ROBERT GORDON UNIVERSITY ABERDEEN

# Assessment Brief - Coursework

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| **Academic Year** | **2023-24** |
| **Semester** | **1** |
| **Module Number** | **CM2607** |
| **Module Title** | **Advanced Mathematics for Data Science** |
| **Assessment Method** | **Coursework** |
| **Deadline (time and date)** | **17th December 2023, 12:00 p.m. (IST)** |
| **Submission** | **Assessment Dropbox in the Module Study Area in Campus Moodle.** |
| **Word Limit** | **N/A** |
| **Use of Generative Artificial Intelligence (AI) text** | **IS NOT authorised** |
| **Module Co-ordinator** | **Prashan Rathnayaka.** |

| What knowledge and/or skills will I develop by undertaking the assessment? |
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| *Differentiation, Integration, sequences, and series with Fourier family with applications to images in python.* |
| **On successful completion of the assessment students will be able to achieve the following Learning Outcomes:**<br>1. *Convert mathematical functions into programmable code* |
| **Please also refer to the Module Descriptor, available from the module Moodle study area.** |

## What is expected of me in this assessment?

### Task(s) – content

**Question 1:**

Given the following periodic function with period $2\pi$

$$f(x) = \begin{cases} x^2 - 1 & ; -\pi \leq x < 0 \\ xe^{-\frac{x}{2}} & ; 0 \leq x \leq \pi \end{cases}$$

a) Obtain the plot for the periodic function $f(x)$ over the domain $[-2\pi, 2\pi]$.
b) Find its Fourier series representation.
c) Plot the obtained Fourier series expansion in part b) showing.
   a. Its first harmonic (i.e., up to terms $a_1$ and $b_1$)
   b. Up to its fifth harmonic (up to terms $a_5$ and $b_5$)
   c. Up to its $150^{th}$ harmonic (up to terms $a_{150}$ and $b_{150}$)
d) Calculate the root mean square error (RMSE) between $f(x)$ and
   a. Its first harmonic
   b. Up to its $5^{th}$ harmonic
   c. Up to its $150^{th}$ harmonic

   Over the domain $[-4\pi, 4\pi]$.
   Plot the RMSE for the $0^{th}$ to $150^{th}$ harmonic in this range. Discuss your results.

**Question 2:**

Demonstrate aliasing in DFT. You can use **np.fft** (NumPy's fast Fourier transform implementation) for your demonstration.
You may use any function for your demonstration. You will be marked on the choice of function and choice and sampling rate, and your explanation for why aliasing occurs in the situation you have chosen.

**Question 3:**

a) Plot the function $\boldsymbol{f(x) = x \sin\left(\frac{x}{4}\right)}$ over the domain $[-6\pi, 3\pi]$.
b) Write the Taylor series expansion for the function $\boldsymbol{f(x) = \sin x}$ at $x = \frac{\pi}{4}$.
c) Plot the result **b)** for up to the first 50 terms of the series.
d) Find an approximation for $\left\{\frac{\pi}{2}\sin\left(\frac{\pi}{4}\right)\right\}$ using the Taylor series obtained in part **b)** and discuss the deviation of this approximation from its actual value using an appropriate measurement.

**Question 4:**

Consider the image given. (Attached in the CW description on Moodle)
a) Find the edges in the image using a 2D Fourier transform.
b) Apply a Gaussian blur to the original image using a 2D Fourier transform.
c) Apply DCT to the original image. Scale image to $\mathbf{240px \times 240px}$ using DCT.
d) Reproduce the common artifacts (ringing and blocking) that occur when an image is compressed.

## What is expected of me in this assessment?

**Question 5:**

Consider the logistic function, given below.

$$f(x) = \frac{1}{1 + e^{-x}}$$

a) Plot the logistic function $f(x)$ with respect to $x$.
b) Plot the derivative of logistic function $f(x)$ with respect to $x$.

c) Plot the functions below with respect to $x$.
   a. $\tan\{\sin^2(3x)\}$
   b. $-x^4 + 2x^2 - 0.5x + 10$
   c. $e^{-x \ln x^2}$
   d. $x^2 \sin(\sin 3x) - 3\cos\left(\cos\left(2x - \frac{\pi}{3}\right)\right)$

d) The logistic function is used in many applications to limit the output to [0,1]. Apply the logistic function to the above functions and plot the result.

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### Task(s) - format

- *Answer all questions.*
- *You may use python with any libraries that you like to answer these questions.*
- ***The assignment should be written using a Python notebook and submitted in the. ipynb file format.***

## How will I be graded?

A number of subgrades will be provided for each criterion on the feedback grid which is specific to the assessment.

The overall grade for the assessment will be calculated using the algorithm below*.

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|---|---|
| **A** | At least 50% of the subgrades to be at Grade A, at least 75% of the subgrades to be at Grade B or better, and normally 100% of the subgrades to be at Grade C or better. |
| **B** | At least 50% of the subgrades to be at Grade B or better, at least 75% of the subgrades to be at Grade C or better, and normally 100% of the subgrades to be at Grade D or better. |
| **C** | At least 50% of the subgrades to be at Grade C or better, and at least 75% of the subgrades to be at Grade D or better. |
| **D** | At least 50% of the subgrades to be at Grade D or better, and at least 75% of the subgrades to be at Grade E or better. |

| How will I be graded? | |
|---|---|
| **E** | At least 50% of the subgrades to be at Grade E or better. |
| **F** | Failing to achieve at least 50% of the subgrades to be at Grade E or better. |
| **NS** | Non-submission. |

*If the word count is above the specified word limit by more than 10% or the submission contains an excessive use of text within tables, the grade for the submission will be reduced to the next lowest grade.

# Feedback grid

| GRADE | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| **DEFINITION / CRITERIA (WEIGHTING)** | **EXCELLENT** Outstanding Performance | **COMMENDABLE/VERY GOOD** Meritorious Performance | **GOOD** Highly Competent Performance | **SATISFACTORY** Competent Performance | **BORDERLINE FAIL** | **UNSATISFACTORY** Fail |
| **Question 1**<br><br>**(Weight 01)** | The student's plot of the periodic function is exceptionally clear, accurate, and well-executed. It effectively captures the behaviour of the function within the specified interval and exhibits a high level of precision. The plot provides a comprehensive visual representation of the function, meeting or exceeding the expected standards. | The student's plot of the periodic function is clear and accurate, and executed. It effectively captures the behaviour of the function within the specified interval and exhibits a high level of precision. The plot provides a comprehensive visual representation of the function | The student's plot of the periodic function is good and executed. It effectively captures the behaviour of the function within the specified interval and exhibits a high level of precision. The plot provides a comprehensive visual representation of the function | The student's plot of the periodic function is good and executed. but it does not capture the behaviour of the function within the specified interval and exhibits a high level of precision. | The student's plot of some other period function. | The student's does not plot of the required periodic function. |
| **Question 2**<br><br>**(Weight 01)** | Selected a highly relevant function that clearly exhibits aliasing when sampled at a specific rate. The chosen sampling rate is meticulously chosen to produce a compelling illustration of aliasing in the DFT. The DFT implementation using NumPy's `np.fft` is flawless and well- | Selected a relevant function that clearly exhibits aliasing when sampled at a specific rate. The chosen sampling rate is meticulously chosen to produce a compelling illustration of aliasing in the DFT. The DFT implementation using NumPy's `np.fft` is flawless and well-documented. Additionally, the student provides a clear and | Selected a function that clearly exhibits aliasing when sampled at a specific rate. The chosen sampling rate is meticulously chosen to produce a compelling illustration of aliasing in the DFT. The DFT implementation using NumPy's `np.fft`. Additionally, the student | Selected a function that exhibits aliasing when sampled at a specific rate. The chosen sampling rate is meticulously chosen to produce a compelling illustration of aliasing in the DFT. The DFT implementation using NumPy's `np.fft`. Additionally, the student provides a explanation of | Selected a function is not aliasing when sampled at a specific rate. Based on the selection the student provides some explanation of why aliasing occurs in the chosen scenario. | Selected a function is not aliasing when sampled at a specific rate and no explanation of why aliasing occurs in the chosen scenario. |

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| | documented. Additionally, the student provides an exceptionally clear and insightful explanation of why aliasing occurs in the chosen scenario, effectively connecting the concept of aliasing to the chosen function and sampling rate. | insightful explanation of why aliasing occurs in the chosen scenario, effectively connecting the concept of aliasing to the chosen function and sampling rate. | provides a explanation of why aliasing occurs in the chosen scenario, effectively connecting the concept of aliasing to the chosen function and sampling rate. | why aliasing occurs in the chosen scenario, effectively connecting the concept of aliasing to the chosen function and sampling rate. | | |
| **Question 3**<br><br>**(Weight 01)** | The student's work demonstrates exceptional excellence in all aspects of the task. They accurately plot the function over the specified domain, providing a clear representation of its behaviour.<br><br>- The plotted result of the Taylor series expansion showcases a thorough understanding of convergence, with up to 50 terms effectively illustrating the approximation's convergence to the actual function.<br><br>- Overall, the work is exceptionally well-organized, free of errors, | - The student's work demonstrates a commendable level of proficiency in most aspects of the task. They accurately plot the function over the specified domain, providing a clear representation of its behaviour.<br><br>- The Taylor series expansion appropriately written, including a substantial number of terms. While the terms are generally accurate, there may be minor errors or omissions in the expansion.<br><br>- The plotted result of the Taylor series expansion visually portrays the convergence of the series reasonably well, although there might be some room for | - The student's work displays a commendable level of competence in addressing the task's requirements. They successfully plot the function over the specified domain, providing a reasonable representation of its behaviour.<br><br>- The Taylor series expansion appropriately written, although it may lack some terms or depth compared to higher-performance levels.<br><br>- The plotted result of the Taylor series expansion demonstrates a basic understanding of convergence, although | - The student's work demonstrates a basic level of competence in addressing the task's requirements. They successfully plot the function over the specified domain, providing a representation of its behaviour.<br><br>- The Taylor series expansion is attempted, but it may lack some terms or detail compared to higher-performance levels.<br><br>- The plotted result of the Taylor series expansion shows a basic understanding of convergence, although | - The student's work falls below the expected standards and demonstrates limited competence in addressing the task's requirements.<br><br>- The plot of the function may be missing, inaccurate, or incomplete.<br><br>- The Taylor series expansion is either not attempted or significantly lacking in terms of terms, accuracy, or completeness.<br><br>- If a plot of the Taylor series expansion is provided, it may not effectively demonstrate convergence or may have | - The student's work is significantly below the expected standards and fails to address the task's requirements.<br><br>- There may be a complete absence of the plot for the function, or the plot may be entirely inaccurate.<br><br>- The Taylor series expansion is not attempted or is entirely absent.<br><br>- If a plot of the Taylor series expansion is provided, it does not effectively demonstrate convergence, may be highly inaccurate, or is missing altogether. |

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| | and effectively communicates complex mathematical concepts. | improvement in terms of clarity.<br><br>- Overall, the work is well-organized and effectively communicates mathematical concepts, with only minor errors or areas for improvement. | there might be room for improvement in clarity and presentation.<br><br>- Overall, the work is adequately organized and communicates mathematical concepts effectively, with some room for refinement and improvement. | there may be room for improvement in clarity or presentation.<br>- Overall, the work is organized to a satisfactory degree and communicates some mathematical concepts, although there is notable room for refinement and improvement." | substantial issues in presentation.<br>- Overall, the work does not meet the minimum standards for the task, lacks organization, and communicates mathematical concepts inadequately." | - Overall, the work is disorganized, incomplete, and fails to communicate mathematical concepts effectively, falling significantly short of the minimum standards for the task." |
| **Question 4**<br><br>**(Weight 01)** | - Accurately and effectively applied 2D Fourier transform to find edges in the image.<br>- Successfully applied a Gaussian blur to the original image using a 2D Fourier transform with minimal loss of image quality.<br>- Accurately applied DCT to the original image.<br>- Resized the image to 240px×240px using DCT with minimal loss of image quality.<br>- Successfully reproduced common artifacts (ringing and blocking) with clear explanations and insights. | - Applied 2D Fourier transform to find edges in the image with minor issues.<br>- Applied Gaussian blur to the original image using a 2D Fourier transform with some minor quality loss.<br>- Applied DCT to the original image with minor issues.<br>- Resized the image to 240px×240px using DCT with some minor loss of image quality.<br>- Reproduced common artifacts (ringing and blocking) with clear explanations, but with some minor omissions or inaccuracies. | - Applied 2D Fourier transform to find edges in the image with noticeable issues.<br>- Applied Gaussian blur to the original image using a 2D Fourier transform with significant quality loss.<br>- Applied DCT to the original image with noticeable issues.<br>- Resized the image to 240px×240px using DCT with significant loss of image quality.<br>- Reproduced common artifacts (ringing and blocking) with some explanations, but with notable inaccuracies or omissions. | - Applied 2D Fourier transform to find edges in the image with major issues or inaccuracies.<br>- Applied Gaussian blur to the original image using a 2D Fourier transform with substantial quality loss.<br>- Applied DCT to the original image with major issues or inaccuracies.<br>- Resized the image to 240px×240px using DCT with significant and noticeable loss of image quality.<br>- Attempted to reproduce common artifacts (ringing and blocking) but with major inaccuracies or incomplete explanations. | - Applied 2D Fourier transform to find edges in the image with critical errors or a complete lack of understanding.<br>- Applied Gaussian blur to the original image using a 2D Fourier transform with severe quality degradation or incorrect application.<br>- Applied DCT to the original image with critical errors or a complete lack of understanding.<br>- Resized the image to 240px×240px using DCT with severe and unacceptable loss of image quality.<br>- Attempted to reproduce common artifacts (ringing and | Did not attempt to apply 2D Fourier transform or any of the specified tasks.<br>- Demonstrated a complete lack of understanding of the concepts involved.<br>- Failed to reproduce common artifacts or provided no relevant information. |

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| | | | | | blocking) but with critical inaccuracies or no clear explanations. | |
| **Question 5**<br><br>**(Weight 01)** | - Accurately plotted the logistic function and its derivative.<br>- Accurately plotted all four specified functions with clear, well-labelled graphs.<br>- Accurately applied the logistic function to the specified functions and plotted the results with clear and well-labelled graphs. | - Plotted the logistic function and its derivative with minor issues or slight inaccuracies.<br>- Plotted all four specified functions with some minor issues or slight inaccuracies.<br>- Applied the logistic function to the specified functions and plotted the results with minor issues or slight inaccuracies. | - Plotted the logistic function and its derivative with noticeable issues or inaccuracies.<br>- Plotted all four specified functions with noticeable issues or inaccuracies.<br>- Applied the logistic function to the specified functions with noticeable issues or inaccuracies. | - Plotted the logistic function and its derivative with significant issues or inaccuracies.<br>- Plotted all four specified functions with significant issues or inaccuracies.<br>- Applied the logistic function to the specified functions with significant issues or inaccuracies. | - Attempted to plot the logistic function and its derivative but with critical errors or a complete lack of understanding.<br>- Attempted to plot the specified functions but with critical errors or a complete lack of understanding.<br>- Attempted to apply the logistic function to the specified functions but with critical errors or a complete lack of understanding. | - Did not attempt to plot the logistic function, its derivative, or the specified functions.<br>- Demonstrated a complete lack of understanding of the concepts involved.<br>- Failed to apply the logistic function to the specified functions or provided no relevant information. |

***Coursework received late will be regarded as a non-submission (NS) and one of your assessment opportunities will be lost.***

## What else is important to my assessment?

### What is the Assessment Word Limit Statement?

It is important that you adhere to the Word Limit specified above. The Assessment Word Limit Statement can be found in Appendix 2 of the RGU Assessment Policy. It provides detail on the purpose, setting and implementation of wordage limits; lists what is included and excluded from the word count; and the penalty for exceeding the word count.

**What's included in the word count?**

The table below lists the constituent parts which are included and excluded from the word limit of a Coursework; more detail can be found in the full Assessment Word Limit Statement. Images will not be allowed as a mechanism to circumvent the word count.

| Excluded | Included |
|---|---|
| Cover or Title Page | Main Text e.g. Introduction, Literature Review, Methodology, Results, Discussion, Analysis, Conclusions, and Recommendations |
| Executive Summary (Reports) or Abstract | Headings and subheadings |
| Contents Page | In-text citations |
| List of Abbreviations and/or List of Acronyms | Footnotes (relating to in-text footnote numbers) |
| List of Tables and/or List of Figures | Quotes and quotations written within "…" |
| Tables – mainly numeric content | Tables – mainly text content |
| Figures | |
| Reference List and/or Bibliography | |
| Appendices | |
| Glossary | |

**What are the penalties?**

The grade for the submission will be reduced to the next lowest grade if:

- The word count of submitted work is above the specified word limit by more than 10%.
- The submission contains an excessive use of text within Tables or Footnotes.

## What else is important to my assessment?

### What is plagiarism?

Plagiarism is "the practice of presenting the thoughts, writings or other output of another or others as original, without acknowledgement of their source(s) at the point of their use in the student's work. All materials including text, data, diagrams or other illustrations used to support a piece of work, whether from a printed publication or from electronic media, should be appropriately identified and referenced and should not normally be copied directly unless as an acknowledged quotation. Text, opinions or ideas translated into the words of the individual student should in all cases acknowledge the original source" (RGU 2022).

### What is collusion?

"Collusion is defined as two or more people working together with the intention of deceiving another. Within the academic environment this can occur when students work with others on an assignment, or part of an assignment, that is intended to be completed separately" (RGU 2022).

For further information please see Academic Integrity.

### What if I'm unable to submit?

- The University operates a Fit to Sit Policy which means that if you undertake an assessment then you are declaring yourself well enough to do so.
- If you require an extension, you should complete and submit a Coursework Extension Form. This form is available on the RGU Student and Applicant Forms page.
- Further support is available from your Course Leader.

### What additional support is available?

- RGU Study Skills provide advice and guidance on academic writing, study skills, maths and statistics and basic IT.
- RGU Library guidance on referencing and citing.
- The Inclusion Centre: Disability & Dyslexia.
- Your Module Coordinator, Course Leader and designated Personal Tutor can also provide support.

### What are the University rules on assessment?

The University Regulation 'A4: Assessment and Recommendations of Assessment Boards' sets out important information about assessment and how it is conducted across the University.