CM 2607 Advanced Mathematics for Data Science

Tutorial No 11

1.

Consider the data points given:

Sampling time	Amplitude
0.1s	0.2
0.2s	0.35
0.3s	0.32
0.4s	0.3

a) What are the values of N and T for this sample?
b) Write the discrete Fourier transform for this sample in matrix form
c) Calculate the discrete Fourier transform for this sample.
d) Write the inverse discrete Fourier transform for this sample in matrix form
e) Calculate the inverse discrete Fourier transform for this sample.

You may use python for the following questions.

2.

a) sample the following function at 100 Hz and 10 Hz , in the range [-π,π] and store these values:
$$f(x) = \sin(x) + 0.25 \sin(50x)$$
b) find the discrete fourier transform of these values (hint: fft)
c) Perform the inverse Fourier transform on the values that you obtained and plot the result. What do you observe?
d) Remove the upper 50% of frequencies from the dft of the 100 Hz signal and plot the inverse dft.
e) Remove frequencies beyond the 50th data point on the dft of the 100 Hz signal and plot the inverse dft.
f) Remove frequencies below the 40th data point on the dft of the 100 Hz signal and plot the inverse dft.
g) Remove the upper and lower 40 data points from the dft of the 100 Hz signal and plot the inverse dft.
h) Remove all data points except the upper and lower 40 data points of the 100 Hz signal and plot the inverse dft.

3.

a) Sample the following function at 100 Hz in the range [-2π, 2π] and store these values:
$$f(x) = \cos(2x) - 0.5\sin(60x)$$
b) Find the discrete cosine transform of the function (hint:dct)
c) Perform the inverse dct on the values you obtained and plot the result.
d) Remove the upper 50% of frequencies and plot the result.
e) Remove the frequencies above the 200th data point and plot the result.

4.

a) Create a 360px * 360 px image of zeros and add several squares/rectangles with values ranging from 1-255.
b) Apply the discrete Fourier transform to the image apply fftshift and show the result. (Hint: fft2)
c) Find the inverse dft of the above and display the resulting image.
d) Remove high frequency components from the image, find the inverse fourier transform, and plot the result. (Hint: in fft, low frequency components are at the centre of the image once fft shift is applied).
e) Remove low frequency components of the image, find the inverse fourier transform, and plot the result. (Refer to the hint above).

5.

a) load image “leaves.jpg” from last week’s tutorial and find its dct (hint: there is no 2D dct function, so apply it along one axis and then the other). Show the result.
b) Remove some of the higher frequencies and display the result. Try changing how much of the higher frequencies you remove and observe the effect it has on the output.
c) Scale the image (hint: you can discard high frequency components)

6.

a) Create a gaussian filter (hint: scipy.signal.gaussian) and find its frequency domain equivalent. Show the result.
b) Load image “leaves.jpg” and find its frequency domain equivalent.
c) Apply the gaussian filter (Hint: A gaussian filter must be convolved with the image it is applied to. However, convolution in the time domain is equivalent to multiplication in the time domain) and display the result.