University of Thessaly

Neuro-Fuzzy Computing

ECE447

2nd Problem Set

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Problem 1

In this exercise we need to find the minimum of the given 2-dimensional function:

$$F(\mathbf{w}) = w_1^2 + w_2^2 + (0.5w_1 + w_2)^2 + (0.5w_1 + w_2)^4 \tag{1}$$

with the Conjugate Gradient (Fletcher-Reeves) method.

Initially, we can conclude that the function $F(w)$ is not in quadratic form because of the term $(0.5w_1 + w_2)^4$. A function is said to be in quadratic form if it can be expressed as a second-degree polynomial where all the terms are either squared terms or cross-products of the variables. The presence of the fourth-degree term $(0.5w_1 + w_2)^4$. makes this function a higher-degree polynomial, specifically a quartic function with respect to $(0.5w_1 + w_2)$, which means it cannot be classified as quadratic.
Also, the independent values in this function are w_1, w_2, because only with them we can manipulate the $F(w)$.

Problem 3

For the given neural network, we have:

- $w^1(0) = -3,\ w^2(0) = -1,$
- $b^1(0) = 2,\ b^2(0) = -1$ and
- input/target pair $\{p = 1,\ t = 0\}$

The output of the first layer is:

$$n^1 = w^1 p + b^1 = (-3)(1) + 2 = -1$$
$$a^1 = \text{Swish}\left(n^1\right) = \text{Swish}\left(-1\right) = \frac{n^1}{1 + e^{-n^1}} = \frac{-1}{1+e} = -0.2689$$

The output of the second layer is then:

$$n^2 = w^2 a^1 + b^2 = (-1)(-0.2689) + (-1) = -0.7311$$
$$a^2 = \text{LReLU}\left(n^2\right) = \text{LReLU}\left(-0.7311\right) = 0.001$$

So, the error calculated is:

$$e = t - a^2 = (1 - (0.001)) = 0.999 \approx 1$$

Now, we can apply back-propagation starting from the second layer:

$$s^2 = -2\text{LReLU}'\left(n^2\right)(t - a)$$