CM 2607 Advanced Mathematics for Data Science

Lecture 04

Integration I

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Indefinite and Definite Integrals

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Content

- Definition
- Integrals for Common functions
- Transformation $x \mapsto px + q$
- Logarithms in Integration
- Definite integrals
- Techniques of Integration

Integration

$$\frac{d}{dx}[f(x)] = F(x) \Rightarrow \int F(x)dx = f(x) + C$$

$C -$ constatnt of integration

$f(x)$ - anti - derivative or integral of $F(x)$

$eg:$

$$\frac{d}{dx}[x^2] = 2x \Rightarrow \int 2xdx = x^2 + C$$

Table of Integrals for Common functions

Function	Integral
$a - \text{constant}$	$ax + c$
$x^n; n \neq -1$	$\dfrac{x^{n+1}}{n+1} + C; n \neq -1$
$1/x$	$\ln\lvert x\rvert + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\sec^2 x$	$\tan x + C$
$\csc^2 x$	$-\cot x + C$

Function	Integral
$\sec x \tan x$	$\sec x + C$
$\csc x \cot x$	$-\csc x + C$
e^x	$e^x + C$
$\dfrac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right) + C$
$\dfrac{1}{a^2 + x^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right) + C$

The Transformation $x \mapsto px + q$

$$\frac{d}{dx}[f(x)] = F(x) \Rightarrow \frac{d}{dx}[f(px \pm q)] = pF(px \pm q)$$

$$\therefore \int F(x)dx = f(x) + C \Rightarrow \int F(px \pm q)dx = \frac{1}{p} f(px \pm q) + C$$

$$eg:$$

$$\int (2x+4)^2 dx = \frac{1}{2}\frac{(2x+4)^3}{3} + C$$

Function	Integral
$(px \pm q)^n; n \neq -1$	$\dfrac{1}{p} \dfrac{(px \pm q)^{n+1}}{n+1} + C$
$\sin(px \pm q)$	$-\dfrac{1}{p} \cos(px \pm q) + C$
$\cos(px \pm q)$	$\dfrac{1}{p} \sin(px \pm q) + C$
$e^{(px \pm q)}$	$\dfrac{1}{p} e^{(px \pm q)} + C$
$\dfrac{1}{px \pm q}$	$\dfrac{1}{p} \ln\lvert px \pm q \rvert + C$

Ex. Integrate with respect to x.

$$i)\,3(2x-7)^{\frac{3}{2}}$$

$$ii)\,\sec^2(3-x)$$

$$iii)\,\frac{5}{\sqrt{9-(2x-3)^2}}$$

$$iv)\,\frac{8}{6-5x}$$

Logarithms in Integration

$$\frac{d}{dx}\ln(f(x)) = \frac{f'(x)}{f(x)} \Rightarrow \int \frac{f'(x)}{f(x)}dx = \ln(f(x)) + C$$

$$eg:$$

$$\int \frac{x+3}{x^2+6x-5}dx = \frac{1}{2}\int \frac{2x+6}{x^2+6x-5}dx$$

$$= \frac{1}{2}\ln\left|x^2+6x-5\right| + C$$

Definite Integrals

$$\int_a^b F(x)dx = [f(x)]_a^b = f(b) - f(a)$$

$eg:$

$$\int_1^3 xdx = \left[\frac{x^2}{2}\right]_1^3 = \frac{9}{2} - \frac{1}{2} = 4$$

$Ex:$

$$i)\int_1^2 \sqrt{5x-1}dx$$

Techniques of Integration

Partial fractions:

Case I: Proper algebraic fractions $\dfrac{p_n(x)}{q_m(x)}; n < m$

$$i)\frac{p_n(x)}{(x\pm a)(x\pm b).....}=\frac{A}{(x\pm a)}+\frac{B}{(x\pm b)}+........$$

$$ii)\frac{p_n(x)}{(x\pm a)^2(x\pm b).....}=\frac{A}{(x\pm a)}+\frac{B}{(x\pm a)^2}+\frac{C}{(x\pm b)}........$$

$$iii)\frac{p_n(x)}{(x^2\pm a)(x\pm b).....}=\frac{Ax+B}{(x^2\pm a)}+\frac{C}{(x\pm b)}+........$$

Partial fractions:

Case II: Improper algebraic functions $\dfrac{p_n(x)}{q_m(x)}; n \geq m$

-Express the fraction as a mixed fraction by division. Then repeat the case I for the proper fraction.

eg : Express in partial fractions.

$i)\ \dfrac{8}{(x-2)(x+5)}$

$ii)\ \dfrac{x^3-4x-5}{x^2-x-6}$

$iii)\ \dfrac{2x+3}{(x^2-2)(x+5)}$

$iv)\ \dfrac{2x+3}{(x-2)^2(x+5)}$

Standard from I

$$\int \frac{p}{ax^2+bx+c}\,dx;\ p,a \neq 0$$

$\text{case I}: b^2-4ac>0$

Use partial fractions and write the anti - derivative using the standard integral

$$\int \frac{1}{px \pm q}\,dx = \tfrac{1}{p}\ln\left|px \pm q\right| + C$$

$\text{case II}: b^2-4ac=0$

write the anti - derivative using the standard integral

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C;\ n \neq -1$$

case III: $b^2 - 4ac < 0$

Write the anti-derivative using the standard integral

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$eg:$

$$i)\int \frac{1}{x^2 + 6x + 8}\,dx$$

$$ii)\int \frac{1}{x^2 + 6x + 9}\,dx$$

$$iii)\int \frac{1}{x^2 + 6x + 25}\,dx$$