

Marketing Spend Optimisation

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October 2024

1 Objective Function

The objective is the maximization of profit over the set of ad channels $i \in [1, \dots, N]$ as a function of the marketing spend c_i , the acquired customers q_i and the lifetime value LTV_i :

$$\begin{aligned} \max_{c_i} \sum_{i=1}^N \pi_i(c_i) &= \max_{c_i} \sum_{i=1}^N q_i(c_i) * LTV_i - c_i \\ \text{s.t. } \sum_{i=1}^N c_i &\leq \text{budget} \end{aligned}$$

with each $c_i \geq 0$

2 Acquisitions Demanded

We represent diminishing returns through linear-log response curves. In that case:

$$q_i(c_i) = b_i * \log(c_i)$$

3 Setting up the Lagrangian Equation

$$L = \sum_{i=1}^N b_i * \log(c_i) * LTV_i - c_i + \lambda * (\text{budget} - \sum_{i=1}^N c_i)$$

4 First Order Conditions

For every channel the following condition applies:

$$\frac{LTV_i * b_i}{c_i} - 1 - \lambda = 0 \quad (1)$$

And the condition for λ

$$\sum_{i=1}^N c_i = budget \quad (2)$$

We solve the system of equations for 2 channels only:

$$(1) \Rightarrow \frac{LTV_1 * b_1}{c_1} = \frac{LTV_2 * b_2}{c_2} \Rightarrow$$

$$(2) \Rightarrow c_2 = budget - c_1$$

Putting (2) into (1):

$$\frac{LTV_1 * b_1}{c_1} = \frac{LTV_2 * b_2}{budget - c_1} \Rightarrow$$

$$\frac{budget - c_1}{c_1} = \frac{LTV_2 * b_2}{LTV_1 * b_1} \Rightarrow$$

$$budget = c_1 \left[1 + \frac{LTV_2 * b_2}{LTV_1 * b_1} \right] \Rightarrow$$

$$c_1 = \frac{budget}{1 + \frac{LTV_2 * b_2}{LTV_1 * b_1}}, c_2 = budget - c_1$$