Deep Convolutional Dictionary Learning

Nikola Janjušević†, Amir Khalilian†, Yao Wang†

†New York University, Tandon School of Engineering

I. Introduction

Convolutional Dictionary Learning:

- Find optimal and data-driven basis for sparse representation
- Convolutional structured dictionary

$$\min_{z,d_j} \frac{1}{2}\| \sum_j d_j \star z_j - y\|_2^2 + \lambda \|z\|_1 , \quad \forall y \in \mathcal{D}$$

- **Solution via iterating between:**
  - Sparse Coding
  - Dictionary Update
- **Slow!**
  - Sparse Code every signal in dataset
  - Dictionary changes every step

- $d_j \rightarrow$ dictionary atom
- $z_j \rightarrow$ sparse code
- $y \rightarrow$ noisy image

Use learnt convolutional sparse coding + dictionary learning to speed-up!

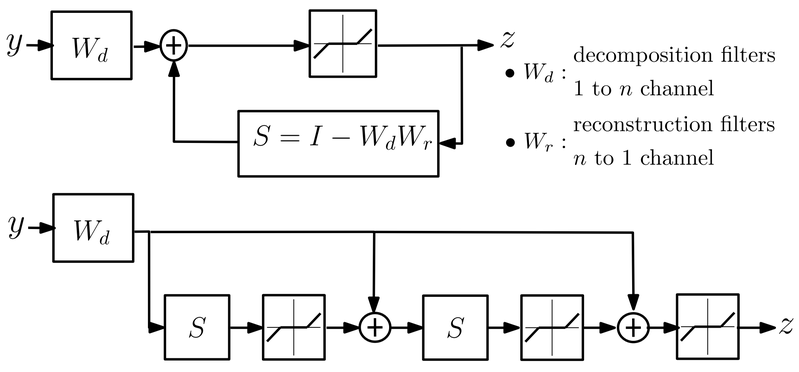
II. Learnt Sparse Coding

ISTA and LISTA

- Each step involves soft-thresholding the residual:

$$z^{k+1} = \mathcal{S}_\lambda \left(z^k - W_d \left(W_r z^k - y \right) \right)$$

- Use deep learning to to accelerate sparse-coding:

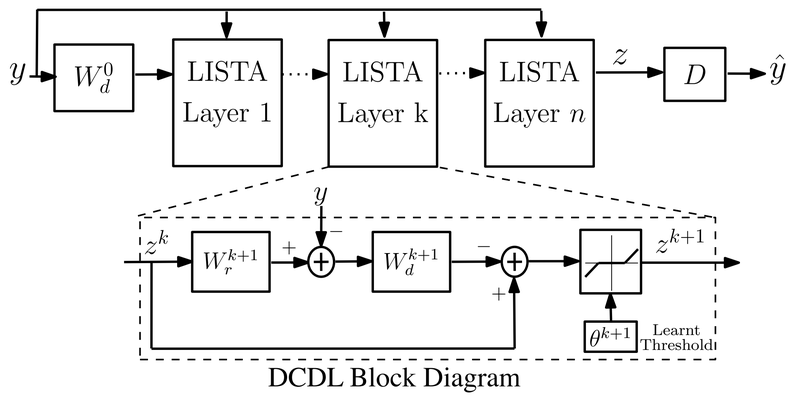


ISTA and LISTA Block Diagram

III. Deep Convolutional Dictionary Learning

LISTA + CDL

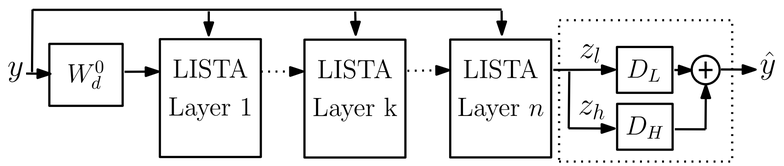
- Learnt sparse coding and dictionary learning trained end-to-end



DCDL Block Diagram

IV. Deep Convolutional Dictionary Learning with Filter Constraints

- More constraints on dictionary elements removes redundancy
  - Only one fixed low-pass channel
  - Other channels constrained to sum to 0 (not low-pass)
  - Norm constraint on dictionary to remove scale ambiguity



FCDCDL Block Diagram

- Update the filters (except for the fixed low-pass channel) and the LISTA parameters by gradient back-propagation.
- Enforce the filter constraint by projection onto constraint set.

$$\min_{z,d_j} \frac{1}{2}\| \sum_j d_j \star z_j - y\|_2^2 + \lambda \|z\|_1$$

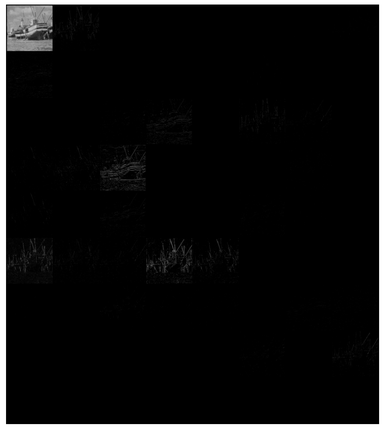
$$\text{subject to: } d_j^T 1 = 0 \text{ and } \|d_j\|_2^2 \leq 1, \ \forall j \text{ not low-pass}$$

Fixing a low-pass channel removes redundancy and allows for multi-resolution processing.

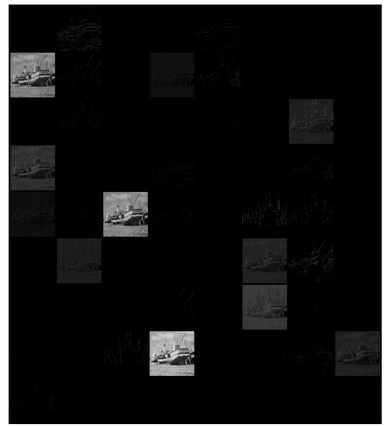
V. Experimental Results

FCDCDL vs. DCDL

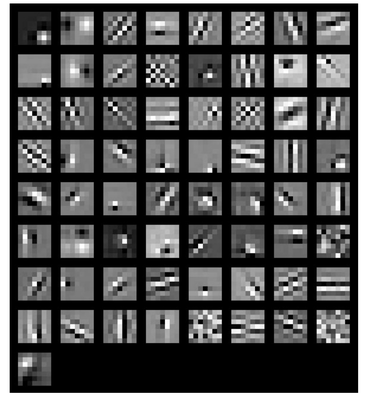
- Improved the sparsity of the codes
- Less redundant and more directional filters



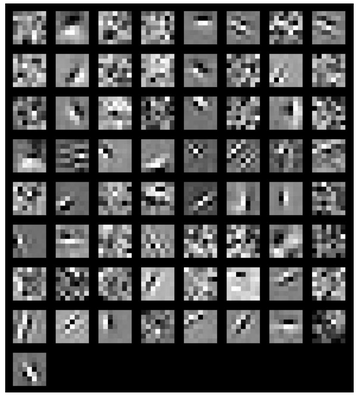
FCDCDL Sparse Codes



DCDL Sparse Codes



FCDCDL Learnt Filters



DCDL Learnt Filters

VI. Conclusion

- Considered end-to-end training of the sparse coding and convolutional dictionary representation.
- Improved the result by constraining the dictionary elements to have one fixed low-pass filter and remaining filters are high pass and have less than unit norm.
- Multi-layer and multi-resolution extensions of the proposed framework will be considered in future work.