CM 1606 Computational Mathematics

Logic

Week 4 | Ganesha Thondilege

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Learning Outcomes

- Covers LO1 for CM1606
- On completion of this lecture, students are expected to be able to:
    - Recognize the logical connectives
    - Discuss the logical reasoning
    - Relate the logical foundations to the emerging field of data science

CONTENT

- Logic - Introduction
- Propositions
- Logical connectives
- Logically equivalence
- Tautology
- Contradiction

Logic - Introduction

- Logic is the study of reasoning
- Specially concerned with whether reason is correct.

Eg: 1) All mathematicians wear sandals

Anyone who wear sandals is an algebraist

Therefore all mathematicians are algebraist

Logic is no help in determining whether any of these statement is true

However if first two statements are true, logic assures that the third statement is also true.

Propositions

- A statement that is either true or false but not both is called a proposition.

Eg: 1) For every positive integer n, there is a prime number larger than n.

2) Earth is the only planet in the universe that has a life.

- Basic building blocks of any theory of logic.

- Denoted by lower case letters such as p, q and r.

- Truth value of the proposition is denoted by T (True) or F (False)

Logical connectives

$\neg$ ~	$\wedge$ &	$\vee$	$\rightarrow$ $\Rightarrow$	$\Leftrightarrow$
NOT Negation	AND Conjunction	OR Disjunction	IF ….. THEN Implication Or Conditional	If and only if Biconditional

Negation

- Negation of p is the proposition not p
- Unary operator

P	~P
T	F
F	T

Conjunction

- $p \wedge q$
- True when both are true only

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

- $p \vee q$
- False when both are False only

P	Q	P V Q
T	T	T
T	F	T
F	T	T
F	F	F

Implication (Conditional)

- If *p* then *q*; $p \rightarrow q$
- *False when p is True, and q is False only*

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

- If and only if (iff) ; $p \leftrightarrow q$
- $p \leftrightarrow q$ is true when both *p* and *q* have same truth values

P	Q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Compound Propositions

- A proposition is said to be primitive, if it cannot be broken down into simple propositions
- Otherwise composite or compound
- Sequence of primitives connected by logical connectives

$$(\ \ ) \gg \sim \gg \wedge \gg \vee \gg \rightarrow \gg \leftrightarrow$$

Example

Represent each sentence given symbolically.

- It is not hot, but it is sunny.
- It is not both hot and sunny.
- It is neither hot nor sunny.

Truth tables for compound propositions

- If the number of primitives = n
- Number of rows of the truth table = 2^n
- Consists with all the possible combinations
- Identify all primitives(atomic propositions)
- Develop the table step by step

Example

Develop truth tables for given compound propositions

$$i)\, p \wedge \sim q$$

$$ii) \sim p \vee q$$

$$iii) \sim p \rightarrow (p \wedge q)$$

$$iv)\, p \rightarrow (q \rightarrow p)$$

$$v) \sim (p \wedge \sim q)$$

Logically equivalence

- Two propositions are said to be logically equivalent if they have identical truth tables.

Eg: Verify that $\sim(p \wedge q) \equiv \sim p \vee \sim q$

Tautology

A compound proposition is called as a **tautology** if all the truth values are true. (Valid Formula)

Eg: Develop the truth tables for following compound propositions

$$i)\, p \vee \sim p$$

$$ii)\, p \rightarrow p \vee q$$

Contradiction

A compound proposition is called as a **Contradiction** if all the truth values are false.

Eg: Develop the truth tables for following compound propositions

$$i)\, p \wedge \sim p$$
$$ii)\, q \wedge \sim (p \rightarrow q)$$

Examples

Check whether the following logical equivalences are correct

i. $(p \vee q) \wedge (\neg p \to \neg q) \equiv p$

ii. $\big((p \to q) \to q\big) \to q \equiv p \to q$