Algos - Reading

Assignment 1

Problem 1-1. a) The oder is $(f_5, f_3, f_4, f_1, f_2)$.

f_2 grows faster than f_1: suppose $n > 4$, then $\log(n) > 2$ so that $\log(n)^n > 2^n$, which grows faster than $n\log(n)$.

b) The order is $(f_1, f_2, f_5, f_4, f_3)$.

Note that f_4 grows faster than f_5 since it has an exponential growth in the exponent, whereas f_5 has polynomial growth.

f_3 grows faster than f_4 since we can rewrite $f_4 = 6006^{(2^n)} = \left(2^{\log_2(6006)}\right)^{2^n} = 2^{\log_2(6006)2^n}$, and 6006^n grows faster than 2^n.

c) The order is $(\{f_2, f_5\}, f_4, f_1, f_3)$.

To find the growth of $f_4 = \binom{n}{\frac{n}{6}}$ note that $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ and suppose that $k < n - k$. From this we have two bounds:

1. $\binom{n}{k} \geq \frac{1}{2}\frac{n!}{(n-k)!} \geq \frac{1}{2}(n-k)^k$

and

2. $\binom{n}{k} \leq \frac{1}{2}\frac{n!}{k!} \leq \frac{1}{2}n^{n-k}$

When $k = \frac{n}{6}$ we see from the latter inequality that f_4 grows slower than n^n.

$(6n)! \sim \sqrt{12\pi n}\left(\frac{6n}{e}\right)^{6n}$ so that f_3 grows faster than f_1.

d) The order is $(f_5, f_2, f_1, f_3, f_4)$.

Simplifying $f_3 = (2^2)^{3n\log(n)} = n^{6n}$

Note that if $\log\left(\frac{f(x)}{g(x)}\right) \to \infty$ as $x \to \infty$ then $\frac{f(x)}{g(x)} \to \infty$ as well, and since $n^2\log(7) - 6n\log(n) \to \infty$ as $n \to \infty$ we have that f_4 grows faster than f_3.

Problem 1-2. (a)

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D.exchange_at(i,j):
    set j to be the larger value
    right_at = D.delete_at(j)
    left_at = D.delete_at(i)
    D.insert_at(i, right_at)
    D.insert_at(j, left_at)
reverse(D,i,k):
    set n = floor(k/2)
    for j in 1 to n:
    D.exchange_at(i, i+k - j)
```

b)

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move(D,i,k,j):
    for n in 0..k-1:
        item = D.delete_at(i+n)
        D.insert_at(j+n-1, item)
```

Problem 1-4. (a)