CM 1606 Computational Mathematics

Tutorial No 05

1) Build the truth tables for the formulae given and establish if they are satisfiable, unsatisfiable, valid, or invalid. Identify the relation between the pairs (iv,v),(iv,vi),(vii,viii), and (ix,x).
   i) $(P \vee \neg Q) \rightarrow P$
   ii) $(Q \rightarrow (\neg P \vee \neg Q)) \rightarrow P$
   iii) $(P \leftrightarrow (P \wedge \neg Q)) \vee \neg P$
   iv) $P \rightarrow (Q \rightarrow P)$
   v) $(\neg P \wedge Q) \rightarrow (Q \rightarrow (\neg P \wedge Q))$
   vi) $\neg(\neg Q \rightarrow (P \rightarrow \neg Q))$
   vii) $(\neg P \rightarrow \neg Q) \rightarrow (Q \rightarrow P)$
   viii) $(\neg(P \wedge Q) \rightarrow \neg Q) \rightarrow (Q \rightarrow (P \wedge Q))$
   ix) $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$
   x) $((P \rightarrow (\neg P \rightarrow Q)) \rightarrow ((P \rightarrow \neg P) \rightarrow (P \rightarrow Q)))$

2) Establish if there is a logical consequence between the given knowledge base KB and the proposition A for each case given below.
   i) $KB=\{(P \rightarrow Q), P\}; A= Q$
   ii) $KB=\{(\neg P \rightarrow Q), \neg P\}; A= Q$
   iii) $KB=\{(\neg P \rightarrow \neg Q), \neg P\}; A= \neg Q$
   iv) $KB=\{(P \rightarrow Q) \vee Q), \neg(P \rightarrow Q)\}; A= Q$
   v) $KB=\{(P \wedge Q), \neg(P \wedge Q)\}; A= \neg Q$
   vi) $KB=\{(P \vee Q), (\neg R \vee P)\}; A= P \vee Q \vee R$
   vii) $KB=\{(\neg(P \rightarrow Q), P)\}; A= (\neg P \vee Q))$
   viii) $KB=\{(\neg(P \wedge \neg Q), P)\}; A=(\neg P \wedge Q))$
   ix) $KB=\{((P \wedge \neg Q), \neg R)\}; A=R$
   x) $KB=\{(P \rightarrow \neg Q), P, (P \wedge \neg R)\}; A=Q$

3) Suppose that when you left the home, you found that your handsfree is not with you. You know that the following statements are true.

i) I was using my tablet in the study room or in the bedroom.

ii) If I was using my tablet in the study room, my handsfree is on the computer table.

iii) Handsfree is not on the computer table.

iv) If I was using my tablet in the bedroom, my handsfree is on the dressing table.

Which of the following is(are) true?

(a) The handsfree is on the dressing table.

(b) The handsfree is not on the dressing table.

(c) I was using the tablet in the bedroom.

(d) I was using the tablet in the study room.

(e) I was not using the tablet in the study room.

4) Construct the tableau and prove these axioms are valid.

i) $P \rightarrow (Q \rightarrow P)$

ii) $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$

iii) $(\neg P \rightarrow \neg Q) \rightarrow (Q \rightarrow P)$

5) Build tableau for the formulae below and determine if they are valid.

i) $(P \vee \neg Q) \rightarrow P$

ii) $((P \wedge Q) \rightarrow P)$

iii) $(Q \rightarrow (\neg P \vee \neg Q)) \rightarrow P$

iv) $(P \leftrightarrow (P \wedge \neg Q)) \vee \neg P$

v) $(\neg P \wedge Q) \rightarrow (Q \rightarrow (\neg P \wedge Q))$

vi) $\neg(\neg Q \rightarrow (P \rightarrow \neg Q))$

vii) $(\neg(P \wedge Q) \rightarrow \neg Q) \rightarrow (Q \rightarrow (P \wedge Q))$

viii) $((P \rightarrow (\neg P \rightarrow Q)) \rightarrow ((P \rightarrow \neg P) \rightarrow (P \rightarrow Q)))$

ix) $((\neg P \vee \neg Q) \rightarrow \neg(P \wedge Q))$

x) $((P \rightarrow Q) \rightarrow (\neg P \vee \neg Q))$

6) I) Discuss how you can use the tableau technique to show that the formulae 'A' given is unsatisfiable with the example A= $((P \vee Q) \wedge (\neg Q \wedge \neg P))$.

II) Similarly show that the negation of the valid formulae in question 5 are unsatisfiable using tableau.