Two's Complement

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A trick to represent negative numbers without an extra symbol.

Consider the following exercise to find (-4): $0-4$.

$$\begin{array}{rcccc} \cdots & 1 & 1 & 1 & \\ \cdots & \cancel{0}^{10} & \cancel{0}^{10} & \cancel{0}^{10} & \cancel{0}^{10} \\ - \cdots & 0 & 0 & 0 & 4 \\ \hline \cdots & 9 & 9 & 9 & 6 \end{array}$$

our row which indicates borrowing

So $-4 = (\cdots 96)$

Check:
$$\begin{array}{rcccc} \cdots & 1 & 1 & 1 & \\ \cdots & 9 & 9 & 9 & 6 \\ & & & & 4 \\ \hline \cdots & 0 & 0 & 0 & 0 \end{array}$$
✓

Now in binary:

What is $4_{10} = 0100_2$

and $-4_{10} = ?$

$$
\begin{array}{rr}
 & \cdots 11 \\
- & \cdots \cancel{0}^{10}\cancel{0}^{10}\cancel{0}^{10}00_2 \\
- & \cdots 00100_2 \\
\hline
 & \cdots 11100_2
\end{array}
$$

$$-4_{10} = (\cdots 1100_2)$$

[<u>check</u>:

$$
\begin{array}{rr}
 & 111 \\
 & \cdots 1100_2 \\
+ & 0100 \\
\hline
 & \cdots 00000
\end{array}
\quad \checkmark
$$

]

- Lets consider a truncated representation: for N bits, the MSB denotes MSB × infinite repeating 1s, ex.

$$(1100)_2 = \cdots 1100$$

So I can add things together.
with finite computation:

ex. $5 + (-4)$:

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 0101_2 \\ + \; \cdots 1100_2 \\ \hline \cdots\cdots 0001 \end{array}$$

$= 0001_2 \longrightarrow 5 + (-4) = 1$ ✓

Ok. So instead of subtracting from zero every time, lets consider that (in finite bits)

$$\begin{array}{r} 0000 \\ -\,0101 \\ \hline \end{array} \iff \begin{array}{r} 10000 \\ -\,0101 \\ \hline \end{array} \iff \begin{array}{r} 1111 + 1 \\ -\,0101 \\ \hline \end{array}$$

and note that theres two cases:

Ⓐ $\begin{array}{r} 1 \\ -\,0 \\ \hline 1 \end{array}$ Ⓑ $\begin{array}{r} 1 \\ -\,1 \\ \hline 0 \end{array}$ No carries involved!

↳ Going to add 1 at the end

see:

$$
\begin{array}{r}
1111 \\
-\ 0101 \\
\hline
1010 + 1 \\
\hline
\end{array}
$$

$-5 = (1011)_2$

Trick: subtracting from zero is equivalent to flip bits & add 1.

Ex. Find -6 in 4 bit 2's comp.

$6_{10} = (0110)_2$

$$
-6_{10} = \begin{array}{r}
\overset{}{1}\overset{}{0}\overset{1}{0}\overset{1}{1} \\
+\ \quad\quad \\
\hline
1010
\end{array}
$$

$= (1010)_2$

What about in reverse?

$$
6_{10} = -(-6_{10}) = \begin{array}{r}
0\overset{1}{1}\overset{1}{0}\overset{}{1} \\
+\ \quad\quad \\
\hline
0110
\end{array}
$$

✓

4 bit 2's Complement

	$S_3 S_2 S_1 S_0$
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-8?	1000
-7	1001
-6	1010
-5	1011
-4	1100
-3	1101
-2	1110
-1	1111

same

-1   +1   +1   +1

$-(1001)_2 = \begin{array}{r} 1 \\ +0110 \\ \hline 111 \end{array}$

1110 ← $-8+6$

1111 ← $-8+7$

⇒ weight factor representation.

$$\boxed{S_3 \times (-2^3) + S_2 \times 2^2 + S_1 \times 2^1 + S_0 \times 2^0}$$

In General: 2's complement

Is a different numbering system for different numbers of bits

MSB has a weight factor of -2^{N-1}, all other bits in position $n = 0, 1, \cdots, N-2$ have wf 2^n

For N bits, we'll have a range of $[-2^{N-1}, 2^{N-1}-1]$