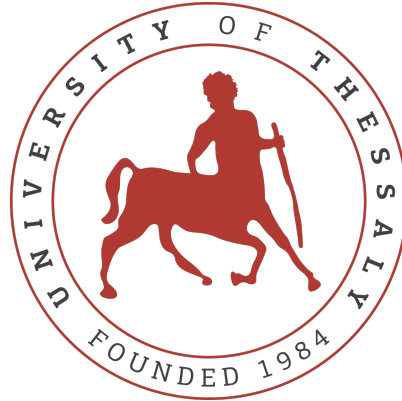


UNIVERSITY OF THESSALY



NEURO-FUZZY COMPUTING

ECE447

2nd Problem Set

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January 19, 2024

Problem 1

In this exercise we need to find the minimum of the given 2-dimensional function:

$$F(\mathbf{w}) = w_1^2 + w_2^2 + (0.5w_1 + w_2)^2 + (0.5w_1 + w_2)^4 \quad (1)$$

with the Conjugate Gradient (Fletcher-Reeves) method.

Initially, we can conclude that the function $F(w)$ is not in quadratic form because of the term $(0.5w_1 + w_2)^4$. A function is said to be in quadratic form if it can be expressed as a second-degree polynomial where all the terms are either squared terms or cross-products of the variables. The presence of the fourth-degree term $(0.5w_1 + w_2)^4$ makes this function a higher-degree polynomial, specifically a quartic function with respect to $(0.5w_1 + w_2)$, which means it cannot be classified as quadratic.

Also, the independent values in this function are w_1, w_2 , because only with them we can manipulate the $F(w)$.

Problem 3

For the given neural network, we have:

- learning rate $LR = 1$,
- $w^1(0) = -3$, $w^2(0) = -1$,
- $b^1(0) = 2$, $b^2(0) = -1$ and
- input/target pair $\{p = 1, t = 0\}$

FIRST ITERATION

Step 1: Calculate first layer's output

$$\begin{aligned} n^1 &= w^1 p + b^1 = (-3)(1) + 2 = -1 \\ a^1 &= \text{Swish}(n^1) = \text{Swish}(-1) = \frac{n^1}{1 + e^{-n^1}} = \frac{-1}{1 + e} = -0.2689 \end{aligned}$$

Step 2: Calculate second layer's output

$$\begin{aligned} n^2 &= w^2 a^1 + b^2 = (-1)(-0.2689) + (-1) = -0.7311 \\ a^2 &= \text{LReLU}(n^2) = \text{LReLU}(-0.7311) = -0.000731 \end{aligned}$$

Step 3: Calculate error

$$e = t - a^2 = (1 - (-0.000731)) = 1.000731$$

Step 4: Calculate sensitivity on second layer

$$s^2 = -2 \text{LReLU}'(n^2) (t - a^2) = -2(0.001)(1.000731) = -0.002001$$

LReLU's derivative is 1 for $x > 0$ and 0.001 for $x < 0$.

Step 5: Calculate sensitivity on first layer using back-propagation

$$s^1 = Swish'(n^1) (w^2)^T s^2 = Swish'(-1) (-1) (-0.002001) = 0.0723 \cdot (-1) \cdot (-0.002001)$$

$$s^1 = 0.000145$$

Step 6: Update weights and biases

$$w^2(1) = w^2(0) - LR s^2 (a^1)^T = -1 - 1(-0.002001)(-0.2689) = -1.0005380689$$

$$b^2(1) = b^2(0) - LR s^2 = -1 - 1(-0.002001) = -0.997999$$

$$w^1(1) = w^1(0) - LR s^1 (a^0)^T = -3 - 1(0.000145)(-1) = -2.999855$$

$$b^1(1) = b^1(0) - LR s^1 = 2 - 1(0.000145) = 1.999855$$

SECOND ITERATIONStep 1:

$$n^1 = w^1 p + b^1 = (-2.999855)(1) + 1.999855 = -1$$

$$a^1 = Swish(n^1) = Swish(-1) = \frac{n^1}{1 + e^{-n^1}} = \frac{-1}{1 + e} = -0.2689$$

Step 2:

$$n^2 = w^2 a^1 + b^2 = (-1.000538)(-0.2689) + (-0.997999) = -0.728954$$

$$a^2 = LReLU(n^2) = LReLU(-0.728954) = -7.2895e-4$$

Step 3:

$$e = t - a^2 = (1 - (-7.2895e-4)) = 1.000728954$$

Step 4:

$$s^2 = -2 LReLU'(n^2) (t - a^2) = -2 (0.001) (1.000728954) = -0.0020014579$$

Step 5:

$$s^1 = Swish'(n^1) (w^2)^T s^2 = Swish'(-1) (-1.0005380689) (-0.0020014579) =$$

$$= 0.0723 \cdot (-1.0005380689) \cdot (-0.0020014579)$$

$$s^1 = 0.0001448423$$

Step 6:

$$w^2(2) = w^2(1) - LR s^2 (a^1)^T = -1.0005380689 - 1(-0.0020014579)(-0.2689) = -1.0010762609$$

$$b^2(2) = b^2(1) - LR s^2 = -0.997999 - 1(-0.0020014579) = -0.9959975421$$

$$w^1(2) = w^1(1) - LR s^1 (a^0)^T = -2.999855 - 1(-0.0020014579)(-1) = -3.0018564579$$

$$b^1(2) = b^1(1) - LR s^1 = 1.999855 - 1(-0.0020014579) = 2.0018564579$$