University of Thessaly

Neuro-Fuzzy Computing

ECE447

1st Problem Set

Alexandra Gianni ID: 3382

Nikos Stylianou ID: 2917

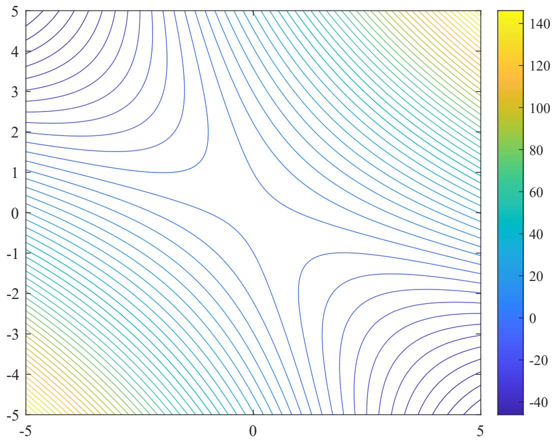
December 11, 2023

1 Problem 1

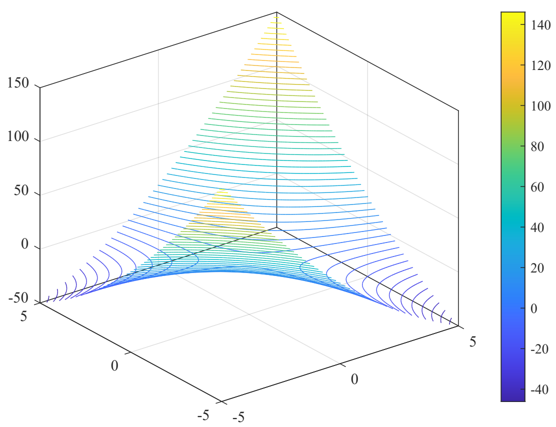
Contour lines of $f(x, y)$ are plotted with the following MATLAB code and are presented in figure 1.

```
function [Z] = plot_contour(start_num, end_num)

        x = linspace(start_num, end_num, 100);
        y = x;
        [X, Y] = meshgrid(x, y);
        Z = X.^2 + 4*X.*Y + Y.^2;
        contour(X, Y, Z, 40);
        xlabel('X');
        ylabel('Y');
end
```



(a) 2D Plot



(b) 3D Plot

Figure 1: Contour lines of $f(x, y)$

A general formula of a quadratic equation is $f(x, y) = ax^2 + 2bxy + cy^2$. Writing our formula in the previous form, we find that $a = 1,\ b = 2,\ c = 1$. Calculation of discriminant can help us calculate the location of function's local minimum/maximum.

$$
\begin{gathered}
D = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} = f_{xx} f_{yy} - f_{xy}^2 = 2 \times 2 - 4^2 = -12 < 0, \quad \text{where} \\
f_{xx} = \frac{\partial^2 f}{\partial x^2} = 2, \quad f_{yy} = \frac{\partial^2 f}{\partial y^2} = 2, \quad f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = 4 D = 2 \times 2 - 4^2 = -12 < 0.
\end{gathered} \tag{1}
$$

So, we only have to find the point where $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are equal to 0. Thus, this point will be a saddle point where gradients in each orthogonal direction are 0, but this point is not either a local minimum or maximum. Specifically:

$$
\begin{cases} \dfrac{\partial f}{\partial x} = 2x + 4y = 0 \\ \dfrac{\partial f}{\partial y} = 4x + 2y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \tag{2}
$$

Thus, the point $(x, y) = (0, 0)$ is the saddle point mentioned before for the function given and this can be justified using the plotted contour lines.

2 Problem 12

In order to evaluate the expression ”`not(A(x) OR B(x))`”, we first have to take a look in how fuzzy logic is different with binary logic at the operation level. In binary logic we have three basic operations: `AND(x,y)`, `OR(x,y)` and `NOT(x)`. But, in fuzzy logic, where a function can have a value in range of $[0...1]$, things are a bit different. Operation `AND(x,y)` of binary is equivalent with `MIN(x,y)` from the fuzzy logic, `OR(x,y)` with `MAX(x,y)` and `NOT(x)` with `1-x`.

We have to find the proper x for which the previous expression has the greater value. Firstly, we will calculate the expression and then find the proper x. To achieve this, we will have to break down our calculations in areas.

Starting with $x \leq 2$, ”$A(x)$ AND $B(x)$” is equal to ”$\max(A(x), B(x))$” $= 1$. So, using De Morgan's law, we have $max\,(A(x), B(x)) = not\,(A(x)\ or\ B(x)) = 0$. The exact same result is obtained at $x \geq 7$.

Things are a bit different in $2 \leq x \leq 7$. Function $A(x)$ starts to fall while $B(x)$ starts to rise. The point where they cross is valuable for defining the expression needed and can be found by solving the equation:

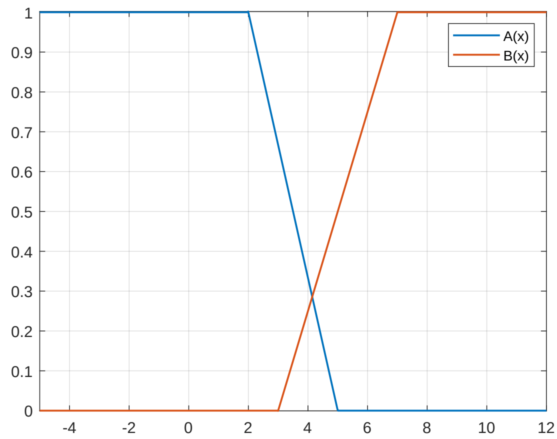


Figure 2: Plot of A(x), B(x)

$$A(x) = B(x) \Leftrightarrow 1 - \frac{x-2}{3} = \frac{x-3}{4} \Rightarrow x = \frac{29}{7}$$

So, for $2 \leq x \leq \frac{29}{7}$, $max\,(A(x), B(x)) = 1 - \dfrac{x-2}{3} = A(x)$, because in this region $A(x)$ is above $B(x)$.

Thus, $not\,(A(x)\ or\ B(x)) = \dfrac{x-2}{3}$.

Using the same logic, we find out that for $\frac{29}{7} \leq x \leq 7$, $not\,(A(x)\ or\ B(x)) = 1 - \dfrac{x-3}{4}$.

Therefore, the expression $f(x) = not\,(A(x)\ or\ B(x))$ is summarized in the following:

$$f(x) = \begin{cases} 0 & x \leq 2, \\ \dfrac{x-2}{3} & 2 \leq x \leq \frac{29}{7}, \\ 1 - \dfrac{x-3}{4} & \frac{29}{7} \leq x \leq 7, \\ 0 & x \geq 7 \end{cases} \tag{3}$$