

Jednodimenzionalne optimizacije

Zadatak I.

Odrediti intervale unimodalnosti funkcija:

$$f_1(x) = (x-1)(x+2)(x-3)(x+5),$$

$$f_2(x) = \max\{(x-1)(x+2), -(x-3)(x+5)\},$$

$$f_3(x) = (x-2)\sin(3x).$$

Odredjivanje intervala unimodalnosti funkcije nije jednostavan problem, pogotovu ako funkcija nije diferencijabilna. Ovde će biti predstavljeno matematički korektno odredjivanje intervala unimodalnosti funkcije $f_1(x)$, a za funkcije $f_2(x)$ i $f_3(x)$ intervali unimodalnosti mogu da se odrede sa grafika.

Navedena su i neka poznata tvrdjenja potrebna za razmatranja unimodalnosti.

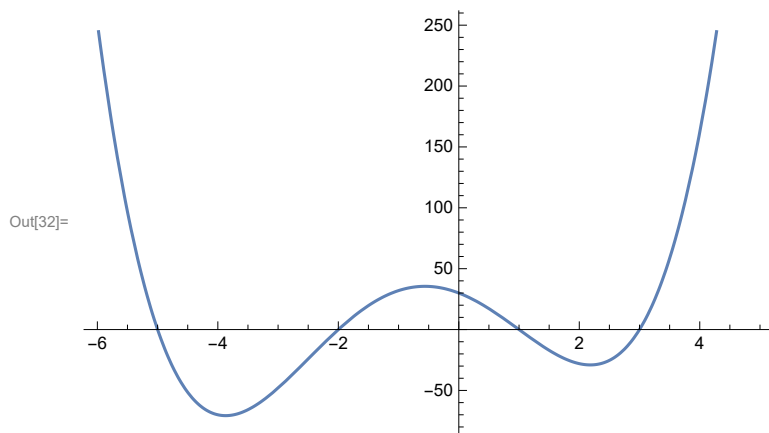
- Tačka c je nula reda $n+1$ puta diferencijabilne funkcije $f(x)$ ako je $f(c) = f'(c) = \dots = f^{(n)}(c) = 0$, $f^{(n+1)}(c) \neq 0$.

- Ako je funkcija $f(x)$ dvaput diferencijabilna u tački c i važi $f'(c) = 0$ i $f''(c) \neq 0$, tada ona dostiže ekstremum u tački c , i to minimum, ako je $f''(c) > 0$, a maksimum, ako je $f''(c) < 0$.

- (Rolova teorema) Neka je funkcija $f(x)$ neprekidna na $[a, b]$, diferencijabilna na (a, b) i neka je $f(a) = f(b)$. Tada postoji $c \in (a, b)$ tako da je $f'(c) = 0$.

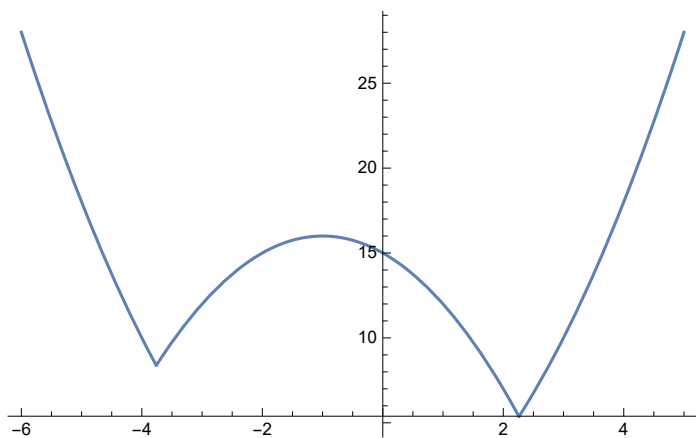
Imajući u vidu da je $f_1(x)$ polinom stepena 4, on je diferencijabilna funkcija na \mathbb{R} , pa zadovoljava uslove Rolove teoreme na svakom od intervala $[-5, -2]$, $[-2, 1]$, $[1, 3]$. Stoga, postoje tačke $c_1 \in (-5, -2)$, $c_2 \in (-2, 1)$, $c_3 \in (1, 3)$ takve da je $f_1'(c_1) = f_1'(c_2) = f_1'(c_3) = 0$. Funkcija $f_1'(x)$, kao polinom stepena 3, ne može da ima više od 3 nule, a c_1 , c_2 i c_3 su različite, one moraju da budu proste, tj. $f_1''(c_1) \neq 0$, $f_1''(c_2) \neq 0$, $f_1''(c_3) \neq 0$. Prema svemu navedenom, na svakom od intervala $[-5, -2]$, $[-2, 1]$, $[1, 3]$ funkcija $f_1(x)$ dostiže po jedan ekstremum, pa su to intervali unimodalnosti funkcije $f_1(x)$.

```
In[31]:= f1[x_] := (x - 1) (x + 2) (x - 3) (x + 5);  
Plot[f1[x], {x, -6, 5}]
```



```
In[33]:= f2[x_] := Max[(x - 1) (x + 2), - (x - 3) (x + 5)];
Plot[f2[x], {x, -6, 5}]
```

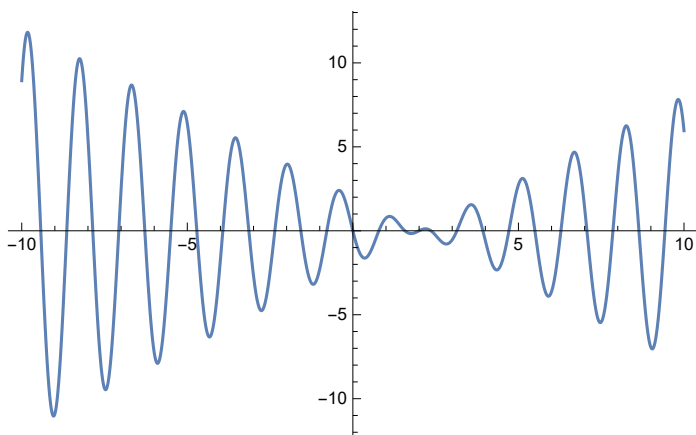
Out[34]=



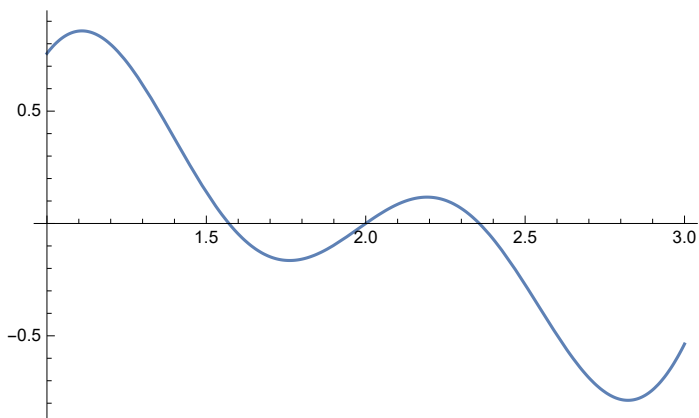
Funkcija $f_3(x) = (x - 2) \sin(3x)$ ima nule u tačkama $x = 2$, $x = \frac{k\pi}{3}$, $k \in \mathbb{Z}$.

```
In[35]:= f3[x_] := (x - 2) Sin[4 x];
Plot[f3[x], {x, -10, 10}]
Plot[f3[x], {x, 1, 3}]
```

Out[36]=



Out[37]=



Zadatak 2.

Metodom ravnomernog pretrazivanja rešiti minimizacioni problem $\min_{1 \leq x \leq 3} (x-1)(x+2)(x-3)(x+5)$.

Rešenje :

```
Clear["Global`*"];
f[x_] := (x - 1) (x + 2) (x - 3) (x + 5);
a = 1.; b = 3.;
(* Metod ravnomernog pretrazivanja sa konstantnim korakom podelom na 10 podintervala *)
n = 9;
delta =  $\frac{b - a}{n + 1}$ ;
Print["interval neodredjenosti: [", a, ",", b, "], korak: ", delta];
A = Table[{a + i * delta, f[a + i * delta]}, {i, 1, n}];
Print[MatrixForm[A]];
fmin = A[[1]];
For[k = 2, k ≤ n, k++, If[A[[k, 2]] ≤ fmin[[2]], fmin = A[[k]]]];
Print["fmin=", fmin[[2]], " za xmin=", fmin[[1]], " greska: |xmin-x*|<", delta];
interval neodredjenosti: [1.,3.], korak: 0.2
```

1.2	-7.1424
1.4	-13.9264
1.6	-19.9584
1.8	-24.8064
2.	-28.
2.2	-29.0304
2.4	-27.3504
2.6	-22.3744
2.8	-13.4784

```
fmin=-29.0304 za xmin=2.2, greska: |xmin-x*|<0.2
```

```
In[49]:= Clear["Global`*"];
f[x_] := (x - 1) (x + 2) (x - 3) (x + 5);
a = 1.; b = 3.;
(* Metod ravnomernog pretrazivanja sa konstantnim korakom *)
(* podela na 10, 10^2, 10^3, ... podintervala *)
For[j = 1, j ≤ 3, j++, n = 2 * 10^j - 1;
  delta =  $\frac{b - a}{n + 1}$ ;
  Print["interval neodredjenosti: [", a, ",", b, "], korak: ", delta];
  A = Table[{a + i * delta, f[a + i * delta]}, {i, 1, n}];
  (* Print[MatrixForm[A]]; *)
  fmin = A[[1]];
  For[k = 2, k ≤ n, k++, If[A[[k, 2]] ≤ fmin[[2]], fmin = A[[k]]]];
  Print["fmin=", fmin[[2]],
    " za xmin=", fmin[[1]], " greska: |xmin-x*|<", delta];
  Print[]
];
```

```
interval neodredjenosti: [1.,3.], korak: 0.1
fmin=-29.0304 za xmin=2.2, greska: |xmin-x*|<0.1
```

```
interval neodredjenosti: [1.,3.], korak: 0.01
fmin=-29.04 za xmin=2.18, greska: |xmin-x*|<0.01
```

```
interval neodredjenosti: [1.,3.], korak: 0.001
fmin=-29.0403 za xmin=2.183, greska: |xmin-x*|<0.001
```

```
In[53]:= Clear["Global`*"];
f[x_] := (x - 1) (x + 2) (x - 3) (x + 5);
a = 1.; b = 3.;
(* Metod ravnomernog pretrazivanja sa promenljivim korakom *)
(* podela na 10 podintervala intervala
   neodredjenosti dobijenog u prethodnoj iteraciji *)
n = 9;
For[j = 1, j ≤ 5, j++,
  delta =  $\frac{b - a}{n + 1}$ ;
  Print["interval neodredjenosti: [", a, ",", b, "], korak: ", delta];
  A = Table[{a + i * delta, f[a + i * delta]}, {i, 1, n}];
  Print[MatrixForm[A]];
  fmin = A[[1]];
  For[k = 2, k ≤ n, k++, If[A[[k, 2]] ≤ fmin[[2]], fmin = A[[k]]]];
  Print["fmin=", fmin[[2]],
    " za xmin=", fmin[[1]], ", greska: |xmin-x*|<", delta];
  Print[];
  a = fmin[[1]] - delta; b = fmin[[1]] + delta
];
```

```
interval neodredjenosti: [1.,3.], korak: 0.2
```

```
( 1.2 -7.1424
  1.4 -13.9264
  1.6 -19.9584
  1.8 -24.8064
   2. -28.
  2.2 -29.0304
  2.4 -27.3504
  2.6 -22.3744
  2.8 -13.4784 )
```

```
fmin=-29.0304 za xmin=2.2, greska: |xmin-x*|<0.2
```

```
interval neodredjenosti: [2.,2.4], korak: 0.04
```

$$\begin{pmatrix} 2.04 & -28.3961 \\ 2.08 & -28.7015 \\ 2.12 & -28.912 \\ 2.16 & -29.0231 \\ 2.2 & -29.0304 \\ 2.24 & -28.9294 \\ 2.28 & -28.7156 \\ 2.32 & -28.3843 \\ 2.36 & -27.9308 \end{pmatrix}$$

fmin=-29.0304 za xmin=2.2, greska: $|x_{\min}-x^*| < 0.04$

interval neodredjenosti: [2.16,2.24], korak: 0.008

$$\begin{pmatrix} 2.168 & -29.033 \\ 2.176 & -29.0387 \\ 2.184 & -29.0402 \\ 2.192 & -29.0375 \\ 2.2 & -29.0304 \\ 2.208 & -29.019 \\ 2.216 & -29.0033 \\ 2.224 & -28.9831 \\ 2.232 & -28.9585 \end{pmatrix}$$

fmin=-29.0402 za xmin=2.184, greska: $|x_{\min}-x^*| < 0.008$

interval neodredjenosti: [2.176,2.192], korak: 0.0016

$$\begin{pmatrix} 2.1776 & -29.0394 \\ 2.1792 & -29.0398 \\ 2.1808 & -29.0401 \\ 2.1824 & -29.0403 \\ 2.184 & -29.0402 \\ 2.1856 & -29.04 \\ 2.1872 & -29.0396 \\ 2.1888 & -29.0391 \\ 2.1904 & -29.0384 \end{pmatrix}$$

fmin=-29.0403 za xmin=2.1824, greska: $|x_{\min}-x^*| < 0.0016$

interval neodredjenosti: [2.1808,2.184], korak: 0.00032

$$\begin{pmatrix} 2.18112 & -29.0402 \\ 2.18144 & -29.0402 \\ 2.18176 & -29.0402 \\ 2.18208 & -29.0402 \\ 2.1824 & -29.0403 \\ 2.18272 & -29.0403 \\ 2.18304 & -29.0403 \\ 2.18336 & -29.0403 \\ 2.18368 & -29.0402 \end{pmatrix}$$

fmin=-29.0403 za xmin=2.18272, greska: $|x_{\min}-x^*| < 0.00032$

```

In[85]:= Clear["Global`*"];
f[x_] := (x - 1) (x + 2) (x - 3) (x + 5);
a = 1.; b = 3.;
(* Metod ravnomernog pretrazivanja sa promenljivim korakom *)
(* podela na 20 podintervala intervala
   neodredjenosti dobijenog u prethodnoj iteraciji *)
n = 19;
For[j = 1, j ≤ 5, j++,
  delta =  $\frac{b - a}{n + 1}$ ;
  Print["interval neodredjenosti: [", a, ", ", b, "], korak: ", delta];
  A = Table[{a + i * delta, f[a + i * delta]}, {i, 1, n}];
  (* Print[MatrixForm[A]]; *)
  fmin = A[[1]];
  For[k = 2, k ≤ n, k++, If[A[[k, 2]] ≤ fmin[[2]], fmin = A[[k]]]];
  Print["fmin=", fmin[[2]],
    " za xmin=", fmin[[1]], ", greska: |xmin-x*| < ", delta];
  Print[];
  a = fmin[[1]] - delta; b = fmin[[1]] + delta
];

```

interval neodredjenosti: [1.,3.], korak: 0.1

fmin=-29.0304 za xmin=2.2, greska: |xmin-x*| < 0.1

interval neodredjenosti: [2.1,2.3], korak: 0.01

fmin=-29.04 za xmin=2.18, greska: |xmin-x*| < 0.01

interval neodredjenosti: [2.17,2.19], korak: 0.001

fmin=-29.0403 za xmin=2.183, greska: |xmin-x*| < 0.001

interval neodredjenosti: [2.182,2.184], korak: 0.0001

fmin=-29.0403 za xmin=2.1828, greska: |xmin-x*| < 0.0001

interval neodredjenosti: [2.1827,2.1829], korak: 0.00001

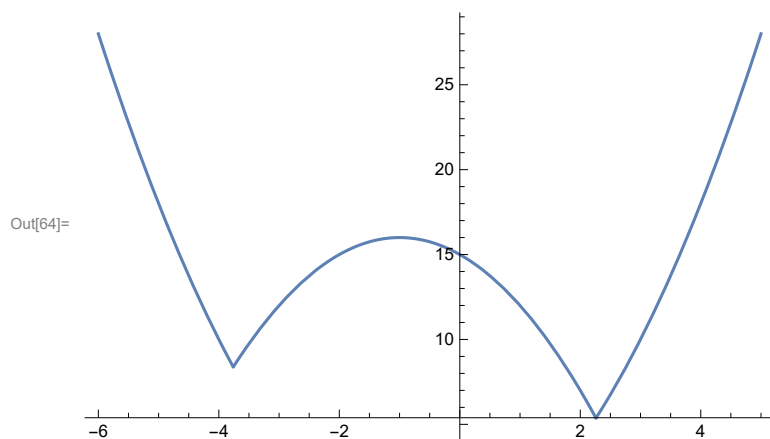
fmin=-29.0403 za xmin=2.18282, greska: |xmin-x*| < 0.00001

Zadatak 3.

Metodom ravnomernog pretrazivanja sa tačnošću 10^{-3} rešiti minimizacioni problem:

a) $\min_{1 \leq x \leq 3} \max \{(x - 1)(x + 2), -(x - 3)(x + 5)\}$; b) $\min_{-4 \leq x \leq -3} \max \{(x - 1)(x + 2), -(x - 3)(x + 5)\}$.

Rešenje :



interval neodredjenosti: [1.,3.], korak: 0.1

fmin=5.59 za xmin=2.3, greska: $|x_{\min}-x^*| < 0.1$

interval neodredjenosti: [2.2,2.4], korak: 0.01

fmin=5.3724 za xmin=2.26, greska: $|x_{\min}-x^*| < 0.01$

interval neodredjenosti: [2.25,2.27], korak: 0.001

fmin=5.3724 za xmin=2.26, greska: $|x_{\min}-x^*| < 0.001$

Zadatak 4.

Metodom zlatnog preseka sa tačnošću 10^{-3} rešiti minimizacioni problem $\min_{1 \leq x \leq 3} (x-1)(x+2)(x-3)(x+5)$.

Rešenje :

```

In[144]:= Clear["Global`*"];
f[x_] := (x - 1) (x + 2) (x - 3) (x + 5);
Phi = N[ $\frac{-1 + \sqrt{5}}{2}$ ];
a = 1.; b = 3.; epsilon = 10-3;
Print["-----"];
Print["  a          b          |b-a|          p          q          f(p)          f(q)"];
Print["-----"];
p = a + (1 - Phi) (b - a);
q = a + Phi (b - a);
BrIteracija = 0;
While[b - a ≥ 2 epsilon,
  BrIteracija = BrIteracija + 1;
  (* NumberForm[expr, {n, f}] prints with approximate real numbers having n digits,
  with f digits to the right of the decimal point. *)
  Print[NumberForm[a, {6, 5}], " ", NumberForm[b, {6, 5}], " ",
    NumberForm[b - a, {6, 5}], " ", p, " ", q, " ", f[p], " ", f[q]];
  If[f[p] ≥ f[q], a = p; p = q; q = a + Phi (b - a),
    b = q; q = p; p = a + (1 - Phi) (b - a)];
];
Print[NumberForm[a, {6, 5}], " ", NumberForm[b, {6, 5}], " ", NumberForm[b - a, {6, 5}]];
xmin =  $\frac{a + b}{2}$ ; fmin = f[xmin];
Print["-----"];
Print["fmin=", fmin, " za xmin=", xmin, " greska: |xmin-x*| < ",  $\frac{b - a}{2}$ ];
Print["broj iteracija: ", BrIteracija];

```

a	b	b-a	p	q	f(p)	f(q)
1.00000	3.00000	2.00000	1.76393	2.23607	-24.0402	-28.9443
1.76393	3.00000	1.23607	2.23607	2.52786	-28.9443	-24.5876
1.76393	2.52786	0.76393	2.05573	2.23607	-28.5272	-28.9443
2.05573	2.52786	0.47214	2.23607	2.34752	-28.9443	-28.0856
2.05573	2.34752	0.29180	2.16718	2.23607	-29.0322	-28.9443
2.05573	2.23607	0.18034	2.12461	2.16718	-28.9299	-29.0322
2.12461	2.23607	0.11146	2.16718	2.1935	-29.0322	-29.0365
2.16718	2.23607	0.06888	2.1935	2.20976	-29.0365	-29.0159
2.16718	2.20976	0.04257	2.18345	2.1935	-29.0403	-29.0365
2.16718	2.19350	0.02631	2.17723	2.18345	-29.0392	-29.0403
2.17723	2.19350	0.01626	2.18345	2.18728	-29.0403	-29.0396
2.17723	2.18728	0.01005	2.18107	2.18345	-29.0402	-29.0403
2.18107	2.18728	0.00621	2.18345	2.18491	-29.0403	-29.0401
2.18107	2.18491	0.00384	2.18254	2.18345	-29.0403	-29.0403
2.18107	2.18345	0.00237	2.18198	2.18254	-29.0402	-29.0403
2.18198	2.18345	0.00147				
fmin=-29.0403 za xmin=2.18271, greska: xmin-x* <0.000733137						
broj iteracija: 15						

Zadatak 5.

Metodom zlatnog preseka sa tačnošću 10^{-3} rešiti minimizacioni problem:

a) $\min_{1 \leq x \leq 3} \max \{(x-1)(x+2), -(x-3)(x+5)\}$; b) $\min_{-4 \leq x \leq -3} \max \{(x-1)(x+2), -(x-3)(x+5)\}$.

Rešenje :

a	b	b-a	p	q	f(p)	f(q)
1.00000	3.00000	2.00000	1.76393	2.23607	8.36068	5.52786
1.76393	3.00000	1.23607	2.23607	2.52786	5.52786	6.91796
1.76393	2.52786	0.76393	2.05573	2.23607	6.66253	5.52786
2.05573	2.52786	0.47214	2.23607	2.34752	5.52786	5.85839
2.05573	2.34752	0.29180	2.16718	2.23607	5.96894	5.52786
2.16718	2.34752	0.18034	2.23607	2.27864	5.52786	5.47084
2.23607	2.34752	0.11146	2.27864	2.30495	5.47084	5.61775
2.23607	2.30495	0.06888	2.26238	2.27864	5.38074	5.47084
2.23607	2.27864	0.04257	2.25233	2.26238	5.42235	5.38074
2.25233	2.27864	0.02631	2.26238	2.26859	5.38074	5.41509
2.25233	2.26859	0.01626	2.25854	2.26238	5.38191	5.38074
2.25854	2.26859	0.01005	2.26238	2.26475	5.38074	5.39385
2.25854	2.26475	0.00621	2.26091	2.26238	5.37264	5.38074
2.25854	2.26238	0.00384	2.26001	2.26091	5.37236	5.37264
2.25854	2.26091	0.00237	2.25945	2.26001	5.37601	5.37236
2.25945	2.26091	0.00147				

fmin=5.37123 za xmin=2.26018, greska: $|x_{\min}-x^*| < 0.000733137$

broj iteracija: 15

a	b	b-a	p	q	f(p)	f(q)
-4.00000	-3.00000	1.00000	-3.61803	-3.38197	9.1459	10.3262
-4.00000	-3.38197	0.61803	-3.76393	-3.61803	8.40325	9.1459
-4.00000	-3.61803	0.38197	-3.8541	-3.76393	9.	8.40325
-3.85410	-3.61803	0.23607	-3.76393	-3.7082	8.40325	8.66563
-3.85410	-3.70820	0.14590	-3.79837	-3.76393	8.62927	8.40325
-3.79837	-3.70820	0.09017	-3.76393	-3.74265	8.40325	8.47789
-3.79837	-3.74265	0.05573	-3.77709	-3.76393	8.4893	8.40325
-3.77709	-3.74265	0.03444	-3.76393	-3.7558	8.40325	8.40556
-3.77709	-3.75580	0.02129	-3.76896	-3.76393	8.43608	8.40325
-3.76896	-3.75580	0.01316	-3.76393	-3.76083	8.40325	8.38299
-3.76393	-3.75580	0.00813	-3.76083	-3.75891	8.38299	8.38843
-3.76393	-3.75891	0.00502	-3.76201	-3.76083	8.39073	8.38299
-3.76201	-3.75891	0.00311	-3.76083	-3.76009	8.38299	8.38189
-3.76083	-3.75891	0.00192				

fmin=8.38314 za xmin=-3.75987, greska: $|x_{\min}-x^*| < 0.000959689$

broj iteracija: 13

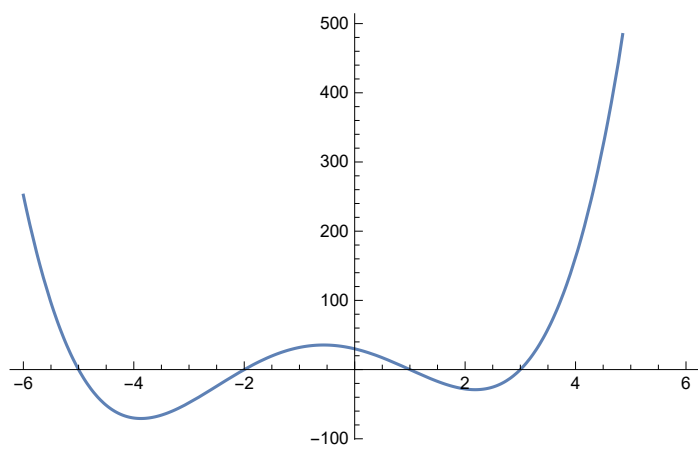
Zadatak 6.

Jednodimenzionalnim simpleks metodom sa tačnošću 10^{-3} rešiti minimizacioni problem:

$\min (x-1)(x+2)(x-3)(x+5)$, uzimajući različite početne vrednosti.

Rešenje :

$$f(x) = (-3+x)(-1+x)(2+x)(5+x)$$



In[404]:=

```

Clear["Global`*"];
f[x_] := (x - 1) (x + 2) (x - 3) (x + 5);
Print["f(x)=", f[x]];
Print["-----"];
epsilon = 10-2; MaxIter = 100;
(* pocetni korak i pocetna tacka *)
(* indikator ind=0:
   uzima vrednost 1 pri prvoj promeni
   smeru posle pocetne i spreca dalje povecanje koraka *)
delta = 0.1;
(* uzimati za x0 redom 0., -1.5, 1.5, -2.5, 2.5, ... *)
x0 = 0.;
xmin = x0; fmin = f[x0]; ind = 0; fx = fmin;
x = x0;
(* utvrdjivanje pocetnog smeru pretrazivanja *)
If[f[x + delta] > fmin,
  Print["potrebna pocetna promena smeru pretrazivanja"];
  delta = -delta];
(*Print["delta(0)=", delta, ",      x(1)=", x, ",      f(1)=", f[x], ",      ind=", ind];*)
BrIteracija = 0;
While[Abs[delta] ≥ epsilon ∧ BrIteracija ≤ MaxIter - 1,
  Print["x(", BrIteracija, ")=", x, ",      f(", BrIteracija, ")=", f[x],
    ",      ind=", ind, ",      xmin=", xmin, ",      delta(", BrIteracija, ")=", delta];
  fx = f[x];
  x = x + delta;
  BrIteracija = BrIteracija + 1;
  If[f[x] ≤ fx,
    If[f[x] ≤ fmin,
      xmin = x; fmin = f[x]];
    If[ind == 0, delta = 2 * delta],
    ind = 1; delta = -delta/4];
];
Print["x(", BrIteracija, ")=", x, ",      f(", BrIteracija, ")=", f[x],
  ",      ind=", ind, ",      xmin=", xmin, ",      delta(", BrIteracija, ")=", delta];
If[ind == 0, Print["Nije pronadjen minimum"],
  Print["-----"];
  Print["fmin=", fmin, "      za xmin=", xmin, ",      greska: |xmin-x*|<", Abs[delta]]];
  Print["broj iteracija:      ", BrIteracija]
]

```

$$f(x) = (-3 + x) (-1 + x) (2 + x) (5 + x)$$

```
-----
x(0)=0.,    f(0)=30.,    ind=0,    xmin=0.,    delta(0)=0.1
x(1)=0.1,   f(1)=27.9531, ind=0,    xmin=0.1,   delta(1)=0.2
x(2)=0.3,   f(2)=23.0391, ind=0,    xmin=0.3,   delta(2)=0.4
x(3)=0.7,   f(3)=10.6191, ind=0,    xmin=0.7,   delta(3)=0.8
x(4)=1.5,   f(4)=-17.0625, ind=0,    xmin=1.5,   delta(4)=1.6
x(5)=3.1,   f(5)=8.6751,  ind=1,    xmin=1.5,   delta(5)=-0.4
x(6)=2.7,   f(6)=-18.4569, ind=1,    xmin=2.7,   delta(6)=-0.4
x(7)=2.3,   f(7)=-28.5649, ind=1,    xmin=2.3,   delta(7)=-0.4
x(8)=1.9,   f(8)=-26.6409, ind=1,    xmin=2.3,   delta(8)=0.1
x(9)=2.,    f(9)=-28.,    ind=1,    xmin=2.3,   delta(9)=0.1
x(10)=2.1,  f(10)=-28.8189,  ind=1,    xmin=2.1,   delta(10)=0.1
x(11)=2.2,  f(11)=-29.0304, ind=1,    xmin=2.2,   delta(11)=0.1
x(12)=2.3,  f(12)=-28.5649, ind=1,    xmin=2.2,   delta(12)=-0.025
x(13)=2.275, f(13)=-28.7486,  ind=1,    xmin=2.2,   delta(13)=-0.025
x(14)=2.25, f(14)=-28.8867,  ind=1,    xmin=2.2,   delta(14)=-0.025
x(15)=2.225, f(15)=-28.9803,  ind=1,    xmin=2.2,   delta(15)=-0.025
x(16)=2.2,  f(16)=-29.0304, ind=1,    xmin=2.2,   delta(16)=-0.025
x(17)=2.175, f(17)=-29.0382,  ind=1,    xmin=2.175,  delta(17)=-0.025
x(18)=2.15, f(18)=-29.0049,  ind=1,    xmin=2.175,  delta(18)=0.00625
-----
```

fmin=-29.0382 za xmin=2.175, greska: |xmin-x*|<0.00625

broj iteracija: 18

Da li je dobijen lokalni ili globalni minimum proverice se promenom startnih tacaka.

$$f(x) = (-3 + x) (-1 + x) (2 + x) (5 + x)$$

potrebna pocetna promena smeru pretrazivanja

```
x(0)=5.,    f(0)=560.,    ind=0,    xmin=5.,    delta(0)=-0.1
x(1)=4.9,   f(1)=506.177, ind=0,    xmin=4.9,   delta(1)=-0.2
x(2)=4.7,   f(2)=408.787, ind=0,    xmin=4.7,   delta(2)=-0.4
x(3)=4.3,   f(3)=251.351, ind=0,    xmin=4.3,   delta(3)=-0.8
x(4)=3.5,   f(4)=58.4375, ind=0,    xmin=3.5,   delta(4)=-1.6
x(5)=1.9,   f(5)=-26.6409, ind=0,    xmin=1.9,   delta(5)=-3.2
```

```

x(6)=-1.3,    f(6)=25.6151,    ind=1,    xmin=1.9,    delta(6)=0.8
x(7)=-0.5,    f(7)=35.4375,    ind=1,    xmin=1.9,    delta(7)=-0.2
x(8)=-0.7,    f(8)=35.1611,    ind=1,    xmin=1.9,    delta(8)=-0.2
x(9)=-0.9,    f(9)=33.4191,    ind=1,    xmin=1.9,    delta(9)=-0.2
x(10)=-1.1,   f(10)=30.2211,    ind=1,    xmin=1.9,    delta(10)=-0.2
x(11)=-1.3,   f(11)=25.6151,    ind=1,    xmin=1.9,    delta(11)=-0.2
x(12)=-1.5,   f(12)=19.6875,    ind=1,    xmin=1.9,    delta(12)=-0.2
x(13)=-1.7,   f(13)=12.5631,    ind=1,    xmin=1.9,    delta(13)=-0.2
x(14)=-1.9,   f(14)=4.4051,    ind=1,    xmin=1.9,    delta(14)=-0.2
x(15)=-2.1,   f(15)=-4.5849,    ind=1,    xmin=1.9,    delta(15)=-0.2
x(16)=-2.3,   f(16)=-14.1669,    ind=1,    xmin=1.9,    delta(16)=-0.2
x(17)=-2.5,   f(17)=-24.0625,    ind=1,    xmin=1.9,    delta(17)=-0.2
x(18)=-2.7,   f(18)=-33.9549,    ind=1,    xmin=-2.7,    delta(18)=-0.2
x(19)=-2.9,   f(19)=-43.4889,    ind=1,    xmin=-2.9,    delta(19)=-0.2
x(20)=-3.1,   f(20)=-52.2709,    ind=1,    xmin=-3.1,    delta(20)=-0.2
x(21)=-3.3,   f(21)=-59.8689,    ind=1,    xmin=-3.3,    delta(21)=-0.2
x(22)=-3.5,   f(22)=-65.8125,    ind=1,    xmin=-3.5,    delta(22)=-0.2
x(23)=-3.7,   f(23)=-69.5929,    ind=1,    xmin=-3.7,    delta(23)=-0.2
x(24)=-3.9,   f(24)=-70.6629,    ind=1,    xmin=-3.9,    delta(24)=-0.2
x(25)=-4.1,   f(25)=-68.4369,    ind=1,    xmin=-3.9,    delta(25)=0.05
x(26)=-4.05,  f(26)=-69.3359,    ind=1,    xmin=-3.9,    delta(26)=0.05
x(27)=-4.,    f(27)=-70.,    ind=1,    xmin=-3.9,    delta(27)=0.05
x(28)=-3.95,  f(28)=-70.4391,    ind=1,    xmin=-3.9,    delta(28)=0.05
x(29)=-3.9,   f(29)=-70.6629,    ind=1,    xmin=-3.9,    delta(29)=0.05
x(30)=-3.85,  f(30)=-70.6809,    ind=1,    xmin=-3.85,    delta(30)=0.05
x(31)=-3.8,   f(31)=-70.5024,    ind=1,    xmin=-3.85,    delta(31)=-0.0125
x(32)=-3.8125, f(32)=-70.5649,    ind=1,    xmin=-3.85,    delta(32)=-0.0125
x(33)=-3.825, f(33)=-70.6156,    ind=1,    xmin=-3.85,    delta(33)=-0.0125
x(34)=-3.8375, f(34)=-70.6543,    ind=1,    xmin=-3.85,    delta(34)=-0.0125
x(35)=-3.85,  f(35)=-70.6809,    ind=1,    xmin=-3.85,    delta(35)=-0.0125
x(36)=-3.8625, f(36)=-70.6952,    ind=1,    xmin=-3.8625,    delta(36)=-0.0125
x(37)=-3.875, f(37)=-70.697,    ind=1,    xmin=-3.875,    delta(37)=-0.0125
x(38)=-3.8875, f(38)=-70.6863,    ind=1,    xmin=-3.875,    delta(38)=0.003125

```

```
fmin=-70.697   za  xmin=-3.875,   greska: |xmin-x*|<0.003125
```

```
broj iteracija: 38
```

$$f(x) = (-3 + x) (-1 + x) (2 + x) (5 + x)$$

```

-----
x(0)=-5.,    f(0)=0.,    ind=0,    xmin=-5.,    delta(0)=0.1
x(1)=-4.9,    f(1)=-13.5169,    ind=0,    xmin=-4.9,    delta(1)=0.2
x(2)=-4.7,    f(2)=-35.5509,    ind=0,    xmin=-4.7,    delta(2)=0.4
x(3)=-4.3,    f(3)=-62.2909,    ind=0,    xmin=-4.3,    delta(3)=0.8
x(4)=-3.5,    f(4)=-65.8125,    ind=0,    xmin=-3.5,    delta(4)=1.6
x(5)=-1.9,    f(5)=4.4051,    ind=1,    xmin=-3.5,    delta(5)=-0.4
x(6)=-2.3,    f(6)=-14.1669,    ind=1,    xmin=-3.5,    delta(6)=-0.4
x(7)=-2.7,    f(7)=-33.9549,    ind=1,    xmin=-3.5,    delta(7)=-0.4
x(8)=-3.1,    f(8)=-52.2709,    ind=1,    xmin=-3.5,    delta(8)=-0.4
x(9)=-3.5,    f(9)=-65.8125,    ind=1,    xmin=-3.5,    delta(9)=-0.4
x(10)=-3.9,    f(10)=-70.6629,    ind=1,    xmin=-3.9,    delta(10)=-0.4
x(11)=-4.3,    f(11)=-62.2909,    ind=1,    xmin=-3.9,    delta(11)=0.1
x(12)=-4.2,    f(12)=-65.8944,    ind=1,    xmin=-3.9,    delta(12)=0.1
x(13)=-4.1,    f(13)=-68.4369,    ind=1,    xmin=-3.9,    delta(13)=0.1
x(14)=-4.,    f(14)=-70.,    ind=1,    xmin=-3.9,    delta(14)=0.1
x(15)=-3.9,    f(15)=-70.6629,    ind=1,    xmin=-3.9,    delta(15)=0.1
x(16)=-3.8,    f(16)=-70.5024,    ind=1,    xmin=-3.9,    delta(16)=-0.025
x(17)=-3.825,    f(17)=-70.6156,    ind=1,    xmin=-3.9,    delta(17)=-0.025
x(18)=-3.85,    f(18)=-70.6809,    ind=1,    xmin=-3.85,    delta(18)=-0.025
x(19)=-3.875,    f(19)=-70.697,    ind=1,    xmin=-3.875,    delta(19)=-0.025
x(20)=-3.9,    f(20)=-70.6629,    ind=1,    xmin=-3.875,    delta(20)=0.00625
-----
fmin=-70.697    za    xmin=-3.875,    greska: |xmin-x*|<0.00625
broj iteracija:    20

```

$$f(x) = (-3 + x) (-1 + x) (2 + x) (5 + x)$$

potrebna pocetna promena smeru pretrazivanja

x(0)=15.,	f(0)=57120.,	ind=0,	xmin=15.,	delta(0)=-0.1
x(1)=14.9,	f(1)=55629.,	ind=0,	xmin=14.9,	delta(1)=-0.2
x(2)=14.7,	f(2)=52733.8,	ind=0,	xmin=14.7,	delta(2)=-0.4
x(3)=14.3,	f(3)=47279.7,	ind=0,	xmin=14.3,	delta(3)=-0.8
x(4)=13.5,	f(4)=37635.9,	ind=0,	xmin=13.5,	delta(4)=-1.6
x(5)=11.9,	f(5)=22788.6,	ind=0,	xmin=11.9,	delta(5)=-3.2
x(6)=8.7,	f(6)=6433.84,	ind=0,	xmin=8.7,	delta(6)=-6.4
x(7)=2.3,	f(7)=-28.5649,	ind=0,	xmin=2.3,	delta(7)=-12.8
x(8)=-10.5,	f(8)=7257.94,	ind=1,	xmin=2.3,	delta(8)=3.2
x(9)=-7.3,	f(9)=1042.12,	ind=1,	xmin=2.3,	delta(9)=3.2
x(10)=-4.1,	f(10)=-68.4369,	ind=1,	xmin=-4.1,	delta(10)=3.2
x(11)=-0.9,	f(11)=33.4191,	ind=1,	xmin=-4.1,	delta(11)=-0.8
x(12)=-1.7,	f(12)=12.5631,	ind=1,	xmin=-4.1,	delta(12)=-0.8
x(13)=-2.5,	f(13)=-24.0625,	ind=1,	xmin=-4.1,	delta(13)=-0.8
x(14)=-3.3,	f(14)=-59.8689,	ind=1,	xmin=-4.1,	delta(14)=-0.8
x(15)=-4.1,	f(15)=-68.4369,	ind=1,	xmin=-4.1,	delta(15)=-0.8
x(16)=-4.9,	f(16)=-13.5169,	ind=1,	xmin=-4.1,	delta(16)=0.2
x(17)=-4.7,	f(17)=-35.5509,	ind=1,	xmin=-4.1,	delta(17)=0.2
x(18)=-4.5,	f(18)=-51.5625,	ind=1,	xmin=-4.1,	delta(18)=0.2
x(19)=-4.3,	f(19)=-62.2909,	ind=1,	xmin=-4.1,	delta(19)=0.2
x(20)=-4.1,	f(20)=-68.4369,	ind=1,	xmin=-4.1,	delta(20)=0.2
x(21)=-3.9,	f(21)=-70.6629,	ind=1,	xmin=-3.9,	delta(21)=0.2
x(22)=-3.7,	f(22)=-69.5929,	ind=1,	xmin=-3.9,	delta(22)=-0.05
x(23)=-3.75,	f(23)=-70.1367,	ind=1,	xmin=-3.9,	delta(23)=-0.05
x(24)=-3.8,	f(24)=-70.5024,	ind=1,	xmin=-3.9,	delta(24)=-0.05
x(25)=-3.85,	f(25)=-70.6809,	ind=1,	xmin=-3.85,	delta(25)=-0.05
x(26)=-3.9,	f(26)=-70.6629,	ind=1,	xmin=-3.85,	delta(26)=0.0125
x(27)=-3.8875,	f(27)=-70.6863,	ind=1,	xmin=-3.8875,	delta(27)=0.0125
x(28)=-3.875,	f(28)=-70.697,	ind=1,	xmin=-3.875,	delta(28)=0.0125
x(29)=-3.8625,	f(29)=-70.6952,	ind=1,	xmin=-3.875,	delta(29)=-0.003125

fmin=-70.697 za xmin=-3.875, greska: |xmin-x*|<0.003125

broj iteracija: 29

$$f(x) = (-3 + x) (-1 + x) (2 + x) (5 + x)$$

```

-----
x(0)=-15.,    f(0)=37440.,    ind=0,    xmin=-15.,    delta(0)=0.1
x(1)=-14.9,   f(1)=36347.5,   ind=0,    xmin=-14.9,   delta(1)=0.2
x(2)=-14.7,   f(2)=34233.3,   ind=0,    xmin=-14.7,   delta(2)=0.4
x(3)=-14.3,   f(3)=30277.9,   ind=0,    xmin=-14.3,   delta(3)=0.8
x(4)=-13.5,   f(4)=23386.7,   ind=0,    xmin=-13.5,   delta(4)=1.6
x(5)=-11.9,   f(5)=13129.9,   ind=0,    xmin=-11.9,   delta(5)=3.2
x(6)=-8.7,    f(6)=2813.42,    ind=0,    xmin=-8.7,    delta(6)=6.4
x(7)=-2.3,    f(7)=-14.1669,   ind=0,    xmin=-2.3,    delta(7)=12.8
x(8)=10.5,    f(8)=13804.7,    ind=1,    xmin=-2.3,    delta(8)=-3.2
x(9)=7.3,     f(9)=3098.83,    ind=1,    xmin=-2.3,    delta(9)=-3.2
x(10)=4.1,    f(10)=189.289,   ind=1,    xmin=-2.3,    delta(10)=-3.2
x(11)=0.9,    f(11)=3.5931,    ind=1,    xmin=-2.3,    delta(11)=-3.2
x(12)=-2.3,   f(12)=-14.1669,  ind=1,    xmin=-2.3,    delta(12)=-3.2
x(13)=-5.5,   f(13)=96.6875,   ind=1,    xmin=-2.3,    delta(13)=0.8
x(14)=-4.7,   f(14)=-35.5509,  ind=1,    xmin=-4.7,    delta(14)=0.8
x(15)=-3.9,   f(15)=-70.6629,  ind=1,    xmin=-3.9,    delta(15)=0.8
x(16)=-3.1,   f(16)=-52.2709,  ind=1,    xmin=-3.9,    delta(16)=-0.2
x(17)=-3.3,   f(17)=-59.8689,  ind=1,    xmin=-3.9,    delta(17)=-0.2
x(18)=-3.5,   f(18)=-65.8125,  ind=1,    xmin=-3.9,    delta(18)=-0.2
x(19)=-3.7,   f(19)=-69.5929,  ind=1,    xmin=-3.9,    delta(19)=-0.2
x(20)=-3.9,   f(20)=-70.6629,  ind=1,    xmin=-3.9,    delta(20)=-0.2
x(21)=-4.1,   f(21)=-68.4369,  ind=1,    xmin=-3.9,    delta(21)=0.05
x(22)=-4.05,  f(22)=-69.3359,  ind=1,    xmin=-3.9,    delta(22)=0.05
x(23)=-4.,    f(23)=-70.,      ind=1,    xmin=-3.9,    delta(23)=0.05
x(24)=-3.95,  f(24)=-70.4391,  ind=1,    xmin=-3.9,    delta(24)=0.05
x(25)=-3.9,   f(25)=-70.6629,  ind=1,    xmin=-3.9,    delta(25)=0.05
x(26)=-3.85,  f(26)=-70.6809,  ind=1,    xmin=-3.85,   delta(26)=0.05
x(27)=-3.8,   f(27)=-70.5024,  ind=1,    xmin=-3.85,   delta(27)=-0.0125
x(28)=-3.8125, f(28)=-70.5649,  ind=1,    xmin=-3.85,   delta(28)=-0.0125
x(29)=-3.825, f(29)=-70.6156,  ind=1,    xmin=-3.85,   delta(29)=-0.0125
x(30)=-3.8375, f(30)=-70.6543,  ind=1,    xmin=-3.85,   delta(30)=-0.0125
x(31)=-3.85,  f(31)=-70.6809,  ind=1,    xmin=-3.85,   delta(31)=-0.0125
x(32)=-3.8625, f(32)=-70.6952,  ind=1,    xmin=-3.8625, delta(32)=-0.0125

```

```

x(33)=-3.875,    f(33)=-70.697,    ind=1,    xmin=-3.875,    delta(33)=-0.0125
x(34)=-3.8875,    f(34)=-70.6863,    ind=1,    xmin=-3.875,    delta(34)=0.003125

```

```

-----
fmin=-70.697    za    xmin=-3.875,    greska: |xmin-x*|<0.003125

```

```

broj iteracija:    34

```

Globalni minimum je $f_{\min} = -70.697$ za $x_{\min} = -3.875$

```

NMinimize[f[t], t]

```

```

{-70.6978, {t -> -3.87062}}

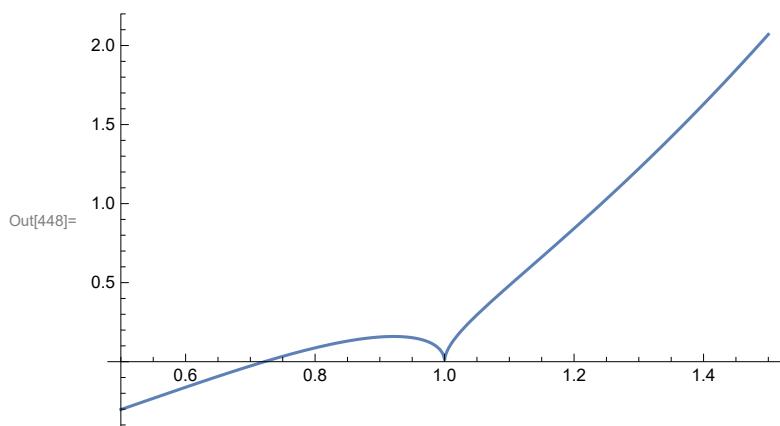
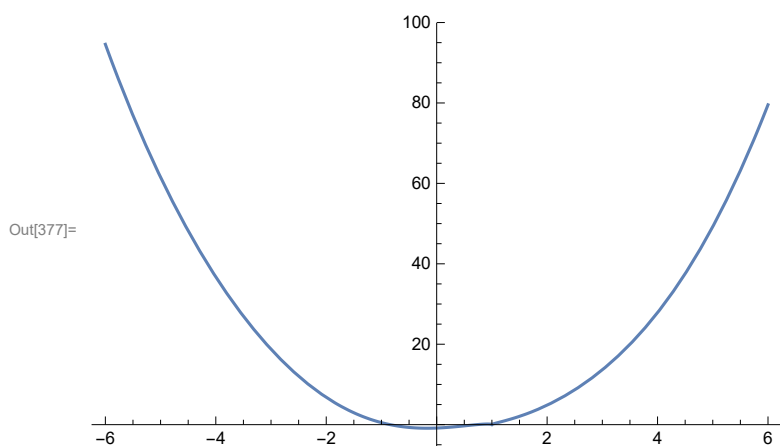
```

Zadatak 7.

Jednodimenzionalnim simpleks metodom sa tačnošću 10^{-3} rešiti minimizacioni problem:

a) $\min \sin(x-1) + x^2 \sqrt{|x-1|}$, b) $\min -x^3$, c) $\max -x^3 \sqrt{|x-1|}$
 uzimajući različite početne vrednosti.

Rešenje :



$$f(x) = x^2 \sqrt{\text{Abs}[-1 + x]} - \text{Sin}[1 - x]$$

potrebna promena smera pretrazivanja

x(0)=0.,	f(0)=-0.841471,	ind=0,	xmin=0.,	delta(0)=-0.1
x(1)=-0.1,	f(1)=-0.880719,	ind=0,	xmin=-0.1,	delta(1)=-0.2
x(2)=-0.3,	f(2)=-0.860942,	ind=1,	xmin=-0.1,	delta(2)=0.05
x(3)=-0.25,	f(3)=-0.879107,	ind=1,	xmin=-0.1,	delta(3)=0.05
x(4)=-0.2,	f(4)=-0.888221,	ind=1,	xmin=-0.2,	delta(4)=0.05
x(5)=-0.15,	f(5)=-0.888635,	ind=1,	xmin=-0.15,	delta(5)=0.05
x(6)=-0.1,	f(6)=-0.880719,	ind=1,	xmin=-0.15,	delta(6)=-0.0125
x(7)=-0.1125,	f(7)=-0.883458,	ind=1,	xmin=-0.15,	delta(7)=-0.0125
x(8)=-0.125,	f(8)=-0.885695,	ind=1,	xmin=-0.15,	delta(8)=-0.0125
x(9)=-0.1375,	f(9)=-0.887422,	ind=1,	xmin=-0.15,	delta(9)=-0.0125
x(10)=-0.15,	f(10)=-0.888635,	ind=1,	xmin=-0.15,	delta(10)=-0.0125
x(11)=-0.1625,	f(11)=-0.889328,	ind=1,	xmin=-0.1625,	delta(11)=-0.0125
x(12)=-0.175,	f(12)=-0.889493,	ind=1,	xmin=-0.175,	delta(12)=-0.0125
x(13)=-0.1875,	f(13)=-0.889126,	ind=1,	xmin=-0.175,	delta(13)=0.003125

fmin=-0.889493 za xmin=-0.175, greska: |xmin-x*|<0.003125

broj iteracija: 13

$$f(x) = x^2 \sqrt{\text{Abs}[-1 + x]} - \text{Sin}[1 - x]$$

potrebna promena smera pretrazivanja

x(0)=10.,	f(0)=300.412,	ind=0,	xmin=10.,	delta(0)=-0.1
x(1)=9.9,	f(1)=292.893,	ind=0,	xmin=9.9,	delta(1)=-0.2
x(2)=9.7,	f(2)=278.189,	ind=0,	xmin=9.7,	delta(2)=-0.4
x(3)=9.3,	f(3)=250.077,	ind=0,	xmin=9.3,	delta(3)=-0.8
x(4)=8.5,	f(4)=198.803,	ind=0,	xmin=8.5,	delta(4)=-1.6
x(5)=6.9,	f(5)=115.27,	ind=0,	xmin=6.9,	delta(5)=-3.2
x(6)=3.7,	f(6)=22.9223,	ind=0,	xmin=3.7,	delta(6)=-6.4
x(7)=-2.7,	f(7)=14.5524,	ind=0,	xmin=-2.7,	delta(7)=-12.8
x(8)=-15.5,	f(8)=976.612,	ind=1,	xmin=-2.7,	delta(8)=3.2
x(9)=-12.3,	f(9)=551.072,	ind=1,	xmin=-2.7,	delta(9)=3.2
x(10)=-9.1,	f(10)=263.799,	ind=1,	xmin=-2.7,	delta(10)=3.2

```

x(11)=-5.9,    f(11)=90.8599,    ind=1,    xmin=-2.7,    delta(11)=3.2
x(12)=-2.7,    f(12)=14.5524,    ind=1,    xmin=-2.7,    delta(12)=3.2
x(13)=0.5,     f(13)=-0.302649,    ind=1,    xmin=0.5,    delta(13)=3.2
x(14)=3.7,     f(14)=22.9223,    ind=1,    xmin=0.5,    delta(14)=-0.8
x(15)=2.9,     f(15)=12.5387,    ind=1,    xmin=0.5,    delta(15)=-0.8
x(16)=2.1,     f(16)=5.51645,    ind=1,    xmin=0.5,    delta(16)=-0.8
x(17)=1.3,     f(17)=1.22117,    ind=1,    xmin=0.5,    delta(17)=-0.8
x(18)=0.5,     f(18)=-0.302649,    ind=1,    xmin=0.5,    delta(18)=-0.8
x(19)=-0.3,    f(19)=-0.860942,    ind=1,    xmin=-0.3,    delta(19)=-0.8
x(20)=-1.1,    f(20)=0.890247,    ind=1,    xmin=-0.3,    delta(20)=0.2
x(21)=-0.9,    f(21)=0.170208,    ind=1,    xmin=-0.3,    delta(21)=0.2
x(22)=-0.7,    f(22)=-0.352783,    ind=1,    xmin=-0.3,    delta(22)=0.2
x(23)=-0.5,    f(23)=-0.691309,    ind=1,    xmin=-0.3,    delta(23)=0.2
x(24)=-0.3,    f(24)=-0.860942,    ind=1,    xmin=-0.3,    delta(24)=0.2
x(25)=-0.1,    f(25)=-0.880719,    ind=1,    xmin=-0.1,    delta(25)=0.2
x(26)=0.1,     f(26)=-0.77384,    ind=1,    xmin=-0.1,    delta(26)=-0.05
x(27)=0.05,    f(27)=-0.810979,    ind=1,    xmin=-0.1,    delta(27)=-0.05
x(28)=-6.38378×10-16,    f(28)=-0.841471,    ind=1,    xmin=-0.1,    delta(28)=-0.05
x(29)=-0.05,    f(29)=-0.864861,    ind=1,    xmin=-0.1,    delta(29)=-0.05
x(30)=-0.1,    f(30)=-0.880719,    ind=1,    xmin=-0.1,    delta(30)=-0.05
x(31)=-0.15,    f(31)=-0.888635,    ind=1,    xmin=-0.15,    delta(31)=-0.05
x(32)=-0.2,    f(32)=-0.888221,    ind=1,    xmin=-0.15,    delta(32)=0.0125
x(33)=-0.1875,    f(33)=-0.889126,    ind=1,    xmin=-0.1875,    delta(33)=0.0125
x(34)=-0.175,    f(34)=-0.889493,    ind=1,    xmin=-0.175,    delta(34)=0.0125
x(35)=-0.1625,    f(35)=-0.889328,    ind=1,    xmin=-0.175,    delta(35)=-0.003125

```

```
-----
fmin=-0.889493    za    xmin=-0.175,    greska: |xmin-x*|<0.003125
```

```
broj iteracija:    35
```

Zadatak 8.

Metodom polovljenja intervala sa tačnošću 10^{-3} rešiti minimizacioni problem

$$\min_{1 \leq x \leq 3} (x-1)(x+2)(x-3)(x+5).$$

Rešenje :

```

In[489]:= Clear["Global`*"];
f[x_] := (x - 1) (x + 2) (x - 3) (x + 5);
a = 1.; b = 3.; epsilon = 10-3;
(*indikator ind=0: uzima vrednost 1 ako se dobije tacno resenje *)
ind = 0;
f1[x_] := D[f[t], t] /. t -> x;
Print["-----"];
Print["  a          b          |b-a|          c= $\frac{a+b}{2}$           f'(c)"];
Print["-----"];
BrIteracija = 0;
While[b - a ≥ 2 epsilon,
  BrIteracija = BrIteracija + 1;
  c =  $\frac{a+b}{2}$ ;
  Print[NumberForm[a, {6, 5}], " ", NumberForm[b, {6, 5}],
    " ", NumberForm[b - a, {6, 5}], " ", c, " ", f1[c]];
  Which[f1[c] > 0, b = c, f1[c] < 0, a = c, f1[c] == 0, ind = 1; Break[]];
];
Print[NumberForm[a, {6, 5}], " ", NumberForm[b, {6, 5}], " ", NumberForm[b - a, {6, 5}]];
xmin =  $\frac{a+b}{2}$ ; fmin = f[xmin];
Print["-----"];
If[ind == 1, Print["tacno resenje: fmin=", f[c], " za xmin=", c],
  Print["fmin=", fmin, " za xmin=", xmin, " greska: |xmin-x*|<",  $\frac{b-a}{2}$ ]];
];
Print["broj iteracija: ", BrIteracija];

```

a	b	b-a	$c = \frac{a+b}{2}$	f' (c)
1.00000	3.00000	2.00000	2.	-11.
2.00000	3.00000	1.00000	2.5	24.75
2.00000	2.50000	0.50000	2.25	4.625
2.00000	2.25000	0.25000	2.125	-3.72656
2.12500	2.25000	0.12500	2.1875	0.311523
2.12500	2.18750	0.06250	2.15625	-1.74158
2.15625	2.18750	0.03125	2.17188	-0.723587
2.17188	2.18750	0.01563	2.17969	-0.208178
2.17969	2.18750	0.00781	2.18359	0.0511358
2.17969	2.18359	0.00391	2.18164	-0.0786551
2.18164	2.18359	0.00195		

fmin=-29.0403 za xmin=2.18262, greska: |xmin-x*| < 0.000976563

broj iteracija: 10

Zadatak 9.

Metodom polovljenja intervala sa tačnošću 10^{-3} rešiti minimizacioni problem $\min_{1 \leq x \leq 9} (x-2)^2 + 1$.

Rešenje :

a	b	b-a	$c = \frac{a+b}{2}$	f' (c)
1.00000	9.00000	8.00000	5.	6.
1.00000	5.00000	4.00000	3.	2.
1.00000	3.00000	2.00000	2.	0.
1.00000	3.00000	2.00000		

tacno resenje: fmin=1. za xmin=2.

broj iteracija: 3

Zadatak 10.

Njutnovim metodom sa tačnošću 10^{-3} rešiti minimizacioni problem $\min_{1 \leq x \leq 3} (x-1)(x+2)(x-3)(x+5)$ uzimajući različite početne vrednosti.

Rešenje :

```
In[519]:= Clear["Global`*"];
f[x_] := (x - 1) (x + 2) (x - 3) (x + 5);
epsilon = 10-3; MaxIter = 30;
(* indikator ind=0: uzima vrednost 1 ako se proces prekine zbog f''(x)=0 *)
ind = 0;
f1[x_] := Simplify[D[f[t], t]] /. t -> x;
f2[x_] := Simplify[D[f1[t], t]] /. t -> x;
Print["f'(x)=", f1[x], ", f''(x)=", f2[x]];
Print["iterativna funkcija: xk+1=", xk -  $\frac{f1[x_k]}{f2[x_k]}$ ];

(* za pocetnu tacku x uzimati redom -3,-2,-1,0,1,2,3 *)
x = -3.; delta = epsilon;
Print["x(0)=", x];
BrIteracija = 0;
While[Abs[delta] ≥ epsilon ∧ BrIteracija < MaxIter,
  BrIteracija = BrIteracija + 1;
  If[f2[x] == 0, ind = 1; Break[]];
  y = x -  $\frac{f1[x]}{f2[x]}$ ;
  delta = y - x;
  Print["x(", BrIteracija, ")=", y, " f'(x)=", f1[y]];
  x = y
];
xopt = y; fopt = f[xopt];
Print["-----"];
If[ind == 1, Print["f''(x)=0, proces prekinut"],
  If[f[x - 10 epsilon] ≥ fopt ∧ f[x + 10 epsilon] ≥ fopt, Print["minimum"],
    If[f[x - 10 epsilon] ≤ fopt ∧ f[x + 10 epsilon] ≤ fopt, Print["maximum"],
      Print["stacionarna tacka"]
    ]
  ];
Print["fopt=", fopt, " za xopt=", xopt, ", greska: |xopt-x*|<", delta];
Print["broj iteracija: ", BrIteracija];
];
```

$$f'(x) = -19 - 30x + 9x^2 + 4x^3, \quad f''(x) = 6(-5 + 3x + 2x^2)$$

$$\text{iterativna funkcija: } x_{k+1} = x_k - \frac{-19 - 30x_k + 9x_k^2 + 4x_k^3}{6(-5 + 3x_k + 2x_k^2)}$$

$$x(0) = -3.$$

$$x(1) = -4.83333 \quad f'(x) = -115.398$$

$$x(2) = -4.12681 \quad f'(x) = -23.0486$$

$$x(3) = -3.89652 \quad f'(x) = -2.10019$$

$$x(4) = -3.87093 \quad f'(x) = -0.0246669$$

$$x(5) = -3.87062 \quad f'(x) = -3.54867 \times 10^{-6}$$

minimum

fopt=-70.6978 za xopt=-3.87062, greska: |xopt-x*|<0.000307827

broj iteracija: 5

Zadatak II.

Dokazati da se za rešavanje problema $\min_{-\infty < x < \infty} x^2 + e^{-x}$ može primeniti Njutnov metod, a zatim odrediti rešenje sa tačnošću 10^{-3} .

Rešenje :

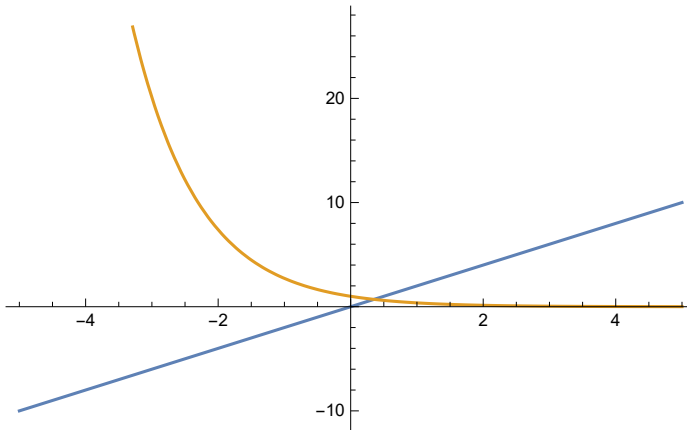
```
In[534]:= Clear["Global`*"];
f[x_] := x^2 + Exp[-x];
f1[x_] := Simplify[D[f[t], t]] /. t -> x;
f2[x_] := Simplify[D[f1[t], t]] /. t -> x;
Print["f'(x)=", f1[x], ",    f''(x)=", f2[x]];
f'(x) = -e-x + 2x,    f''(x) = 2 + e-x
```

Njutnov metod može da se primeni sa bilo kojom početnom tačkom jer je $f''(x) \neq 0$ na $(-\infty, \infty)$.

Osim toga, $f''(x) > 0$, pa je $f(x)$ konveksna funkcija. Minimum se nalazi u tački za koju je $f'(x) = 0$, tj. u zajedničkoj tački funkcija $\phi(x) = 2x$ i $\psi(x) = e^{-x}$.

```
In[539]:= (* Lokalizacija nula *)
Plot[{2 x, Exp[-x]}, {x, -5, 5}]
```

Out[539]=



Jedina zajednička tačka se nalazi u intervalu (0, 1), pa za početnu tačku Njuntnovog metoda može da se uzme, na primer, $x = 0$.

```
In[540]:= epsilon = 10-3;
Print["iterativna funkcija:  $x_{k+1} = x_k - \frac{f1[x_k]}{f2[x_k]}$ "];

x = 0.;
Print["x(0)=", x];
y = x -  $\frac{f1[x]}{f2[x]}$ ;
BrIteracija = 0;
While[Abs[y - x] ≥ epsilon,
  BrIteracija = BrIteracija + 1;
  x = y;
  Print["x(", BrIteracija, ")=", x];
  y = x -  $\frac{f1[x]}{f2[x]}$ ;
];
xmin = y; fmin = f[xmin];
Print["-----"];
Print["fmin=", fmin, " za xmin=", xmin, ", greska: |xmin-x*| < ", Abs[y - x]];
Print["broj iteracija: ", BrIteracija + 1];
```

iterativna funkcija: $x_{k+1} = x_k - \frac{-e^{-x_k} + 2x_k}{2 + e^{-x_k}}$

$x(0) = 0.$

$x(1) = 0.333333$

$x(2) = 0.351689$

fmin=0.827184 za xmin=0.351734, greska: $|x_{\min} - x^*| < 0.0000443794$

broj iteracija: 3

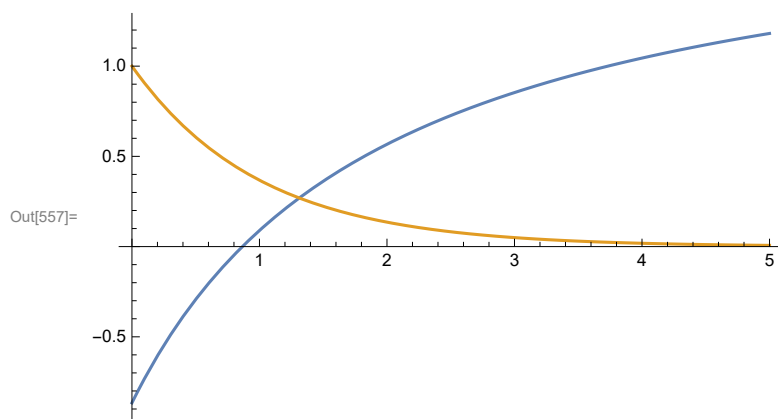
Zadatak 12.

Ispitati da li se za rešavanje problema $\max_{x>0} x\sqrt{3} - x^2 - (x+3)e^{-x} + 6$ može primeniti Njutnov metod, a zatim odrediti rešenje sa tačnošću 10^{-3} .

Rešenje :

$f'(x) = -\sqrt{3} + 2x - e^{-x}(2+x)$, $f''(x) = 2 + e^{-x}(1+x)$

$f''(x) > 0 \Rightarrow f(x)$ je konveksna na $(0, \infty)$



iterativna funkcija: $x_{k+1} = x_k - \frac{-\sqrt{3} + 2x_k - e^{-x_k}(2+x_k)}{2 + e^{-x_k}(1+x_k)}$

$x(0) = 1.$

$x(1) = 1.30547$

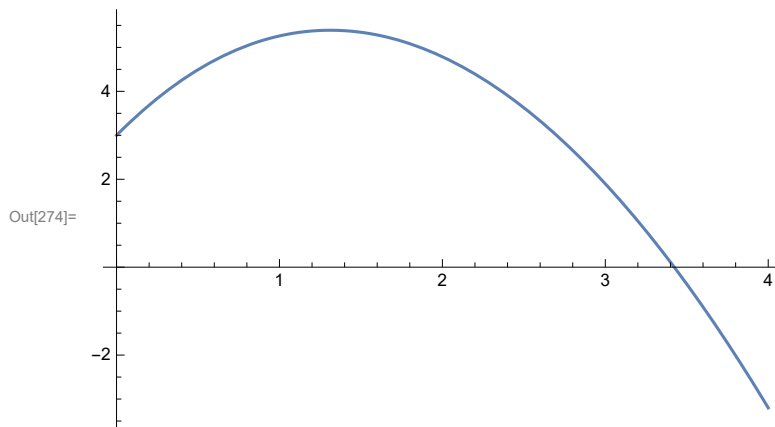
$x(2) = 1.31196$

fmin=-5.38997 za xmin=1.31197, greska: $|x_{\min} - x^*| < 2.84325 \times 10^{-6}$

broj iteracija: 3

gmax=5.38997 za xmin=1.31197

In[274]:= Plot[-f[x], {x, 0, 4}]



Zadatak 13.

Neka je $f(x) = |x^2 - 1.5| |x| - 2|x| - x$.

a) Ako postoje globalni ekstremumi funkcije $f(x)$, tj. $\min_{-\infty \leq x \leq \infty} f(x)$ i $\max_{-\infty \leq x \leq \infty} f(x)$, odrediti ih sa tačnošću $5 \cdot 10^{-3}$.

b) Odrediti $\min_{-4 \leq x \leq -1.5} f(x)$ i $\max_{0 \leq x \leq 2} f(x)$ sa tačnošću $5 \cdot 10^{-4}$.

Zadatak 14.

Data je funkcija $f(x) = \frac{x + \log x}{1 + \log^2 x}$.

a) Ispitati koliko lokalnih ekstremuma ima funkcija $f(x)$ i odrediti intervale unimodalnosti.

b) Da li postoje globalni ekstremumi funkcije $f(x)$?

c) Sa tačnošću 10^{-3} odrediti najmanju vrednost u kojoj funkcija $f(x)$ dostiže lokalni minimum.

Zadatak 15.

Odrediti rešanje problema $\min_{-\infty < x < \infty} (x^2 + 2)e^{-x} + e^{x-2} + 4x^2 - x + 3$ sa tačnošću 10^{-5} .