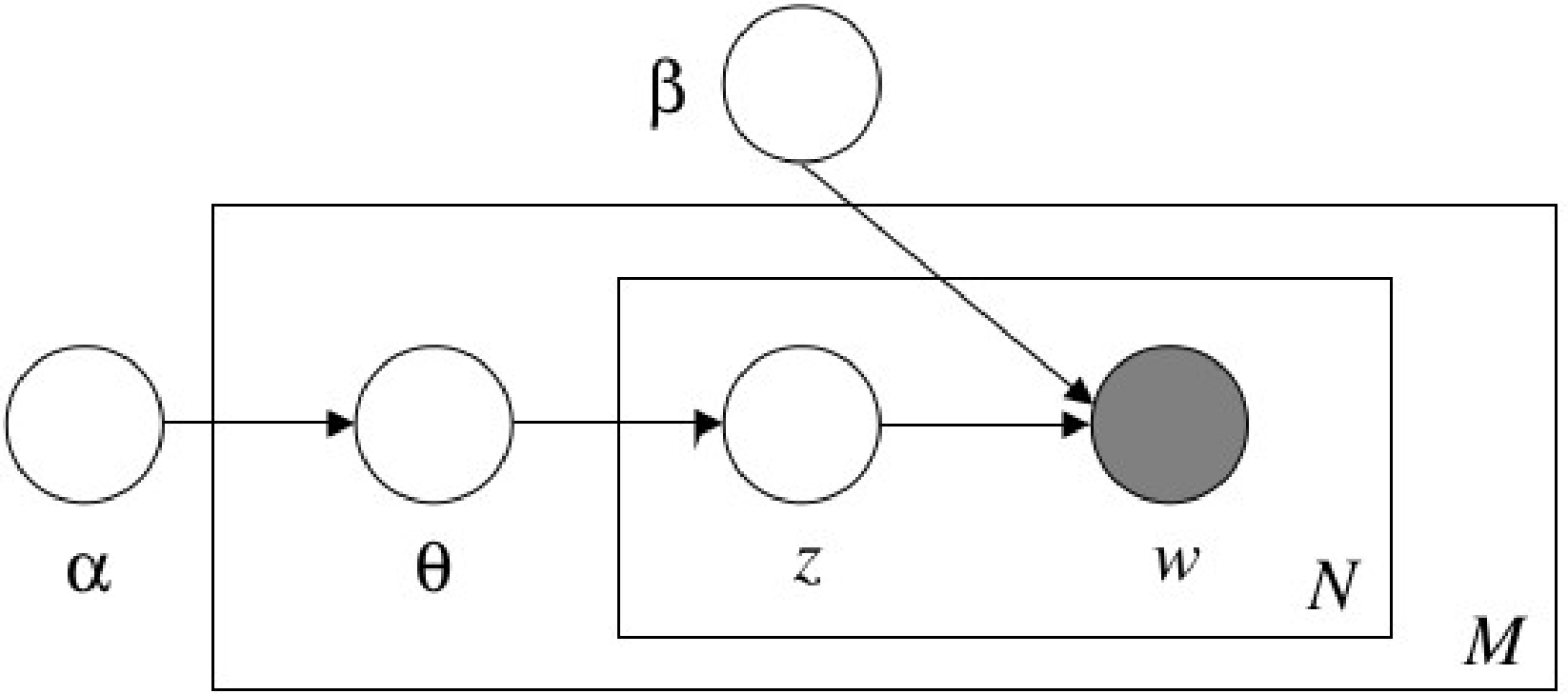
Implementation of LDA

LDA

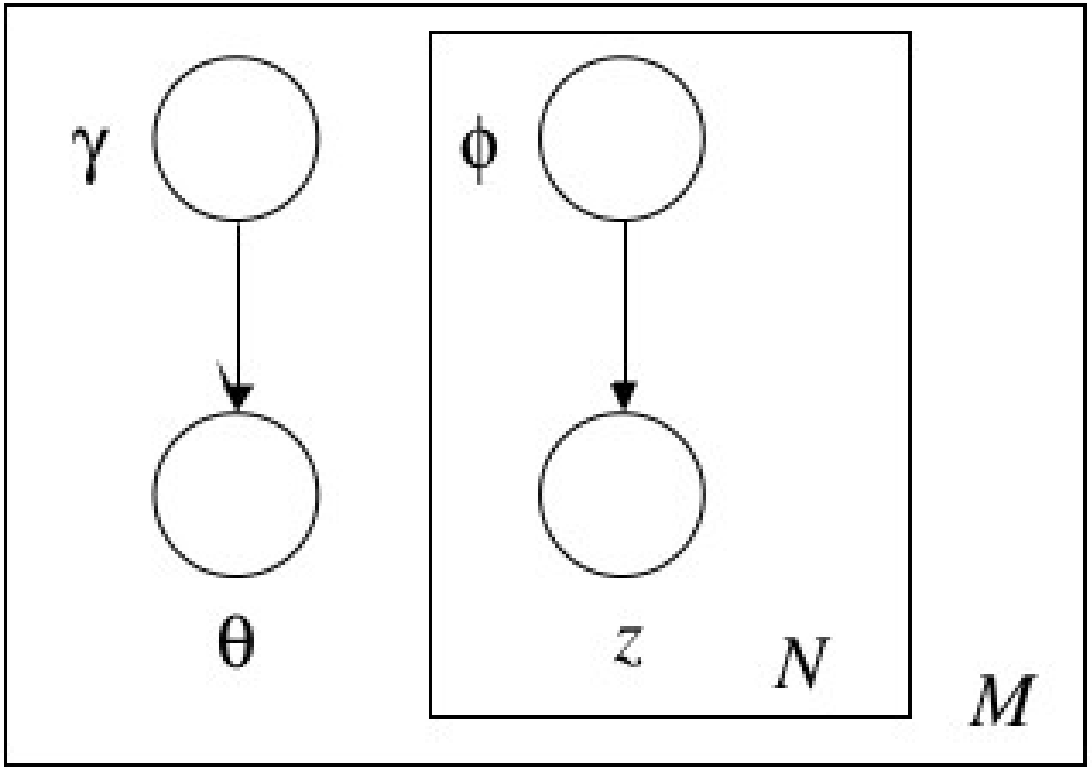


Intractable

Posterior distribution of hidden variables given the document

$$p(\theta, z|w, \alpha, \beta) = \frac{p(\theta, z, w|\alpha, \beta)}{p(w|\alpha, \beta)}$$

Variational distribution



$$q(\theta, z|\gamma, \phi) = q(\theta|\gamma) \prod_{n=1}^{N} q(z_n|\phi_n)$$

Optimization Problem

$$(\gamma', \phi') = \underset{(\gamma, \phi)}{argmin}\left(q(\theta, z|\gamma, \phi)||p(\theta, z|w, \alpha, \beta)\right)$$

Other perspective

$$p(z|w)=\frac{p(w|z)\,p(z)}{p(w)}$$

$$P(\mathbf{w}|\mathbf{z}) = \left(\frac{\Gamma(W\beta)}{\Gamma(\beta)^W}\right)^T \prod_{j=1}^{T} \frac{\Pi_w \Gamma(n_j^{(w)} + \beta)}{\Gamma(n_j^{(\cdot)} + W\beta)},$$

$$P(\mathbf{z}) = \left(\frac{\Gamma(T\alpha)}{\Gamma(\alpha)^T}\right)^D \prod_{d=1}^{D} \frac{\Pi_j \Gamma(n_j^{(d)} + \alpha)}{\Gamma(n_\cdot^{(d)} + T\alpha)},$$

Gibb's sampling

$$P(z_i = j | \mathbf{z}_{-i}, \mathbf{w}) \propto \frac{n_{-i,j}^{(w_i)} + \beta}{n_{-i,j}^{(\cdot)} + W\beta} \frac{n_{-i,j}^{(d_i)} + \alpha}{n_{-i,\cdot}^{(d_i)} + T\alpha},$$