

CM 2607 Advanced Mathematics for Data Science

Tutorial No 02

Q1) Find the first derivative of the functions given.

I. $y = (3x - 2)^2 + 5x$

II. $y = 4(x - 2)^2 + 3(x - 2) + 1$

III. $y = -3e^{x^2+5x} + 1$

IV. $y = 5e^{x^2+1}(\sin 3x)$

V. $y = (x - 2)^2(3 \sin 2x - e^x)$

VI. $y = \sqrt{3x^2 - 5}$

VII. $y = \frac{3 \sin(x^2 + \frac{\pi}{3})}{e^x}$

VIII. $y = e^{-3x} \ln(x^2 + 1)$

IX. $y = \frac{4}{1+e^{-x}}$

X. $y = (\sin^3 x + e^{-4x})^5$

Q2) If u, v , and w are functions of x and $y = uvw$ show that

$$\frac{dy}{dx} = uv \frac{dw}{dx} + wu \frac{dv}{dx} + vw \frac{du}{dx} \text{ using the product rule twice.}$$

Q3) Given that $x^3 - \frac{3}{2}x^2 - 6x + 5 = y$, find the value/values of x for which $\frac{dy}{dx} = 0$.

Q4) Given that $y = \sin 3x$, show that $y'' + 9y = 0$.

Q5) Given that $y = \sin x + \cos x$, find y', y'', y''' and $y^{(4)}$. Hence find $y^{(11)}$.

Q6) For the function $y = \frac{x^4}{4} - \frac{x^3}{2} - 9x^2 + 6$ find the values of x which $y'' = 0$ and evaluate y'' at $x = -3$.

Q7) Probability of passing the CM2607 Advanced Mathematics module follows a logistic function with respect to the hours of studying as given.

$$P(t) = \frac{1}{1+e^{-(4 \cdot 1 + 1 \cdot 5t)}} \text{ where } t \text{ is the number of hours}$$

Find $P'(1), P'(2.25), p'(4.75)$. Interpret each result.

Q8) Use logarithmic differentiation to find $\frac{dy}{dx}$ for the given functions.

I. $y = (x^3 + 3x)e^{-5x} \tan 2x$

II. $y = \frac{3e^{-4x}(x^2-5)^3}{\sin^3 2x}$

III. $y = \frac{x^4 \sin x^3}{(x^3-5)}$

Q9) Find the Hessian matrix for the functions and evaluate at the points given.

I. $h = x^3 + 2xy - y^3$ at $(-1,1)$

II. $p = (3x - 2y)(x + 3y)$ at $(0,1)$

III. $q = \sin(4x - 3y)$ at $(\frac{\pi}{2}, \frac{\pi}{2})$

Q10) Find $\frac{dy}{dx}$ for the given implicit functions.

I. $xy + \sin y = 2x$

II. $e^{x+y} = x^3 y^3$

III. $\sin(x^2 + y) = y^3 e^x$