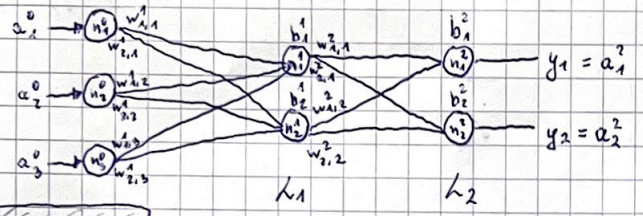
$x = [0.5, 0.5, 0.1] = [a_1^0, a_2^0, a_3^0]$; $\hat{y} = [0.9, 0.1]$; $\eta = 0.1$

$w_{1,1}^1 = 0.1$, $w_{2,1}^1 = 0.4$, $w_{1,2}^1 = 0.2$, $w_{2,2}^1 = 0.1$, $w_{1,3}^1 = 0.5$, $w_{2,3}^1 = 0.5$, $b_1^1 = 0.2$, $b_2^1 = 0.2$

$w_{1,1}^2 = 0.6$, $w_{2,1}^2 = 0.5$, $w_{1,2}^2 = 0.1$, $w_{2,2}^2 = 0.2$, $b_1^2 = 0.2$, $b_2^2 = 0.1$



FORWARD

n_1^1:

$z_1^1 = w_{1,1}^1 * a_1^0 + w_{1,2}^1 * a_2^0 + w_{1,3}^1 * a_3^0 + b_1^1 = (0.1*0.5) + (0.2*0.5) + (0.5*0.1) + 0.2 =$

$= 0.05 + 0.1 + 0.05 + 0.2 = 0.4$

$a_1^1 = \delta(z_1^1) = \frac{1}{1+e^{-0.4}} = 0.5987$

n_2^1:

$z_2^1 = w_{2,1}^1 * a_1^0 + w_{2,2}^1 * a_2^0 + w_{2,3}^1 * a_3^0 + b_2^1 = (0.4*0.5) + (0.1*0.5) + (0.5*0.1) + 0.2 =$

$= 0.2 + 0.05 + 0.05 + 0.2 = 0.5$

$a_2^1 = \delta(z_2^1) = \delta(0.5) = 0.6225$

} layer 1

n_1^2:

$z_1^2 = w_{1,1}^2 * a_1^1 + w_{1,2}^2 * a_2^1 + b_1^2 = (0.6*0.5987) + (0.1*0.6225) + 0.2 =$

$= 0.35922 + 0.06225 + 0.2 = 0.6251$

$a_1^2 = \delta(z_1^2) = \delta(0.6251) = 0.6506 = y_1$

n_2^2:

$z_2^2 = w_{2,1}^2 * a_1^1 + w_{2,2}^2 * a_2^1 + b_2^2 = (0.5*0.5987) + (0.2*0.6225) + 0.1 =$

$= 0.2994 + 0.1245 + 0.1 = 0.5239$

$a_2^2 = \delta(z_2^2) = \delta(0.5239) = 0.6281 = y_2$

} layer 2 (output)

$$L = \frac{1}{2}\left[(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2\right] = \frac{1}{2}\left[(0.6506 - 0.9)^2 + (0.6281 - 0.1)^2\right] =$$

$$= \frac{[0.0622 + 0.2789]}{2} = \frac{0.3411}{2} = 0.1705$$

BACKWARD

$\delta_1^2 = \frac{\partial L}{\partial a_1^2} \cdot \sigma'(z_1^2) = (y_1 - \hat{y}_1) * \sigma'(0.6251) = (-0.2494) * 0.2273 = -0.0567$

$\delta_2^2 = \frac{\partial L}{\partial a_2^2} \cdot \sigma'(z_2^2) = (y_2 - \hat{y}_2) * \sigma'(0.5239) = 0.5281 * 0.2336 = 0.1234$

} layer 2

$\delta_1^1 = w_{1,1}^2 * \delta_1^2 * \sigma'(z_1^1) + w_{2,1}^2 * \delta_2^2 * \sigma'(z_1^1) = 0.6 * (-0.0567) * 0.24026 + 0.5 * 0.1234 * 0.24026 =$

$= 0.01482 - 0.00817 = 0.0067$

(For layer 1 we need only that value in this case)

$\frac{\partial L}{\partial w_{1,1}^1} = \delta_1^1 * a_1^0 = 0.0067 * 0.5 = 0.0034$ ⇒ change in weight

New value of $w_{1,1}^1$ after the backpropagation is:

$$w_{1,1}^1 = w_{1,1}^1 - \eta * \frac{\partial L}{\partial w_{1,1}^1} = 0.1 - 0.1 * 0.0034 = 0.1 - 0.00034 = 0.0997$$