

EXERCISE 2 – Dynamic Structural Analysis

Consider a Carbon/Epoxy beam, fixed on the left side ($w=\theta=0$) and on the right side free only in rotation ($w=0$), with a length of $L=400$ mm, cross-section thickness $h=2$ mm, width $w=30$ mm, elastic modulus $E=147$ GPa, Poisson's ratio 0.275, and density $\rho=1578$ kg/m³.

For this beam, construct the equivalent discrete system of dynamic equations using 12 finite elements, based on the kinematic assumptions of Timoshenko theory with nodal degrees of freedom $\{w, \theta\}$.

1. Calculate the consistent mass matrix.
2. Compute and present the natural frequencies of the system.
3. Present the first five mode shapes.
4. Construct the equivalent dynamic system using modal superposition, using first one and then the first three mode shapes. Compute, at 3/4 of the beam's length:
 - a) The forced vibration with harmonic excitation $F(t)=0.1\sin(\omega t)$ N applied at the midpoint ($L/2$) of the beam, where ω is the average of the first two natural frequencies.
 - b) The transient response due to an impulse force $F(t)=0.1\delta(t)$ N applied at the midpoint ($L/2$).
5. Consider viscous damping with damping matrix proportional to the finite element stiffness matrix as defined in theory $[C = a * K_e]$.
6. Repeat question 2 for $a_1 = 0.5\%$ (proportional damping).
7. Determine the poles and compute the right and left eigenvectors.
8. Find the transient response of the system for part (4a) using the central difference method of direct integration. Investigate the solution behavior for large and small time step sizes, $\Delta t_1 = T_1/5$ and $\Delta t_2 = T_{\max}/5$ respectively, where T_1 and T_{\max} are the periods of the first and last mode shapes of the system.

Submission Date: On the day of the course examination as stated in the exam schedule.

* Initial displacement, velocity, and acceleration conditions are zero.