CM 1606 Computational Mathematics

Matrices II

Week 8 | Ganesha Thondilege













Learning Outcomes

- Covers LO1 for CM 1606
- On completion of this lecture, students are expected to be able to:
 - Recognize how matrices are used to store and process data
 - Discuss the applications of Matrices in Machine Learning and Deep Learning







CONTENT

- Determinant Higher order
- Diagonal method for 3rd order
- Cofactor matrix
- Adjoint matrix
- Inverse
- Application Simultaneous equations







Determinants of Third Order

- Select a raw or a column for the expansion
- Find Minors for every element of the selected raw(column)
- Find cofactors and then the determinant
- For higher order determinant



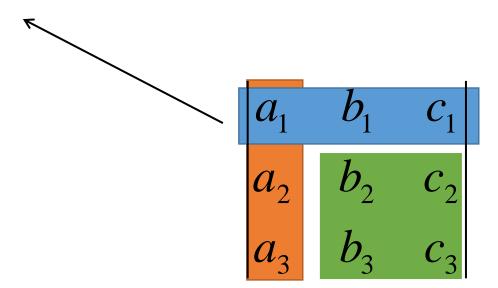




Determinants of Third Order ctd.

Minor of
$$a_1$$
 is $\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$

Try to get other minors....









Determinants of Third Order ctd.

- Cofactor of a_{ij}^{th} element = $(-1)^{i+j}$ Minor of a_{ij}
- Place signs, $(-1)^{i+j}$

Expansion over 1st raw

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$







Determinants of Third Order - Example

Expanding by 1st row

$$\begin{vmatrix} 2 & 1 & 3 \\ 6 & 0 & -2 \\ -1 & 4 & 5 \end{vmatrix} = 2 \begin{vmatrix} 0 & -2 \\ 4 & 5 \end{vmatrix} - 1 \begin{vmatrix} 6 & -2 \\ -1 & 5 \end{vmatrix} + 3 \begin{vmatrix} 6 & 0 \\ -1 & 4 \end{vmatrix}$$
$$= 2(0 - (-8)) - 1(30 - 2) + 3(24 - 0) = 16 - 28 + 72 = 60$$

Expanding by 3rd column

$$\begin{vmatrix} 2 & 1 & 3 \\ 6 & 0 & -2 \\ -1 & 4 & 5 \end{vmatrix} = 3 \begin{vmatrix} 6 & 0 \\ -1 & 4 \end{vmatrix} - (-2) \begin{vmatrix} 2 & 1 \\ -1 & 4 \end{vmatrix} + 5 \begin{vmatrix} 2 & 1 \\ 6 & 0 \end{vmatrix}$$
$$= 3(24 - 0) + 2(8 - (-1)) + 5(0 - 6) = 72 + 18 - 30 = 60$$





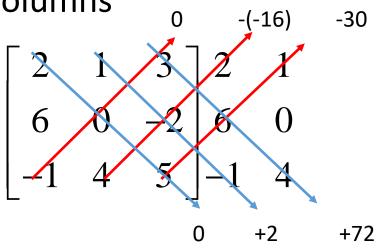


Diagonal Method for third order

Only for third order matrices

$$\begin{vmatrix} 2 & 1 & 3 \\ 6 & 0 & -2 \\ -1 & 4 & 5 \end{vmatrix}$$

Find the multiplication of diagonal entries as given, by adding 1st two columns as next two columns



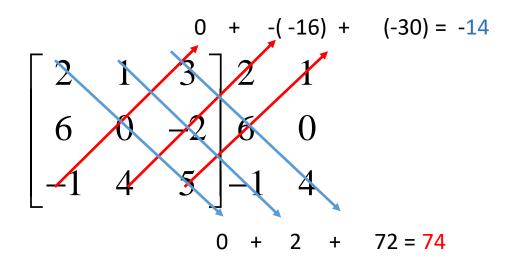






Diagonal Method for third order ctd.

Sum of diagonal multiplications separately



Determinant = 74-14=60





Determinants of Third Order ctd.

Evaluate the following determinants

$$i. \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ -3 & 4 & 1 \end{vmatrix}$$

i.

$$\begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ -3 & 4 & 1 \end{vmatrix}$$
 ii.
 $\begin{vmatrix} -3 & 4 & 2 \\ 2 & 1 & -4 \\ 0 & 9 & 0 \end{vmatrix}$



Cofactor Matrix

• Cofactor Matrix of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix}$

The cofactor for each element of matrix A with the sign

$$A_{11} = \begin{vmatrix} 4 & 5 \\ 0 & 6 \end{vmatrix} = 24$$
 $A_{12} = -\begin{vmatrix} 0 & 5 \\ 1 & 6 \end{vmatrix} = 5$ $A_{13} = \begin{vmatrix} 0 & 4 \\ 1 & 0 \end{vmatrix} = -4$

$$A_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 6 \end{vmatrix} = -12$$
 $A_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 6 \end{vmatrix} = 3$ $A_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = 2$

$$A_{31} = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = -2 \qquad A_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = -5 \qquad A_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} = 4$$
11/21/2020 | A₃₁ = | A₃₂ = | A₃₃ = | A₃₃ = | A₃₄ = | A₃₅ = | A₃₅ = | A₃₆ = | A₃₇ = | A₃₇ = | A₃₈ = |







Adjoint matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix}$$

• Cofactor Matrix of
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix}$$
 is given by $\begin{bmatrix} 24 & 5 & -4 \\ -12 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix}$

- Adjoint matrix is denoted as adj(A)
- Defined as







Inverse of a Matrix

- Denoted as A⁻¹
- If AB = BA = I, then $B = A^{-1}$; and $A = B^{-1}$
- Ex:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 1 & 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 6 & -2 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Then

Ans: Note that
$$AB = BA = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Can you show the







Properties

•
$$(AB)^{-1} = B^{-1}A^{-1}$$

- $(A^T)^T = A$ and $(\lambda A)^T = \lambda A^T$
- $\bullet \quad (A+B)^T = A^T + B^T$
- $(AB)^T = B^T A^T$







Inverse

- Written as $A^{-1} = \frac{1}{|A|} adj(A)$ Where $|A| \neq 0$.
- For a matrix A of order 2 inverse can be written easily as

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$$adj(A)_{\text{CM1606 Computational Mathematics}}$$







Inverse for third order matrices

Inverse matrix of
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix}$$
 is given by:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} 24 & 5 & -4 \\ -12 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix}^{T} = \frac{1}{22} \begin{bmatrix} 24 & -12 & -2 \\ 5 & 3 & -5 \\ -4 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 12/11 & -6/11 & -1/11 \\ 5/22 & 3/22 & -5/22 \\ -2/11 & 1/11 & 2/11 \end{bmatrix}$$







Solutions for simultaneous equations

eg.
$$2x + 5y = 1$$
$$x + 3y = 4$$

$$\Rightarrow \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$
 (matrix representation of the system)

This is of the type Ax = b, where

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}, \quad x = \begin{pmatrix} x \\ y \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$







Solutions for simultaneous equations ctd.

$$Ax = b$$

multiply both sides by A⁻¹

$$A^{-1}Ax = A^{-1}b$$

$$(A^{-1}A)x = A^{-1}b$$

$$I x = A^{-1}b$$

$$x = A^{-1}b$$

Then, the solution of the system can be obtained by $A^{-1}b$

Then,
$$x = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} -17 \\ 7 \end{pmatrix}$$

i.e.
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -17 \\ 7 \end{pmatrix} \rightarrow x = -17, \quad y = 7.$$



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Exercises

Solve the given systems using inverse

$$i) \quad 2x - 3y = -8$$
$$3x + 4y = 5$$

ii)
$$x+2y+z=4$$

 $3x-4y-2z=2$
 $5x+3y+5z=-1$

iii)
$$2x_1 - x_2 + 3x_3 = 2$$
$$x_1 + 3x_2 - x_3 = 11$$
$$2x_1 - 2x_2 + 5x_3 = 3$$