University of Thessaly

Neuro-Fuzzy Computing

ECE447

1st Problem Set

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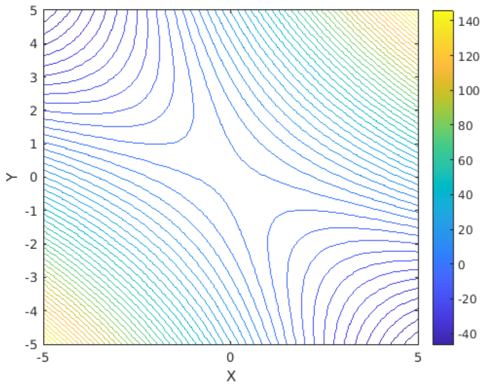
1 Problem 1

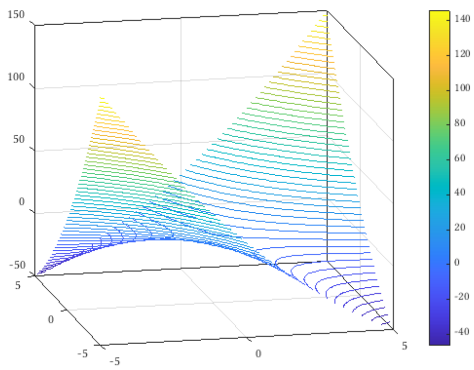
Contour lines of $f(x, y)$ given are produced with the following MATLAB code and are presented in figure 1.

```
function [Z] = plot_contour(start_num, end_num)

        x = linspace(start_num, end_num, 100);
        y = x;
        [X, Y] = meshgrid(x, y);
        Z = X.^2 + 4*X.*Y + Y.^2;
        contour(X, Y, Z, 40);
        xlabel('X');
        ylabel('Y');
end
```



(a) 2D Plot



(b) 3D Plot

Figure 1: Contour lines of $f(x, y)$

A general formula of a quadratic equation is $f(x, y) = ax^2 + 2bxy + cy^2$. Writing our formula in the previous form, we find that $a = 1,\ b = 2,\ c = 1$. Calculation of discriminant can help us calculate the location of function's local minimum/maximum.

$$D = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} = f_{xx}f_{yy} - f_{xy}^2 = 2 \times 2 - 4^2 = -12 < 0, \quad \text{όπου}$$
$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = 2, \quad f_{yy} = \frac{\partial^2 f}{\partial y^2} = 2, \quad f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = 4D = 2 \times 2 - 4^2 = -12 < 0. \tag{1}$$

So, we only have to find the point where $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are equal to 0. Thus, this point will be a saddle point where gradients in each orthogonal direction are 0, but this point is not either a local minimum or maximum. Specifically:

$$\begin{cases} \frac{\partial f}{\partial x} = 2x + 4y = 0 \\ \frac{\partial f}{\partial y} = 4x + 2y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \tag{2}$$

Thus, the point $(x, y) = (0, 0)$ is the saddle point mentioned before for the function given and this can be justified using the plotted contour lines.