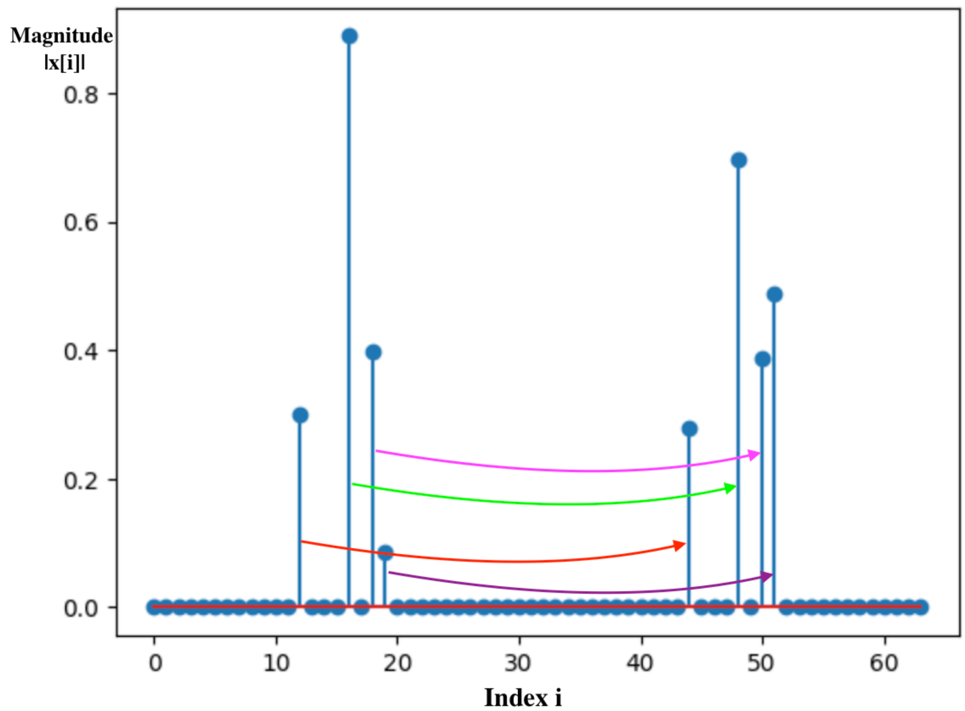
Compressed sensing of structured sparse signals



Description

In a certain application, we want to compressively sense a sparse signal $\boldsymbol{x} \in \mathbb{R}^N$, whose i^{th} entry is x_i. Here, we know that the support of the non-zero components in $\boldsymbol{x}$ exhibit a special structure. Specifically, if $x_i \neq 0$ for $i \leq N/2$, then $x_{i+N/2} \neq 0$. Equivalently, the support of the second half of $\boldsymbol{x}$ follows the same pattern as the support of its first half. The goal of this project is to exploit this known support structure together with sparsity to recover such signals from a compressed representation.

The vector of M CS measurements in our setup is given by

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{v},$$

where $\boldsymbol{A}$ is an $M \times N$ CS matrix and $\boldsymbol{v}$ is vector of white Gaussian noise samples. The entries of $\boldsymbol{A}$ are drawn from the standard Gaussian Distribution $\mathcal{N}(0,1)$ and then the colums are normalized to have unit ℓ_2 norm. In this project, we use $N = 64$.

Dataset

Please check the .ipynb notebook here that creates a dataset of such structured sparse signals.

Methodology and Evaluation

Conduct a brief literature study to orient yourself on this problem. Develop an algorithm to recover the structured sparse signal. Your method should exploit the support structure across two halves of $\boldsymbol{x}$ during reconstruction.

Compare your developed method with a standard CS-recovery counterpart that does not exploit the support structure given in this problem.

The methods must be evaluated on the "Project_Structured_Sparsity.npy" dataset using performance metrics such as reconstruction error and computation time. These metrics must be studied as a function of the number of measurements, i.e., M, and the signal-to-noise ratio.

We encourage you to think about new research questions around this topic, e.g., new optimization formulations, adaptation of greedy methods, choice of the measurement matrix. A good way to get started with this problem is to review existing literature on standard joint sparse recovery formulation [1] where the measurement model considered is

$$\boldsymbol{z_1} = \boldsymbol{B}\widetilde{\boldsymbol{x}}_1 + \boldsymbol{v}_1;\ \boldsymbol{z_2} = \boldsymbol{B}\widetilde{\boldsymbol{x}}_2 + \boldsymbol{v}_2.$$

Here, $\widetilde{\boldsymbol{x}}_1$ and $\widetilde{\boldsymbol{x}}_2$ are sparse vectors that are non-zero at the same indices.

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