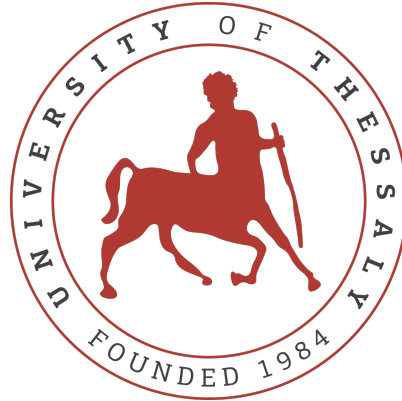


UNIVERSITY OF THESSALY



NEURO-FUZZY COMPUTING

ECE447

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## 2<sup>nd</sup> Problem Set

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January 20, 2024

## Problem 1

In this exercise we need to find the minimum of the given 2-dimensional function:

$$F(\mathbf{w}) = w_1^2 + w_2^2 + (0.5w_1 + w_2)^2 + (0.5w_1 + w_2)^4 \quad (1)$$

with the Conjugate Gradient (Fletcher-Reeves) method.

Initially, we can conclude that the function  $F(w)$  is not in quadratic form because of the term  $(0.5w_1 + w_2)^4$ . A function is said to be in quadratic form if it can be expressed as a second-degree polynomial where all the terms are either squared terms or cross-products of the variables. The presence of the fourth-degree term  $(0.5w_1 + w_2)^4$  makes this function a higher-degree polynomial, specifically a quartic function with respect to  $(0.5w_1 + w_2)$ , which means it cannot be classified as quadratic.

Also, the independent values in this function are  $w_1, w_2$ , because only with them we can manipulate the  $F(w)$ .

## Problem 3

For the given neural network, we have:

- learning rate  $LR = 1$ ,
- $w^1(0) = -3$ ,  $w^2(0) = -1$ ,
- $b^1(0) = 2$ ,  $b^2(0) = -1$  and
- input/target pair  $\{p = 1, t = 0\}$

### FIRST ITERATION

Step 1: Calculate first layer's output

$$n^1 = w^1 p + b^1 = (-3)(1) + 2 = -1$$

$$a^1 = \text{Swish}(n^1) = \text{Swish}(-1) = \frac{n^1}{1 + e^{-n^1}} = \frac{-1}{1 + e} = -0.2689$$

Step 2: Calculate second layer's output

$$n^2 = w^2 a^1 + b^2 = (-1)(-0.2689) + (-1) = -0.7311$$

$$a^2 = \text{LReLU}(n^2) = \text{LReLU}(-0.7311) = -0.000731$$

Step 3: Calculate error

$$e = t - a^2 = (0 - (-0.000731)) = 0.000731$$

Step 4: Calculate sensitivity on second layer

$$s^2 = -2 \text{LReLU}'(n^2) (t - a^2) = -2(0.001)(0.000731) = -1.462\text{e-}6$$

*LReLU's derivative is 1 for  $x > 0$  and 0.001 for  $x < 0$ .*

Step 5: Calculate sensitivity on first layer using back-propagation

$$s^1 = Swish'(n^1) (w^2)^T s^2 = Swish'(-1) (-1) (-1.462e-6) = 0.0723(-1)(-1.462e-6) \\ s^1 = 1.0570e-7$$

Step 6: Update weights and biases

$$w^2(1) = w^2(0) - LR s^2 (a^1)^T = -1 - 1(-1.462e-6)(-0.2689) \approx -1 \\ b^2(1) = b^2(0) - LR s^2 = -1 - 1(-1.462e-6) \approx -1 \\ w^1(1) = w^1(0) - LR s^1 (a^0)^T = -3 - 1(1.0570e-7)(-1) \approx -3 \\ b^1(1) = b^1(0) - LR s^1 = 2 - 1(1.0570e-7) \approx 2$$

Since there were no changes on the biases and weights, the next iteration will not change the parameters of the given neural network, but we will calculate them anyway.

SECOND ITERATIONStep 1:

$$n^1 = w^1 p + b^1 = (-3)(1) + 2 = -1 \\ a^1 = Swish(n^1) = Swish(-1) = \frac{n^1}{1 + e^{-n^1}} = \frac{-1}{1 + e} = -0.2689$$

Step 2:

$$n^2 = w^2 a^1 + b^2 = (-1)(-0.2689) + (-1) = -0.7311 \\ a^2 = LReLU(n^2) = LReLU(-0.7311) = -0.000731$$

Step 3:

$$e = t - a^2 = (0 - (-0.000731)) = 0.000731$$

Step 4:

$$s^2 = -2 LReLU'(n^2) (t - a^2) = -2 (0.001) (0.000731) = -1.462e-6$$

Step 5:

$$s^1 = Swish'(n^1) (w^2)^T s^2 = Swish'(-1) (-1) (-1.462e-6) = 0.0723(-1)(-1.462e-6) \\ s^1 = 1.0570e-7$$

Step 6:

$$w^2(1) = w^2(0) - LR s^2 (a^1)^T = -1 - 1(-1.462e-6)(-0.2689) \approx -1 \\ b^2(1) = b^2(0) - LR s^2 = -1 - 1(-1.462e-6) \approx -1 \\ w^1(1) = w^1(0) - LR s^1 (a^0)^T = -3 - 1(1.0570e-7)(-1) \approx -3 \\ b^1(1) = b^1(0) - LR s^1 = 2 - 1(1.0570e-7) \approx 2$$