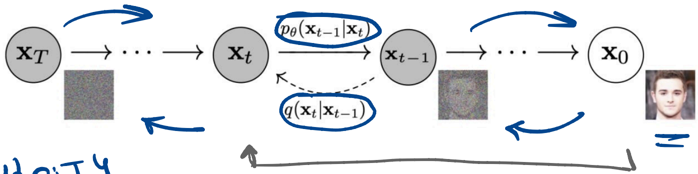
1



$\mathcal{N}(0; I)$

$$q(x_t|x_{t-1}) = \mathcal{N}(x_t \mid \sqrt{\beta_t}\, x_{t-1}; \sqrt{1-\beta_t}\, I)$$

$$x_t = x_{t-1}\cdot\sqrt{\beta_t} + \sqrt{1-\beta_t}\cdot\varepsilon, \quad \varepsilon \sim \mathcal{N}(0; I)$$

$$q(x_t|x_0) = \mathcal{N}(x_t \mid \sqrt{\bar\alpha_t}\cdot x_0; (1-\bar\alpha_t) I)$$

$$x_t = \sqrt{\bar\alpha_t}\cdot x_0 + \sqrt{1-\bar\alpha_t}\,\varepsilon$$

$x_T \sim \mathcal{N}(0; I)$ $\quad x_{t-1} = p_\theta(x_{t-1}|x_t)$

$t = T \dots 1$ $\quad (T = 1000)$

$$p_\theta(x_{t-1}|x_t) = \mathcal{N}(x_{t-1} \mid \mu_\theta(x_t; t); \beta_t I)$$

DDPM: $\varepsilon_\theta(x_t; t) = \arg\min_\theta \mathbb{E}_{\substack{t\sim U[0,T] \\ x_0\sim p_{data}(x_0) \\ x_t \sim q(x_t|x_0)}} \|\varepsilon_\theta(x_t; t) - \varepsilon\|^2$

$$x_t = \sqrt{\bar\alpha_t}\cdot x_0 + \sqrt{1-\bar\alpha_t}\,\varepsilon$$

$$x_1 = \sqrt{\bar\alpha_1}\, x_0 + \sqrt{1-\bar\alpha_1}\,\varepsilon$$

x_T $\quad x_t$ $\quad x_1$ $\quad x_0$

$\alpha_1 \simeq 1 \mapsto x_1 \simeq x_0$

$x_T = \varepsilon$

② Репараметризация через скор функцию

1) 1. $q(x) \leftrightarrow \nabla \log q(x)$ Скор. функция

2. $q(x) = const \cdot \exp\{-\frac{(x-\mu)^2}{2\sigma^2}\}$

$\log q(x) = \log const + \{-\frac{(x-\mu)^2}{2\sigma^2}\}$

$\nabla \log q(x) = -\frac{\cancel{2}(x-\mu)}{\cancel{2}\sigma^2} = -\frac{x-\mu}{\sigma^2}$

2) 1. $q(x_t) = \int q(x_t, x_0)\,dx_0 = \int q(x_t|x_0)\,q(x_0)\,dx_0$

$\nabla \log q(x_t) \leftrightarrow \varepsilon_\theta(x_t, t)$

2. $\mathbb{E}_{t \sim U[0,T]} \|s_\theta(x_t, t) - \nabla \log q(x_t)\|^2 \to \min_\theta$

$x_t \sim q(x_t)$ → не вычислимо

$||$

3. $\mathbb{E}_{t \sim U[0,T]} \|s_\theta(x_t, t) - \nabla \log q(x_t|x_0)\|^2 \to \min_\theta$

$(x_0, x_t) \begin{cases} x_0 \sim q(x_0) \\ x_t \sim q(x_t|x_0) \end{cases}$

4. $q(x_t|x_0) \sim \mathcal{N}\{x_t | \sqrt{\bar{\alpha}_t}\, x_0, (1-\bar{\alpha}_t) I\}$

$\nabla \log q(x_t|x_0) = -\frac{x_t - \sqrt{\bar{\alpha}_t}\, x_0}{1-\bar{\alpha}_t}$

3. $$\mathbb{E}_{\substack{t\sim U[0,T] \\ x_0\sim q(x_0) \\ x_t\sim q(x_t|x_0)}} \| s_\theta(x_t,t) - \nabla \log q(x_t|x_0)\|^2 \to \min_\theta$$

4. $$q(x_t|x_0) \sim \mathcal{N}\{x_t \,|\, \sqrt{\bar\alpha_t}\, x_0, (1-\bar\alpha_t) I\}$$

$$\nabla \log q(x_t|x_0) = -\frac{x_t - \sqrt{\bar\alpha_t}\, x_0}{1-\bar\alpha_t}$$

5. $$\mathbb{E}_{\substack{t\sim U[0,T] \\ x_0\sim q(x_0) \\ x_t\sim q(x_t|x_0)}} \left\| s_\theta(x_t,t) + \frac{x_t - \sqrt{\bar\alpha_t}\, x_0}{1-\bar\alpha_t} \right\|^2 \to \min_\theta$$

6. $$x_t = \sqrt{\bar\alpha_t}\, x_0 + \sqrt{1-\bar\alpha_t}\,\varepsilon \mapsto \varepsilon = \frac{x_t - \sqrt{\bar\alpha_t}\, x_0}{\sqrt{1-\bar\alpha_t}}$$

7. $$\mathbb{E}_{\substack{t\sim U[0,T] \\ x_0\sim q(x_0) \\ x_t\sim q(x_t|x_0)}} \left\| s_\theta(x_t,t) + \frac{\varepsilon}{\sqrt{1-\bar\alpha_t}} \right\|^2 \to \min_\theta$$

8. $$\mathbb{E}_{\substack{t\sim U[0,T] \\ x_0\sim q(x_0) \\ x_t\sim q(x_t|x_0)}} \| \varepsilon_\theta(x_t,t) - \varepsilon\|_2^2 \to \min_\theta$$

DDPM

9. $$\mathbb{E}_{\substack{t\sim U[0,T] \\ x_0\sim q(x_0) \\ x_t\sim q(x_t|x_0)}} \cancel{\frac{1}{1-\bar\alpha_t}} \| -\sqrt{1-\bar\alpha_t}\, s_\theta(x_t,t) - \varepsilon\|^2 \to \min_\theta$$

10. $$\mathbb{E}_{\substack{t\sim U[0,T] \\ x_0\sim q(x_0) \\ x_t\sim q(x_t|x_0)}} \| \varepsilon_\theta(x_t, t) - \varepsilon \|_2^2 \to \min_\theta$$

DDPM

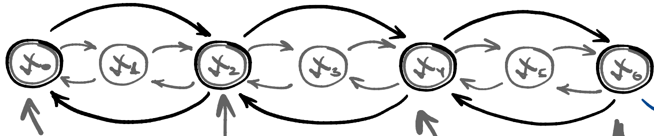
$$\mathbb{E}_{\substack{t\sim U[0,T] \\ x_0\sim q(x_0) \\ x_t\sim q(x_t|x_0)}} \cancel{\frac{1}{1-\bar{\alpha}_t}} \| -\sqrt{1-\bar{\alpha}_t}\, s_\theta(x_t, t) - \varepsilon \|^2 \to \min_\theta$$

$\hookrightarrow$ $$\varepsilon_\theta(x_t, t) = -\sqrt{1-\bar{\alpha}_t}\, s_\theta(x_t)$$

$$s_\theta(x_t, t) = -\frac{\varepsilon_\theta(x_t, t)}{\sqrt{1-\bar{\alpha}_t}} \simeq \nabla \log q(x_t)$$

$$\mu_\theta(x_t, t)$$

③



$T = 6$

$\to \mathcal{N}\{0, I\}$

$\varepsilon_\theta(x_t \mid t), \quad t = \overline{0,6}$

1. $i = \overline{0,3}$: $\quad \tau(0) = 0, \quad \tau(1) = 2, \quad \tau(2) = 4, \quad \tau(3) = 6$

2. $$q^{new}(x_i) = q(x_{\tau(i)}) = \int q(x_{\tau(i)} \mid x_0)\, q(x_0)\, dx_0 = \int q^{new}(x_i \mid x_0)\, q(x_0)\, dx_0$$

3. $$q^{new}(x_i \mid x_0) = q(x_{\tau(i)} \mid x_0) = \mathcal{N}\{x_{\tau(i)} \mid \sqrt{\bar{\alpha}_{\tau(i)}}\, x_0, (1 - \bar{\alpha}_{\tau(i)}) I\}$$

$$\hookrightarrow \bar{\alpha}_i^{new} = \bar{\alpha}_{\tau(i)} \quad \leftrightarrow \quad \alpha_i^{new} = ?$$

4. $$\bar{\alpha}_i^{new} = \prod_{k=1}^{i} \alpha_k^{new} = \prod_{k=1}^{i-1} \alpha_k^{new} \cdot \alpha_i^{new} = \bar{\alpha}_{i-1}^{new} \cdot \alpha_i^{new}$$

$$\hookrightarrow \alpha_i^{new} = \frac{\bar{\alpha}_i^{new}}{\bar{\alpha}_{i-1}^{new}} = \frac{\bar{\alpha}_{\tau(i)}}{\bar{\alpha}_{\tau(i-1)}}$$

5. $x_{t-1} \sim p_\theta(x_{t-1}|x_t) = \mathcal{N}\{x_{t-1} | \mu_\theta(x_t|t); \tilde{\beta}_t I\}$

$$x_{t-1} = \underbrace{\sqrt{\frac{1-\bar{\alpha}_{t-1}}{1-\bar{\alpha}_t}(1-\alpha_t)}}_{\sqrt{\tilde{\beta}_t}} \cdot \underbrace{\varepsilon}_{\mathcal{N}\{0;I\}} + \underbrace{\frac{1}{\sqrt{\alpha_t}}\left\{x_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}}\varepsilon_\theta(x_t;t)\right\}}_{\mu_\theta(x_t;t)}$$

$\downarrow$

$$x_{i-1} = \sqrt{\frac{1-\bar{\alpha}_{i-1}^{new}}{1-\bar{\alpha}_i^{new}}(1-\alpha_i^{new})}\,\varepsilon$$

$$+ \frac{1}{\sqrt{\alpha_i^{new}}}\left\{x_i - \frac{1-\alpha_i^{new}}{\sqrt{1-\bar{\alpha}_i^{new}}}\varepsilon_\theta\left(\underline{\underline{x_{\tau(i)}}}; \underline{\underline{\tau(i)}}\right)\right\}$$

$$\nabla \log q(x_t) = -\frac{\varepsilon_\theta(x_t;t)}{\sqrt{1-\bar{\alpha}_t}}$$

$$q^{new}(x_i) = q(x_{\tau(i)}) \rightarrow \nabla \log q^{new}(x_i) = \nabla \log q(\underline{\underline{x_{\tau(i)}}})$$

$$\cancel{-}\frac{\varepsilon_\theta^{new}(x_i;i)}{\cancel{\sqrt{1-\bar{\alpha}_i^{new}}}} = \cancel{-}\frac{\varepsilon_\theta(x_{\tau(i)};\tau(i))}{\cancel{\sqrt{1-\bar{\alpha}_{\tau(i)}}}}$$

$\hookrightarrow$ $\varepsilon_\theta^{new}(x_i|i) = \varepsilon_\theta(x_{\tau(i)}|\tau(i))$