# CM 1606 Computational Mathematics

## Set Theory

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# Learning Outcomes

- Covers LO1 for CM1606
- On completion of this lecture, students are expected to be able to:
  - Identify the set-theoretical approach suited for situations to deal with different types or degrees of a specific type of data

# CONTENT

- Terminology
- Subset
- Powerset
- Set Operations
- Intervals

# Terminology

- A **Set** is a well-defined collection of objects.
- The objects are called the elements or members.
- Notation:
  - Sets are usually denoted by capital letters *A B, C,………*

    Ex: *Z* – Set of Integers

    *B* – Integers between 1 and 50 which are divisible by 4
- Set builder notation, $\{x|c(x)\}$

  Ex: $A = \{a \in \mathbb{Z} \mid -2 \leq a \leq 5\}$

# Terminology ctd.

- Elements of sets are written within curly brackets and separated by a comma

Ex:

$$Z = \{\dots\dots\dots\dots -3, -2, -1, 0, 1, 2, 3, 4, \dots\dots\dots\}$$

$$B = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48\}$$

- Cardinality

The number of elements in a set

Denoted by |A| or n(A)

Finite set - |A| Finite

Infinite set - |A| Infinite

# Terminology ctd.

- ‘Belongs to’ and ‘doesn’t belong to’

  If A={a, b, c} then $a \in A$ and $d \notin A$

- Universal Set – U or Ɛ
- Empty Set (Null Set), $\phi$ or $\{\ \}$

  Ex:

  $$A = \{\text{Prime numbers between 3 and 5}\}$$

  $$A = \{\ \} \text{ or } A = \phi$$

# Subsets

- For any two nonempty sets A and B,

  If all the elements of A also in B, then $A \subseteq B$

- Identities:

  $$A \Rightarrow \varphi \subseteq A \subseteq U$$

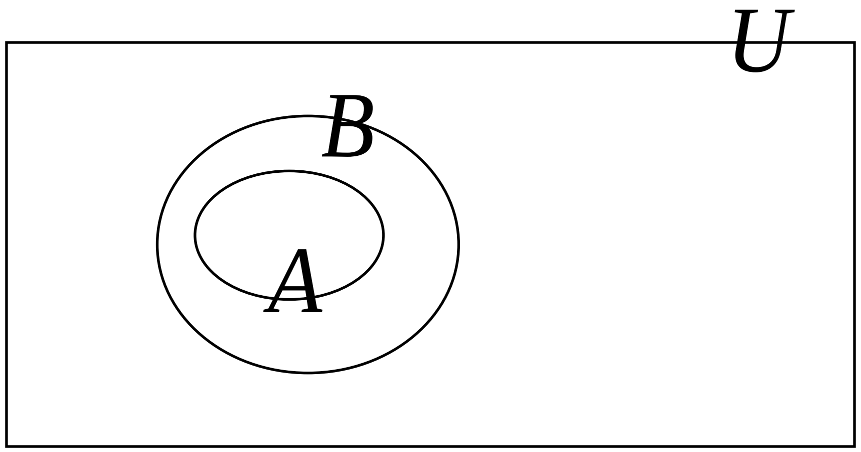
  $$A \Rightarrow A \subseteq A$$

  $A = B$ if and only if $B \subseteq A$ and $A \subseteq B$,

  $A$ and $B$ contains same elements

- Proper subset

  $$A \subset B$$




$A \subseteq B$

# Powerset

- The set which contains all the subsets of a given set
- Denoted as P(A)

Ex: $If \;\; X = \{1, 2\} \text{ then } P(X) = \{\{\;\}, \{1\}, \{2\}, \{1, 2\}\}$

$or$

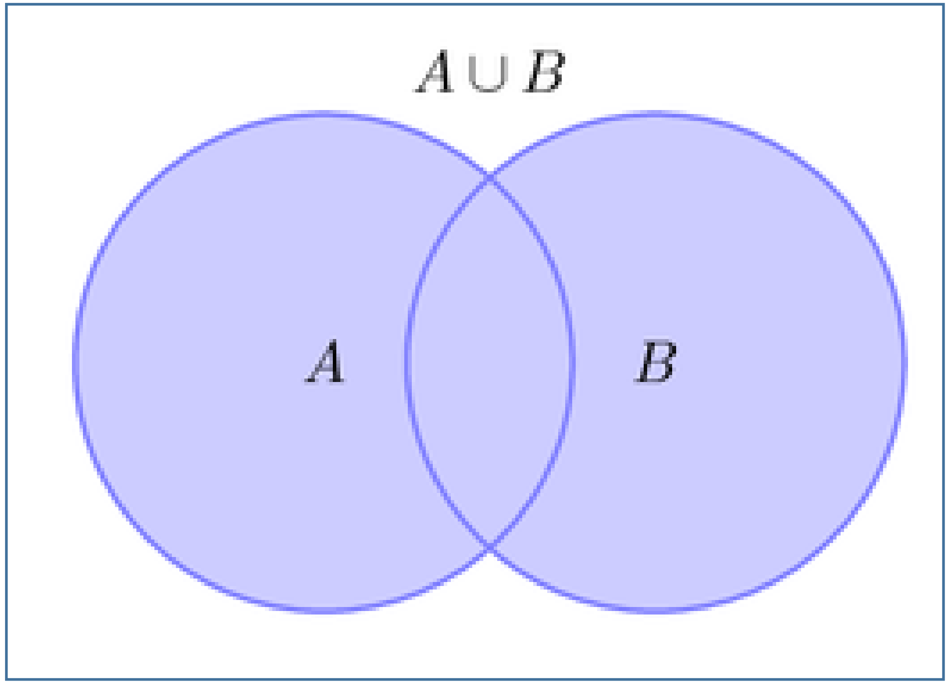
$P(X) = \{\varphi, \{1\}, \{2\}, X\}$

- $|P(A)| = 2^{|A|}$

# Set Operations

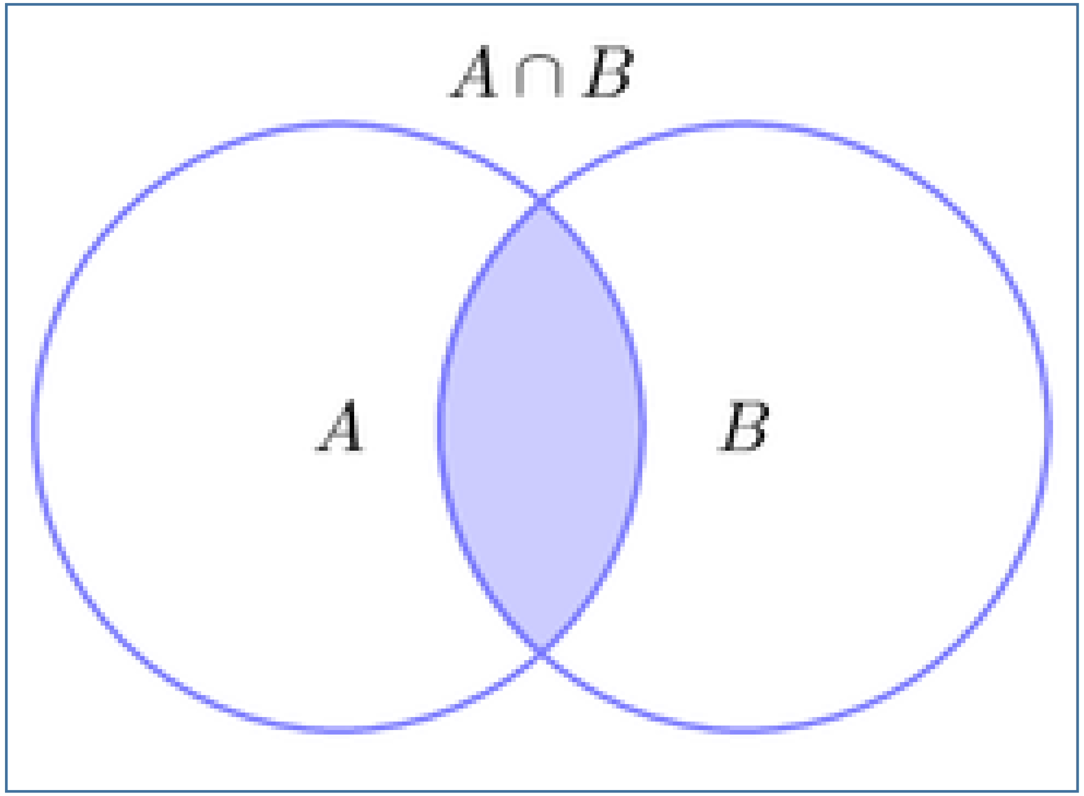
# Union

- Denoted as $A \cup B$
- Defined as $\{x \mid x \in A \text{ or } x \in B\}$
- Simply contains all the elements of A and B



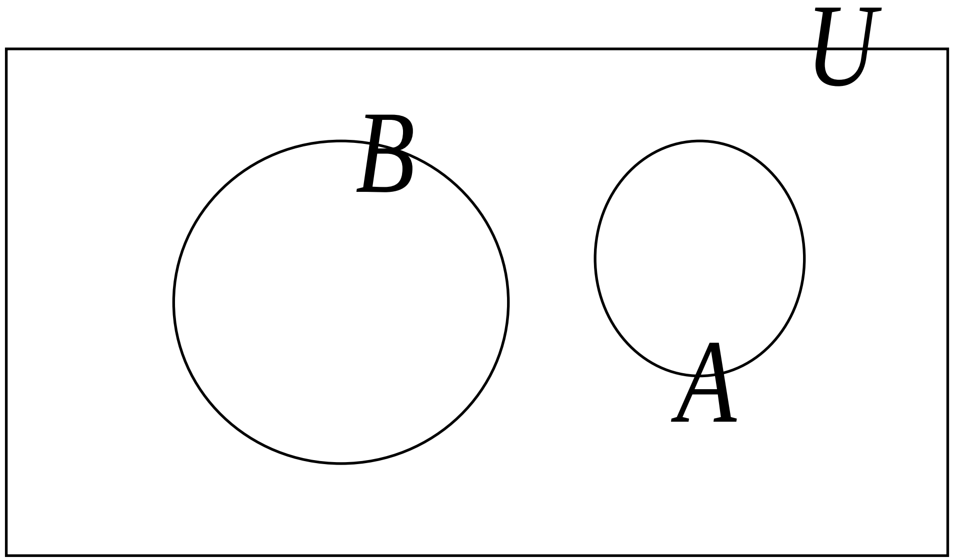
# Intersection

- Denoted as $A \cap B$
- Defined as $\{x \mid x \in A \text{ and } x \in B\}$
- Common elements for both A and B



# Disjoint sets

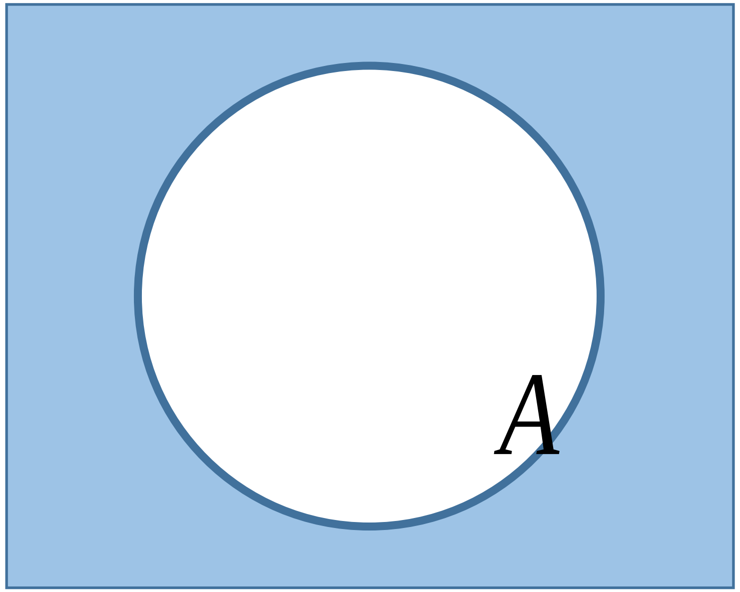
- $A$ and $B$ are disjoint if they do not have elements in common




$$A \cap B = \varnothing$$

# Complement

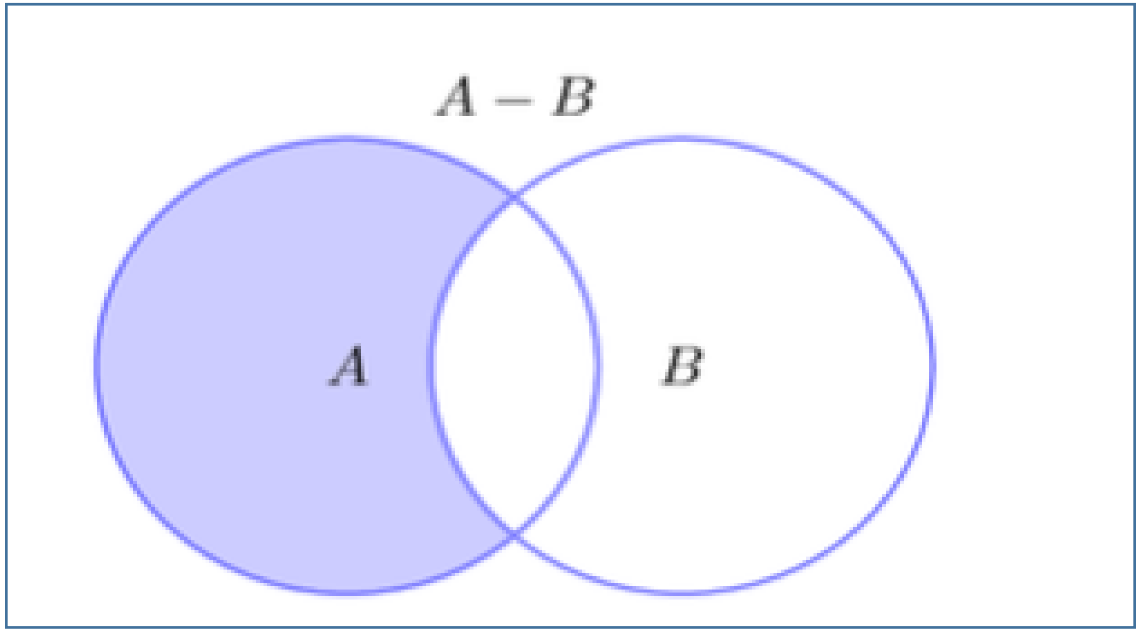
- Denoted as $A' \quad or \quad A^c \quad \text{or} \quad \bar{A}$
- Contains all the elements of $U$ but not in $A$
- $\{x \in U \mid x \notin A\}$




$A'$

# Difference

- Denoted as A-B
- Contains all the elements in A but not in B
- Relative complement of B with respect to the set A
- $\{x \mid x \in A \text{ and } x \notin B\}$



# Laws of set Theory

Commutative Laws

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

Distributive Laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

# Laws of set Theory

## Identity Laws

$$A \cup \varnothing = A$$

$$A \cap U = A$$

$$A \cup U = U$$

$$A \cap \varnothing = \varnothing$$

## Complement Laws

$$A \cup A^c = U$$

$$A \cap A^c = \varnothing$$

$$U^c = \varnothing$$

$$\varnothing^c = U$$

## Involution Law

$$(A^c)^c = A$$

## De Morgan's Law

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

# Cartesian Product

For any two nonempty sets *A* and *B*

- Denoted as

$$A \times B$$

- Defined as

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

*Ex: A={1,2} and B={a,b,c}*

# Intervals

Open Interval

$$(a,b)=\{x\in\mathbb{R}\,|\,a<x<b\}$$

Closed Interval

$$[a,b]=\{x\in\mathbb{R}\,|\,a\le x\le b\}$$

Open begining Interval

$$(a,b]=\{x\in\mathbb{R}\,|\,a<x\le b\}$$

Open ended Interval

$$[a,b)=\{x\in\mathbb{R}\,|\,a\le x<b\}$$

# Exercises

Express each interval in the set builder notation

$i)\ (-2,3)$

$ii)\ (5,10]$

$iii)\ [-1,8)$

$iv)\ [-3,\infty)$

$v)\ (-\infty,5]$

$vi)\ (-\infty,\infty)$

# Exercises

$U = \{x \in \mathbb{Z} \mid 0 < x < 16\}$, $A$ is the set of even numbers

$B$ is the set of odd numbers, $C$ is the set of prime numbers

Write the elements for each set given.

$A \cap B, A \cap C, B \cap C$

$A \cup B, A \cup C, B \cup C$

$A^c, B^c, C^c$

$A - B, B - C, C - B$

$(B - C) \times (C - B)$