CM 1606 Computational Mathematics

Relations

Week 2| Ganesha Thondilege

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Learning Outcomes

- Covers LO1 for CM1606
- On completion of this lecture, students are expected to be able to:
  - Recall cartesian product and discuss ordered pairs
  - Identify relations and inverse relations
  - Describe composition of relations
  - Explain types of relations

CONTENT

- Cartesian product Review
- Ordered pairs
- Relations
- Inverse of a Relation
- Composition of Relations
- Types of Relations

Cartesian Product

For any two nonempty sets *A* and *B*, the cartesian product is

- Denoted as

$$A \times B$$

- Defined as

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

Ex: A={a, b} and B={1, 2}

Ordered Pairs

- For any two sets A and B, set of ordered pairs ρ , from A to B

Defined as

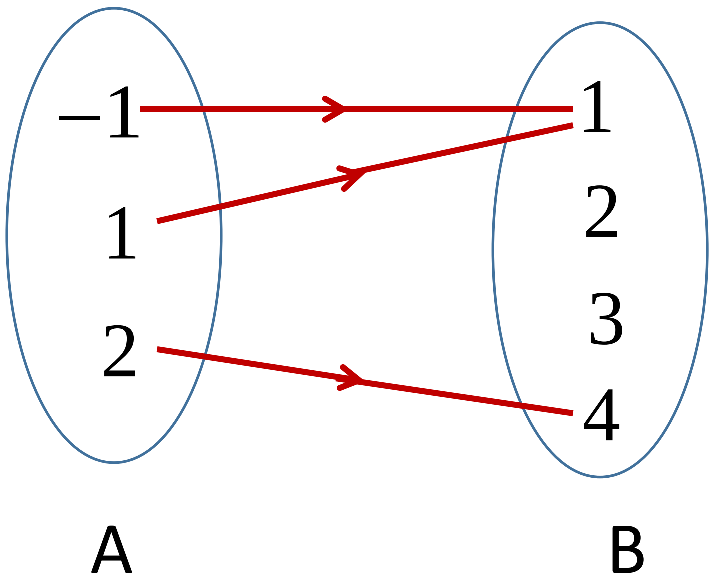
$$\rho = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

Note that {a, b}= {b, a}. But (a, b) $\neq$ (b, a)

- Always ρ is a subset of $A \times B$

Ordered Pairs ctd.

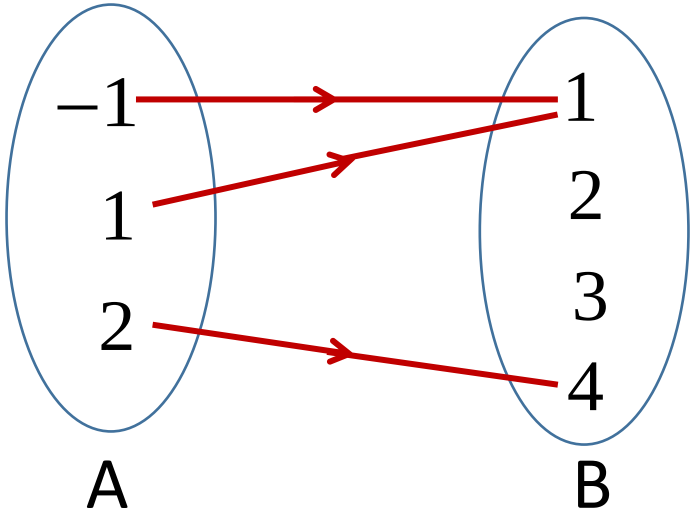
- A={-1,1,2} and B={1,2,3,4}



$$\rho = \{(-1,1),(1,1),(2,4)\} = \{(a,b) \mid a \in A \text{ and } b \in B \text{ and } b = a^2\}$$

Relations

- A Relation is a set of ordered pairs
- ρ is a relation from A to B is a subset of $A \times B$



$$\rho = \{(-1,1), (1,1), (2,4)\} = \{(a,b) \mid a \in A \text{ and } b \in B \text{ and } b = a^2\}$$

Relation on a set

- For the set A, if ρ is a relation from A to itself then

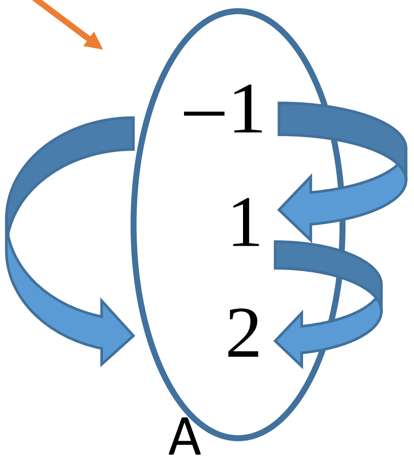
ρ is a relation on A and $\rho \subseteq A \times A$

- Domain

$$D(\rho) = \{x \mid \exists y, (x, y) \in \rho\}$$

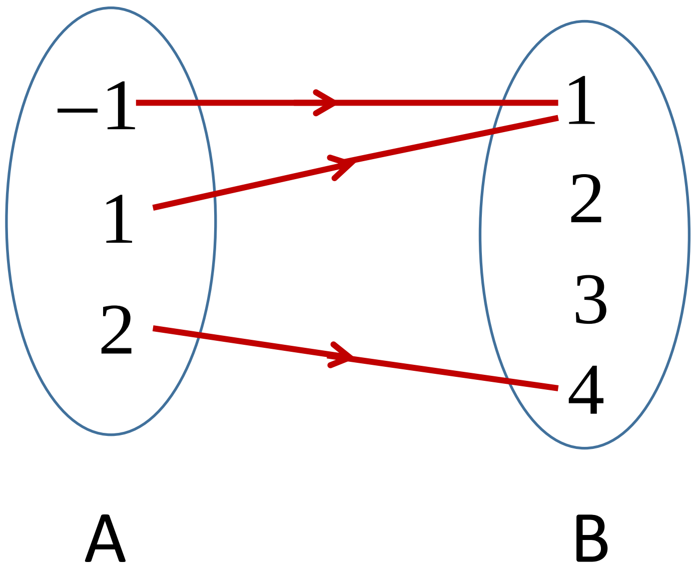
- Range

$$R(\rho) = \{y \mid \exists x, (x, y) \in \rho\}$$



Domain and Range

$$\rho = \{(-1,1),(1,1),(2,4)\}$$



$$D(\rho) = \{-1,1,2\} \qquad R(\rho) = \{1,4\}$$

Inverse of a Relation

- For the relation ρ , inverse relation of ρ

  Denoted as ρ^{-1}

  Defined as
  $$\rho^{-1} = \{(x, y) \mid (y, x) \in \rho\}$$

Note that $\left(\rho^{-1}\right)^{-1} = \rho$

Ex: $\rho = \{(-1,1), (1,1), (2,4)\}$

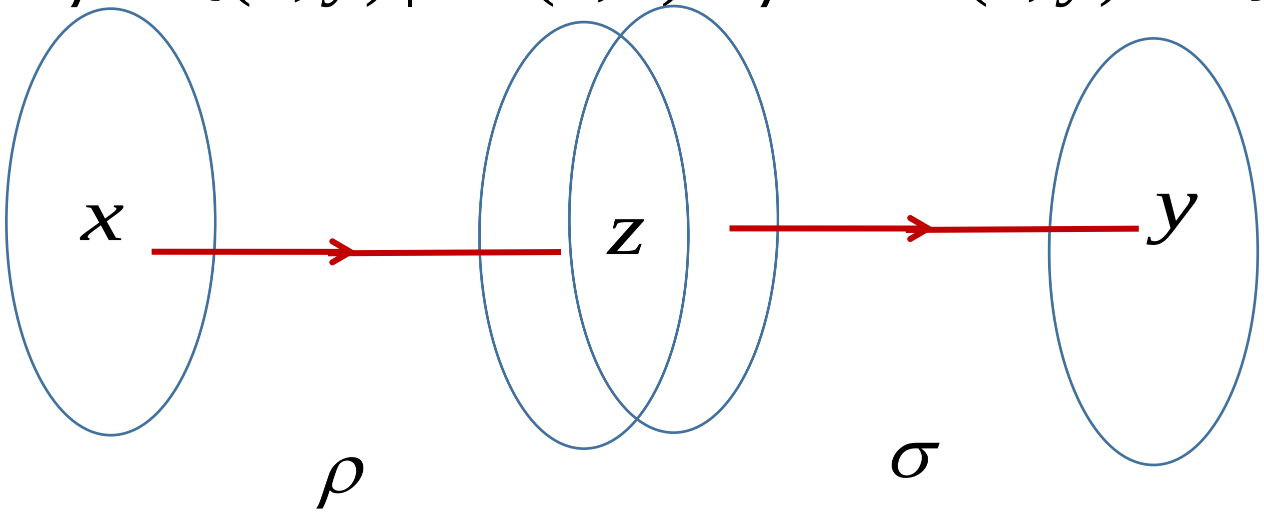
Then
$$\rho^{-1} = \{(1,-1), (1,1), (4,2)\}$$

Composition of Relations

- For two relations ρ and σ , their composition $\sigma \circ \rho$ is

Defined as

$$\sigma \circ \rho = \{(x, y) \mid \exists z \ (x, z) \in \rho \text{ and } (z, y) \in \sigma\}$$



$$z = R(\rho) = D(\sigma)$$

Example

- Consider two relations

$$\rho = \{(1,a),(2,b),(3,c),(4,d),(5,g)\} \text{ and}$$

$$\sigma = \{(a,2),(b,1),(b,2),(e,3),(g,4)\}$$

Find $\sigma \circ \rho$ and $\rho \circ \sigma$

Types of relations

- Reflexive relation

  For every x in the domain of ρ, the ordered pair $(x, x) \in \rho$

- Symmetric relation

  For every $(x, y) \in \rho$, then $(y, x) \in \rho$

- Transitive relation

  For every (x, y) and $(y,z) \in \rho$, then $(x, z) \in \rho$

- Equivalence relation

  ρ satisfies all three properties reflexive, symmetric and transitive

$D(\rho) = R(\rho)$

Example

Consider the relation $\rho = \{(a,a),(b,b),(c,c),(d,d),(e,e),(f,f),$

$(a,b),(b,a),(b,c),(c,b),(a,c),(c,a),(d,e),(e,d)\}$

Check whether ρ is a

- Reflexive
- Symmetric
- Transitive
- Equivalence relation.
- By considering each element of $D(\rho)$, sketch the graphical representation for the relation and interpret