University of Thessaly


Neuro-Fuzzy Computing

ECE447

# 2nd Problem Set

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# Problem 1

In this exercise we need to find the minimum of the given 2-dimensional function:

$$F(\mathbf{w}) = w_1^2 + w_2^2 + (0.5w_1 + w_2)^2 + (0.5w_1 + w_2)^4 \tag{1}$$

with the Conjugate Gradient (Fletcher-Reeves) method.

Initially, we can conclude that the function $F(w)$ is not in quadratic form because of the term $(0.5w_1 + w_2)^4$. A function is said to be in quadratic form if it can be expressed as a second-degree polynomial where all the terms are either squared terms or cross-products of the variables. The presence of the fourth-degree term $(0.5w_1 + w_2)^4$. makes this function a higher-degree polynomial, specifically a quartic function with respect to $(0.5w_1 + w_2)$, which means it cannot be classified as quadratic.
Also, the independent values in this function are $w_1, w_2$, because only with them we can manipulate the $F(w)$.

# Problem 3

For the given neural network, we have:

- learning rate $LR = 1$,
- $w^1\,(0) = -3,\ w^2\,(0) = -1$,
- $b^1\,(0) = 2,\ b^2\,(0) = -1$ and
- input/target pair $\{p = 1,\ t = 0\}$

*FIRST ITERATION*

Step 1: Calculate first layer's output

$$n^1 = w^1 p + b^1 = (-3)(1) + 2 = -1$$
$$a^1 = Swish\left(n^1\right) = Swish\,(-1) = \frac{n^1}{1 + e^{-n^1}} = \frac{-1}{1 + e} = -0.2689$$

Step 2: Calculate second layer's output

$$n^2 = w^2 a^1 + b^2 = (-1)(-0.2689) + (-1) = -0.7311$$
$$a^2 = LReLU\left(n^2\right) = LReLU\,(-0.7311) = -0.000731$$

Step 3: Calculate error

$$e = t - a^2 = (1 - (-0.000731)) = 1.000731$$

Step 4: Calculate sensitivity on second layer

$$s^2 = -2\ LReLU'\left(n^2\right)\left(t - a^2\right) = -2\,(0.001)\,(1.000731) = -0.002001$$

*LReLU's derivative is* $1$ *for* $x > 0$ *and* $0.001$ *for* $x < 0$.

Step 5: Calculate sensitivity on first layer using back-propagation

$$s^1 = Swish^{'}\left(n^1\right)\left(w^2\right)^T s^2 = Swish^{'}\left(-1\right)\left(-1\right)\left(-0.002001\right) = 0.0723 \cdot (-1) \cdot (-0.002001)$$
$$s^1 = 0.000145$$

Step 6: Update wheights and biases

$$w^2(1) = w^2(0) - LR\, s^2(a^1)^T = -1 - 1(-0.002001)(-0.2689) = -1.0005380689$$
$$b^2(1) = b^2(0) - LR\, s^2 = -1 - 1(-0.002001) = -0.997999$$
$$w^1(1) = w^1(0) - LR\, s^1(a^0)^T = -3 - 1(0.000145)(-1) = -2.999855$$
$$b^1(1) = b^1(0) - LR\, s^1 = 2 - 1(0.000145) = 1.999855$$

*SECOND ITERATION*

Step 1:

$$n^1 = w^1 p + b^1 = (-2.999855)(1) + 1.999855 = -1$$
$$a^1 = Swish\left(n^1\right) = Swish\left(-1\right) = \frac{n^1}{1+e^{-n^1}} = \frac{-1}{1+e} = -0.2689$$

Step 2:

$$n^2 = w^2 a^1 + b^2 = (-1.000538)(-0.2689) + (-0.997999) = -0.728954$$
$$a^2 = LReLU\left(n^2\right) = LReLU\left(-0.728954\right) = -7.2895\text{e}{-4}$$

Step 3:

$$e = t - a^2 = (1 - (-7.2895\text{e}{-4})) = 1.000728954$$

Step 4:

$$s^2 = -2\, LReLU^{'}\left(n^2\right)\left(t - a^2\right) = -2\left(0.001\right)\left(1.000728954\right) = -0.0020014579$$

Step 5:

$$s^1 = Swish^{'}\left(n^1\right)\left(w^2\right)^T s^2 = Swish^{'}\left(-1\right)\left(-1.0005380689\right)\left(-0.0020014579\right) =$$
$$= 0.0723 \cdot (-1.0005380689) \cdot (-0.0020014579)$$
$$s^1 = 0.0001448423$$

Step 6:

$$w^2(2) = w^2(1) - LR\, s^2(a^1)^T = -1.0005380689 - 1(-0.0020014579)(-0.2689) = -1.0010762609$$
$$b^2(2) = b^2(1) - LR\, s^2 = -0.997999 - 1(-0.0020014579) = -0.9959975421$$
$$w^1(2) = w^1(1) - LR\, s^1(a^0)^T = -2.999855 - 1(-0.0020014579)(-1) = -3.0018564579$$
$$b^1(2) = b^1(1) - LR\, s^1 = 1.999855 - 1(-0.0020014579) = 2.0018564579$$