# Declarative Modeling

Winter Term 2016/2017
Lecture Slides

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# Rough Roadmap

1. Background
   - Grounding
   - Solving
2. Modeling
   - Satisfiability
   - Optimization
   - Incrementality
3. Applications
   - Puzzles
   - Planning
   - Scheduling

## Resources

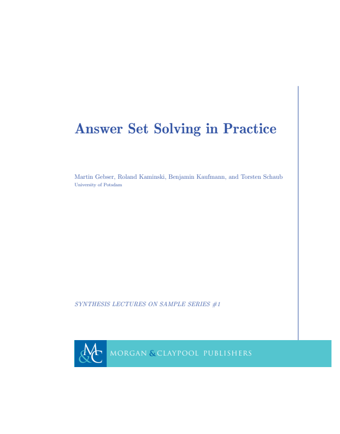
- Course material
  - `https://moodle.cs.uni-potsdam.de/course/view.php?id=39`
  - `https://potassco.org/teaching/`
  - `https://potassco.org/support/`
- Systems
  - *clasp*
  - *gringo*
  - *clingo*

`https://potassco.org`

# The Potassco Book

https://potassco.org/book/

1. Motivation
2. Introduction
3. Basic modeling
4. Grounding
5. Characterizations
6. Solving
7. Systems
8. Advanced modeling
9. Conclusions



# Administrative Information

- Lecture: 2x (at the beginning)
- Project: implementation and presentation (at end of semester)
- Credits: 6

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  - Area: "Praktische Informatik" / "Angewandte Informatik" / "Wahlfrei"
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# Background: Overview

# Overview

# Human versus Computational Intelligence?


Picture from Wikimedia Commons

vs.

Picture from Wikimedia Commons

| | | | |
|---|---|---|---|
| Speed | ✔ | Speed | ✘ |
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| Intuitions | | Intuitions | |
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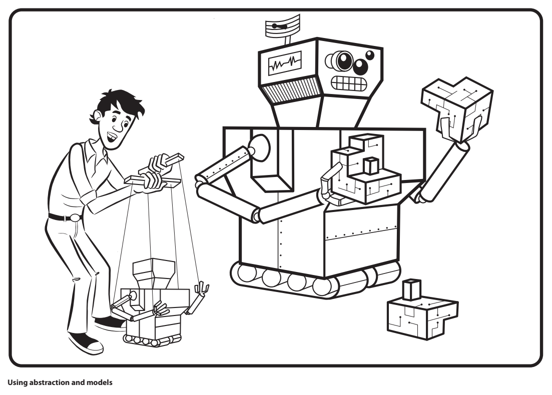
# Human plus Computational Intelligence!




Picture from Computational Thinking Illustrated

**KRR = Knowledge + Representation + Reasoning**

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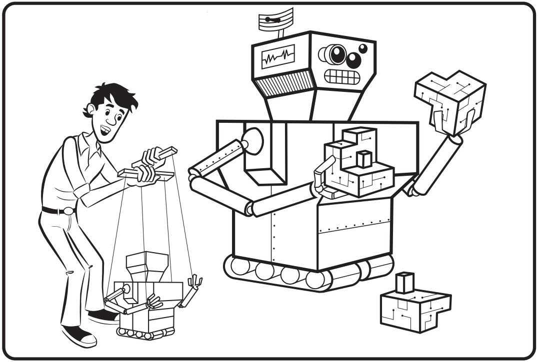
Using abstraction and models

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# Illustration: Pigeonhole Principle

## One small step for man

Can one put 10 pigeons into 9 holes such that no pigeons share a hole?

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## One small step for man

Can one put $n{+}1$ pigeons into $n$ holes such that no pigeons share a hole?

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# Illustration: Pigeonhole Principle

## One worst case for machines

Can one put $n+1$ pigeons into $n$ holes such that no pigeons share a hole?

### "Naive" Formulation

| Each pigeon requires a hole | No pigeons share a hole | | |
|---|---|---|---|
| $p_{1,1} \vee p_{1,2} \vee \cdots \vee p_{1,n}$ | $\neg p_{1,1} \vee \neg p_{2,1}$ | $\ldots$ | $\neg p_{1,n} \vee \neg p_{2,n}$ |
| $p_{2,1} \vee p_{2,2} \vee \cdots \vee p_{2,n}$ | $\neg p_{1,1} \vee \neg p_{3,1}$ | $\ldots$ | $\neg p_{1,n} \vee \neg p_{3,n}$ |
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☞ Runtime of (resolution-based) solvers:

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### "Clever" Formulation

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\end{array}
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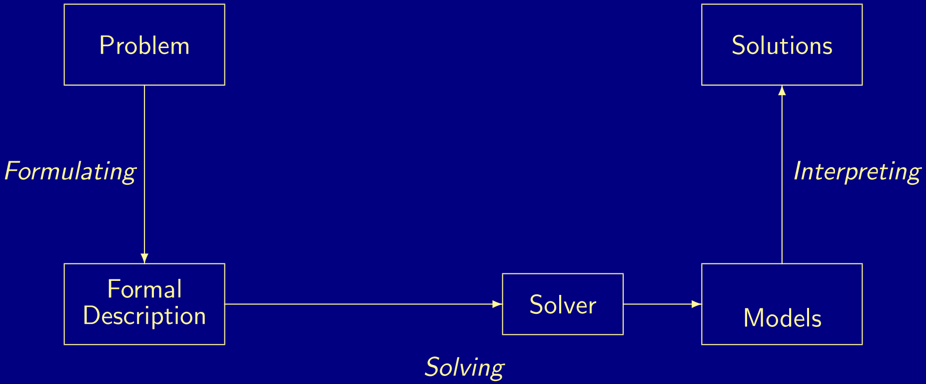
# Overview

# Declarative Problem Solving

with SAT



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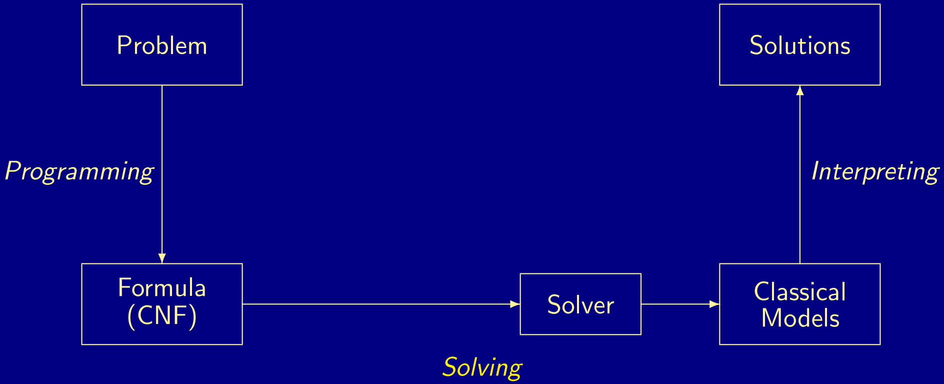
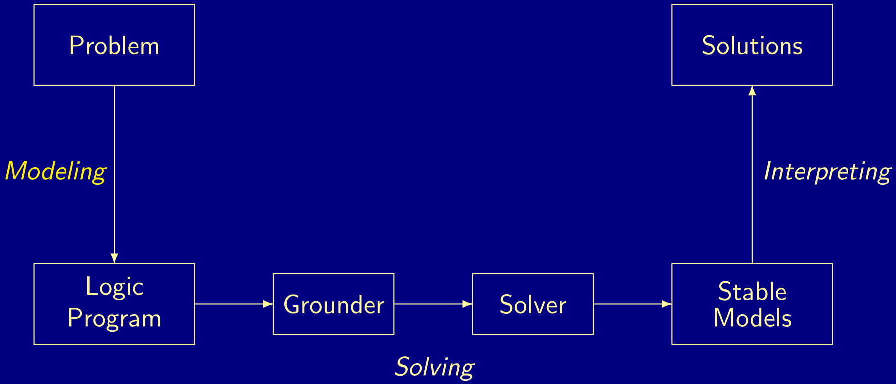


# Declarative Problem Solving

## with ASP



## Modeling Constructs

- Variables `p(X) :- q(X).`
- Conditional literals `p :- q(X) : r(X).`
- Disjunction `p(X) ; q(X) :- r(X).`
- Integrity constraints `:- q(X), p(X).`
- Choice `2 { p(X,Y) : q(X) } 7 :- r(Y).`
- Aggregates `s(Y) :- r(Y), 2 #sum{ X : p(X,Y), q(X) } 7.`
- Optimization
  - Weak constraints `:~ q(X), p(X,C). [C,X]`
  - Statements `#minimize { C,X : q(X), p(X,C) }.`

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- Aggregates `s(Y) :- r(Y), 2 #sum{ X : p(X,Y), q(X) } 7.`
- Multi-objective optimization
  - Weak constraints `:~ q(X), p(X,C). [C@42,X]`
  - Statements `#minimize { C@42,X : q(X), p(X,C) }.`

# Reasoning Modes

- Satisfiability
- Optimization
- Enumeration†
- Projection†
- Intersection‡
- Union‡

- and combinations of them
- via single- or multi-threading

† without solution recording

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# Grounding Basics

Task Instantiate first-order rules (encoding) relative to facts (instance)

## Approach

- Head atom(s) of a rule (or fact) are derivable if all positive elements of the rule body are derivable
- Iterative instantiation of derivable atoms and resulting rule bodies, starting from facts, yields all relevant ground rules

☞ Semi-naive Evaluation

## Safety Requirement

Any variable of a rule must appear (outside of arithmetic expressions)
- globally in some positive element of the rule body or
- locally on the right-hand side of ":" in a condition

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# Predicate Classification

## Built-in Predicates

- Term comparisons
  - `f(X1,Y1) != f(X2,Y2)`, `(X1,Y1) < (X2,Y2)`, etc.
  - Can be viewed as negative elements, not binding variables to values
- Term assignments
  - `X = Y+1`, `f(X,Y) = f(XX+1,YY-1)`, `X = #min{ Y : p(Y) }`, etc.
  - Bind (global) variables on one side, if those on other side are bound

## Domain Predicates (including built-ins)

- Predicates whose atoms neither
  - occur in heads of choice rules or depend, transitively, on them nor
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## Solving Basics

Task Find some (optimal) stable model of a propositional logic program

### Approach

- Consider atoms, rule bodies, and aggregates as propositional variables
- Unit propagation (extended to aggregates, unfounded sets, and optimize statements) yields deterministic consequences
- Decision guesses some literal when fixpoint is partial and conflict-free
- Conflict-driven learning records nogood and directs backjumping from deadend

### Two Sides to Every Story

Find some stable model (quickly)

Build some refutation (quickly), includes optimality proofs

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Task Find some (optimal) stable model of a propositional logic program

## Approach

- Consider atoms, rule bodies, and aggregates as propositional variables
- Unit propagation (extended to aggregates, unfounded sets, and optimize statements) yields deterministic consequences
- Decision guesses some literal when fixpoint is partial and conflict-free
- Conflict-driven learning records nogood and directs backjumping from deadend

## Two Sides to Every Story

Satisfiability Find some stable model (quickly)

Unsatisfiability Build some refutation (quickly), includes optimality proofs

# The Modeling and Solving Process

## Problem Comprehension

1. Create a working encoding
2. Verify correctness on small (toy) instances

## Scaling up

1. Compact constraint formulation
2. Reduce (magnitude of) instantiation size
   Desirable Linear to instance size (terms in facts)
3. Reduce (magnitude of) instantiation time
   Desirable Few discarded instantiations (eg. due to built-in predicates)
4. Incorporate more knowledge
   Conceivable Redundant constraints, symmetry breaking, etc.

## Final Punch (only)

1. Tweak solver parameters
2. Increase computing power (multi-cores, clusters, etc.)

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# Modeling Methodology: Overview

# Running Example: Latin Square

**Given:** An $N \times N$ board

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

represented by facts:

```
square(1,1). ... square(1,6).
square(2,1). ... square(2,6).
square(3,1). ... square(3,6).
square(4,1). ... square(4,6).
square(5,1). ... square(5,6).
square(6,1). ... square(6,6).
```

**Wanted:** Assignment of $1, \dots, N$

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 3 | 4 | 5 | 6 | 1 |
| 3 | 3 | 4 | 5 | 6 | 1 | 2 |
| 4 | 4 | 5 | 6 | 1 | 2 | 3 |
| 5 | 5 | 6 | 1 | 2 | 3 | 4 |
| 6 | 6 | 1 | 2 | 3 | 4 | 5 |

represented by atoms:

```
num(1,1,1) num(1,2,2) ... num(1,6,6)
num(2,1,2) num(2,2,3) ... num(2,6,1)
num(3,1,3) num(3,2,4) ... num(3,6,2)
num(4,1,4) num(4,2,5) ... num(4,6,3)
num(5,1,5) num(5,2,6) ... num(5,6,4)
num(6,1,6) num(6,2,1) ... num(6,6.5)
```

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num(6,1,6) num(6,2,1) ... num(6,6.5)
```

# Linearizing Inequality Tests

## A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

```
gringo latin0.lp | wc
```

1054728 6288392 21030438

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gringo latin1.lp | wc
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227336 1199112 4773202

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occ(1,X-1,Y,    N) :- num(X,Y,N), square(X-1,Y).
occ(0,X,  Y-1,  N) :- num(X,Y,N), square(X,Y-1).
occ(D,X-D,Y+D-1,N) :- occ(D,X,Y,N), square(X-D,Y+D-1).
:- num(X,Y,N), occ(D,X,Y,N).
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```
gringo latin1.lp | wc
```

227336 1199112 4773202

# Using Aggregates

## Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum{ X : square(X,n) }.

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occ(1,X,N,C) :- X = 1..n, N = 1..n, C = #count{ Y : num(X,Y,N) }.
occ(0,Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ X : num(X,Y,N) }.
:- occ(D,Z,N,C), C != 1.

% DISPLAY
#show num/3. #show sigma/1.
```

```
gringo latin2.lp | wc        gringo latin3.lp | wc
```

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gringo latin2.lp | wc gringo latin3.lp | wc

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#show num/3. #show sigma/1.
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% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occ(1,X,N,C) :- X = 1..n, N = 1..n, #count{ Y : num(X,Y,N) }=C, C=0..n.
occ(0,Y,N,C) :- Y = 1..n, N = 1..n, #count{ X : num(X,Y,N) }=C, C=0..n.
:- occ(D,Z,N,C), C != 1.

% DISPLAY
#show num/3. #show sigma/1.
```

☞ Internal transformation by *gringo*

# Using Aggregates

## Another Latin square encoding

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% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occ(1,X,N,C) :- X = 1..n, N = 1..n, C = #count{ Y : num(X,Y,N) }.     ✗
occ(0,Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ X : num(X,Y,N) }.     ✗
:- occ(D,Z,N,C), C != 1.

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49160 375816 2189400

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% DISPLAY
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```
gringo latin2.lp | wc
```

366600 6089736 34019251

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49160 375816 2189400

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% TEST
:- X = 1..n, N = 1..n, #count{ Y : num(X,Y,N) } != 1.
:- Y = 1..n, N = 1..n, #count{ X : num(X,Y,N) } != 1.


% DISPLAY
#show num/3.
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# Breaking Symmetries

## The ultimate Latin square encoding?

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:- Y = 1..n, N = 1..n, #count{ X : num(X,Y,N) } != 1.


% DISPLAY
#hide. #show num/3.
```

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:- Y = 1..n, N = 1..n, #count{ X : num(X,Y,N) } != 1.


% DISPLAY
#hide. #show num/3.
```

☞ Many symmetric solutions (mirroring, rotation, value permutation)

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:- Y = 1..n, N = 1..n, #count{ X : num(X,Y,N) } != 1.


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#hide. #show num/3.
```

☞ Easy and safe to fix a full row/column!

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:- Y = 1..n, N = 1..n, #count{ X : num(X,Y,N) } != 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

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```

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☞ Let's compare enumeration speed!

# Breaking Symmetries

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```
gringo -c n=5 latin3.lp | clasp -q 0
```

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% DISPLAY
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Models : 161280    Time : 2.078s
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% DISPLAY
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```

```
gringo -c n=5 latin4.lp | clasp -q 0
```

```
Models : 161280    Time : 2.078s
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# Breaking Symmetries

## The ultimate Latin square encoding?

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% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, #count{ Y : num(X,Y,N) } != 1.
:- Y = 1..n, N = 1..n, #count{ X : num(X,Y,N) } != 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin4.lp | clasp -q 0
```

```
Models : 1344      Time : 0.024s
```

# Term Order

- Investigating terms along an order often helps to encode compactly

☞ Linking successive terms

## Example

```
item(i1).    item(i2).
prop(i1,p1). prop(i2,p1).
prop(i1,p2). prop(i2,p2).
prop(i1,p3). prop(i2,p3).
prop(i1,p4).

prop(P) :- prop(I,P). % Projection

next(P1,P2) :- prop(P1), prop(P2), P1 < P2, P <= P1 : prop(P), P < P2.

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# Implementing Counters

- Aggregates do not always fit (eg. comparisons among many objects)

☞ Exact outcome is often unnecessary!

☞ Interval outcome is often easier to handle

## Example continued

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{ has(I,P) } :- prop(I,P).

count(I,P1,0)   :- item(I), init(P1), not has(I,P1).
count(I,P1,1)   :- item(I), init(P1), has(I,P1).
count(I,P2,N)   :- count(I,P1,N), next(P1,P2), not has(I,P2).
count(I,P2,N+1) :- count(I,P1,N), next(P1,P2), has(I,P2).

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# Encode With Care!

1. Create a working encoding
   - Q1: Do you need to modify the encoding if the facts change?
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# Some Hints on (Preventing) Debugging

## Kinds of errors

- Syntactic ☞ follow error messages by the grounder
- Semantic ☞ (most likely) encoding/intention mismatch

## Ways to identify semantic errors (early)

- Develop and test incrementally
  - Prepare toy instances with "interesting features"
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- Compare the encoded to the intended meaning
  - Check whether the grounding fits (use `gringo --text`)
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## Grounding

- Monitor time spent by and output size of *gringo*
  1. System tools (eg. `time(gringo [...] | wc)`)
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☞ Once identified, reformulate "critical" logic program parts

## Solving

- Check solving statistics (use `clasp --stats`)

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# Optimization Problems: Overview

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  - Specification limits
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## Soft Constraints

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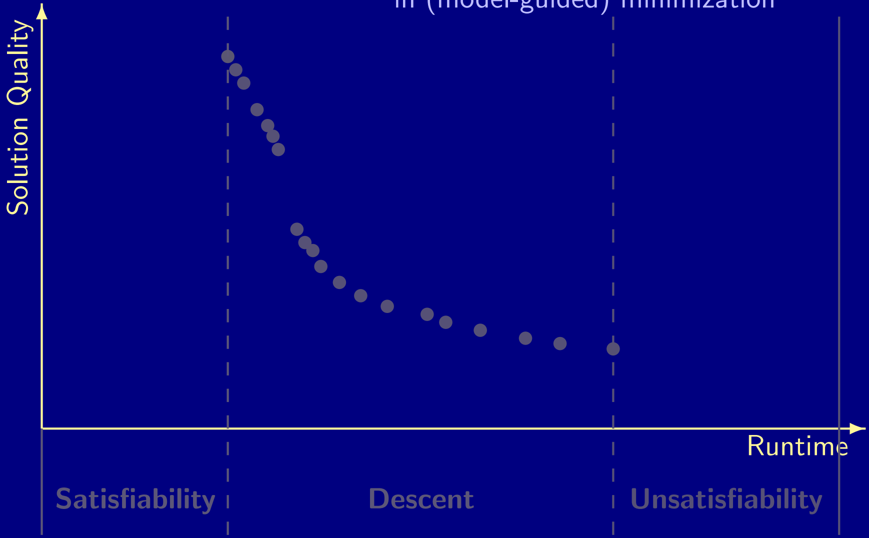
in (model-guided) minimization



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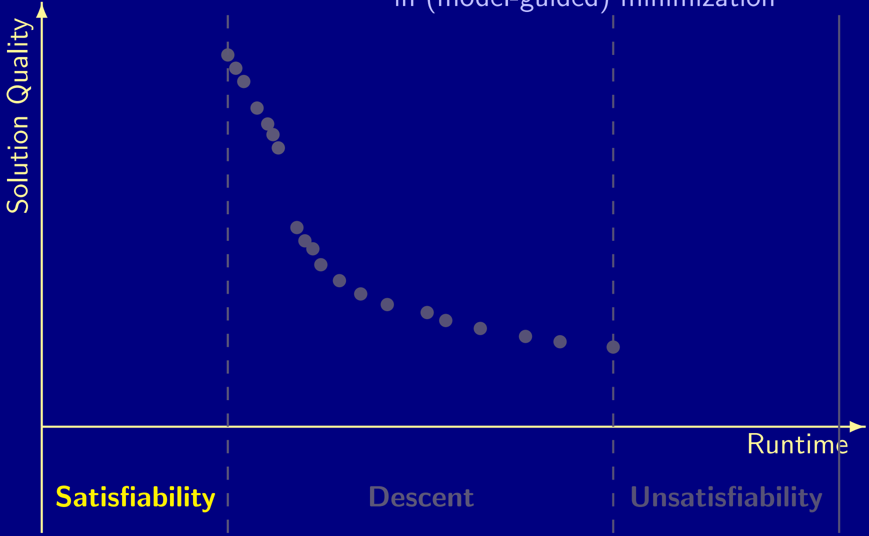

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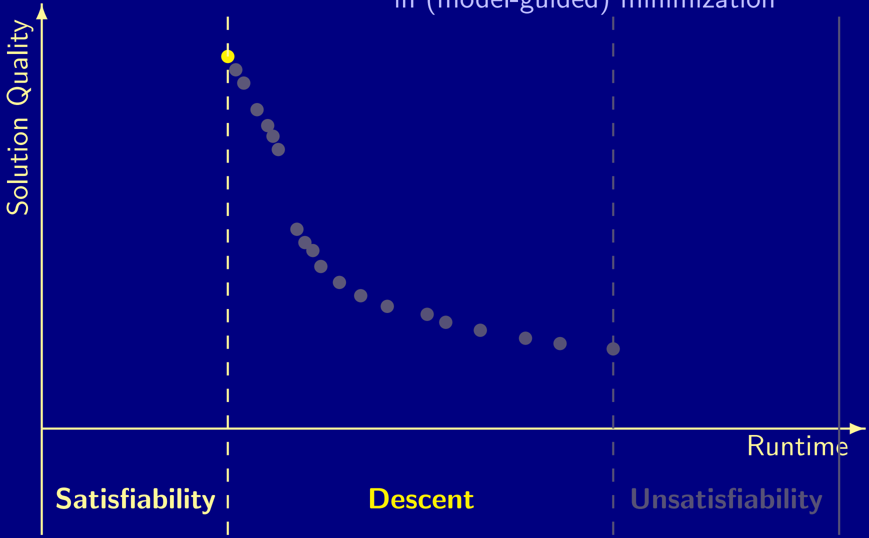

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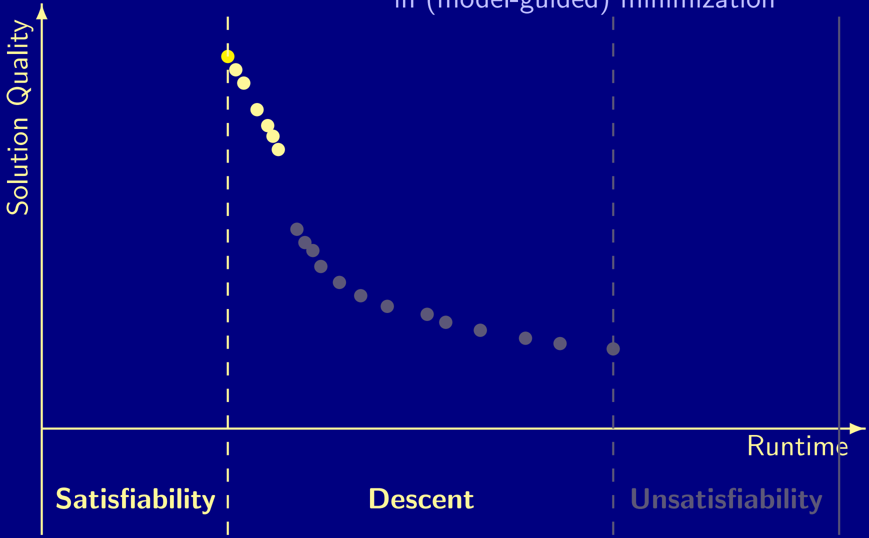

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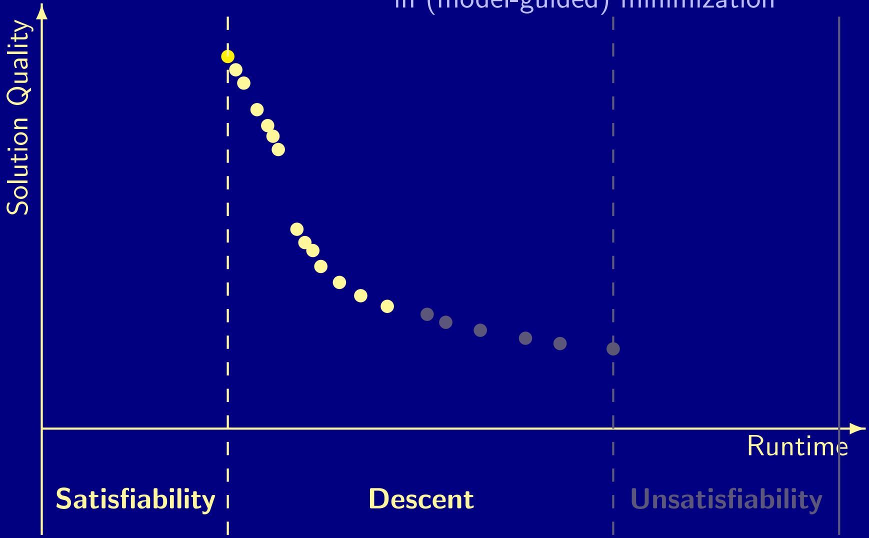

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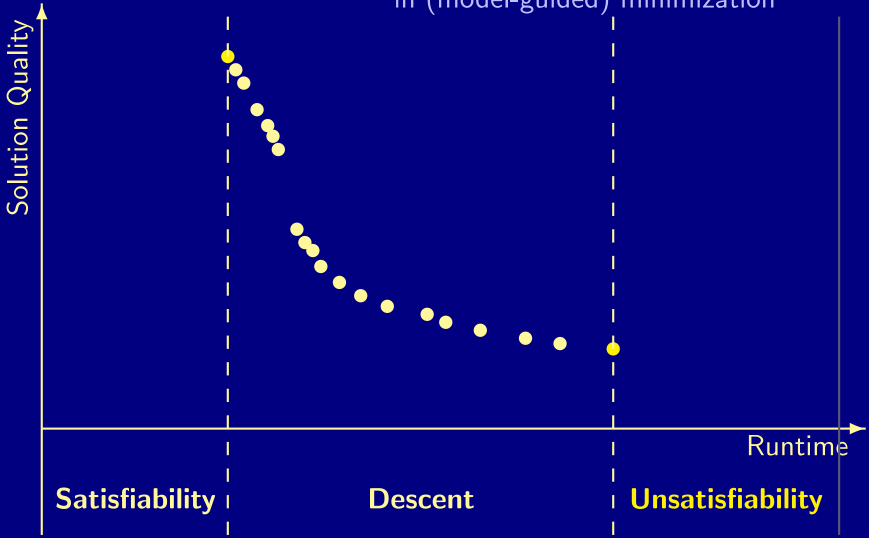

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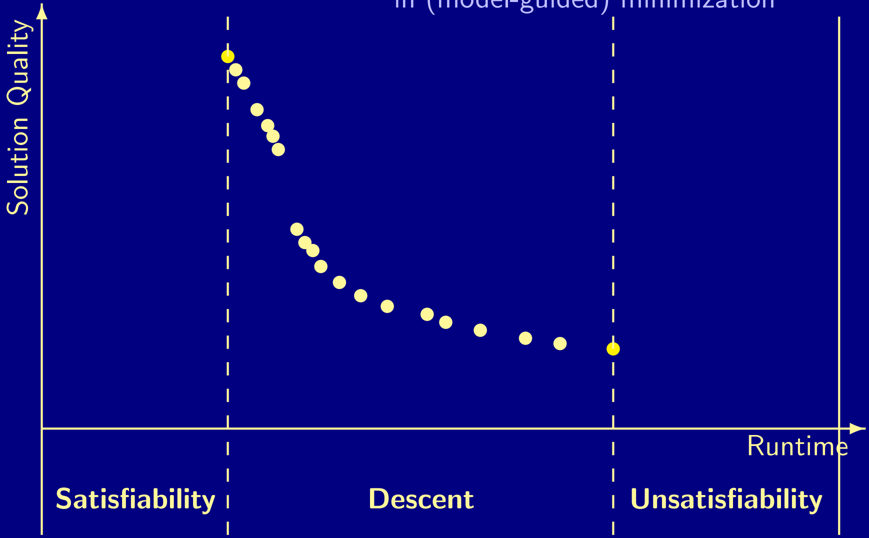

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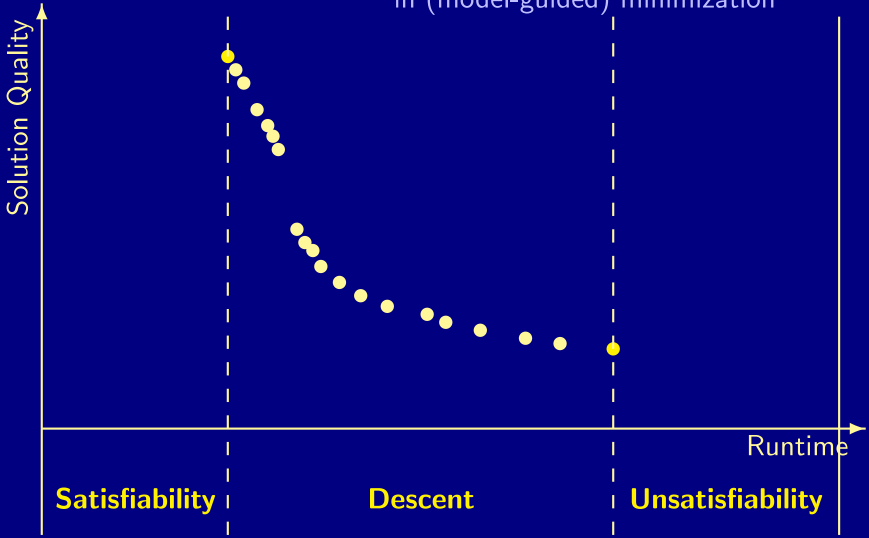

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- ☞ Try "aggressive" search strategies (*clasp* options like `--opt-strategy`, `--opt-heuristic`, and/or `--restart-on-model`)
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☞ Try "aggressive" search strategies (*clasp* options like `--opt-strategy`, `--opt-heuristic`, and/or `--restart-on-model`)

☞ Abstract from particular candidate solutions
- General lower bounds
- Symmetry breaking
- Encoding methods

# Overview

# Pigeonhole Principle Revisited

When we have to select at least $n/2$ out of $n$ items, how many do we need?

"Expert knowledge": $n/2$!

## Naive Encoding

```
#const n = 24.
n/2 { select(1..n) }.
#minimize{ 1,I : select(I) }.
```

## Performance?

```
Answer: 1
select(13) select(14) select(15) select(16) select(17) select(18) ...
Optimization: 12
OPTIMUM FOUND

Time        : 7.351s (Solving: 7.35s 1st Model: 0.00s Unsat: 7.35s)
Choices     : 1831336
Conflicts   : 1830166
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# Counting Revisited

When we have to select at least $n/2$ out of $n$ items, how many do we need?

Let's count them!

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```
#const n = 24.
n/2 { select(1..n) }.
count(I,1)     :- select(I).
count(I+1,N)   :- count(I,N), I < n.
count(I+1,N+1) :- count(I,N), select(I+1).
:- not count(n,n/2).
#minimize{ 1,I : select(I) }.
```

## Performance? ✔

```
Time        : 0.008s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
Choices     : 105
Conflicts   : 67
```

# Counting Revisited

When we have to select at least $n/2$ out of $n$ items, how many do we need?
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## Localizing Penalties

- Counting over $n$ items (to minimize) requires $O(n^2)$ rules

### Global Selection + Global Counting

```
#const n = 42.
n/3 { select(1..n) }.
#minimize{ 1,I : select(I) }.
```

# Localizing Penalties

- Counting over $n$ items (to minimize) requires $O(n^2)$ rules
  ☞ Often penalties can be attributed to subgroups

## Local Selection + Global Counting

```
#const n = 42.
1 { select(3*I-D) : D = 0..2 } :- I = 1..n/3.
% 1 { select(1), select(2), select(3) }.
% 1 { select(4), select(5), select(6) }.
% 1 { ... }.
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## Performance?

```
Optimization: 14
Time        : 16.640s (Solving: 16.64s 1st Model: 0.00s Unsat: 16.64s)
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## Performance?

```
Time        : 0.030s (Solving: 0.02s 1st Model: 0.00s Unsat: 0.01s)
Conflicts   : 14
Variables   : 1778
Constraints : 5180
```

# Localizing Penalties

- Counting over $n$ items (to minimize) requires $O(n^2)$ rules
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```
Time        : 0.003s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
Conflicts   : 14
Variables   : 140
Constraints : 266
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# Localizing Penalties

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# Overview

# Example: Traveling Sales-Person (TSP)

## Task

Given a (directed) graph with positive edge costs,
find a round trip with minimum accumulated edge costs.

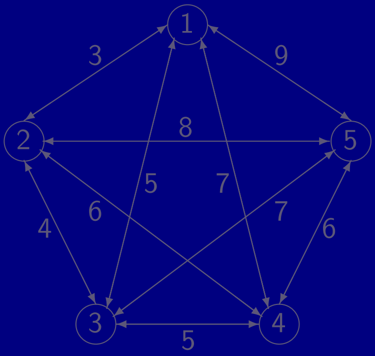
## (Complete) Graph Gadget: graph.lp

```
#const n = 5.

node(1..n).

cost(X,Y,2*Y-X) :- X = 1..n-1,
                   Y = X+1..n.
cost(Y,X,C)     :- cost(X,Y,C).

edge(X,Y) :- cost(X,Y,C).
```



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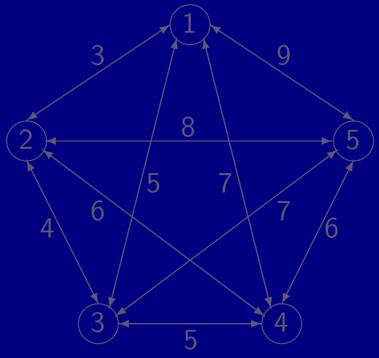

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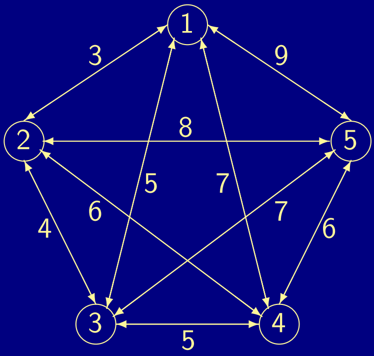

# Taking TSP Literally

## Straightforward Encoding: tsp0.lp

```
% GENERATE: Precisely one outgoing and incoming edge per node
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

% DEFINE + TEST: Each node must be reached from starting node
reached(X) :- X = #min{ Y : node(Y) }.
reached(Y) :- reached(X), cycle(X,Y).
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% OPTIMIZE: Minimize accumulated edge costs of round trip
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```
gringo -c n=5 graph.lp tsp0.lp | clasp --stats
```

```
Models       : 1
Optimization: 27
Time         : 0.001s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
Choices      : 41
Conflicts    : 39
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Time         : 60.017s (Solving: 60.01s 1st Model: 0.00s Unsat: 57.61s)
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**Ideas for improvements?**

# Mind the Gap(s)!

- Every node requires some outgoing (and incoming) edge

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# Mind the Gap(s)!

- Every node requires some outgoing (and incoming) edge
  - ☞ Gaps to minimum outgoing edge cost provide penalty per node!

## Elaborate Encoding: `tsp1.lp`

```
% GENERATE: Precisely one outgoing and incoming edge per node

% DEFINE + TEST: Each node must be reached from starting node

% OPTIMIZE: Minimize accumulated edge costs of round trip
cost(X,C) :- cost(X,Y,C).

next(X,C1,C2) :- cost(X,C1), cost(X,C2), C1 < C2, C <= C1 : cost(X,C), C < C2.

gap(X,C1,C2-C1) :- next(X,C1,C2), cycle(X,Y), cost(X,Y,C2).
gap(X,C1,C2-C1) :- next(X,C1,C2), gap(X,C2,G).

#minimize{ G,X,C : gap(X,C,G) }.
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# Mind the Gap(s)!

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```
% GENERATE: Precisely one outgoing and incoming edge per node

% DEFINE + TEST: Each node must be reached from starting node

% OPTIMIZE: Minimize accumulated edge costs of round trip
cost(X,C) :- cost(X,Y,C).

next(X,C1,C2) :- cost(X,C1), cost(X,C2), C1 < C2, C <= C1 : cost(X,C), C < C2.

gap(X,C1,C2-C1) :- next(X,C1,C2), cycle(X,Y), cost(X,Y,C2).
gap(X,C1,C2-C1) :- next(X,C1,C2), gap(X,C2,G).

#minimize{ G,X,C : gap(X,C,G) }.
```

# Mind the Gap(s)!

- Every node requires some outgoing (and incoming) edge
  - ☞ Gaps to minimum outgoing edge cost provide penalty per node!

## Elaborate Encoding: `tsp1.lp`

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```
gringo -c n=12 graph.lp tsp1.lp | clasp --stats
```

```
Models       : 1
Optimization: 111
Time         : 60.017s (Solving: 60.01s 1st Model: 0.00s Unsat: 57.61s)
Conflicts    : 7053956
```

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Models       : 1
Optimization: 20
Time         : 0.206s (Solving: 0.20s 1st Model: 0.00s Unsat: 0.20s)
Conflicts    : 20201
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gringo -c n=16 graph.lp tsp1.lp | clasp --stats
```

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Models       : 1
Optimization: 20
Time         : 0.206s (Solving: 0.20s 1st Model: 0.00s Unsat: 0.20s)
Conflicts    : 20201
```

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```
gringo -c n=16 graph.lp tsp1.lp | clasp --stats
```

```
Models       : 1
Optimization: 28
Time         : 31.926s (Solving: 31.91s 1st Model: 0.00s Unsat: 31.91s)
Conflicts    : 1991585
```

# Overview

# Optimization Phenomena

- Addressing candidate solutions (unfiltered) makes optimization hard
  - ☞ Consider candidates' properties, but not candidates themselves
  - ☞ Orient baseline at hard constraints, to not optimize the empty set
- Sharing penalties (utilities) uniformly may sacrifice problem structure
  - ☞ Attribute penalties to subgroups, when there is a known partition
  - ☞ Consider local baselines for subgroups, in case they are divergent
- Transferring preference relationships to the propositional level is useful
  - ☞ Counting abstracts from particular items, which need no distinction
  - ☞ Chaining along gaps relates diverse quantitative values, if available

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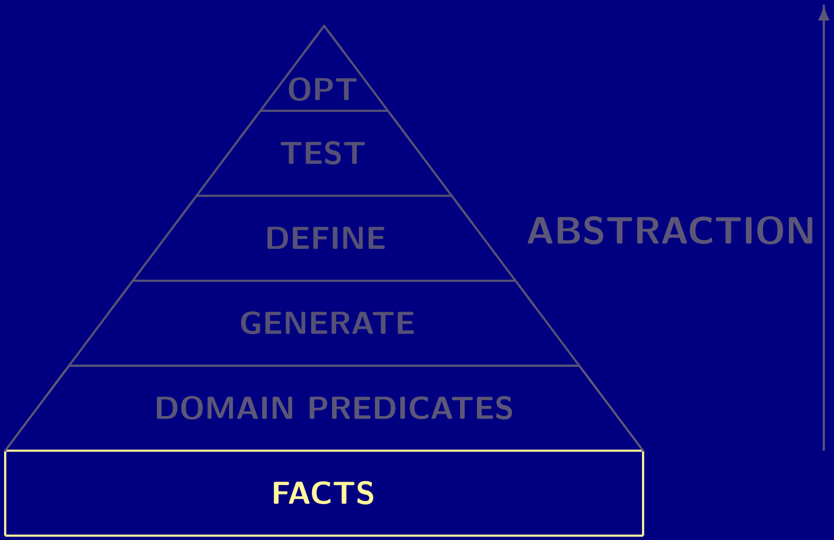
# Walk Like an Egyptian



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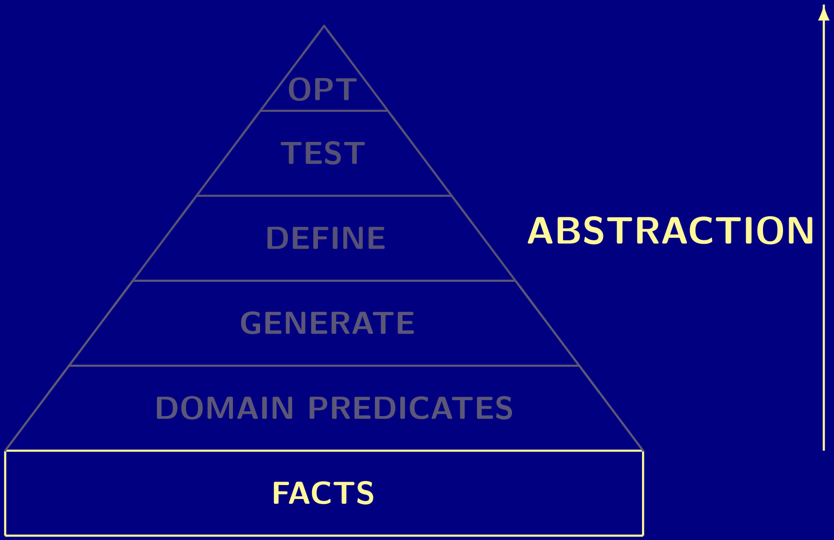

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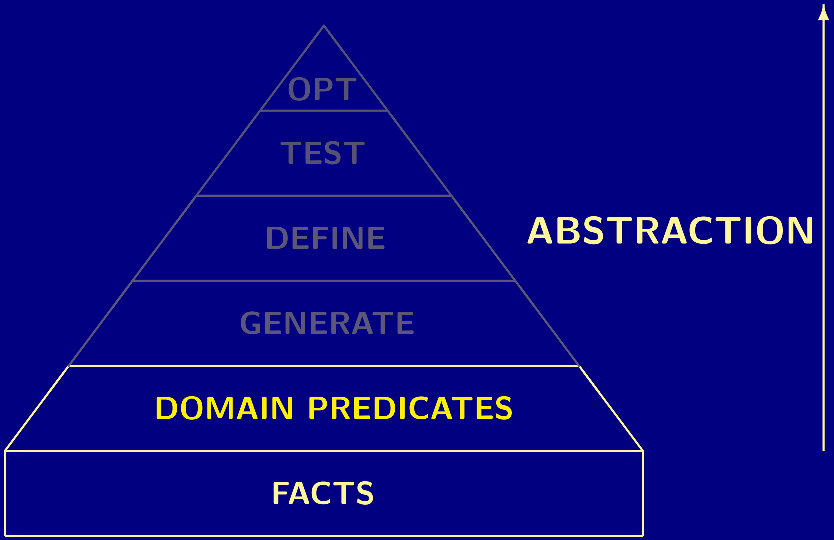

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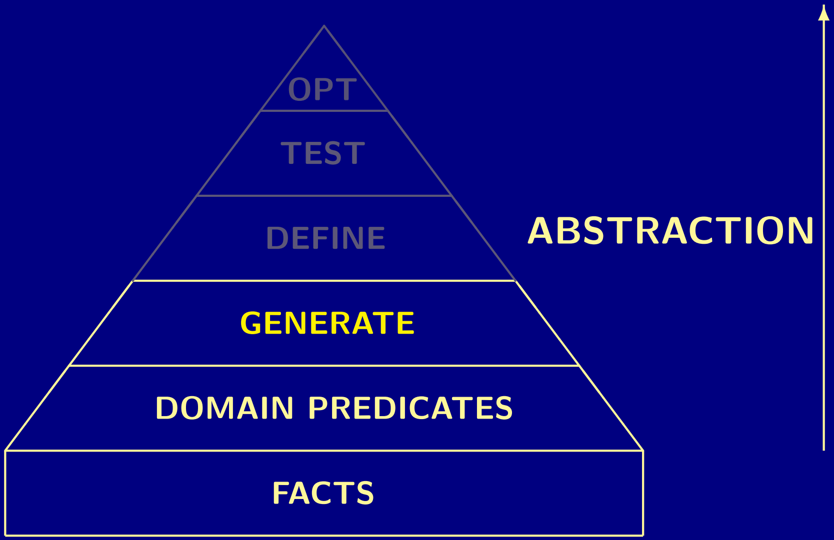

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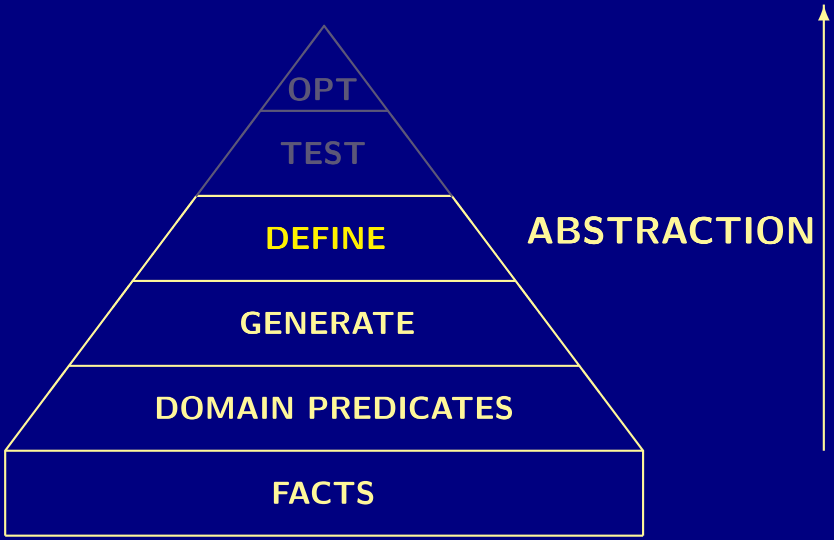

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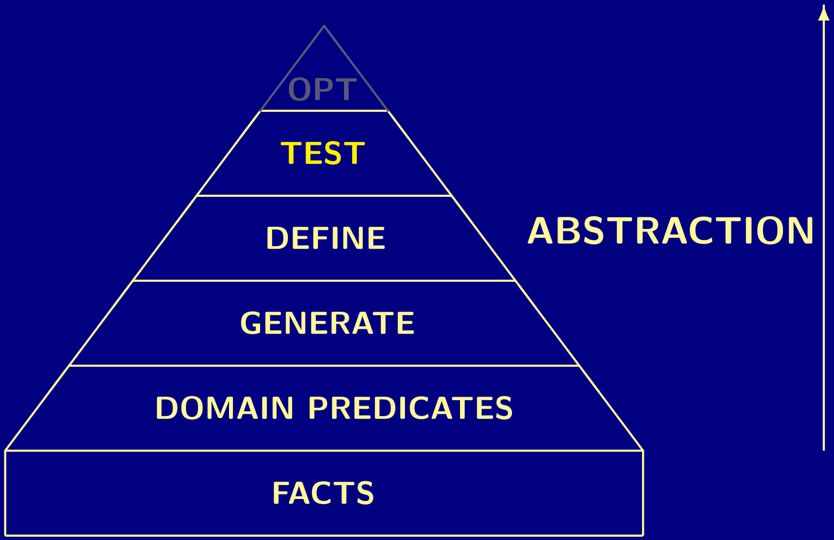

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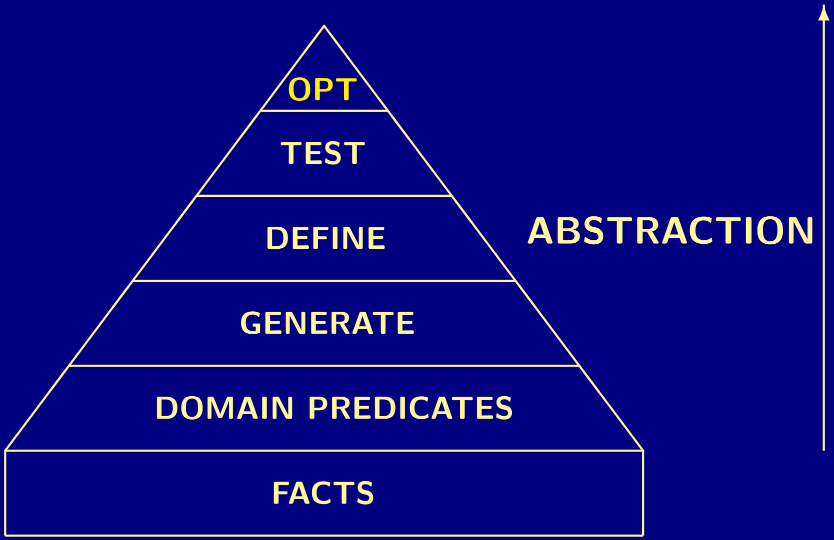

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