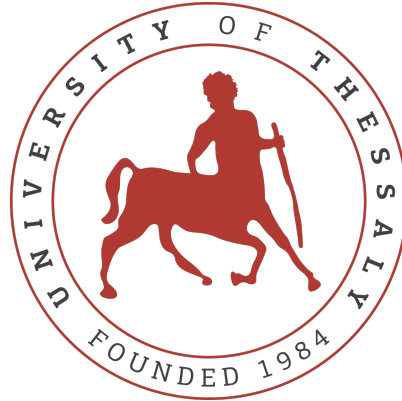


UNIVERSITY OF THESSALY



NEURO-FUZZY COMPUTING

ECE447

1st Problem Set

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1 Problem 1

Contour lines of $f(x, y)$ are plotted with the following MATLAB code and are presented in figure 1.

```
function [Z] = plot_contour(start_num , end_num)

    x = linspace(start_num , end_num , 100);
    y = x;
    [X, Y] = meshgrid(x, y);
    Z = X.^2 + 4*X.*Y + Y.^2;
    contour(X, Y, Z, 40);
    xlabel('X');
    ylabel('Y');
end
```

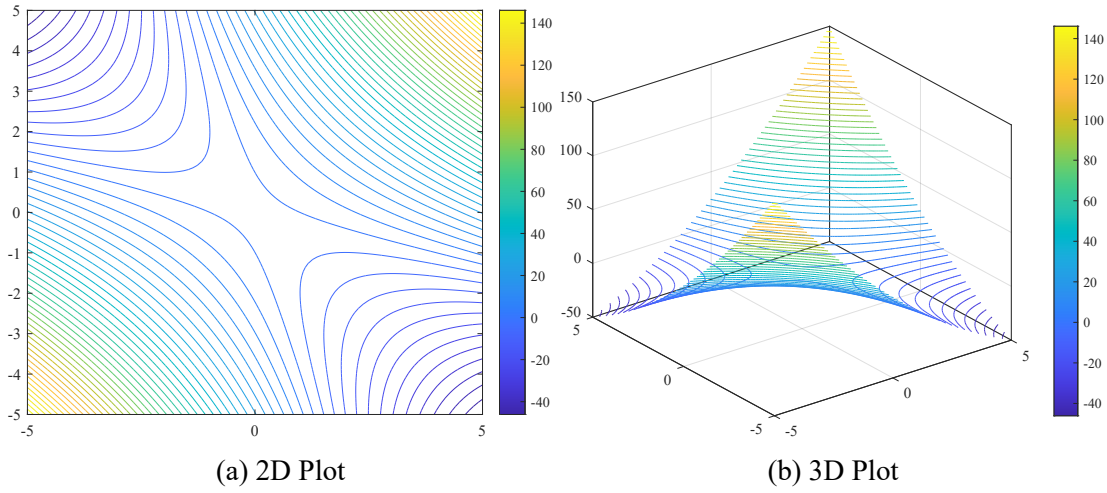


Figure 1: Contour lines of $f(x, y)$

A general formula of a quadratic equation is $f(x, y) = ax^2 + 2bxy + cy^2$. Writing our formula in the previous form, we find that $a = 1$, $b = 2$, $c = 1$. Calculation of discriminant can help us calculate the location of function's local minimum/maximum.

$$D = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} = f_{xx}f_{yy} - f_{xy}^2 = 2 \times 2 - 4^2 = -12 < 0, \quad \text{where} \quad (1)$$

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = 2, \quad f_{yy} = \frac{\partial^2 f}{\partial y^2} = 2, \quad f_{xy} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) = 4D = 2 \times 2 - 4^2 = -12 < 0.$$

So, we only have to find the point where $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are equal to 0. Thus, this point will be a saddle point where gradients in each orthogonal direction are 0, but this point is not either a local minimum or maximum. Specifically:

$$\begin{cases} \frac{\partial f}{\partial x} = 2x + 4y = 0 \\ \frac{\partial f}{\partial y} = 4x + 2y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \quad (2)$$

Thus, the point $(x, y) = (0, 0)$ is the saddle point mentioned before for the function given and this can be justified using the plotted contour lines.

2 Problem 12

In order to evaluate the expression "not ($A(x)$ OR $B(x)$)", we first have to take a look in how fuzzy logic is different with binary logic at the operation level. In binary logic we have three basic operations: AND(x, y), OR(x, y) and NOT(x). But, in fuzzy logic, where a function can have a value in range of $[0...1]$, things are a bit different. Operation AND(x, y) of binary is equivalent with MIN(x, y) from the fuzzy logic, OR(x, y) with MAX(x, y) and NOT(x) with $1-x$.

We have to find the proper x for which the previous expression has the greater value. Firstly, we will calculate the expression and then find the proper x . To achieve this, we will have to break down our calculations in areas.

Starting with $x \leq 2$, " $A(x)$ AND $B(x)$ " is equal to " $\max(A(x), B(x))$ " = 1. So, using De Morgan's law, we have $\max(A(x), B(x)) = \text{not}(A(x) \text{ or } B(x)) = 0$. The exact same result is obtained at $x \geq 7$.

Things are a bit different in $2 \leq x \leq 7$. Function $A(x)$ starts to fall while $B(x)$ starts to rise. The point where they cross is valuable for defining the expression needed and can be found by solving the equation

$$A(x) = B(x) \Leftrightarrow 1 - \frac{x-2}{3} = \frac{x-3}{4} \Rightarrow x = \frac{29}{7}$$

So, for $2 \leq x \leq \frac{29}{7}$, $\max(A(x), B(x)) = 1 - \frac{x-2}{3} = A(x)$, because in this region $A(x)$ is above $B(x)$.

Thus, $\text{not}(A(x) \text{ or } B(x)) = \frac{x-2}{3}$.

Using the same logic, we find out that for $\frac{29}{7} \leq x \leq 7$, $\text{not}(A(x) \text{ or } B(x)) = 1 - \frac{x-3}{4}$.

Therefore, the expression $f(x) = \text{not}(A(x) \text{ or } B(x))$ is summarized in the following:

$$f(x) = \begin{cases} 0 & x \leq 2, \\ \frac{x-2}{3} & 2 \leq x \leq \frac{29}{7}, \\ 1 - \frac{x-3}{4} & \frac{29}{7} \leq x \leq 7, \\ 0 & x \geq 7 \end{cases} \quad (3)$$

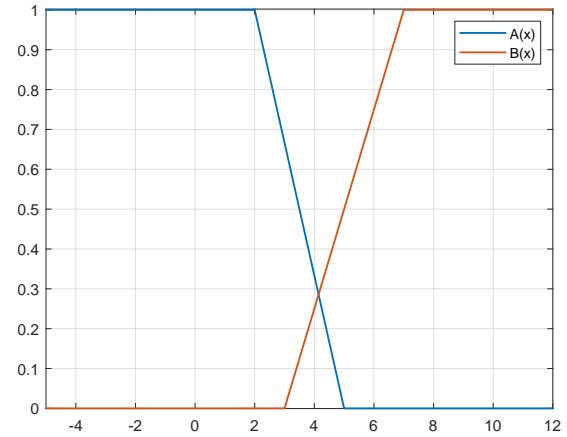


Figure 2: Plot of $A(x)$, $B(x)$