

## CM 2607 Advanced Mathematics for Data Science

### Tutorial No 01

Q1) Find the derivative of the given functions below and explain about the graphical explanation of the answer.

- I.  $y = x$
- II.  $y = -x$
- III.  $y = 2x$
- IV.  $y = \text{constant}$
- V.  $y = mx + c$  for two non zero constants  $m$  and  $c$

Q2) Find the first derivative of the following functions.

- I.  $-x^{0.5}$
- II.  $\sqrt{x}$
- III.  $\sqrt[4]{x}$
- IV.  $\frac{1}{\sqrt{x}} - 9x$
- V.  $4\sqrt{x^{-3}} + 3x^2 - 2$

Q3) Find the gradient of the graphs of each of the functions at given points when possible.

- I.  $x^3$  at  $x = -1, x = 0$  and  $x = 3$
- II.  $-2\sqrt{x}$  at  $x = 0$  and  $x = 9$
- III.  $x^2 - 4x + 9$  at  $x = 0, x = 4$  and  $x = 2$

Discuss how the sign of the gradient changes when  $x=0$  and  $x=4$

- IV.  $-x^2 + 4x - 9$  at  $x = 0, x = 4$  and  $x = 2$

Discuss how the sign of the gradient changes when  $x=0$  and  $x=4$

- V. **Self-Study activity:**

Search on the types of curves given and sketch them. Explain how the gradient varies in each type.

- a. Concave upward decreasing
- b. Concave upward increasing
- c. Concave downward decreasing

d. Concave downward increasing

Explain these using the two functions  $x^2 - 4x + 9$  and  $-x^2 + 4x - 9$  by considering different  $x$  values.

VI. Sketch the graph of  $y = |x|$ . Explain about the regions where the gradient of the function  $y = |x|$  exists and does not exist.

Q4) Consider the graph of the function  $y = 3x^3 + 2x^2 - 5x + 6$ .

- i) Find the gradient of the function  $y$ .
- ii) Find the gradient at  $x=0$
- iii) Determine at what point/points the function has a zero gradient.

Q5) Find at what point the functions  $y = 2x^2 - 12x + 12$  and  $y = x^2 - 10x + 31$  have the same gradient.

Q6) Consider the function  $y = ax^3 + bx^2 + cx + d$  where  $a, b, c$  and  $d$  are constants.

- i) Find the gradient of the function
- ii) Given that the graph of the function passes through the point  $(0,1)$  and gradients at  $x = 1, x = -2$ , and  $x = 0$  are 5, -2 and 2 respectively, find the values of  $a, b, c$  and  $d$ .
- iii) Calculate the gradient of  $y$  at  $x = 3$ .