CM 2607 Advanced Mathematics for Data Science

Tutorial No 03

Q1) Find $\frac{dy}{dx}$ for the function $y = 3x^3 - 2x + 7$ and evaluate at x= -1. Interpret the result.

Q2) Calculate the gradient of the straight line joining the points $(1,1) and\ (-3,-15)$. Compare the result with the first derivative of the function $y = 4x - 3$ at any arbitrary point.

Q3) Consider the function $f(x) = 2x^3 - 3x^2 + 6x - 3$.

I. Find $f'(x)$
II. Show that the derivative function of $f(x)$ has its minimum when $x = 0.5$.
III. Find the minimum value of $f'(x)$
IV. Find the equation of the tangent line drawn to the function $f(x)$ at $x = -1$.
V. Find the equation of the normal line drawn to the function $f(x)$ at $x = -1$.
VI. Find the equation of the normal line to the curve $f(x)$ at $x = 0$.

Q4) Consider the parametric function given.

$$x = sin\, t\left(e^{cos\, t} - 2\, cos\, 4t - sin^5\left(\frac{t}{12}\right)\right)$$

$$y = cos\, t\left(e^{cos\, t} - 2\, cos\, 4t - sin^5\left(\frac{t}{12}\right)\right)$$

I. Find $\frac{dy}{dx}$
II. Plot the parametric function for $t = [0,12\pi]\ as\ \delta t = 1\ /\ 16$.

Q5) Find the critical points of the functions given and determine the nature of the critical points.

I. $y = 2x + e^{-2x}$
II. $y = x^3$
III. $y = \frac{1}{3}x^3 + 2x^2 + 7$
IV. $y = x^3 - x$
V. $y = 3x^4 + 4x^3 - 12x^2 + 60$

Q6) Determine the Hessian matrix for the following functions.

Self-study: Discuss how you can use the Hessian matrix to determine the nature of the critical points. Plot the functions.

I. $f(x,y) = x^2 + y^2$
II. $f(x,y) = -x^2 - y^2$
III. $f(x,y) = x^2 - y^2$
IV. $f(x,y) = 2x^2 + 5xy + 3y^2$
V. $f(x,y) = 3x^2 + 7xy + 5y^2$
VI. $f(x,y) = -3x^2 + 7xy - 5y^2$
VII. $f(x,y) = 4 + x^3 + y^3 - 3xy$

Q7) **Challenge: Self-Study**: Explain about the statements given below.

I. $x = a$ is a root of the function $y = f(x)$.
II. There is a root for the function $y = f(x)$ in between $x = x_1, and\ x = x_2$.
III. Plot the function $f(x) = x^3 + 2x^2 - 5x - 1$ over $[-4,4]$.
IV. Using the above two results show that, there exists a root for the function $f(x) = x^3 + 2x^2 - 5x - 1$ in between $x = 1\ and\ x = 2$.
V. Study about the iterative methods to approximate the root lies between $x = 1\ and\ x = 2$ for the function $f(x) = x^3 + 2x^2 - 5x - 1$.
VI. Develop an algorithm to approximate the root for iv) using the selected iterative method.
VII. Explain the results to the class. Make sure that you have a well-prepared summary to present to the audience. It may be an individual work or group work. If it is a groupwork make sure to mention the team members.