

# CM 1606 Computational Mathematics

---

## Matrices I

Week 7 | Ganesha Thondilege

# Learning Outcomes

---

- Covers LO1 for CM 1606
- On completion of this lecture, students are expected to be able to:
  - Recognize how matrices are used to store and process data
  - Discuss the applications of Matrices in Machine Learning and Deep Learning

# CONTENT

---

- Introduction
- Types of Matrices
- .Addition and Subtraction
- Multiplication
- Transpose
- Determinant – Order 2
- Matrix properties

# Matrix

- A matrix is any doubly subscripted array of elements arranged in rows and columns.

$$\mathbf{A} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & \dots & a_{2n} \\ \dots & \dots & \dots \\ a_{m1} & \dots & a_{mn} \end{bmatrix}_{m \times n}$$

Any element is a real number

$a_{ij}$ ;  $i$  – row number and  $j$  – column number

# Types of Matrices

Row matrix

$$A = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}_{1 \times n}$$

Column matrix

$$B = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_m \end{bmatrix}_{m \times 1}$$

Square matrices

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}_{n \times n}$$

*where*

$$m = n$$

# Equal Matrices

---

Example:  $A = \begin{bmatrix} 1 & 0 \\ -4 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given that  $A = B$ , find  $a$ ,  $b$ ,  $c$  and  $d$ .

if  $A = B$ , then  $a = 1$ ,  $b = 0$ ,  $c = -4$  and  $d = 2$ .

# Zero Matrix

---

- Every element of a matrix is zero, it is called a zero matrix, i.e.,

$$A = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & 0 \end{bmatrix}$$

# Lower and Upper Triangular Matrices

- Upper triangular Matrix

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & & a_{2n} \\ \vdots & & \ddots & \\ 0 & 0 & & a_{nn} \end{bmatrix}$$

- Lower triangular Matrix

$$\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & & 0 \\ \vdots & & \ddots & \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix}$$



# Diagonal Matrices

- All off diagonal entries are zero

$$D = \begin{bmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & a_{nn} \end{bmatrix}$$

Simply

$$D = \text{diag}[a_{11}, a_{22}, \dots, a_{nn}]$$

# Identity Matrix

- If all diagonal entries are 1 for a diagonal matrix, the matrix is called identity matrix.
- Properties:  $AI = IA = A$

Examples of identity matrices:  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  and  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

# Matrix addition and subtraction

- Applied for same order matrices

$$\{C_{ij}\} = \{A_{ij}\} \pm \{B_{ij}\}$$

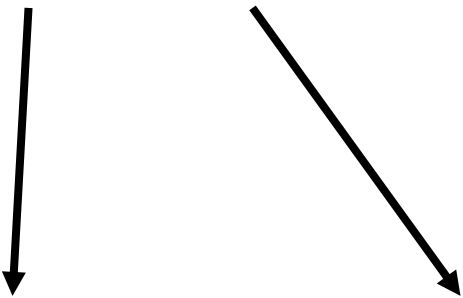
- If  $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$  and  $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$  then

$$C = \begin{bmatrix} a_{11} \pm b_{11} & a_{12} \pm b_{12} \\ a_{21} \pm b_{21} & a_{22} \pm b_{22} \end{bmatrix}$$

# Matrix Multiplication

---

Suppose matrices A and B have these dimensions:



$[m \times n]$  and  $[p \times q]$

# Matrix Multiplication

---

Matrices A and B can be multiplied if:

$$[m \times n] \text{ and } [p \times q]$$



$$n = p$$

# Matrix Multiplication

The resulting matrix will have the dimensions:

$$[m \times n] \text{ and } [p \times q]$$

$m \times q$

# Matrix Multiplication

- Computation  $A \times B = C$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$AB \neq BA$  in general

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$C = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \end{bmatrix}$$

# Matrix Multiplication

- Computation  $A \times B = C$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$[3 \times \textcircled{2}]$ 
 $\textcircled{[2]} \times 3]$

A and B can be multiplied

$$C = \begin{bmatrix} 2*1+3*1=5 & 2*1+3*0=2 & 2*1+3*2=8 \\ 1*1+1*1=2 & 1*1+1*0=1 & 1*1+1*2=3 \\ 1*1+0*1=1 & 1*1+0*0=1 & 1*1+0*2=1 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 8 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$[3 \times 3]$



# Transpose of a Matrix

---

- Obtained by interchanging the rows and columns of a matrix  $A$
- Denoted as  $A^T$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

# Symmetric Matrices

---

- A matrix  $A$  such that  $A^T = A$  is called symmetric

Example:  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & -5 \\ 3 & -5 & 6 \end{bmatrix}$  is symmetric.

- $A^T + A$ ?

# Determinant – Order 2

---

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

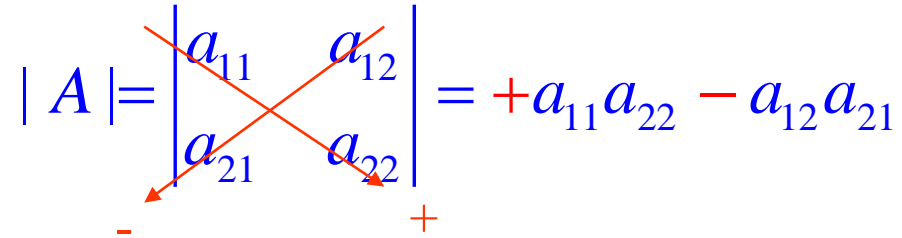
- Determinant of  $A$ , denoted  $|A|$  or  $\det(A)$ ,
- Is a real number and can be evaluated by

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

# Determinant – Order 2

- Only for order 2 matrices

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = +a_{11}a_{22} - a_{12}a_{21}$$



Ex:  $A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \times 4 - 2 \times 3 = -2$$

# Properties

- Following properties are true for determinants of any order.

1. If every element of a row (column) is zero,

e.g.,  $\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = 1 \times 0 - 2 \times 0 = 0$ , then  $|A| = 0$ .

2.  $|A^T| = |A|$  ← determinant of a matrix =  
that of its transpose

3.  $|AB| = |A||B|$

# Exercises

---

1) Suppose  $A = \begin{pmatrix} -1 & 3 \\ 2 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & 2 \\ -2 & 0 \end{pmatrix}$ ,  $C = \begin{pmatrix} 5 & 4 \\ -1 & 1 \end{pmatrix}$ .

Evaluate the following.

*i.*  $A + 2B - C$

*ii.*  $AB$  and  $BA$

*iii.*  $A^T, A^T B$  and  $(A + A^T)B$

*iv.*  $|A|, |B|, |C|$  and  $|ABC|$

*v.*  $A - B, A + B$  and  $A^2 - B^2$

# Matrix properties

---

- $A+B=B+A$
- $A+(B+C)=(A+B)+C$
- $a(A+B)=aA+aB$  ;  $a$ -constant
- $(ab)A=a(bA)=b(aA)$  ;  $a,b$ - constants

# Matrix properties

---

- $(AB)C = A(BC) = ABC$
- $A(B+C) = AB + AC$
- $A(aB) = a(AB) = (aA)B$
  
- $(A^T)^T = A$
- $(A+B)^T = A^T + B^T$
- $(AB)^T = B^T A^T$