Measuring the Astronomical Unit by Analyzing Redshift in Spectra of Distant Stars

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1 Introduction

The challenge for the 2020 United States Association of Young Physicists' Tournament is to measure the length of the Astronomical Unit. The Astronomical Unit (AU) is the average distance from the Earth to the Sun. While the problem statement suggests deriving the AU by observing the transit of Venus, I used a different approach. I decided to look at redshift data from the spectra of a distant star to determine the Earth's orbital velocity around the Sun and then apply this to calculate the AU.

2 Identifying the Reference Star

I first needed to identify a star to use for the analysis. The target star needed to have two spectral observations spaced six months apart, with one observation taken when Earth is moving towards the star, and one when Earth is moving away. Using two observations spaced six months apart yields the largest velocity difference between the two observations in the direction of the reference star. Additionally, both spectral observations must be taken by the same instrument with the same wavelength range. To find a star and spectra that would suit my needs, I ran a SQL query on NASA's Exoplanet Archive's Celestial Body database, which had spectral observations at the precision I needed. The query identified one star, HD185144, that met the requirements.

3 Data Reduction

The spectral measurements for HD185144 were taken by the HIRES instrument at the Keck Observatory in Hawaii. The spectra were recorded with an echelle grating, a type of diffraction grating that disperses light into a 2D laddered pattern as shown in Figure 1 on Page 2. In order to measure redshift I needed to reduce the echelle spectrographs into graphs of Relative Flux vs. Wavelength.

The next step was to convert the Makee (Mauna Kea Echelle Extraction) data reduction pipeline, originally written for Linux FORTRAN compilers, to work with a g77 compiler, in order to produce the graphs.

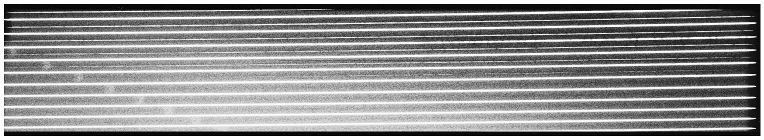

Figure 1: An echelle spectrograph taken by the HIRES Instrument

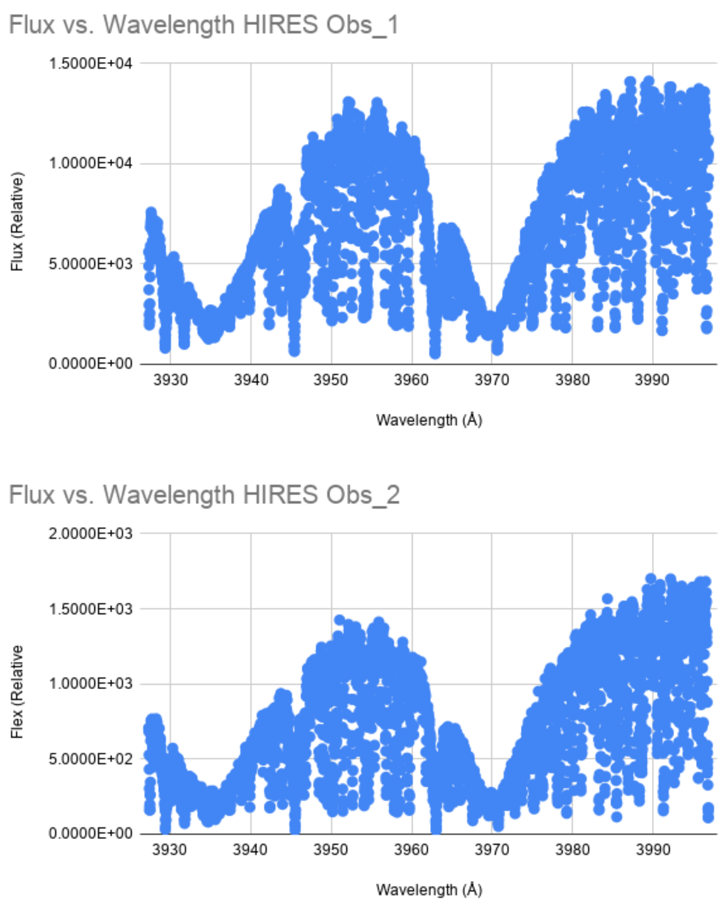

Figure 2: Reduced Spectral Graphs

4 Calculating the AU

After reducing the spectral data and graphing flux vs. wavelength to determine prominent spectral features, I was able to observe differences in the wavelength of the same spectral features between the two measurements. Using this difference, I calculated the redshift given in Equation 1, where λ_1 is the wavelength of a spectral feature in observation 1 and λ_2 is the wavelength of the same spectral feature in observation 2.

$$z = \frac{\lambda_2 - \lambda_1}{\lambda_1} \tag{1}$$

Our measurements yield a value of $z = 2.8923 \cdot 10^{-5}$.

Given that $z = \frac{v}{c}$, where $c =$ the speed of light and v is the velocity difference between two observations of an object with redshift z, $v = zc$. Thus, v_{HD185144}, the velocity difference between the Earth at observation 1, and observation 2, is given by the formula:

$$v_{\mathrm{HD185144}} = zc \tag{2}$$

The two spectra were captured on 6/24/2008 and 12/31/2009. At the times they were taken, the ecliptic longitudes of the Sun were 93.34748487° and 279.6390296°, respectively. If $\theta_{\mathrm{HD185144}} =$ HD185144's ecliptic longitude and $\theta_{\mathrm{Sun}} =$ the ecliptic longitude of the Sun, the formula to calculate the angle between the direction of the velocity of Earth and HD185144 on the ecliptic plane is given by the formula:

$$\theta_{\mathrm{offset}_{\mathrm{ecliptic}}} = \theta_{\mathrm{HD185144}_{\mathrm{lat}}} - (\theta_{\mathrm{Sun}} + 90) \tag{3}$$

Thus, $\theta_{\mathrm{offset}_{\mathrm{ecliptic}_{\mathrm{obs1}}}} = 153.0255239°$ and $\theta_{\mathrm{offset}_{\mathrm{ecliptic}_{\mathrm{obs2}}}} = 20.68293136°$, as shown in figures 3 and 4 on pages 3 and 4 respectively:

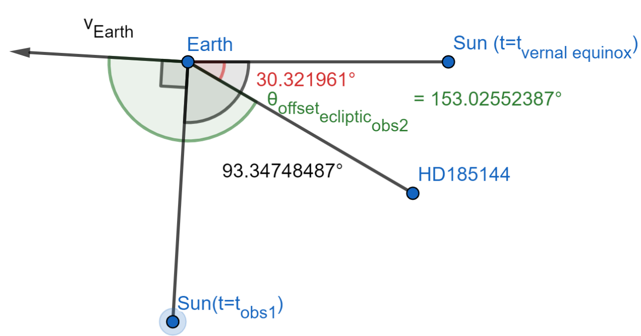

Figure 3: Diagram of $\theta_{\mathrm{offset}_{\mathrm{ecliptic}_{\mathrm{obs1}}}}$.
Note: Figure is not to scale, and is a visualization on the ecliptic plane.

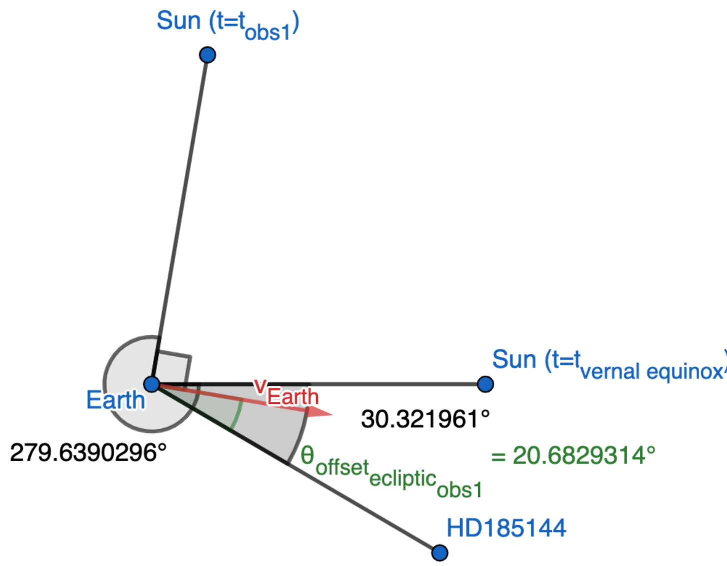

Figure 4: Diagram of $\theta_{\text{offset}_{\text{ecliptic}_{\text{obs2}}}}$.
Note: Figure is not to scale, and is a visualization on the ecliptic plane.

HD185144 also has an ecliptic latitude of 80.917843°. Thus, the velocity of the Earth's orbit around the Sun is simply:

$$v_{\text{Earth}} = \frac{(z \cdot c)}{\left(\cos\left(\theta_{\text{offset}_{\text{ecliptic}_{obs1}}}\right) - \cos\left(\theta_{\text{offset}_{\text{ecliptic}_{obs2}}}\right)\right) \cdot \cos\left(\theta_{\text{ecliptic}_{\text{lat}}}\right)} \tag{4}$$

Given that the period of the Earth's orbit, T_{Earth} is 31556952 seconds, and the Earth's orbit is circular, the radius r_{Earth} can be written as:

$$r_{\text{Earth}} = \frac{v_{\text{Earth}} T_{\text{Earth}}}{2\pi} \tag{5}$$

5 Results

Using the equations described above, I calculated an AU value of $1.5112836468 \cdot 10^{11}$ meters, with an error of 0.0102% compared to the actual AU.