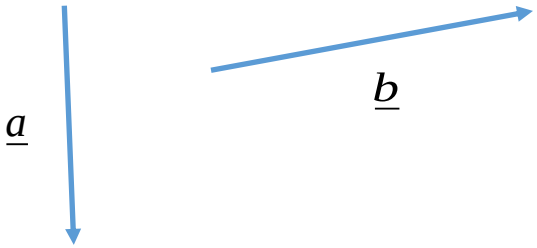
CM 1606 Computational Mathematics

Tutorial No 09

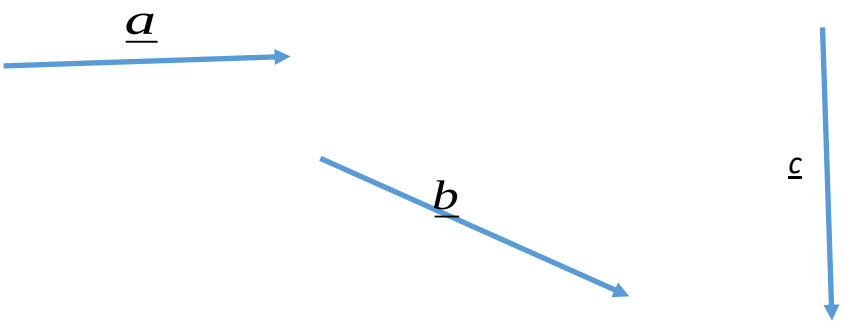
1) Use the vector addition laws to find the addition of given vectors.

i)



ii)

iii)



2) Identify the position vector for the points given and find the magnitude of each position vector. Sketch all the vectors in the same XOY plane.

$A(-3,4)$

$B(0,2)$

$C(-2,0)$

$D(3,-2)$

$E(4,5)$

3) Given that $\underline{a}=2\underline{i}+3\underline{j}-4\underline{k}$, $\underline{b}=-2\underline{i}+\underline{j}+\underline{k}$ and $\underline{c}=\underline{i}+\underline{j}$ find the following.

i) $-\underline{a}+2\underline{b}$

ii) $\underline{a}+0.5\underline{c}$

iii) $\underline{a}-\underline{b}+\underline{c}$

iv) $-2\underline{a}+3\underline{c}$

4) Find the scalar product between each pair of vectors given below.

i) $2i+3j-4k, 2i+j+k$

ii) $-i+3j+k, 2i-j$

iii) $i+3j, -3i+j+5k$

iv) $5i+4j-k, 4i+5j-k$

v) $0.5i+2j-k, 2i-3.5j$

5) Determine if the following vectors are perpendicular, parallel and same direction, parallel and opposite direction or neither.

i) $2i-j, -\frac{1}{2}i+\frac{1}{4}j$

ii) $6i-2j-k, 2i+5j+2k$

iii) $3i-4j+2k, 5j+2k$

iv) $3i-2j+k, 9i-6j+3k$

6) If $\vec{a}=2i-j+k$ and $\vec{b}=i+3j+2k$ find the following.

i) $\vec{a}.\vec{b}$

ii) $\vec{a}\times\vec{b}$

iii) $(2\vec{a}-\vec{b}).\vec{b}$

iv) $(\vec{a}-\vec{b}).\vec{a}$

v) $(\vec{a}-\vec{b}).(\vec{a}+\vec{b})$

vi) $(\vec{a}\times\vec{b}).3\vec{a}$

vii) $\vec{a}.(\vec{a}\times\vec{b})$

viii) $(\vec{a}\times\vec{b}).(\vec{b}\times\vec{a})$

7) Given that $\vec{a}=i-3j+2k$ and $\vec{b}=-2i+j+k$, find the direction cosines of the following.

i) $\vec{a}$

ii) $\vec{b}$

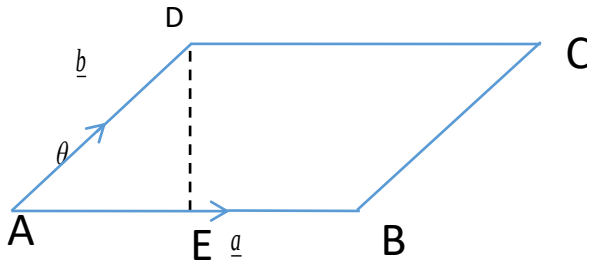
iii) $\vec{a}+2\vec{b}$

iv) $\vec{a}\times\vec{b}$

v) $\vec{b}\times\vec{a}$

8) Determine the total surface area and the volume of the parallelepiped where three adjacent lines are represented by $\underline{a} = \underline{i} + \underline{j} + \underline{k}$, $\underline{b} = 2\underline{i} - 3\underline{j}$ and $\underline{c} = -\underline{i} + 2\underline{j} - \underline{k}$.

Hint: For a parallelogram ABCD with the two vectors $\underline{a}$ and $\underline{b}$ along the adjacent sides AB and AD, its are is given by



Area of the parallelogram ABCD= $AB \times DE = |\underline{a}|\ |\underline{b}| \sin\theta = |\underline{a} \times \underline{b}|$

Volume of a parallelepiped:

For a parallelepiped where three adjacent lines are represented by three vectors $\underline{a}$, $\underline{b}$, $\underline{c}$ its volume is given by

$$\text{Volume} = \text{Height} \times \text{Area of the bottom}$$
$$= |\underline{a}|\ \cos\theta\ \ |\underline{b} \times \underline{c}|$$
$$= |\underline{a}|\ \ |\underline{b} \times \underline{c}| \cos\theta = \underline{a} \cdot (\underline{b} \times \underline{c})$$

9) Write the following using summation convention

$i)(x^1)^1 + (x^1)^2 + (x^1)^3 + \ldots\ldots\ldots + (x^1)^n$

$ii)(x^1)^3 + (x^2)^3 + (x^3)^3 + \ldots\ldots\ldots + (x^n)^3$

10) Write the tensor contained in $x_{pq}.x_{qr}$ if n=2