CM 2607 Advanced Mathematics for Data Science

Tutorial No 07

1) List the first five terms of the sequences defined by:
   a. $a_n = 3n + 5$
   b. $a_0 = 0, a_1 = 1, a_n = a_{n-1} + 0.5 * a_{n-2}$
   c. $a_n = 2(3^2 - 1)$
   d. $a_0 = 12, a_n = \begin{cases} 3a_{n-1} + 1 & if\ a_{n-1}\ is\ odd \\ \frac{a_{n-1}}{2} & if\ a_{n-1}\ is\ even \end{cases}$
   e. $a_n = 2 + (n-1) * 4$
   f. $a_n = 2 \times (-1^{n-1})$
2) Find the sum of the first 10 terms for the following arithmetic sequences:
   a. a = 1, d = 0.5
   b. a = -5, d = 3
   c. a = 0, d = -2
3) Find the sum of the fist 5 terms for the following geometric sequences:
   a. a = 1, r = 2
   b. a = 2, r = -3
   c. a = 1, r = 5
4) Determine whether these sequences are monotonic and/or bounded.
   a. $a_n = \frac{1}{n}$
   b. $a_n = a_{n-1} \times -1.5,\ \ a_0 = 1$
   c. $a_n = a_{n-1} + 2, a_0 = 1$
   d. $a_n = \begin{cases} a_{n-1} + a_{n-2}, & if\ n\ is\ even \\ a_{n-1} - a_{n-2}, & if\ n\ is\ odd \end{cases}$
   e. $a_n = -0.9 \times a_{n-1},\ \ a_0 = 1$
5) Identify whether these sequences converge, diverge, or does not converge or diverge. If they converge, find the limits of these sequences.
   a. $a_n = \frac{n+1}{n+2}$
   b. $a_n = a_{n-1} + 3,\ \ a_0 = 1$
   c. $a_n = n^2 - 2n + 1$
   d. $a_n = n \cdot e^n$
   e. $a_n = n \cdot \cos\left(\frac{n}{2}\right)$
   f. $a_n = \frac{1}{2n} \cdot (\sin(n + \pi) + 1)$
   g. $a_n = |-2 * n + 5|$
   h. $a_n = \frac{\cos(n)}{n}$
   i. $a_n = \frac{\sin(2n) + \cos(3n+1)}{n}$
   j. $a_n = \frac{\sin(n) + 1}{n^2}$

6) Plot the first 100 terms of the following sequences. You may use python.

a. $a_n = 0.1n^2 - 10n + 5$

b. $a_n = a_{n-1} + 0.5a_{n-1}, a_0 = 1, a_1 = 1$

c. $a_n = \sum_{m=0}^{m=n-1} \frac{a_m}{n-m}, a_0 = 1$

d. $$a_n = \begin{cases} \frac{a_{n-1}}{3} & if\ n\ is\ divisible\ by\ 3 \\ & a_0 = 3 \\ 2a_{n-1} - 1 & otherwise \end{cases}$$

e. $$a_n = \begin{cases} \frac{a_{n-1}}{-5}, & a_{n-1} > 5 \\ 1.5a_{n-1}, & -5 \geq a_{n-1} \geq 5, \\ a_{n-1} \times -2, & a_{n-1} < -5 \end{cases} \quad a_0 = 1$$

f. $a_n = \sin\left(\frac{\pi n}{12}\right)$

g. $a_n = \frac{n+1}{0.1n^2 - 5x + 2.5}$