Tutorial 03

1. A machine is operated by two workers. There are sixteen workers available. How many possible teams of two workers are there?

2. A factory has 52 machines. Two of these have been given an experimental modification. In the first week after this modification, problems are reported with thirteen of the machines. What is the probability that both of the modified machines are among the thirteen with problems assuming that all machines are equally likely to give problems,?

3. A factory has 52 machines. Four of these have been given an experimental modification. In the first week after this modification, problems are reported with thirteen of the machines. What is the probability that exactly two of the modified machines are among the thirteen with problems assuming that all machines are equally likely to give problems?

4. A random number generator produces sequences of independent digits, each of which is as likely to be any digit from 0 to 9 as any other. If X denotes any single digit, find $\mathsf{E}(X)$.

5. A hand-held calculator has a clock cycle time of 100 nanoseconds; these are positions numbered $0, 1, \ldots, 99$. Assume a flag is set during a particular cycle at a random position. Thus, if X is the position number at which the flag is set.

   $$\mathsf{P}(X = k) = \frac{1}{100} \qquad k = 0, 1, 2, \ldots, 99.$$

   Evaluate the average position number $\mathsf{E}(X)$, and σ, the standard deviation.

   (Hint: The sum of the first k integers is $k(k+1)/2$ and the sum of their squares is: $k(k+1)(2k+1)/6$.)

6. Concentric circles of radii 1 cm and 3 cm are drawn on a circular target radius 5 cm. A darts player receives 10, 5 or 3 points for hitting the target inside the smaller circle, middle annular region and outer annular region respectively. The player has only a 50-50 chance of hitting the target at all but if he does hit it he is just as likely to hit any one point on it as any other. If $X =$ 'number of points scored on a single throw of a dart' calculate the expected value of X.

Answers