Marketing Spend Optimisation

Nick Gavriil

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1 Objective Function

The objective function is the maximization of profit over the set of ad channels $i\epsilon[1, ..., N]$ as a function of the marketing spend c_i, with q_i being the acquired customers through channel i:

$$max_{c_i} \sum_{i=1}^{N} \pi_i(c_i) = max_{c_i} \sum_{i=1}^{N} q_i(c_i) * LTV_i - c_i$$

$$s.t. \sum_{i=1}^{N} c_i <= budget$$

with each $c_i \geq 0$

2 Acquisitions Demanded

We can represent diminishing returns through linear-log response curves. In that case:

$$q_i(c_i) = b_i * log(c_i)$$

3 Setting up the Lagrangian Equation

$$L = \sum_{i=1}^{N} b_i * log(c_i) * LTV_i - c_i + \lambda * (budget - \sum_{i=1}^{N} c_i)$$

4 First Order Conditions

For every channel the following condition applies:

$$\frac{LTV_i * b_i}{c_i} - 1 - \lambda = 0 \, (1)$$

And the condition for λ

$$\sum_{i=1}^{N} c_i = budget \, (2)$$

For 2 channels:

$$(1) \Rightarrow \frac{LTV_1 * b_1}{c_1} = \frac{LTV_2 * b_2}{c_2} \Rightarrow$$

$$(2) \Rightarrow c_2 = budget - c_1$$

Putting (2) into (1):

$$\frac{LTV_1 * b_1}{c_1} = \frac{LTV_2 * b_2}{budget - c_1} \Rightarrow$$

$$\frac{budget - c_1}{c_1} = \frac{LTV_2 * b_2}{LTV_1 * b_1} \Rightarrow$$

$$budget = c_1[1 + \frac{LTV_2 * b_2}{LTV_1 * b_1}] \Rightarrow$$

$$c_1 = \frac{budget}{1 + \frac{LTV_2 * b_2}{LTV_1 * b_1}}, c_2 = budget - c_1$$