

CM 2607 Advanced Mathematics for Data Science

Tutorial No 10

1.

Consider the following function:

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 0, & 0 \leq x < \frac{\pi}{2} \\ 1, & \frac{\pi}{2} \leq x \leq \pi \end{cases}$$

- a) Calculate the coefficients of the Fourier sine series for this function.
- b) Write the Fourier sine series for this function up to the first 5 terms.
- c) Calculate the coefficients of the Fourier cosine series for this function.
- d) Write the Fourier cosine series for this function up to the first 5 terms.
- e) Calculate the coefficients for the Fourier series of this function.
- f) What are the differences between the Fourier sine series, Fourier cosine series, and the Fourier series of the function?

2.

- a) Give a function that would be represented accurately with a Fourier cosine series. Justify why it would be represented accurately by a Fourier cosine series.
- b) Calculate the Fourier cosine series for this function for up to five terms.
- c) Calculate the Fourier series of this function for up to five terms and compare the result.

3.

Find the Fourier transform for the following function using the formula:

$$f(x) = \begin{cases} e^t & -\infty \leq t \leq 0 \\ e^{-t} & 0 < t \leq \infty \end{cases}$$

4.

Find the Fourier transform for the following function using the formula:

$$f(t) = \begin{cases} t, & 0 \leq t \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

5.

Find the Fourier transform for the following function using the formula:

$$f(t) = \begin{cases} 1, & -1 \leq t < 0 \\ 2, & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

6.

Find the inverse Fourier transform of the following function:

$$F(\omega) = \begin{cases} 1, & -1 \leq \omega \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

7.

Use the Fourier transform pairs given and the properties of Fourier transforms to determine Fourier transforms of the following functions:

a)

$$f(t) = 2\delta(t) + 1$$

b)

$$f(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

c)

$$f(t) = \begin{cases} 2, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$$

d)

$$f(t) = \begin{cases} 0, & t \leq 0 \\ 1, & 0 < t \leq 1 \\ 2, & t > 1 \end{cases}$$

e)

$$f(t) = 1 + \cos(t) - \frac{\cos(3t)}{3}$$

8.

Use the Fourier transform pairs given and the properties of Fourier transforms to determine the inverse Fourier transforms of the following functions:

a)

$$F(\omega) = 3\delta(\omega) + 1$$

b)

$$F(\omega) = \delta(\omega - 2) + \delta(\omega + 2)$$

c)

$$F(\omega) = 2 \operatorname{sinc}\left(\frac{\omega}{2\pi}\right) e^{-i\omega}$$

d)

$$F(\omega) = \delta(\omega - 2) + 1$$

e)

$$F(\omega) = \delta(\omega - 1) + \delta(\omega + 3) - \delta(\omega - 5) - \delta(\omega + 4)$$

9.

Introduction to image processing using python.

Task 1

- a) Create a 360px * 360 px image in python. Place several squares on the image. Display the image.
- b) Display the image in black and white.
- c) Crop the image to 240 px * 240 px.
- d) Flip the image in both the x and y axes.

Task 2

- a) Import the given image into python (leaves4.jpg) and display it.
- b) Display the image in black and white.
- c) Rotate the image 90° clockwise (hint: Use a rotation matrix) and display the result.
- d) Display the image within a circle of Centre 180, radius 180 (make the rest of the pixels 0).