

## CM 2607 Advanced Mathematics for Data Science

### Tutorial No 06

Q1) Evaluate the given double integrals.

I.  $\int_0^1 \int_0^x x^2 + y^2 \, dy \, dx$

II.  $\int \int_R (x + y) \, dy \, dx$  Where  $R$  is the region bounded by  $x = 0, x = 3, y = 2x, y = x + 2$

III.  $\int \int_R 2xy \, dy \, dx$  Where  $R$  is the quadrant of the circle  $x^2 + y^2 = 4$  such that  
 $x \geq 0$  and  $y \geq 0$

IV.  $\int_0^2 \int_{y^2}^y (2 + xy^2) \, dx \, dy$

Q2) Evaluate the following integrals.

$$\int_0^{\pi/2} \int_{a(1-\cos\theta)}^a r^2 \, dr \, d\theta$$

Q3) Find the area enclosed by the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$ .

Q4) Find the area enclosed by the curve  $y^2 = 2(2 - x)$  and  $y = 2 - x$  straight line.

Q5) Find the area between the circle  $x^2 + y^2 = 4$  and the straight line  $y = -x + 2$  in the first quadrant.

Q6) Evaluate the following triple integrals.

I.  $\int \int \int_R x + y + z \, dx \, dy \, dz$  where  $R: 0 \leq x \leq 1, 1 \leq y \leq 2, 2 \leq z \leq 3$

II.  $\int_0^{\ln 2} \int_0^x \int_0^{x+y} e^{x+y+z} \, dz \, dy \, dx$

Q7) Find the volume of the tetrahedron bounded by the planes  $x = 0, y = 0, z = 0$  and  $x + y + z = 4$ .

Q8) Find the constant  $c$  such that the function

$$f(x) = \begin{cases} cx^2; & 0 < x < 4 \\ 0; & \text{otherwise} \end{cases} \quad \text{is a probability density function and compute}$$

$\Pr(1 < X < 2)$ .

Q9) A random variable  $X$  follows the probability density function

$$f(x) = \frac{c}{4+x^2} \text{ where } -\infty < x < \infty.$$

- I. Find the value of  $c$ .
- II. Find the probability that  $X^2$  lies between the values 1 and 4.

Q10) Consider the probability density function given.

$$f(x) = \begin{cases} cx^2; & 1 \leq x \leq 2 \\ cx; & 2 < x < 3 \\ 0; & \text{otherwise} \end{cases}$$

- I. Find the constant  $c$
- II.  $\Pr(X > 2)$
- III.  $\Pr\left(\frac{1}{2} < X < \frac{3}{2}\right)$