CM 2607 Advanced Mathematics for Data Science

Tutorial No 09

1.

Consider the following function:

$$f(x) = \begin{cases} 1, & 0 < x \leq \pi \\ 0, & -\pi \leq x \leq 0 \end{cases}$$

a) Is this an odd function or an even function?
b) Which terms of the Fourier series of this function would you expect to be zero?
c) Calculate the Fourier series coefficients for this function.
d) Write down the Fourier series for this function for the first five terms (up to a_4)

2.

Consider the following function:

$$f(x) = \begin{cases} x, & 0 < x \leq \pi \\ -x, & -\pi \leq x \leq 0 \end{cases}$$

a) Is this an odd function or an even function?
b) Which terms of the Fourier series of this function would you expect to be zero?
c) Calculate the Fourier series coefficients for this function.
d) Write down the Fourier series for this function for the first five terms (up to a_4)

3.

Consider the following function:

$$f(x) = \begin{cases} 1 - x, & -1 < x \leq 0 \\ x, & 0 \leq x \leq 1 \end{cases}$$

a) Is this an odd function or an even function?
b) Which terms of the Fourier series of this function would you expect to be zero?
c) Calculate the Fourier series coefficients for this function.
d) Write down the Fourier series for this function for the first five terms (up to a_4)

4.

Consider the following function:

$$f(x) = \begin{cases} 0, & -1 \leq x < 0 \\ x, & 0 \leq x \leq 1 \end{cases}$$

a) Is this an odd function or an even function? Would any of the terms of this Fourier series be zero?
b) Calculate the Fourier series coefficients for this function.
c) Write down the Fourier series for this function for the first five terms (up to a_4)

5.

Consider the following function:

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 0, & 0 \le x < \frac{\pi}{2} \\ 1, & \frac{\pi}{2} \le x \le \pi \end{cases}$$

a) Calculate the coefficients of the Fourier sine series for this function.
b) Write the Fourier sine series for this function up to the first 5 terms.
c) Calculate the coefficients of the Fourier cosine series for this function.
d) Write the Fourier cosine series for this function up to the first 5 terms.
e) Calculate the coefficients for the Fourier series of this function.
f) What are the differences between the Fourier sine series, Fourier cosine series, and the Fourier series of the function?

6.

a) Give a function that would be represented accurately with a Fourier cosine series. Justify why it would be represented accurately by a Fourier cosine series.
b) Calculate the Fourier cosine series for this function for up to five terms.
c) Calculate the Fourier series of this function for up to five terms and compare the result.

7.

You may use python for this question.

Consider the following function:

$$f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ x^2, & 0 < x \le \pi \end{cases}$$

a) Find the coefficients for the Fourier series of this function.
b) Find the Fourier series of this function for up to 10 terms (a_9 and b_9)
c) Plot this function.
d) Plot the Fourier series of this function up to:
    a. 5 terms
    b. 10 terms
    c. 100 terms