lower limit of the fatigue life for a 10 % probability of failure, at a confidence level of 95 %, is estimated from equation (5), taking $k_{(0,1;\ 0,95;\ 6)}$ as 2,755 as given in Table B.1:

$$\hat{x}_{(10)} = 4{,}915 - (2{,}755 \times 0{,}109) = 4{,}615$$

or

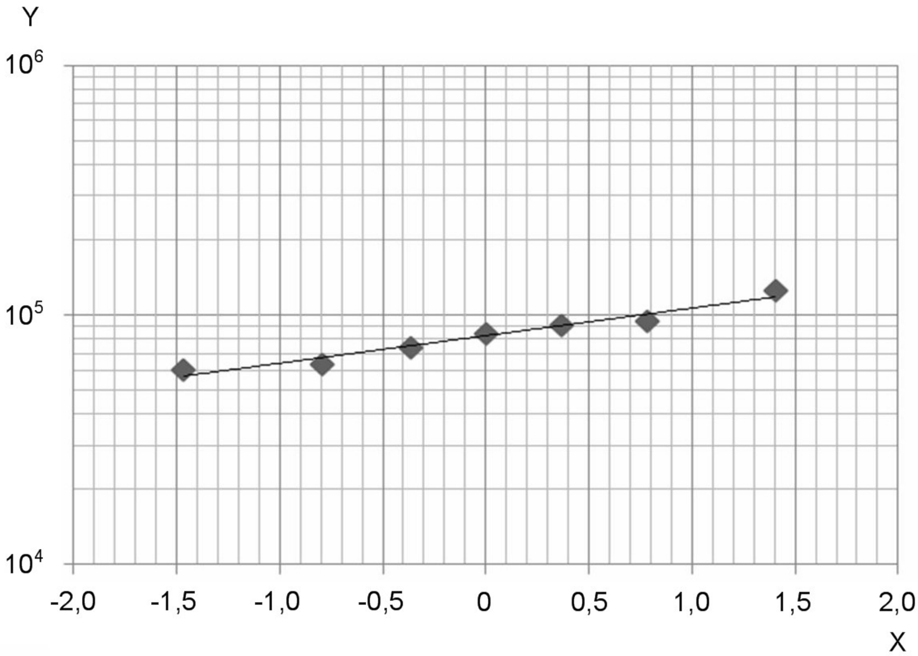
$$\hat{N}_{(10)} = 10^{4{,}615} = 4{,}1210 \times 10^{4} \quad \text{cycles}$$

Finally, the median life is $10^{4{,}915} = 82224$ cycles

Table A.1 — Example of fatigue life data at a specific stress

Specimen number	Fatigue life	Log of fatigue life	Rank	Number of standard deviations*
i	N_i, cycles	$x_i = \log N_i$	$(i - 0{,}5)/n$	z
1	$6{,}05 \times 10^4$	4,782	0,0714	-1,46523
2	$6{,}31 \times 10^4$	4,800	0,2143	-0,79164
3	$7{,}39 \times 10^4$	4,869	0,3571	-0,36611
4	$8{,}46 \times 10^4$	4,927	0,5000	0,00000
5	$9{,}11 \times 10^4$	4,960	0,6429	0,36611
6	$9{,}37 \times 10^4$	4,972	0,7857	0,79164
7	$1{,}25 \times 10^5$	5,097	0,9286	1,46523

*Number of standard deviations, z, from the Normal Probability Table. For the rank, the estimation function of specimen number i and sample size n, it is the corresponding z value. The rank is the probability value, in terms of z, that is plotted against the logarithm of fatigue life.



Key

X cumulative normal probability, Z

Y cycles to failure

Figure A.1 — Example of a cumulative normal probability plot for the fatigue life data given in Table A.1

A.2 Examples of statistical estimation of fatigue strength

A.2.1 Staircase method

When using the staircase method, specimens are tested sequentially under increasing stresses until a failure occurs or under decreasing stresses until a non-failure is observed. An example of a set of data is given in Table A.2. From the beginning, the first and second non-failure in terms of stress are not counted, for the test start is defined at the first stress level which is preferably close to the estimated mean strength. Therefore the first valid data is 500 MPa in Table A.2. In this test, there are seven failures and eight non-failures. The failure observation is therefore the one considered in the analysis. Only three stress levels are considered in the analysis, as shown in Table A.3, with S_0 = 500 MPa and stress step d = 20 MPa. The number of the relevant observation, f_i, is given in the third column of the table. The values of A, B, C and D are as follows:

A = 7; B = 11; C = 7; D = 0,571

The mean and the standard deviation of the fatigue strength are calculated from equations (7) and (8), as follows:

$$\hat{\mu}_y = 500 + 20(7/7 - 1/2) = 510 MPa$$

$$\hat{\sigma}_y = -1,62x20(0,571 + 0,029) = 19,4 MPa$$

Table A.2 — Example of staircase test data

Stress	**Sequence number of specimen**																
S_i MPa			**1**				**5**					**10**					**15**
540									X								X
520				X				O		X		X				O	
500			O		X		O				O		X		O		
480		O*				O								O			
460	O*																

X for failure
O for non-failure
* not counted (see the discussion in the first paragraph of A.2).

Table A.3 — Analysis of the data in Table A.2

Stress	**Level**	**Values**		
S_i MPa	i	f_i	if_i	$i^2 f_i$
540	2	2	4	8
520	1	3	3	3
500	0	2	0	0
Sum	—	7	7	11
		C	A	B

The lower limit of the fatigue strength for a probability of failure of 10 % is calculated from equation (9) at a confidence level of 95 %. The value of the appropriate coefficient, $k_{(0,1,\ 0,95,\ 6)}$, taken from Table B.1, is 2,755.

$$\hat{y}_{(10)} = 510 - (2{,}755 \times 19{,}4)$$
$$= 456 \text{ MPa}$$

In this example, the stress step d is close enough to the estimated standard deviation and D is greater than 0,3.

A.2.2 Modified staircase method

This example is based on the same fatigue test data as in A.2.1, but only for sequence numbers 1 to 6. The set of data used is given in Table A.4. The standard deviation of the fatigue strength is 19,4 MPa with a number of degrees of freedom of 6, as calculated above. The test was conducted with a stress step of 20 MPa which is close enough to the standard deviation.

The mean fatigue strength is calculated from the data, using equation (10), as follows:

$$\hat{\mu}_y = (500 + 520 + 500 + 480 + 500 + 520 + 540)/7$$
$$= 508{,}6 \text{ MPa}$$

The lower limit of the fatigue strength for a probability of failure of 10% is calculated from equation (9), at a confidence level of 95 % and taking a value for $k_{(0,1;\ 0,95;\ 6)}$ of 2,755 from Table B.1, as follows:

$$\hat{y}_{(10)} = 508{,}6 - (2{,}755 \times 19{,}4)$$
$$= 455{,}2 \text{ MPa}$$

Table A.4 — Example of modified staircase test data

Parameter	**Test sequence**						
i	1	2	3	4	5	6	7
S_i MPa	500	520	500	480	500	520	540
observation	O	X	X	O	O	O	[a]

X for failure

O for non-failure

[a] Test not actually carried out (stress level calculated from previous value).

A.3 Examples of statistical estimation of *S-N* curve

A.3.1 Example of force-controlled fatigue life data

A set of ten data items is given in Table A.5 and Table A.6 as an example of force-controlled fatigue life data at different stress levels. Calculate the linear regression coefficients b_1 and b_0 using the equations (16) and (17) for the average line of the linear fatigue response model according to (13), respectively equation (15). Then calculate from these results of the regression analysis the standard deviation according equation (18).

Table A.5 — Force-controlled fatigue life data at different stress levels and regression analysis

Specimen number	Stress level	Fatigue life	Log of stress level	Log of fatigue life		
i	S_i, MPa	N_i, cycles	$X_i = \log S_i$	$Y_i = \log N_i$	$X_i \cdot Y_i$	$X_i^{\ 2}$
1	900	$27{,}7 \times 10^3$	2,954243	4,442480	13,124163	8,727549
2	850	$47{,}6 \times 10^3$	2,929413	4,677607	13,702670	8,581495
3	800	$38{,}9 \times 10^3$	2,903090	4,589950	13,325037	8,727931
4	800	$86{,}2 \times 10^3$	2,903090	4,935507	14,328222	8,727931
5	750	$114{,}5 \times 10^3$	2,875061	5,058805	14,544376	8,265977
6	700	$132{,}1 \times 10^3$	2,845098	5,120903	14,569471	8,094583
7	700	$161{,}2 \times 10^3$	2,845098	5,207365	14,815464	8,094583
8	650	$596{,}3 \times 10^3$	2,812913	5,775465	16,245882	7,912482
9	600	$955{,}3 \times 10^3$	2,778151	5,980140	16,613733	7,718124
10	600	$368{,}5 \times 10^3$	2,778151	5,566437	15,464405	7,718124
n = 10	—	—	ΣX_i = 28,624315	ΣY_i = 51,354659	$\Sigma(X \cdot Y)_i$ = 146,733422	$\Sigma(X_i^{\ 2})$ = 81,968780

Note n in this context represents only the number of failures in the data.

Final parameters

$$b_1 = \frac{\sum_{i=1}^{n}(X_i \cdot Y_i) - \dfrac{\sum_{i=1}^{n} X_i \cdot \sum_{i=1}^{n} Y_i}{n}}{\sum_{i=1}^{n}\left(X_i^2\right) - \dfrac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}} = \frac{146{,}733422 - \dfrac{28{,}624315 \cdot 51{,}354659}{10}}{81{,}968780 - \dfrac{28{,}624315^{\ 2}}{10}} = -7{,}900038$$

and

$$b_0 = \frac{1}{n}\left(\sum_{i=1}^{n} Y_i - b_1 \cdot \sum_{i=1}^{n} X_i\right) = \frac{1}{10} \cdot \left(51{,}354659 - (-7{,}900038) \cdot 28{,}624315\right) = 27{,}748783$$

of the linear fatigue response model (equation (15))

$$\hat{Y}_i = -b_0 + b_1 X_i$$

Hence, the average S-N line (P = 50 %) is

$$\log_{10}(N) = b_0 + b_1 \log_{10}(S) = 27{,}748783 - 7{,}900038 \cdot \log_{10}(S)$$

Table A.6 — Example of calculation the standard deviation of the fatigue life data of Table A.5

Specimen number	Stress level	Fatigue life	S-N line			
i	S_i, MPa	N_i, cycles	$(N_{(50)})_i$	$(N/N_{(50)})_i$	$Y_i = \log\,(N/N_{(50)})_i$	$Y_i{}^2 = [\log\,(N/N_{(50)})_i]^2$
1	900	$27{,}7 \times 10^3$	$25{,}713 \times 10^3$	1,077270	0,032325	0,001045
2	850	$47{,}6 \times 10^3$	$40{,}389 \times 10^3$	1,178541	0,071345	0,005090
3	800	$38{,}9 \times 10^3$	$65{,}202 \times 10^3$	0,596606	-0,224312	0,050316
4	800	$86{,}2 \times 10^3$	$65{,}202 \times 10^3$	1,322042	0,121245	0,014700
5	750	$114{,}5 \times 10^3$	$108{,}565 \times 10^3$	1,054667	0,023116	0,000534
6	700	$132{,}1 \times 10^3$	$187{,}241 \times 10^3$	0,705508	-0,151498	0,022952
7	700	$161{,}2 \times 10^3$	$187{,}241 \times 10^3$	0,860923	-0,065036	0,004230
8	650	$596{,}3 \times 10^3$	$336{,}249 \times 10^3$	1,773389	0,248804	0,061903
9	600	$955{,}3 \times 10^3$	$632{,}824 \times 10^3$	1,509583	0,178857	0,031990
10	600	$368{,}5 \times 10^3$	$632{,}824 \times 10^3$	0,582311	-0,234845	0,055152
n = 10	—	—	—	—	$\Sigma Y_i = 0{,}000$	$\Sigma(Y_i{}^2) = 0{,}247912$

Note $(\log N - \log N_{(50)})_i = \log(N/N_{(50)})_i$

With the results of the average S-N line the standard deviation (equation (18))

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n-p}}$$

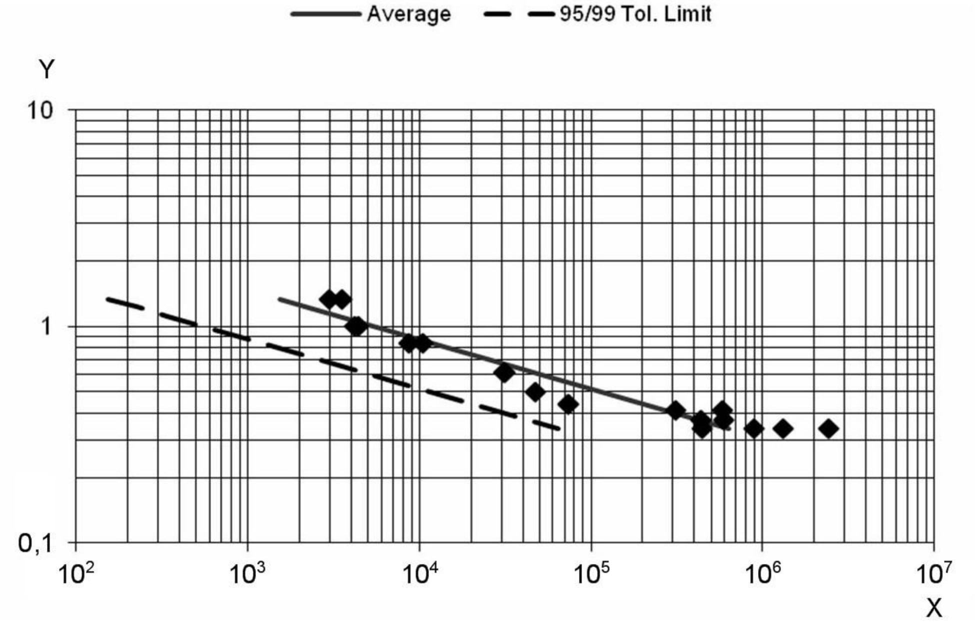
Hence:

$$\hat{\sigma} = \sqrt{\frac{1}{n-p} \cdot \sum_{i=1}^{n}\left(\log\left(\frac{N}{N_{(50)}}\right)_i - \sum_{i=1}^{n}\left(\log\left(\frac{N}{N_{(50)}}\right)_i\right)\right)^2} = \sqrt{\frac{0{,}247912}{10-2}} = 0{,}176$$

A.3.2 Example of strain–controlled fatigue life data

Table A.7 — Strain-controlled low cycle fatigue results

Specimen number	Strain range %	Cycles to failure
1	1,34	3 534
2	1,34	3 002
3	1,01	4 174
4	1,01	4 442
5	0,84	10 477
6	0,84	8 758
7	0,61	31 476
8	0,61	30 990
9	0,50	47 000
10	0,44	73 387
11	0,44	73 280
12	0,41	309 600
13	0,41	583 300
14	0,37	595 800
15	0,37	433 620
16	0,34	2 400 800
17	0,34	1 315 800
18	0,34	895 280
19	0,34	443 930

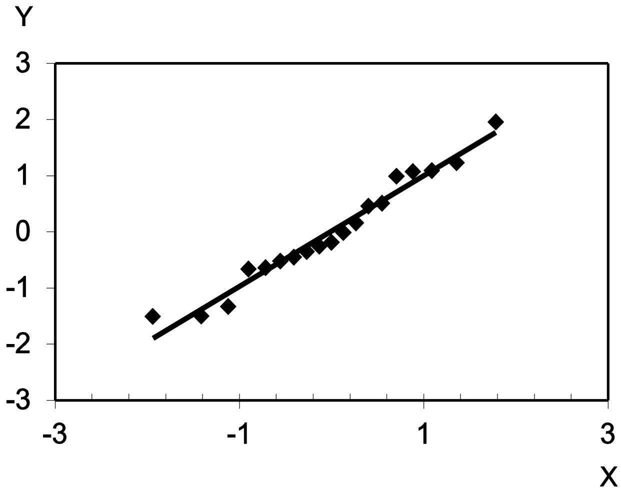


Key

X cycles to failure

Y strain range, %

Figure A.2 — Results for the linear model

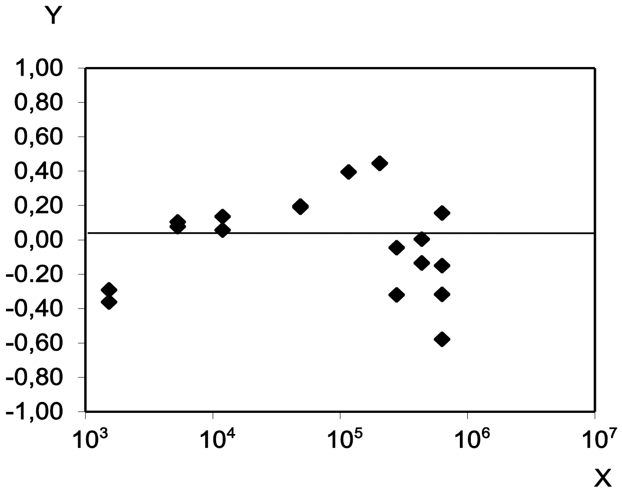


Key

X cumulative normal probability, *Z*

Y standardized residuals

Figure A.3 — Cumulative normal probability plot for the linear model

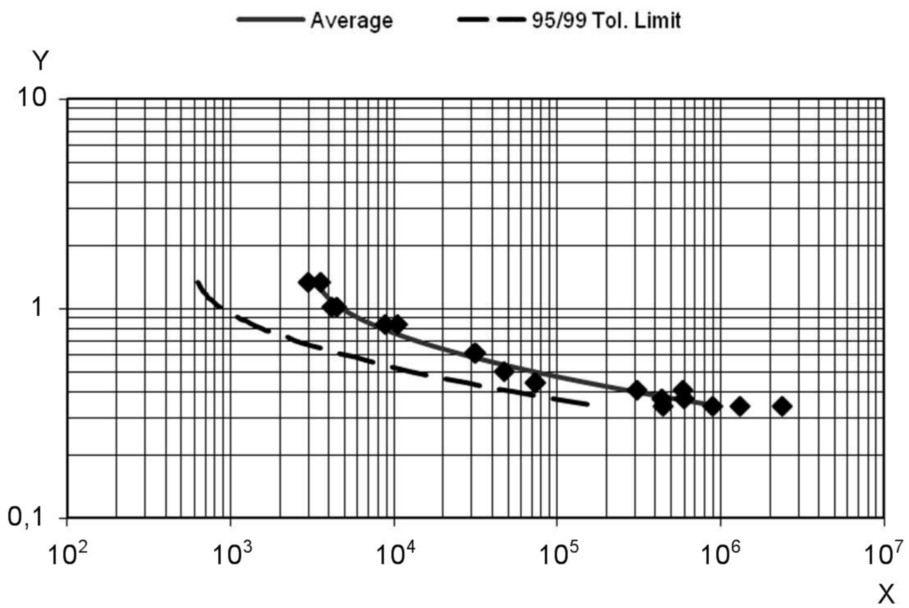


Key

X predicted life

Y standardized residuals

Figure A.4 — Residuals plot for the linear model

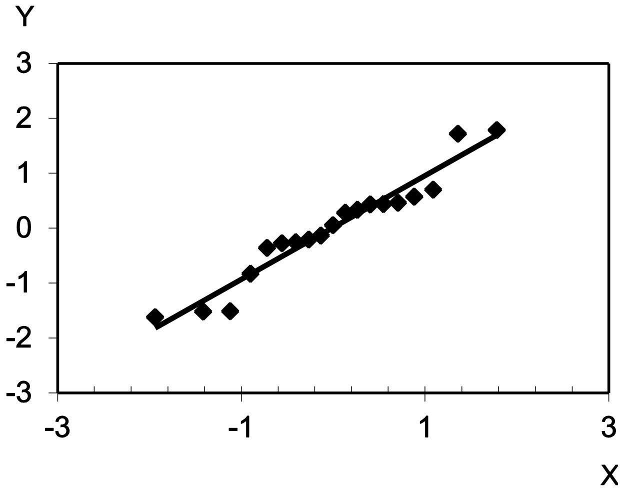


Key

X cycles to failure

Y strain range, %

Figure A.5 — Results for the quadratic model

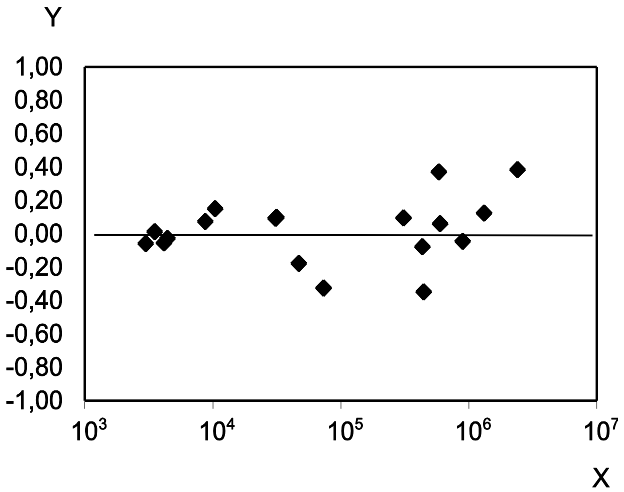


Key

X cumulative normal probability, Z

Y standardized residuals

Figure A.6 — Cumulative normal probability plot for the linear model



Key

X predicted life

Y standardized residuals

Figure A.7 — Residuals plot for the quadratic model

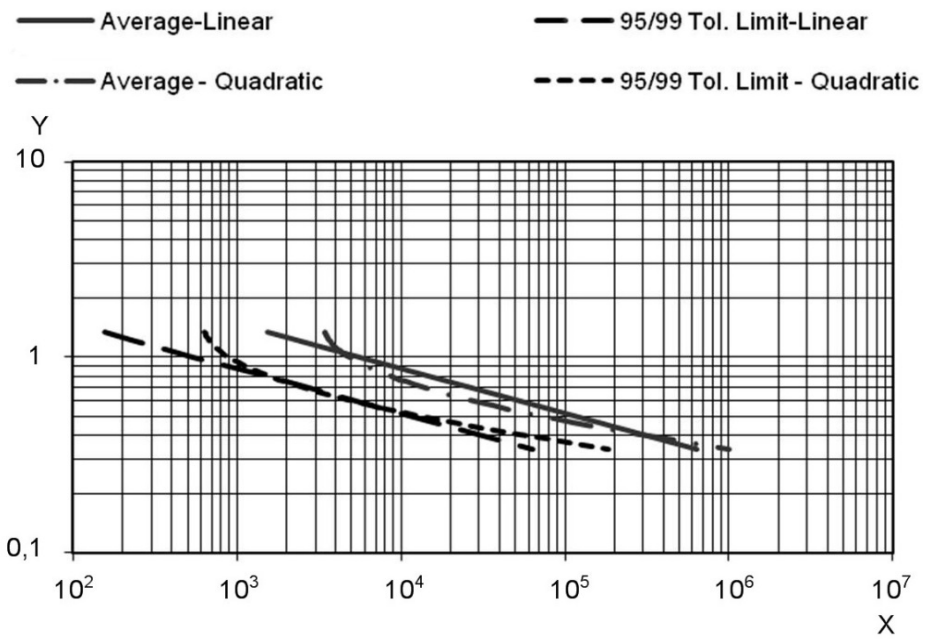
Pertinent calculations:

	Std. Dev.	R^2	**General linear test**	**Critical F* value**	**Probability**
			<u>F calculated</u>		
Linear model	0,295 5	0,912			
Quadratic model	0,2151	0,952	8,02	4,49	0,012

* Critical F value based upon an α level = 0,05 and 1 and 16 degrees of freedom for the quadratic model. The 16 degrees of freedom are used for the denominator (ν_2) for the integration of the F distribution. The

degrees of freedom for the numerator (ν_2) = 1. Table B.2 contains the critical F values. More extensive tables can be found in Reference [11] or in many other statistical textbooks.

Figure A.8 provides a graphical comparison of the linear and quadratic models.



Key

X cycles to failure

Y strain range, %

Figure A.8 — Comparison of the results for the linear and quadratic models

A review of the *S-N* plots suggests that the fit of the quadratic model to the data is better relative to the linear estimate. Both normal probability plots are reasonable and do not suggest any deviant (outlying) results. They also suggest, in either case, the results are log-normally distributed. The residual plot for the linear model demonstrates the classic case where a straight line is not capturing quadratic response. The residuals for the quadratic model populate the plot much more uniformly. Finally, the standard deviation for the quadratic model is lower than for the linear estimate, while the R^2 value are the quadratic relationship is higher than for the linear. The general-linear-test calculations also indicate the addition of the quadratic term is significantly improving (in a statistical sense) the data evaluation. Finally, by comparison of the final predicted models for each demonstrate the results are practically different. As a result of all the analysis and evaluation, the quadratic model is the relationship of choice for the reported data.

Final model parameters

b_0	3,685 06
b_1	−1,968 38
b_2	6,332 15
Std. Dev.	0,215 1

A.4 Example of an experimental plan to develop *S-N* fatigue data

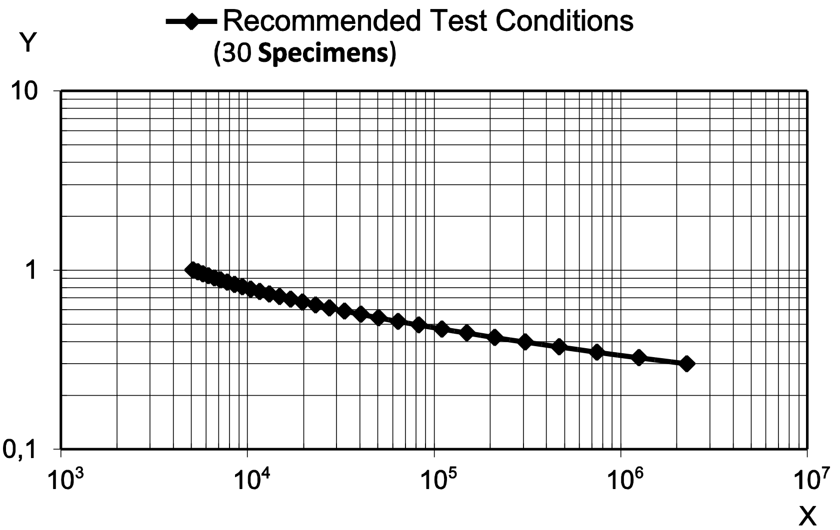
The following example illustrates the process of establishing the preliminary test conditions. Assume 30 specimens will be tested and the maximum value of strain to be considered is 1.00 % and the minimum value is 0.30 %. Since 30 specimens are to be used, the increment is $(\log_{10}[\log_{10}(10)] - \log_{10}[\log_{10}(3)])/29$. The increment is in terms of 10 and 3 as opposed to 1 and 0,3, as the first logarithm of each of these latter values is negative and the second logarithm cannot be taken. So, multiplying each strain level by 10 permits the process. Then candidate strain values are calculated by incrementing the strain level and taking appropriate antilogarithms. Then, a best estimate (guess) of the fatigue properties are obtained and plotted on *S-N* coordinates and the candidate strain (or stress) values plotted on the curve to display anticipated life response. The program should be executed by first generating a few specimens to learn how well anticipated behaviour matches observed response. The experimental plan should be adjusted accordingly. After testing several additional specimens, modification may still be in order. This process should be continued until the results are completed. This will ensure that at the end of the experiment the life response is measured over the strain or stress range of interest.

Assume:

Thirty test specimens	
Maximum strain range of interest:	1,0
Minimum strain range of interest:	0,30
R ratio:	1,0
Strain increment	$(\log_{10}[\log_{10}(10)] - \log_{10}[\log_{10}(3)])/29 = -0{,}011\ 08$

Strain values

1,00, 0,94, 0,89, 0,84, 0,80, 0,76, 0,72, 0,69, 0,65, 0,62, 0,60, 0,57, 0,54, 0,52, 0,50, 0,48, 0,46, 0,44, 0,43, 0,41, 0,40, 0,38, 0,37, 0,36, 0,35, 0,34, 0,33, 0,32, 0,31, 0,30



Key

X cycles to failure

Y strain range, %

Figure A.9 — Plot of candidate experimental conditions with anticipated fatigue lives

Annex B
(informative)

Statistical tables

Table B.1 — Coefficient $k_{\left(P, 1-\alpha, \nu\right)}$ for the one-sided tolerance limit for a normal distribution

Number of degrees of freedom ν	**Probability**, P (%)								
	10		5			1		0,1	
	Confidence level, $100 - \alpha$ (%)								
	90	95	75	90	95	90	95	90	95
2	4,258	6,158	3,805	5,310	7,655	7,340	10,55	9,651	13,86
3	3,187	4,163	2,617	3,957	5,145	5,437	7,042	7,128	9,215
4	2,742	3,407	2,149	3,400	4,202	4,666	5,741	6,112	7,501
5	2,494	3,006	1,895	3,091	3,707	4,242	5,062	5,556	6,612
6	2,333	2,755	1,732	2,894	3,399	3,972	4,641	5,201	6,061
7	2,219	2,582	1,617	2,755	3,188	3,783	4,353	4,955	5,686
8	2,133	2,454	1,532	2,649	3,031	3,641	4,143	4,772	5,414
9	2,065	2,355	1,465	2,568	2,911	3,532	3,981	4,629	5,203
10	2,012	2,275	1,411	2,503	2,815	3,444	3,852	4,515	5,036
11	1,966	2,210	1,366	2,448	2,736	3,370	3,747	4,420	4,900
12	1,928	2,155	1,328	2,403	2,670	3,310	3,659	4,341	4,787
13	1,895	2,108	1,296	2,363	2,614	3,257	3,585	4,274	4,690
14	1,866	2,068	1,267	2,329	2,566	3,212	3,520	4,215	4,607
15	1,842	2,032	1,242	2,299	2,523	3,172	3,463	4,164	4,534
16	1,820	2,001	1,220	2,272	2,486	3,136	3,415	4,118	4,471
17	1,800	1,974	1,200	2,249	2,453	3,106	3,370	4,078	4,415
18	1,781	1,949	1,182	2,228	2,423	3,078	3,331	4,041	4,364
19	1,765	1,926	1,166	2,208	2,396	3,052	3,295	4,009	4,319
20	1,750	1,905	1,151	2,190	2,371	3,028	3,262	3,979	4,276
21	1,736	1,887	1,138	2,174	2,350	3,007	3,233	3,952	4,238
22	1,724	1,869	1,125	2,159	2,329	2,987	3,206	3,927	4,204
23	1,712	1,853	1,113	2,145	2,309	2,969	3,181	3,904	4,171
24	1,702	1,838	1,103	2,132	2,292	2,952	3,158	3,882	4,143
25	1,657	1,778	1,093	2,080	2,220	2,884	3,064	3,794	4,022

Note For welding fatigue practices current values used are a failure probability of 5 % with a confidence level, $100 - \alpha$, of 75.

Table B.2 — Values of $F_{\left(1-\alpha,\ v1,\ v2\right)}$ at a confidence level, 100 - α, of 0.95

Number of degrees of freedom						
v_2	v_1					
	1	2	3	4	5	6
1	161	200	216	225	230	234
2	18,5	19,0	19,2	19,2	19,3	19,3
3	10,1	9,55	9,28	9,12	9,01	8,94
4	7,71	6,94	6,59	6,39	6,26	6,16
5	6,61	5,79	5,41	5,19	5,05	4,95
6	5,99	5,14	4,76	4,53	4,39	4,28
7	5,59	4,74	4,35	4,12	3,97	3,87
8	5,32	4,46	4,07	3,84	3,69	3,58
9	5,12	4,26	3,86	3,63	3,48	3,37
10	4,96	4,10	3,71	3,48	3,33	3,22
11	4,84	3,98	3,59	3,36	3,20	3,09
12	4,75	3,89	3,49	3,26	3,11	3,00
13	4,67	3,81	3,41	3,18	3,03	2,92
14	4,60	3,74	3,34	3,11	2,96	2,85
15	4,54	3,68	3,29	3,06	2,90	2,79
16	4,49	3,63	3,24	3,01	2,85	2,74
17	4,45	3,59	3,20	2,96	2,81	2,70
18	4,41	3,55	3,16	2,93	2,77	2,66
19	4,38	3,52	3,13	2,90	2,74	2,63
20	4,35	3,49	3,10	2,87	2,71	2,60
21	4,32	3,47	3,07	2,84	2,68	2,57
22	4,30	3,44	3,05	2,82	2,66	2,55
23	4,28	3,42	3,03	2,80	2,64	2,53
24	4,26	3,40	3,01	2,78	2,62	2,51
25	4,24	3,39	2,99	2,76	2,60	2,49
26	4,23	3,37	2,98	2,74	2,59	2,47
27	4,21	3,35	2,96	2,73	2,57	2,46
28	4,20	3,34	2,95	2,71	2,56	2,45
29	4,18	3,33	2,93	2,70	2,55	2,43
30	4,17	3,32	2,92	2,69	2,53	2,42

Bibliography

[1] BS 3518-5:1966, *Methods of fatigue testing — Guide to the application of statistics*

[2] JSME S 002:1981, *Standard Method of Statistical Fatigue Testing*

[3] NF A 03-405:1991, *Metallic products — Fatigue tests — Statistical treatment of data*

[4] WEIBULL W. *Fatigue Testing and the Analysis of Results*. Pergamon Press, 1961

[5] NISHIJIMA S. *Statistical Fatigue Properties of Some Heat-Treated Steels for Machine Structural Use.* STP 744 (1981), pp. 75-88, ASTM

[6] ASTM E739-91*, Standard Practice for Statistical Analysis of Linear or Linearized Stress-Life (S-N) and Strain-Life (∑-N) Fatigue Data*

[7] DIXON W.J., MOOD A.M. A Method for Obtaining and Analyzing Sensitivity Data. *J. Am. Stat. Assoc.* 1948, **43**, pp. 109-126

[8] BROWNLEE, K.A., HODGES, J.L., JR, and MURRAY ROSENBLATT, The Up-and-Down Method with Small Samples. *J. Am. Stat. Assoc.* 1953, **48**, pp. 262-277

[9] SHAPIRO S.S. *How to Test Normality and Other Distributional Assumptions*. American Society for Quality Control, Milwaukee: Vol. 3, 1986

[10] BASTENAIRE, F., POMEY, G., and RABBE, P., Etude statistique des durées de vie en fatigue et des courbes de Wöhler de cinq nuances d'acier. *Mémoires scientifiques de la revue de métallurgie*, **68** 1971, pp. 645-664

[11] BASTENAIRE F. *New Method for the Statistical Evaluation of Constant Stress Amplitude Fatigue Test Results,* STP 511 (1972), ASTM

[12] NETER J., WASSERMAN W., KUTNER M.H. *Applied Linear Statistical Models*, (1985), pp. 123-132, Irwin, Homewood, Il, USA

[13] DRAPER, Norman and SMITH, Harry*. Applied Regression Analysis*. Wiley, New York, NY, Second Edition, 1981

[14] STROHMEYER, C.E. The Determination of Fatigue Limits under alternating Stress Conditions. Proc. Roy. Soc. A90, 1914, pp. 411-425