CM 2607 Advanced Mathematics for Data Science

Tutorial No 08

1. Find the n^{th} partial sum of the following series:
    a. Arithmetic series where a = 1.5, d = 0.5, for n = 10
    b. Arithmetic series where a = 0, d = -2, for n = 15
    c. Geometric series where a = 2, r = -1 for n = 20
    d. Geometric series where a = 1, r = 5, for n = 12
    e. $\sum_{r=1}^{5}(r+1)$
    f. $\sum_{r=1}^{10}(r^2-3r+5)$
    g. $\sum_{r=1}^{8}(r^3-3r)$
    h. $\sum_{r=1}^{10}\frac{2}{r^2+3r+2}$
    i. $\sum_{r=1}^{20}\frac{2}{r^2+2r}$
    j. $\sum_{r=1}^{15}\frac{3}{r^2+5r+6}$
2. Determine if the following series converge. Find their sum to infinity if they converge.
    a. Arithmetic series where a = 2, d = 0.5
    b. Arithmetic series where a = 0.1, d = 0
    c. Geometric series where a = 1, r = 0.5
    d. Geometric series where a = 2, r = -2.5
    e. $\sum_{r=1}^{\infty}(r^2-2r)$
    f. $\sum_{r=1}^{\infty}(r^3-3)$
    g. $\sum_{r=1}^{\infty}\frac{2}{n}$
    h. $\sum_{r=1}^{\infty}\frac{1}{r(r+1)}$
    i. $\sum_{r=1}^{\infty}\frac{n+1}{n+2}$
    j. $\sum_{r=1}^{\infty}\frac{\sin(r)+2}{n}$
3. For the given functions,
    - Calculate the Taylor series expansion around point x = a to three terms (up to the f”(x) term).
    - Calculate the value of the function at point x = b based on the Taylor series expansion
    - Calculate the true value of the function at point b
    - Compare to the result obtained by the given Taylor series expansion.

a. $f(x) = x^2 - 2x, \ \ a = 2, b = 5$

b. $f(x) = \sin\left(3x - \frac{\pi}{3}\right), a = \frac{\pi}{2}, b = \pi$

c. $f(x) = \ e^{-3x}, a = 1, b = 2$

d. $f(x) = \ x^3 - 3x^2 + 5, \quad a = 3, b = 5$

e. $f(x) = \frac{\sin(x+\pi)}{x}, \quad a = \pi, b = \frac{\pi}{6}$

4. Find the Maclaurin series expansion for the following functions for up to three terms. Use the expansion to determine the value of the function at point a. Compare the results with the true value.

a. $f(x) = \sin(x) + \cos(2x), \qquad a = \frac{\pi}{2}$

b. $f(x) = \frac{x}{x+2}, \quad a = 1$

c. $f(x) = xe^{-2x}, \quad a = 2$

d. $f(x) = x \cdot \sin(x) + 2, \ \ a = \pi$

e. $f(x) = \frac{5}{(x+3)^2} - x, \quad a = 2$

5. Consider the following function:

$$f(x) = \ x^2 \cdot \sin(0.5x)$$

a. Plot the function for $x = [5, -5]$

b. Find the Maclaurin series expansion of the given function for

i. 2 terms

ii. 5 terms

iii. 10 terms

c. Plot the Maclaurin series expansions that you have calculated for the same range.

You may use python for this question.

6. Consider the following function:

$$f(x) = x \cdot \sin\left(x - \frac{\pi}{3}\right)$$

a. Plot the function for $x = [10,20]$

b. Find the Taylor series expansion for the given function for

i. 2 terms

ii. 5 terms

iii. 10 terms

At $x = 15$.

c. Plot the Taylor series expansions that you have calculated for the same range.

You may use python for this question.