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Seminar 1b

Ultra low temperatures

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Abstract

In this seminar the motivation for ultra-low temperature physics is briefly presented. A few of the most frequently used techniques for reaching ultra-low temperatures (below 1K) - helium dilution refrigeration, nuclear adiabatic demagnetization and magneto-optical trapping of atoms are described.

Contents

ln	ntroduction	2
1	The history of low temperature physics	3
2	Temperature from a thermodynamical point of view	
3	Temperature in statistical physics	
4	Cryogenic gases	5
5	Helium dilution refrigeration 5.1 Properties of Liquid ³ He - ⁴ He Mixtures	
6	Adiabatic demagnetization 6.1 Adiabatic demagnetization of paramagnetic salts	
7	Ultra cold atoms 7.1 Cooling molecules	
8	Conclusion	12
\mathbf{R}_{i}	References	

Introduction

Because life as we know it only exists in a fairly limited temperature range, we are often not aware of the importance of temperature or what temperature actually even really is. Another thing we don't realize is that low temperature physics is one of the rare fields, where we managed to surpass nature, managing to go to much lower temperatures than normally found in nature (The coldest spot in the known universe is in the Boomerang Nebula and has a temperature of 1 K [1].).

There are two main reasons for trying to go to lower and lower temperatures. The first is the desire to do more precise measurements. One of the examples are atomic clocks, another would be precise measurements of an atomic structure, where thermal vibrations make it almost impossible the achieve atomic resolution. The second, but not any less important reason is the search for new properties of matter, that are screened by thermal noise at usual temperatures, sometimes even finding new exciting states of matter like superconductivity, superfluidity, BEC...

I will start with a brief history of low temperature physics and later on focus on two important techniques of cooling - adiabatic demagnetization and dilution refrigeration. In the end I will try to shortly present the concept of cold atom experiments. The main motivation to try to cool clouds of atoms or even molecules instead of a solid sample to low temperatures is the fact that this enables simulations of solid matter physics at extremely low temperatures below the μ K range.

1 The history of low temperature physics

The field now known as low or ultra low temperature physics started with cryogenics in the beginning of the 19th century when Faraday managed to liquefy ammonia. Until the end of the century scientists managed to liquefy and eventually solidify air, nitrogen and oxygen, reaching a temperature below 100 K for the first time in 1883. The lowest temperature in logarithmic scale and the year it was achieved are represented in figure 1.

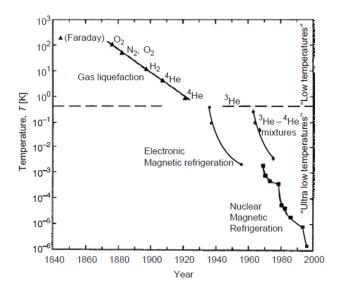


Figure 1: A timeline of the lowest temperatures achieved as a function of time [2].

There were two main reasons for trying to reach lower temperatures. The more practical one was trying to find a way to transport meat to Europe, while scientists were curious about the existence of "permanent gases" — substances that don't exist in liquid or solid form. This led to liquefaction and solidification of hydrogen by James Dewar in 1898. He reached a temperature of 20 K and later 13 K. At about the same time a new element, helium, was discovered. The race to liquefy this last element was won by Heike Kamerlingh-Onnes who liquefied helium at 4.2 K in 1908. This opened a completely new field of low temperature physics and led to the discovery of superconductivity in 1911. Later he managed to reach 0.83 K by pumping vapor above a bath of boiling ⁴He. The record minimum temperature was lowered even further in the forthcoming years, but scientists had to develop magnetic refrigeration methods and helium dilution refrigerators to go lower than 0.5 K. Nowadays magneto-optical and gravito-magnetic traps are used to cool atoms down to even lower temperatures. However, these methods cannot, unlike the previous ones mentioned, cool a sample - here we directly cool a group of atoms.

Using helium dilution refrigeration and two stages of nuclear adiabatic demagnetization a sample of rhodium has been cooled to 0.1 nK in Sweden in 2010, which is also the lowest temperature a sample has ever been cooled to [5].

2 Temperature from a thermodynamical point of view

If we want to talk about ultra low temperatures, we must first remember the definition of temperature. It consists of much more than just describing the warmth or cold of objects we encounter and it took a long while for this unit to be defined properly. The first definition of temperature was based on thermodynamic laws. The Zeroth law of thermodynamics states: "If two systems are in equilibrium with a third, than they are all in thermal equilibrium with each other." From this law, we can define temperature as a thermodynamic parameter or variable,

that is the same for two systems that are in thermal equilibrium. Taking in account the First and Kelvin's formulation of the Second thermodynamic law, we can get a quantized definition of temperature from describing the Carnot cycle. We can proof that for a reversible heat engine operating between temperatures θ_1 and θ_2 in a Carnot cycle, the ratio of heat Q_1 taken at the higher temperature θ_1 and the heat Q_2 given at a lower temperature θ_2 is proportional to the ratio of functions of these two temperatures as seen below:

$$\frac{Q_1}{Q_2} = \frac{\Phi(\theta_1)}{\Phi(\theta_2)} \rightarrow \frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$
 (2.0.1)

Lord Kelvin established, that these functions didn't depend on anything but the temperature so he used this relation to define the *thermodynamic temperature*. To make this scale relative, we need a fixed point. He used Guy-Lussac's law, which states that temperature is proportional to the volume of an ideal gas at constant pressure and extrapolated the measurements to -273,15° C also known as the absolute zero.

Another thermodynamic parameter that can be defined using the Carnot cycle is entropy. If we apply equation 2.0.1 to a case, where the two adiabatics lie very close together, we get

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \rightarrow \frac{dQ}{T} = dS \ .$$
 (2.0.2)

We can show, that $\frac{dQ}{T}$ is independent of temperature at various temperatures and is typical for two neighboring adiabatics, thereby defining entropy, denoted by S in the above equation.

3 Temperature in statistical physics

A more microscopical approach gives us a definition of temperature using the Poisson-Boltzman distribution.

$$p(\varepsilon) = q(\varepsilon)e^{-\varepsilon/k_BT} \tag{3.0.3}$$

where $p(\varepsilon)$ is the probability that the system's energy will be between ε and $\varepsilon + \Delta \varepsilon$ at a certain temperature T, where $g(\varepsilon)$ is the density of states in this system. We can see from this equation, that k_BT represents the characteristic energy of the system.

From this we can derive the equipartition theorem which states, that the average energy of a system \overline{E} at a certain temperature is the sum of $\frac{1}{2}k_BT$ over all degrees of freedom, where T is the average temperature of the system.

$$\overline{E} = \sum_{i=1}^{n} \frac{1}{2} k_B T , \qquad (3.0.4)$$

where n is the number of degrees of freedom of a system. In the case of a diatomic molecule, there are three degrees of fredom: translation, rotation and vibration. Therefore the average energy of a molecule at a certain temperature is $\frac{3}{2}k_BT$.

We can also use statistical physics to get a microscopic definition of entropy, known as the Gibbs definition:

$$S = k_B \log(\Gamma) . (3.0.5)$$

This defintion states that the entropy of a system S is a function of the number of specific ways in which the thermodynamic system can be arranged Γ .

Using the equipartition theorem and Gibbs's definition of entropy we can define an effective temperature for decoupled degrees of freedom or subsets of particles in a system. For example, we can define the temperature of nuclear spins from their change of entropy. So when talking about temperature we should always keep in mind, that the temperature always represents an average temperature of what we measure and that the temperatures of certain decoupled degrees of freedom (like spins) can be very different of the temperature of a sample.

4 Cryogenic gases

Every temperature range requires a different approach to cooling. For cooling samples under the temperature range achievable by ordinary or industrial freezers we mostly use cryogenic gases. Here we use the latent heat needed for the element to go from a liquid to a gaseous state to cool our sample. This also sets the limitations - when we can not get a transition from solid or liquid to gas, our cooling power drops significantly because materials have very small heat capacitance at low temperatures. Some of the most used cryogenic gases are presented in table 1. If we keep pumping the vapours away, thereby lowering the pressure, we can get the phase transition to occur at lower temperatures. The lowest limit we can reach this way is 300mK, by pumping the vapours from aboove liquid ³He. For lower temperatures, we need new methods.

Element/compound	Boiling temperature
Ammonia	−35.5 °C
CO_2	$-78.5^{\circ}\mathrm{C}$
O_2	−183 °C
N_2	$-196^{\circ}\mathrm{C}$
⁴ He	$4.23\mathrm{K}$
⁴ He	$3.19\mathrm{K}$

Table 1: Boiling temperatures of some cryogenic gases.

5 Helium dilution refrigeration

Unlike cooling with liquid helium ${}^{3}\text{He}$ or ${}^{4}\text{He}$ this method doesn't use the latent heat from evaporating the refrigerant, but the heat of mixing of the two isotopes. For the dilution refrigerator, the cooling power corresponds to the enthalpy ΔH of mixing of the two isotopes, which is given by the integral of the differences of the specific heats of the two phases

$$\Delta H = \int \Delta C dT. \tag{5.0.6}$$

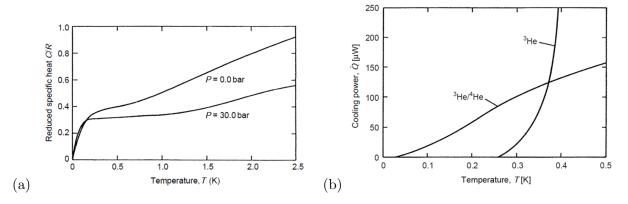


Figure 2: (a) Temperature dependence of reduced specific heat of ³He at two different pressures. (b) Comparison of cooling power of liquid ³He and a ³He - ⁴He dilution refrigerator [2].

Because the specific heats of 3 He and 4 He are proportional to T at low temperatures as can be seen in figure 2(a), we end up with a temperature dependence of

$$\dot{Q} \propto \dot{n}\Delta H \propto T^2 \tag{5.0.7}$$

which gives us much more efficient cooling at low temperatures than evaporation as seen on figure 2(b).

It was first proposed by H. London, G.R. Clarke and E. Mendoza in 1962. A group in Leiden built the first machine and reached 0.22 K in 1965. Only one year later temperatures down to 25 mK were reached in Dubna. The lowest temperature achieved by this method is 2 mK.

Helium dilution refrigeration is a very important method for reaching temperatures in the micro Kelvin range, because it, unlike adiabatic demagnetization, enables continuous cooling under 0.3 K (the lowest temperature that can be achieved using liquid ³He). It is used as an independent cooling method or to precool the sample before some other cooling method is used. To understand how the refrigeration process works, we must first learn something about the properties of liquid ³He-⁴He mixtures.

5.1 Properties of Liquid ³He - ⁴He Mixtures

³He is a lighter isotope of ⁴He but the lack of one neutron changes more than just the mass. When cooled to low temperatures we notice the big differences that are present on the quantum level. The total spin of a ⁴He atom is 0, making it behave as a boson, whereas ³He has spin 1/2 and is thus a fermion. We can notice this if we cool the element the mixture under 2.177 K. At that temperature liquid ⁴He undergoes a phase transition and becomes a super-fluid that obeys Bose-Einstein statistics. ³He stays a normal liquid until temperatures below 1 mK, where it too becomes a super-fluid but with different properties due to the fact that it is a fermion.

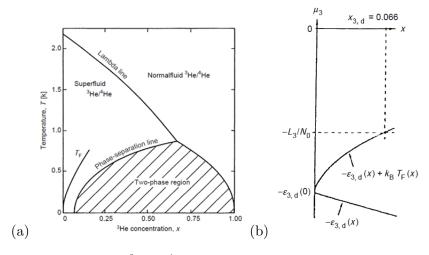


Figure 3: (a) Phase diagram of a ³He - ⁴He mixture at saturated vapor pressure. (b) Chemical potential of ³He in the diluted (⁴He) phase. [2].

Another important difference between the two isotopes is their mass. As we know, nothing can really ever stand still according to the Heisenberg principle, even at 0 K. We can either determine its position or velocity, but never both. This leaves an uncertainty about the position of the atom or molecule, called the zero point motion. It is not hard to see, that at a certain energy, the velocity of a ³He atom is going to be larger than that of a ⁴He atom, as ³He is lighter, and as a consequence also the zero point motion of ³He will be larger.

When we mix the two isotopes we would expect them to form a pure ${}^{3}\text{He}$ state floating on top of the pure ${}^{4}\text{He}$ phase, because as we know ${}^{3}\text{He}$ is lighter. But that is not the case. If we take a look at an ${}^{3}\text{He}$ atom in the concentrated ${}^{3}\text{He}$ phase consisting of N_{0} atoms, each ${}^{3}\text{He}$ atom is bound with an energy $\varepsilon_{3,c}$

$$\varepsilon_{3,c} = -\frac{L_3}{N0} \tag{5.1.1}$$

where L_3 is the latent heat needed to evaporate the $^3\mathrm{He}$ liquid, while the chemical potential is

$$\mu_{3,c} = -L_3. \tag{5.1.2}$$

A ³He atom in the ⁴He phase will have lower energy, because the zero point motion of ⁴He is smaller and therefore the ⁴He "pushes back" less, giving the ³He atom more space than it would have in the ³He phase. (Here we can assume the concentration of ³He atoms x is practically zero.) The ³He is therefore more bound if it is in the ⁴He phase and its energy $\varepsilon_{3,d}(x=0)$ is lower.

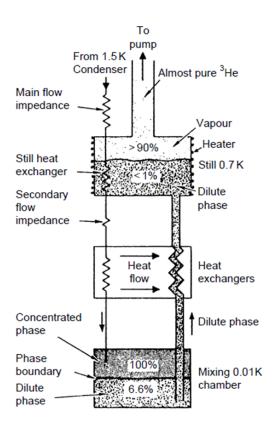
When we increase the concentration of 3 He atoms x in the 4 He phase, we have to take in account the fact that they are fermions so they have to obey the Pauli exclusion principle. Their chemical potential can be written as

$$\frac{\mu_{3,d}(x)}{N_0} = -\varepsilon_{3,d}(x) + k_{\rm B}T_{\rm F}(x)$$
 (5.1.3)

where $T_{\rm F}$ is the Fermi temperature, which is proportional to $x^{\frac{3}{2}}$. Eventually we reach the equilibrium concentration at x = 6.6% as can be seen in figures 3 (a) and 3 (b).

5.2 The refrigeration process

The cooling is based on the heat of mixing of the two isotopes. As we have seen in the phase diagram above, the ³He - ⁴He mixture undergoes a phase separation when sufficiently cooled, separating in an almost purely ³He phase and a dilute phase. The working fluid in this process if ³He, which is circulated by vacuum pumps, as can be seen in figure 4.



First ³He is cooled to about 1.3 K by the "1 K bath", which is ⁴He boiling at a lower temperature due to lower pressure. This stage liquefies the ³He and removes the heat of condensation. After that ³He enters the main flow impedance - a capillary with large flow resistance. Here it is cooled by the still, to $500 - 700 \,\mathrm{mK}$. It continues through a second impedance, followed by one side of counterflow heat exchangers, where it is cooled by a cold flow of ³He. Finally it enters the mixing chamber - the coldest place of the entire device. Here we have the concentrated ³He phase and the diluted phase in equilibrium, separated by a phase boundary. ³He is pumped from the concentrated to the diluted phase. Because this is an endothermic process it provides the useful cooling. The ³He than leaves the chamber in the dilute phase. On its way up it cools the downward flowing ³He until it enters the still. In the still liquid ³He flows through the super-fluid ⁴He, which is at

Figure 4: A schematic diagram of a dilution refrigerator [2].

The pressure in the still is low, about 10 Pa, so the vapor is practically pure ³He and the partial pressure of ³He is much bigger than that of ⁴He. This way the pumps create an osmotic

pressure difference, which drives more ³He from the concentrated to the diluted phase in the mixing chamber. After that the pumps compress the ³He to a few hundred milibar and feed it back into the beginning of the cycle.

6 Adiabatic demagnetization

6.1 Adiabatic demagnetization of paramagnetic salts

Before the development of helium dilution refrigeration, adiabatic demagnetization of paramagnetic salts was used in this temperature range. It was suggested in 1926 when P. Debye and W. F. Giauque independently suggested using the magnetic disorder entropy of electronic magnetic moments in paramagnetic salts for cooling. The first successful experiment was conducted in 1933, when a group at Berkeley first managed to reach a temperature of 0.53 K, followed in a few months by the group in Leiden who cooled their sample down to 0.27 K.

For this cooling method one needs a sample with paramagnetic ions, which are not too close to each other - mostly salts with a lot of bound water are used so the ions are spaced about 10 Å. That way the interactions among them have energies ϵ_m that are much smaller than k_BT at most relevant temperatures so we can assume that their magnetic moments are independent.

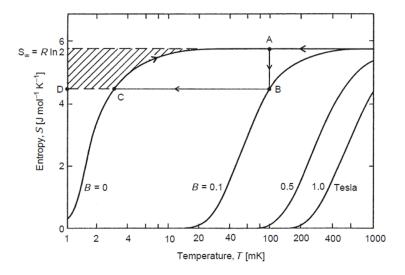


Figure 5: Entropy of the paramagnetic salt CMN as a function of temperature at various magnetic fields [2].

Therefore their total angular momentum J can have 2J+1 different orientations with respect to the magnetic field. The contribution to the entropy per mole s is consequently

$$s = R\log(2J + 1), (6.1.1)$$

where R is the gas constant. At low temperatures which are of interest in this seminar, the contribution is of the order of a joule per mole of refrigerant and is much larger than the contributions of the lattice and the conducting electrons which we will therefore neglect.

The cooling process is a so called "one-shot" technique and can be described in three steps. First we magnetize the sample, while keeping it at a constant temperature T_i by absorbing the heat in the precooling bath (from A to B in fig. 5). The released heat during this isothermal process is given by

$$Q(T_{\rm i}) = nT_{\rm i} \int_0^{B_{\rm i}} \left(\frac{\partial s}{\partial B}\right)_{T_{\rm i}} dB = nT_{\rm i} \int_0^{B_{\rm i}} \left(\frac{\partial M}{\partial T}\right)_B dB \tag{6.1.2}$$

The external magnetic field causes all angular momenta to align, lowering the entropy.

Isolating the sample from the precooling bath we then lower the magnetic field adiabatically (from B to C in fig. 5) so it can not exchange any heat with it's surroundings, therefore the angular momenta remain pointing in the same direction even when the external field is almost zero. This means their entropy is now very low, while there is almost no external field, meaning their temperature got much lower. Because the free magnetic entropy is just a function of the ratio of the magnetic energy to the thermal energy, and we know the entropy does not change during an adiabatic process we can conclude that the ratio between the external field and the temperature of the sample must remain constant.

$$S(B_{\rm i}/T_{\rm i}) = S(B_{\rm f}/T_{\rm f}) \quad \Rightarrow \quad \frac{T_{\rm f}}{B_{\rm f}} = \frac{T_{\rm i}}{B_{\rm i}}.$$
 (6.1.3)

We must be aware though, that $T_{\rm f}$ is just the temperature of the ions, and not that of the conducting electrons or the lattice. To cool those, the ions interact with the conducting electrons which in turn cool the lattice - therefore the final temperature is significantly higher than the initial temperature of the ions.

The cooling power of the salt after the demagnetization to $B_{\rm f}$ is given by

$$Q(B_{\rm f}) = n \int_{T_{\rm f}}^{\infty} T\left(\frac{\partial S}{\partial T}\right)_{B_{\rm f}} dT \tag{6.1.4}$$

and is smaller than the heat of magnetization, because it happens at constant entropy and a lower temperature. We can see this in the phase diagram in figure 5, where the energy removed from the sample is presented by the hatched area.

This method is fairly limited, because our initial assumption $\epsilon_{\rm m} \ll k_{\rm b}T$ only works at temperatures higher than 100 mK. Below this limit, the paramagnetic salt becomes either diamagnetic or ferromagnetic. Because the method doesn't enable continuous cooling and is limited to a relatively small range, it is almost not used any more and has been replaced by helium dilution refrigeration. The basic concept remains important for the understanding of nuclear adiabatic demagnetization, which enables reaching temperatures as low as $0.1\,\mathrm{nK}$

6.2 Nuclear adiabatic demagnetization

One of the most important differences is that the magnitude of magnetic moments of nuclei is approximately 1000 times smaller than that of the electronic configuration of paramagnetic salts. Therefore they have much lower interaction energies and are less prone to spontaneous self-alignment. On the other hand, this also means we need to precool the sample to much lower temperatures and use larger magnetic fields to align the sample in the first place. This method enables reaching temperatures in the picokelvin range.

We need to keep in mind that the final temperature of the nuclear spins is going to be much lower than the final temperature of our sample. As early as in 2000 achieving temperatures under 100 pK has been reported, but that was just the temperature of the nuclear spins. The lowest temperature a sample was cooled to by this method is 100 pK [5].

7 Ultra cold atoms

Cooling a sample to temperatures below the microkelvin range, becomes increasingly challenging. Another method to see effects that happen at lower temperatures are simulations, which can be performed with a newly discovered state of matter - a Bose-Einstein condensate (BEC). Because this is a completely different field, with more than enough material for a seminar on its own I will just cover the basic concepts, while presenting the experiment that is being set up at the Jožef Stefan Institute.

The system used to cool the atoms is a closed, ultra high vacuum system. The source of the atoms (in this case cesium) is an oven, which emits Cs atoms when heated. These atoms fly out in various directions, with speeds of about 240 m/s. We would like to eventually catch them in the trap, so we have to first drastically reduce their speeds and energies, which we achieve by laser cooling.

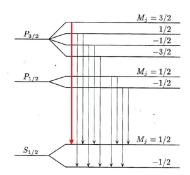


Figure 6: Hyperfine structure of an alkali atom, with a closed transition marked in red [7].

To understand this, we first have to remember the energy levels in a cesium atom. In this, so called hyperfine structure we can find transitions that are essentially closed. The (dipole) selection rules prevent the electron from decaying to a different state than it was excited from. An example of such a transition is marked with red on figure 6. Such a state can be used for cooling. To cool the atom, we shine a laser beam in the opposite direction of the beams of atoms. Photons with the right energies will be absorbed, thereby slowing the atom a little. Shortly after that, the atom emits a photon with the same frequency, but in a random direction. If we repeat this process often enough, the random "kicks" from the emitted photons will average out to zero, effectively inhibiting the forward motion of the atom.

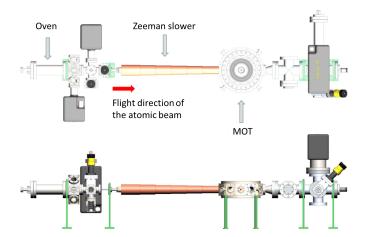


Figure 7: A schematic representation of the cold atom experiment being set up at the Jožef Stefan Institute. (by dr. Erik Zupanič)

Here we neglected an important factor - the Doppler effect which presents a problem, because suddenly our constant laser frequency can only be absorbed by a very narrow part of the velocity spectrum. This can be neutralized by a Zeeman slower. We already know, the energy difference between states in an atom can be changed if magnetic field is applied. The Zeeman slower is essentially just a coil, designed in a way that the produced external magnetic field B(x) satisfies the equation:

$$\omega + kv(x) = \omega_0 + m_l \frac{\mu_B}{\hbar} B(x)$$
 (7.0.1)

where ω is the frequency of the laser, v(x) the speed of the atom cloud as a function of distance x, ω_0 the laser frequency needed to get the atoms into the desired excited state and m_l the projection of the third component of spin on the axis of quantization [8].

This way, the energy shifts will be just right, that the atoms will "see" the right frequency laser light at all points along the way to the trap, absorbing the maximal amount of photons and slowing down to energies that equal temperatures of the order of microkelvins. These already quite cool atoms now enter the magneto optical trap (MOT), where a set of three perpendicular lasers, together with a magnetic field creates a harmonic potential in which the atoms are confined. Because we are still cooling atoms with photon absorption, there is a minimum temperature that can theoretically be reached this way - the temperature that is equivalent to the energy of one photon absorption. An atom of mass m, can therefore only be cooled to

$$T_{min} = \frac{\hbar^2 k^2}{mk_B}. (7.0.2)$$

For cesium this temperature is about 200 nK. It turns out that the temperatures achieved this way experimentally are approximately ten times larger.

Further cooling is only possible by evaporative cooling - slightly lowering the potential, so the atoms with the largest energies can escape, thereby lowering the cloud's average energy or temperature.

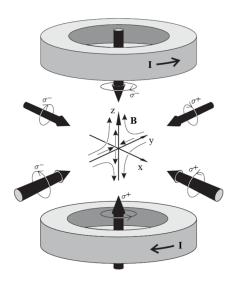


Figure 8: Schematic view of a magneto-optical trap (MOT) [7].

It might be worth mentioning that the temperature cannot be measured in some kind of standard way in this case. One of the ways of determining the temperature of the cloud is to suddenly release the trap. We can then calculate some kind of average temperature from how fast the cloud spreads after it transforms from a BEC to a normal phase, by using the Boltzmann statistics.

The first successful experiments where a BEC was created was conducted in 1995 by Eric Cornell and Carl Wiemann with Rb atoms, and later that year by Wolfgang Ketterle with Na atoms for which they received a Nobel prize in 2001. Since then there has been quite some development and temperatures under $0.5\,\mu\mathrm{K}$ are achieved on a regular basis nowadays. The current record has been set at about $100\,\mathrm{pK}$.

7.1 Cooling molecules

Many attempts have been made to cool molecules, because their electric dipoles enable simulating many interesting structures at extremely low temperatures. First they tried to cool

molecules with sympathetic cooling - letting the precooled molecules (for example with expansion) thermalize with atoms that have been cooled by trapping. Later they tried making molecules from BEC condensates. The first success in this field were Rb₂ molecules, that also created the first observed BEC. They could also observe and measure the interactions between the molecule and the BEC and ultra precisely measure the molecular binding energy. [9]

A group at Harvard managed to directly trap approximately 10^8 molecules cooled to $400 \,\mathrm{mK}$, which was the first successful experiment of this kind. In 2014 scientists at Yale University managed to cool SrF molecules with a trap similar to a MOT, making them form a BEC at a temperature of about $2.5 \,\mathrm{mK}$. [6]

7.2 NASA cold atom lab - the CAL experiment

NASA is currently working on a program where they will cool atoms in space, avoiding the pull of gravity. This reduces the need for trapping and enables cooling to temperatures approximately 1000 times lower than we have achieved on Earth. They have successfully cooled atoms in laboratory conditions to 200 nK with their device and intend to launch the experiment to space in December 2016 [10].

8 Conclusion

Low temperature physics is a well developed field with many exciting results in the last few years. The most common techniques have been presented, together with their advantages and limitations. Achieving lower and lower temperatures enables us to observe consequences of quantum effects on a macroscopic scale, discover new states of matter and do very precise measurements. Therefore this field provides key information for understanding of the world around us.

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