Interleaver Design for LDPC-partial polar codes based on EXIT analysis

Xueting Zhang*, Yingzhuang Liu*, Shaoping Chen† and Chen Hu*
*Dept. of Electronic Information and Communications, Huazhong University of Science and Technology, Wuhan, China.
†Dept.of Electronics and Information Engineering, South-Central University For Nationalities, Wuhan, China.
* Email: {deppy124@hust, liuyz@mail.hust}.edu.cn, huchen1015@gmail.com
† Email: spchen@scuec.edu.cn.

***Abstract*—The concatenation of polar codes with LDPC codes is a promising approach to enhance the performance of polar codes at finite lengths while suppressing the error floor of LDPC codes. The polarized bit-channels of polar coding at finite lengths exhibit different qualities, i.e., some are almost noiseless while others are noisy. This implies that unbalanced protection is needed in the design of concatenating polar codes with other codes. To this end, a scheme of concatenatng polar codes with outer irregular LDPC codes for the protection of intermediate channels of polar codes is proposed in this paper. Between two constituent codes, a specifically designed interleaver is employed such that the overall error probability is minimized. The extrinsic information transfer (EXIT) analysis and a low-complexity heuristic method are proposed for the interleaver design. The soft cancellation (SCAN) and belief propagation(BP) algorithms are employed for the decoding of concatenated codes. Simulations are provided to validate its advantages in terms of excellent error correction capability and low decoding complexity.**

***Index Terms*—Concatenated codes, Polar codes, Irregular LDPC codes, Interleaver design, SCAN decoding**

I. Introduction

Polar codes, invented by Arikan [1], are the codes proved theoretically to achieve the capacity of binary-input discrete memoryless channels (B-DMCs). In addition, polar codes are equipped with low complexity algorithms for encoding and decoding. Thus, polar codes have been chosen by the 3rd Generation Partnership Project(3GPP) as the enhanced mobile broadband(eMBB) control channel coding, which shows great potential in wireless communication systems [2], [3]. However, the performance of finite-length polar codes cannot rival those of LDPC codes at the same length, due to the bit-error propagation problem of successive cancellation (SC) decoding and partial channel polarization. To further improve the performance of SC decoding, BP and SCAN decoding were proposed at the cost of a tolerably higher complexity [4], [5]. Besides, concatenating polar codes with other good codes, e.g., LDPC codes, provides a favourable approach to improve the performance of finite-length polar codes. Many schemes of polar and LDPC concatenated codes have been proposed, where advantages of both codes can be exploited, e.g., low-complexity decoding and capacity-approaching capability, and their weaknesses can be mitigated, i.e., the error floor of LDPC codes and relatively poor performance of finite-length polar codes. In [6], [7], serial LDPC-polar codes were introduced to lower the error floor of LDPC codes for optical transport network, which have also been proved to be effective in secure coding schemes. In [8], C.Yu et al. compared polar-LDPC codes with RS-LDPC codes and proved its technical feasibility in 5G or ultra-high-speed wireless communications [9]. A novel construction of LDPC-partial polar codes was proposed in [10], where short LDPC codes were employed to enhance the reliability of intermediate bit-channels in polar codes such that a better performance could be achieved at the cost of affordable rate loss. Moreover, Abbas et al. proposed a strategy for selecting information bit-channels based on small leaf set size to improve the performance of LDPC-partial polar codes [11]. Instead of protecting partial bits in polar codes with short-length LDPC codes, Liu et al. proposed a scheme to protect polar codewords with long-length LDPC codes and presented a refined hardware implementation, which showed a significant improvement [12].

Inspired by the good performance of LDPC-partial polar codes, an irregular LDPC-polar concatenated code with a specially designed interleaver is introduced in this paper. As a crucial element in the concatenated coding scheme, the interleaver plays an important role in the design of concatenated codes. In our scheme, the interleaver is designed to balance the information exchanged between constituent codes based on EXIT analysis and degree distributions. For EXIT-based information flow arrangement, LDPC encoded bits carrying more information are transmitted over worse subchannels in polar coding and those carrying less information pass through better subchannels. Although EXIT analysis provides relatively accurate estimates of the information that each bit conveys, it introduces a computational burden. To simplify the design, a simple yet accurate enough heuristic design rule, called degree-based analysis, is also proposed. The LDPC encoded bits with higher degrees, corresponding to the bits with more information in the EXIT-based method, pass through worse subchannels in polar coding and vice versa. In this way, the information exchanged between polar codes and LDPC codes can be balanced to boost the error correction ability of the concatenated code.

The rest of the paper is organized as follows. Section II introduces the basic theory about polar codes and LDPC codes,

978-1-5386-5005-9/18/$31.00 ©2018 IEEE

as well as SCAN decoding. The proposed irregular LDPC-partial polar code is presented in Section III, where the EXIT-based and degree-based analysis are introduced for interleaver design. Section IV presents simulation results, and finally, conclusions are drawn in Section V.

II. Preliminary

A. Polar codes

The polar code is derived from a phenomenon called "channel polarization" discovered by Arikan [1]. The polarized subchannels are obtained by performing a series of combining and splitting operations on N independent and identical B-DMCs. For N to be large enough, the polarized subchannels tend to approach either noise-free (good channels) or purely noisy (bad channels). By transmitting the information bits over the good channels while transmitting frozen bits over bad channels, polar codes can achieve the capacity of the B-DMCs. The main task of constructing polar codes is to select the reliable subchannels for data transmission, by computing their Bhattacharyya parameters [13] or using the Gaussian Approximation [14]. As a soft-output version of SC decoding, the high-level architecture of SCAN decoding is basically identical with that of SC decoding. The main difference is the updated process of messages from the information vector to the received vector in the Tanner graph of polar codes. For SC decoding, the update of information-to-received messages is simple additions and transmissions of hard decisions of nodes, while the computation of soft log-likelihood ratios(LLR) is required in the updated process for SCAN decoding. The introduced soft LLRs in SCAN decoding significantly improve the error correction ability of SC decoding with tolerable complexity [4].

B. LDPC codes

The LDPC code is a linear block code characterized by a sparse parity-check matrix. According to their degree distributions, LDPC codes can be divided into two groups: regular and irregular codes. The regular LDPC codes have uniform degrees for all variable nodes(v-nodes) and check nodes(c-nodes), while degrees vary in irregular codes, which could be specified by the following polynomials of degree distributions:

$$\lambda(x) = \sum_{i\geq 2}^{d_v} \lambda_i x^{i-1}, \rho(x) = \sum_{i\geq 2}^{d_c} \rho_i x^{i-1}$$

where λ_i and ρ_i denote the fraction of edges connected to v-nodes and c-nodes of degree i, respectively. It is universally acknowledged that irregular codes have a better performance than regular ones. Despite of its capacity-approaching performance and low-complexity iterative decoding, LDPC codes suffer from the error floor problem, which could be a hurdle in the application where the extremely low error rate is required. Error floor is mainly caused by trapping sets in the high signal-to-noise (SNR) regime and can be mitigated with various schemes, e. g., construction of high-girth codes [15] or multi-stage BP decoding [16]. Also, concatenation of LDPC code with other linear codes provides an effective and practical way to lower the error floor of LDPC codes [17].

III. The Proposed Scheme

In this section, the system model for concatenating polar codes with irregular LDPC codes based on LDPC-partial polar codes is introduced. Then the interleaver design is addressed.

A. System Model

The proposed scheme based on LDPC-partial polar codes is illustrated in Fig.1. At the transmitter, the information sequence $\mathbf{u}$ is divided into two parts $(\mathbf{u}_l, \mathbf{u}_p)$. Firstly, $\mathbf{u}_l$ is encoded using LDPC encoder. Then the obtained LDPC codeword $\mathbf{x}_l$ passes through the designed interleaver, which allocates one subchannel for each bit in $\mathbf{x}_l$. Finally, the interleaved LDPC codeword $\mathbf{x}_l^{'}$ is combined with the rest information bits $\mathbf{u}_p$ to obtain the polar codeword $\mathbf{x}$ through the polar encoder.

Note that the interleaved LDPC codeword $\mathbf{x}_l^{'}$ transmits through intermediate subchannels in the polar coding. The intermediate subchannels were first introduced in [10] and denoted relatively less reliable subchannels for transmission of non-frozen bits, corresponding to those nodes with less information or higher Bhattacharyya parameters in the information set(intermediate bits for short). To provide reliable protection of intermediate bits, an irregular LDPC code with a relatively large minimum distance is preferred in the design of LDPC-polar constituent codes. In this paper, the irregular LDPC codes are constructed by progressive edge-growth(PEG) methods with recommended degree distributions in [18]. Other algorithms to construct short-length LDPC codes with relatively large minimum distance may also be favorable to the design of concatenated codes, which can be further explored.

The decoding of irregular LDPC-partial polar codes is briefly described as follows.

1) At the receiver, the received codeword $\mathbf{y}$ is firstly decoded by SCAN algorithm;
2) If any intermediate bit is decoded, pause the SCAN decoding and output the soft decoded LLR $L_p(\widehat{\mathbf{u}})$;

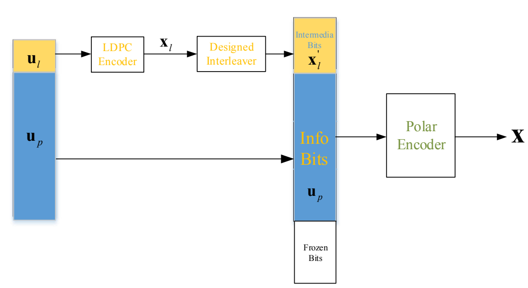

Fig. 1. The construction of irregular LDPC-partial polar code

3) According to the indices of intermediate channels, the LLR $L_l(\widehat{\widehat{\mathbf{u}}})$ is extracted from $L_p(\widehat{\widehat{\mathbf{u}}})$ and then reordered by the deinterleaver for LDPC decoding;
4) After LDPC decoding for I_l iterations and interleaving, the updated LLR $L_l(\widehat{\mathbf{u}})$ is inputted into the polar decoder as extrinsic messages of intermediate bits in the information vector;
5) Go to step 2 and continue SCAN decoding algorithm until all the information bits of polar codes are decoded. When the polar codes is decoded completely, then a cycle of iterations ends; If the maximum number of cycles is reached, stop and make decisions $\widehat{\mathbf{u}}$ according to the output of SCAN decoding $L_p(\widehat{\widehat{\mathbf{u}}})$.

Note that only a few number of iterations I_l is needed to control the computational complexity and the extrinsic influence of LDPC codes, since LDPC decoding proceeds whenever an intermediate bit is decoded.

B. Interleaver Design

In the proposed scheme, the subchannels of polar codes have varying reliabilities while the information that each bit of irregular LDPC codes conveys is different. In the interleaver design, we may take advantage of these features such that the proposed scheme can achieve good error correction performance.

1) EXIT-based design: To counteract the performance of inferior intermediate nodes in polar coding, the interleaver is designed according to the rule that the LDPC encoded bits carrying more information are assigned to worse intermediate subchannels for polar encoding. The information that each LDPC encoded bit conveys can be estimated by computing EXIT functions for each node during iterations of decoding [19][20]. The detailed process is introduced in the following.

Assume that the AWGN channel has noise variance δ^2 and an all-zero codeword is transmitted. The received LLR message L_y conditioned on transmitted bit x is a Gaussian variable with mean $\mu_y = \frac{2}{\delta^2}$ and variance ${\delta_y}^2 = \frac{4}{\delta^2}$. Denote $J(\delta_y)$ as the mutual information $I(X; L_y)$ between x and L_y , we have

$$\begin{aligned} J(\delta_y) &= I(X; L_y) \\ &= 1 - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi{\delta_y}^2}} e^{-\frac{(l-{\delta_y}^2/2)^2}{2{\delta_y}^2}} \log_2(1+e^{-l})dl \end{aligned} \tag{1}$$

$J(\delta_y)$ and its inverse function $J^{-1}(I)$ can be simplified as (2) and (3) [19].

According to EXIT functions for irregular LDPC code, the mutual information of individual nodes can be estimated as follows [20].

Suppose that v-node m has degree d_v and denote μ as the set of the indices of its neighboring check nodes ($|\mu| = d_v$). The extrinsic information function of v-node m propagated to c-node n at the l-th iteration is given by

$$I^{(l)}_{E_{v_m->c_n}} = J(J^{-1}(I_y) + \sum_{\mu:j\neq n} J^{-1}(I^{(l-1)}_{A_{v_j}})) \tag{4}$$

where $I^{(l-1)}_{A_{v_j}}$ is the extrinsic information function of its connected c-node at the previous iteration and I_y denotes the received information from the channel.

Suppose that c-node n has degree d_c and denote γ as the set of the indices of its neighboring v-nodes ($|\gamma| = d_c$). The extrinsic information function of the Gaussian output message of c-node n propagated to v-node m at the l-th iteration is given by

$$I^{(l)}_{E_{c_n->v_m}} = \frac{1}{\ln 2}\sum_{i=1}^{\infty}\frac{1}{(2i-1)(2i)}\prod_{\gamma:j\neq m}[\Phi_i(J^{-1}(I^{(l-1)}_{A_{c_j}}))] \tag{5}$$

where $I^{(l-1)}_{A_{c_j}}$ is the extrinsic information function of its connected v-node at the previous iteration and $\Phi_i(\delta)$ is given by

$$\Phi_i(\delta) = \int_{-1}^{+1} \frac{2t^{2i}}{(1-t^2)\sqrt{2\pi\delta^2}} e^{-\frac{(\ln\frac{1+t}{1-t}-\delta^2/2)^2}{2\delta^2}} dt \tag{6}$$

Note that in EXIT functions for LDPC codes, $I_{A_c} = I_{E_v}, I_{E_c} = I_{A_v}$. After certain iterations, the information an individual node conveys could be estimated as the mutual information of the node from (4) and (5) [20]. Those LDPC encoded bits conveying more information will be connected to worse intermediate subchannels so that unfavourable performance of inferior intermediate nodes can be compensated.

2) Degree-based design: The EXIT-based design is computationally intensive, so we propose a heuristic design rule, which has low complexity yet not causing much performance loss. The idea behind the heuristic design is simple and intuitive: the LDPC encoded bits with higher degrees are assigned to worse intermediate subchannels of polar codes.

The v-nodes of the LDPC codeword are sorted by their degrees from the highest to the lowest. Then those v-nodes with the same degree are sorted by the degree of their neighbouring c-nodes in an ascending manner. In short, the LDPC encoded bits are sorted by their degrees and degrees of their neighbouring c-nodes. For example, in Fig.2, where three v-nodes $\{v_1, v_2, v_3\}$ and their partial tanner graphs are shown, v_1 and v_3 are sorted ahead of v_2 due to their higher degrees. For v_1 and v_3 have the same degree, they are sorted by the degree of neighboring c-nodes. The sorted result is $\{v_3, v_1, v_2\}$. Then v-nodes with higher degrees, amounting to the nodes with more information in EXIT-based design, are transmitted through worse intermediate subchannels in the polar coding.

In conclusion, the LDPC encoded bits with more information or higher degrees are assigned to inferior intermediate subchannels in polar coding at the transmitter. At the receiver, those favorable LDPC nodes will provide more reliable LLRs to worse bit-channels in polar decoding, and vice versa. This implies that the unreliable information in both decoders can be offset in the information exchange during the turbo-like decoding of the concatenated codes. In this way, a better error correction performance can be achieved for the proposed scheme.

$$J(\delta) \approx \begin{cases} -0.0421061\delta^3 + 0.209252\delta^2 - 0.00640081\delta & 0 < \delta < 1.6363 \\ 1 - e^{0.00181491\delta^3 - 0.142675\delta^2 - 0.0822054\delta + 0.0549608} & 1.6363 < \delta < 10 \\ 1 & \delta \geq 10 \end{cases} \tag{2}$$

$$J^{-1}(I) \approx \begin{cases} 1.09542I^2 + 0.214217I + 2.33727I^{1/2} & 0 \leq I < 0.3646 \\ -0.706692\ln[0.386013(1-I)] + 1.75017I & 0.3646 < I < 1 \end{cases} \tag{3}$$

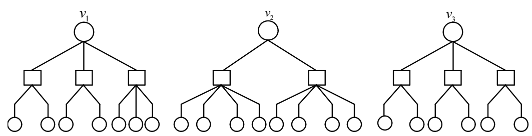

Fig. 2. Partial tanner graphs of three variable nodes$\{v_1, v_2, v_3\}$

IV. Performance analysis

In this section, the proposed scheme is compared with some existing interleavers and LDPC-polar concatenated codes. The following simulations are provided to validate its performance. The additive white Gaussian noise (AWGN) channel is assumed and the algorithm for constructing polar codes is based on Bhattacharyya parameters to get the indices of good and intermediate subchannels. SCAN decoder and BP decoder are applied for polar and LDPC decoding, respectively.

A. Comparison with other interleavers

We first verify the performance effect of the proposed interleaver design, which is compared with common interleavers, including a random interleaver, a block interleaver, and a interleaver in the 3GPP standard. For the fairness of comparison, (4096,2663) polar code is concatenated with (155,62) irregular LDPC code of maximum v-node degree $d_{v_{\max}} = 9$. Thus, the length and rate of the concatenated codes are 4096 and 0.63. The number of iterations in BP decoding for LDPC codes is set to be $I_l = 1$ and the maximum number of cycles is 2.

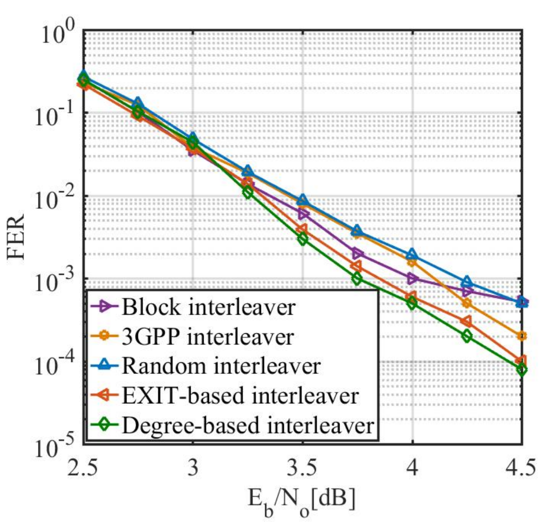

Fig. 3. The comparison of different interleavers

As shown in Fig.3, the interleaver design plays a significant role in lowering the error floor and enhancing the performance of the concatenated codes. Due to the trapping sets, the performance curve becomes gradually flattened as SNR increases for the block interleaver, showing its weakness in mitigating the effects of trapping sets, while the error floor does not appear after 3.75dB for other interleavers. In addition, the proposed EXIT-based interleaver performs better than random and 3GPP interleavers when SNR>3.25dB and is surpassed by the degree-based interleaver, e.g., about 0.1dB gain at FER of 10^{-3}. This suggests that carefully designed interleavers are required in the construction of irregular LDPC-polar concatenated nodes such that the error floor can be efficiently lowered. Considering that the performance gap is close and EXIT-based analysis will introduce a heavy computational burden, the degree-based interleaver achieves a good tradeoff between performance and complexity.

B. Comparison with other concatenated codes

The frame error rate (FER) performance comparison of the proposed scheme with exsiting LDPC-polar concatenated schemes is shown in Fig.4, including (1) a serial LDPC-polar in [6], the concatenation of (1024,682) polar code with (2048,1024) LDPC code; (2) a LDPC-partial polar code based on leafset (leafset code for short) in [11], the concatenation of (2048,717) polar code with (78,39) LDPC code; (3) a merged BP decoding of LDPC-polar codes in [12], the concatenation of (1024,682) polar code with (2048,1024) LDPC code. The SCAN decoding of polar code with the same length 2048 and rate 1/3 is added. In our proposed concatenated codes, (2048,717) polar codes are concatenated with (78,39) irregular LDPC codes, constructed according to the following degree distributions: $\lambda(x) = 0.3419x + 0.2564x^2 + 0.1196x^3 + 0.2821x^5$ and $\rho(x) = 0.0427x^4 + 0.8974x^5 + 0.0599x^6$. The maximum number of iterations in BP decoding for LDPC codes is set to be $I_l = 1$ and the maximum number of cycles is 2 for our proposed algorithm. For other concatenated codes, BP decoding are employed for the decoding of polar code and LDPC codes and the maximum number of iterations is set as 60.

It is observed that the proposed irregular LDPC-partial polar code with specially designed interleavers outper-

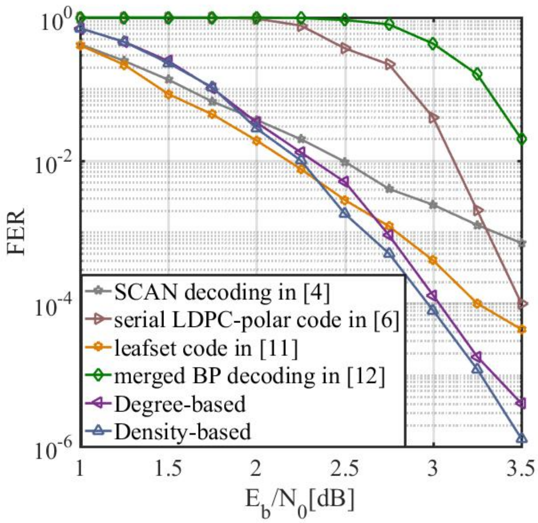

Fig. 4. The FER performance comparison

forms other concatenated codes when SNR is relatively high(SNR>2.5dB). The serially concatenated code performs poorly in this experiment, due to the lack of information exchanged between constituent codes. Even though information-exchange exists in the merged BP decoding, the inaccuracy of information exchanged degrades the performance of the concatenated code. In the low SNR regime (SNR<2.5dB), the leafset code performs better than others, while it is surpassed by our proposed concatenated codes in the relatively high SNR regime (SNR>2.5dB), e.g. about 0.2dB gain at FER of 10^{-4}. Considering that the leafset code has gradually slow performance improvement, the proposed schemes may achieve the best performance tradeoff.

V. Conclusion

The construction of irregular LDPC-partial polar concatenated codes with a specifically designed interleaver is presented in this paper. The irregularity of LDPC codes makes more room for concatenation design and provides better-unbalanced protection capability, enabling these proposed schemes to capture the capacity-achieving property of polar codes and lower the error floor of LDPC codes. A carefully designed interleaver is inserted between polar codes and irregular LDPC codes by manipulating the information flow exchanged between constituent codes. The EXIT analysis is employed for the optimal interleaver design. To reduce the computationally intensive calculation of EXIT analysis, a low-complexity degree-based method is proposed for the interleaver design. It achieves a good tradeoff between performance and complexity. Simulation results are provided to validate the advantages of the proposed scheme.

Acknowledgment

This work was supported by The National Natural Science Fund under Grant 61231007 and 61571467.

References

[1] E.Arikan, “Channel polarization: a method for constructing capacity achieving codes for symmetric binary-input memoryless channels,” IEEE Transactions on Information Theory, vol. 55, no. 7, pp. 3051-3073, July 2009.

[2] 3rd Generation Partnership Project (3GPP), “Technical specification group radio access network,” 3GPP TS 38.212 V.15.0.0, 2017.

[3] X. Ge, B. Yang, J. Ye, G. Mao, C.-X. Wang and T. Han, “Spatial Spectrum and Energy Efficiency of Random Cellular Networks,” IEEE Transactions on Communications, vol. 63, no. 3, pp. 1019 - 1030, March 2015.

[4] U. U. Fayyaz and J. R. Barry, “Low-Complexity Soft-Output Decoding of Polar Codes,” IEEE Journal on Selected Areas in Communications, vol. 32, no. 5, pp. 958-966, May 2014.

[5] E.Arikan, “Polar codes: A pipelined implementation,” In Proc. 4th International Symposium on Broad. Commun. ISBC 2010.

[6] A.Eslami, H. Pishro-Nik, “On finite-length performance of polar codes: stopping sets, error floor, and concatenated design,” IEEE Transactions on Communications, vol.61, no.3, pp. 919-929, Mar 2013.

[7] Y. Zhang, A. Liu, C. Gong, G. Yang and S. Yang, “Polar-LDPC Concatenated Coding for the AWGN Wiretap Channel,” IEEE Communications Letters, vol. 18, no. 10, pp. 1683-1686, Oct. 2014.

[8] C. Yu, Z. H. Lin, T. W. Hsu, M. W. Chen and Y. A. Chen, “Lower bit-error-rate polar-LDPC concatenated coding for wireless communication systems,” in IEEE 6th Global Conference on Consumer Electronics (GCCE), Nagoya, 2017, pp. 1-2.

[9] X. Ge, K. Huang, C.-X. Wang, X. Hong, and X. Yang, “Capacity analysis of a multi-cell multi-antenna cooperative cellular network with co-channel interference, IEEE Transactions on Wireless Communications, vol. 10, no. 10, pp. 3298-3309, Oct. 2011.

[10] J. Guo, M. Qin, A. Guillen iFabregas, et al., “Enhanced belief propagation decoding of polar codes through concatenation,” in Proc. IEEE International Symposium on Information Theory (ISIT), 2014, pp. 2987-2991.

[11] S. M. Abbas, Y. Fan, J. Chen and C. Y. Tsui, “Concatenated LDPC-polar codes decoding through belief propagation,” in IEEE Int. Symp. on Circuits and Systems (ISCAS), Baltimore, MD, 2017, pp. 1-4.

[12] J. Liu, S. Jing, X. You and C. Zhang, “A merged BP decoding algorithm for polar-LDPC concatenated codes,” in International Conference on Digital Signal Processing (DSP), London, 2017, pp. 1-5.

[13] Ido Tal, “How to Construct Polar Codes,” IEEE Transactions on Information Theory, vol. 59, no. 10, pp. 6562-6582, Oct 2013.

[14] Peter Trifonov, “Efficient Design and Decoding of Polar Codes,” IEEE Transactions on Communications, vol. 60, no. 11, pp. 1-7, Nov 2012.

[15] Y. Wang, S. C. Draper and J. S. Yedidia, “Hierarchical and High-Girth QC LDPC Codes,” IEEE Transactions on Information Theory, vol. 59, no. 7, pp. 4553-4583, July 2013.

[16] X. Zhang and S. Chen, “A Two-Stage Decoding Algorithm to Lower the Error-Floors for LDPC Codes,” IEEE Communications Letters, vol. 19, no. 4, pp. 517-520, April 2015.

[17] Sang Hoon Lee, Jia Ae Seok and Eon Kyeong Joo, “Serial concatenation of LDPC and turbo code for the next generation mobile communications,” in Second IFIP International Conference on Wireless and Optical Communications Networks, 2005, pp. 425-427.

[18] T. J. Richardson, M. A. Shokrollahi and R. L. Urbanke, “Design of capacity-approaching irregular low-density parity-check codes,” IEEE Transactions on Information Theory, vol. 47, no. 2, pp. 619-637, Feb 2001.

[19] S. ten Brink, G. Kramer, A. Ashikhmin, “Design of Low-Density Parity-Check Codes for Modulation and Detection,” IEEE Transactions on Communications, vol.52, no. 4, pp. 670-678, 2004.

[20] E. Sharon, A. Ashikhmin, S. Litsyn, “Analysis of Low-Density Parity-Check Codes Based on EXIT Functions,” IEEE Transactions on Communications, vol. 54, no. 8, pp. 1407-1414, 2006.