Two Approaches to Forecasting GBP/USD: Molodtsova-Papell and Kalman Filters

Econ 409: Forecasting Asset Prices

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1 Introduction

For our final project we present two separate strategies for forecasting the British Pound to U.S. Dollar (GBP/USD) exchange rate in hopes of developing two different viable trading strategies. Firstly, we use a linear model — a modification of the Fisher Equation in line with the methodology followed by Moldotsova and Papell (2012) — to leverage any theoretical connection among exchange rates, inflation, and industrial production. Second, we use a Kalman filter to provide a non-linear estimate on the interest rate differential (IRD) between the U.S. and U.K.. Since the Kalman filter is a recursive estimation technique, we hope to leverage its real-time efficiency to produce estimates as data would come in at a daily frequency.

To simulate a real-world scenario as closely as possible, we segment our data into a training set (everything through the end of 2019) and a deployment set (2020 onwards). With this set-up, our model will in no way be influenced by data it would not have had access to in a real setting. Therefore, we expect our strategies to perform worse than much of what was presented in-class or in homework assignments as those were often trained on the entire data. We proceed by describing the modified Molodtsova-Papell (MP) strategy first, then follow with the modified Kalman strategy. Lastly, we present our forecasting metrics and discuss what improvements would have been made, time permitting. All figures (heatmaps & equity curves) are presented in the appendix (section 5.1).

2 Molodtsova-Papell Strategy with PCE

In this section, we present our first approach based on the model described by Molodtsova & Papell (2012), which utilizes the Taylor-rule fundamentals. However we make two primary changes: first, in order to estimate our model to forecast the GBP/USD exchange rate, we rely on the Personal Consumption Expenditures (PCE) price index, rather than the Consumer Price Index (CPI) traditionally employed in Taylor-rule approximations. The former is preferred by the U.S. Federal Reserve, and we chose it assuming that it would therefore align better with U.S. monetary policy decisions. Second, since GDP data is only produced quarterly, we utilize industrial production data, the Industrial Production Index (IPI) as a proxy.

We first bring all three sets of our data: the GBP/USD exchange rate, PCE, and IPI, up to time t, aggregating at the monthly level so that our strategy will update positions on the last day of each month. We then tune our hyperparameters, h_{us} and h_{uk}, as the window-sizes for the respective countries when calculating our output gap for industrial production (see section 5.2). We then utilized OLS to estimate the exchange rate change, and calculated the training error as mean squared error (MSE), which we used to pick our optimal window-sizes seen in Figure 2 as two months for both the UK and the US.

We then deployed our model on our deployment set, the data from 2020 onwards, utilizing our chosen hyperparameters, following the strategy we defined in class as follows:

- When $\Delta\hat{s}_{t+1} \geq 0$, we go long.
- Otherwise, when $\Delta\hat{s}_{t+1} < 0$, we go short.
- We only exit a position when an opposite signal is issued.

where $\Delta \hat{s}_{t+1}$ is the estimated log change in the GBP/USD exchange rate for time $t + 1$. The deployed strategy results in an equity curve which delivers us a final profit of approximately 5% as seen in figure 3.

In order to assess the performance of our model against the random-walk, we utilize the Diebold-Mariano (DM) and Clark-West (CW) tests. The test statistics are $T_{DM} \approx 0.11$ and $T_{CW} \approx 0.35$, respectively. Since $T_{DM} < 1.65$, we fail to reject the null hypothesis that our model does not outperform the random walk. Similarly, $T_{CW} < 1.65$, so we again fail to reject the null hypothesis that our model does not outperform the random walk. Ultimately, this model does not seem to perform well against the random walk, so this would not be a useful strategy to successfully forecast the GBP/USD exchange rate.

3 Kalman Filter Strategy

In this section, we present our second approach that implements a "manual" Kalman filter to estimate the IRD between the U.S. and the U.K. and applies the *Econ 409 Strategy* to forecast directional movements in the GBP/USD exchange rate. In particular, we build up the Kalman filter without using the exponentially weighted moving average function from Python's Pandas package. As far as tuning hyperparameters, we tune two elements of the Kalman filter: Process Noise (Q), and Measurement Noise (R), and one element of the trading strategy — the window size for our rolling mean/standard deviation (T). These chosen elements of the Kalman filter are essential in determining the Kalman gain K_t at any given time t. Details on the "manual" Kalman filter can be found in the appendix (section 5.3).

As previously mentioned, we apply the *Econ 409 Strategy* to determine when we go long or short on the GBP/USD exchange rate. Firstly, we define the "surprise" of our Kalman filter as the difference between the data and our prediction. The strategy relies on a series of hypothesis tests that compare the current mean $\bar{\varepsilon}_t$ (the mean of the surprises up to time t) against the sample mean $\hat{\mu}_t$ (the mean of the surprises on a rolling window of size T) The null hypotheses are that the current mean and sample mean at a given time t have opposite signs (if one is negative, then the other is positive and vice versa). We define our trading strategy entry positions based on failing to reject the null hypothesis at a given time t as follows:

- When $\bar{\varepsilon}_t < 0$ and $\hat{\mu}_t > 0$, we go long
- When $\bar{\varepsilon}_t > 0$ and $\hat{\mu}_t < 0$, we go short
- Otherwise, we take a neutral position and do nothing. Positions are state contingent such that we either go long, go short, or do nothing every period.

We tune our hyperparameters (Q, R, and T) for this strategy on the training set (Dec. 2003 - Dec. 2019) using the Sharpe Ratio as our performance metric. We see the results of the optimization process in figure 4 where it is clear that the optimal $Q^* \approx 0.046$ and $R^* \approx 0.17$. The optimal rolling window $T^* = 5$ was found by optimizing over the set {5, 10, 15, 20, 25, 30}.

We then apply the optimal hyperparameters $(Q^*, R^*, T^*) = (0.046, 0.17, 5)$ to our testing data (Jan. 2020 - Dec. 2023). The deployed strategy results in an equity curve which delivers us a final profit of approximately 11.3%, seen in figure 5.

Since this strategy provides directional forecasts, we test it using the Binomial test and Weighted Binomial test: the test statistics are $T_B \approx 1.68$ and $T_{WB} \approx 0.477$, respectively. Since, $T_B > 1.65$, we can reject the hypothesis that our directional forecasts are uncorrelated with the realized directional changes. This suggests that our directional forecasts may be successfully capturing the realized appreciation or depreciation of exchange rates. However, since $T_{WB} < 1.65$, we fail to reject the null that our directional forecasts are capturing the big movements of the realized appreciation or depreciation of exchange rates. In essence, our model does seem to capture some of the realized directional changes in the exchange rate; however, when weighted for big/small changes then this is no longer true.

4 Final Metrics

Ultimately, we can see in the table below that both of our strategies largely fail to outperform the HFRI. We see lower geometric average monthly returns, annualized returns, Sharpe-ratios, and a smaller percentage of winning months — generally a bad combination, indicating that these strategies were not at all effective. However, we can see that our strategies are "riskier" — with larger standard deviations and annualized standard deviations, and large max drawdowns, particularly for our Kalman filter. In this sense, we would typically want to see a higher payout for a riskier strategy, but this was not the case for our deployment set. Going forward, this could be an area that we would want to explore further in terms of setting different lengths for our training versus our deployment set, as potentially training on less or more data could better impact our returns.

In terms of the regression metrics, we can see that our values for beta are closer to 0 than the HFRI, indicating that our strategies are less correlated with the S&P 500. However, this is not a surprise, as since our strategies have poor performance, we would not expect them to be closely correlated with the market.

Given more time, we would like to focus our attentions on a number of potential modifications to these strategies. As mentioned before, we could better choose the size of the training data and try to determine more of an ideal length in order to achieve better performance. In addition, we could have generally considered a wider variety of values for the hyperparameters that we tuned — or choose alternative metrics on which to tune them. For example tuning our Molodtsova-Papell strategy on Sharpe-ratio instead of MSE. For our Molodtsova-Papell strategy in particular, we could have created a strategy that nowcasted PCE inflation rather than relying on its delayed release, as we were making a number of assumptions when we relied on the random walk to shift our data to corresponding time periods. This alternate approach could potentially have allowed us to respond more dynamically to new information, particularly as we see generally poor performance in the first half of our deployment set throughout 2020 — likely caused by the COVID-19 pandemic — and would have allowed us to capture these changes in real-time. Meanwhile for our Kalman-filter strategy, we could have considered a multivariate filter that predicts multiple variables simultaneously in order to develop a more robust strategy that would take into account other macroeconomic factors.

	HFRI5FWC	MP Strategy	Kalman Filter
Geo. Average Monthly	0.41	0.10	0.24
Std. Deviation	1.70	2.57	2.50
High Month	5.29	4.57	6.79
Low Month	-7.95	-5.99	-5.27
Annualized Return	5.02	1.48	2.98
Annualized STD	5.90	8.91	8.67
Sharpe Ratio	0.61	-0.02	0.17
% of Winning Mo.	65.65	50.00	62.50
Max Drawdown	20.72	97.08	167.88

	B3	MP Strategy	Kalman Filter
Alpha	0.15	0.06	0.03
Beta	0.31	0.04	0.21
Mnt. R-Squared	0.62	0.01	0.23
Correlation	0.79	0.08	0.48
Up Alpha	0.33	0.39	0.70
Up Beta	0.26	-0.03	0.05
Up R-Squared	0.34	0.00	0.01
Down Alpha	0.12	0.21	1.79
Down Beta	0.32	0.09	0.55
Down R-Squared	0.40	0.01	0.45

Figure 1: Strategy Performance Metrics

Note: Risk-Free Rate Used = 1.47%

5 Appendix

5.1 Figures

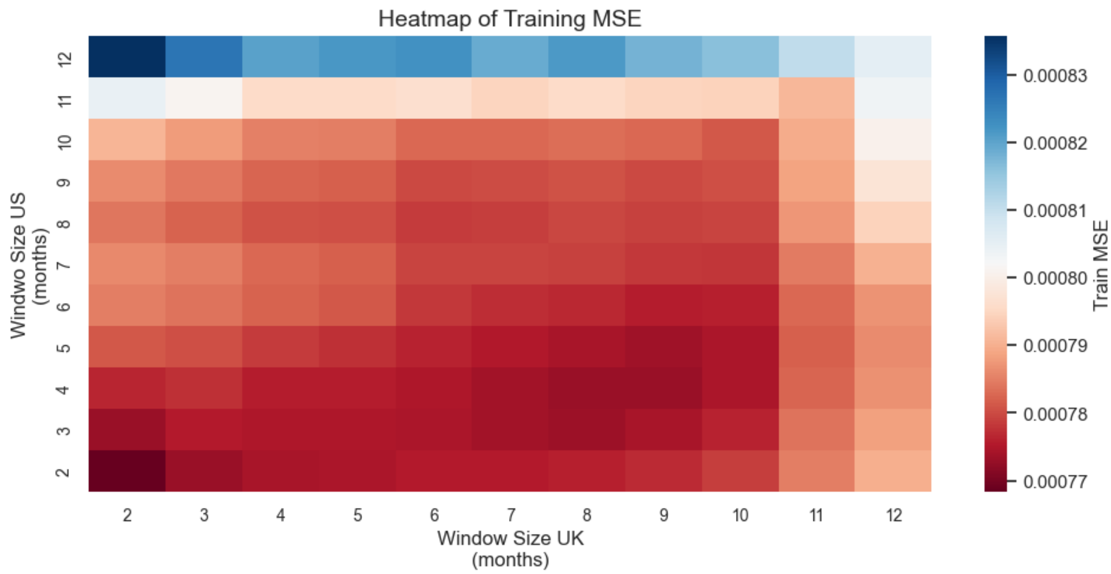

Figure 2: Heatmap for Molodtsova-Papell Strategy

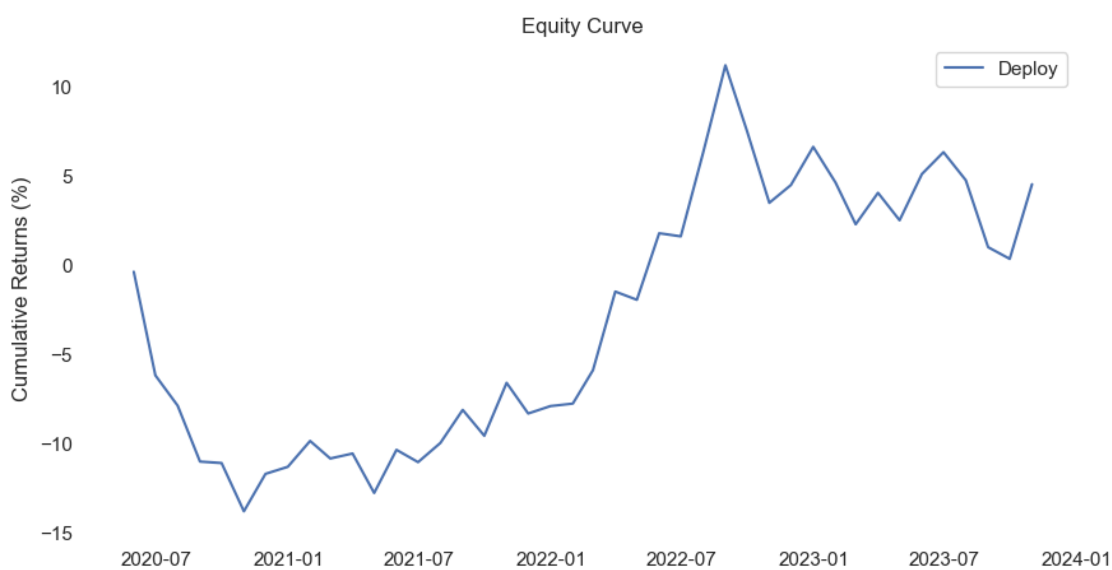

Figure 3: Equity Curve for Molodtsova-Papell Strategy

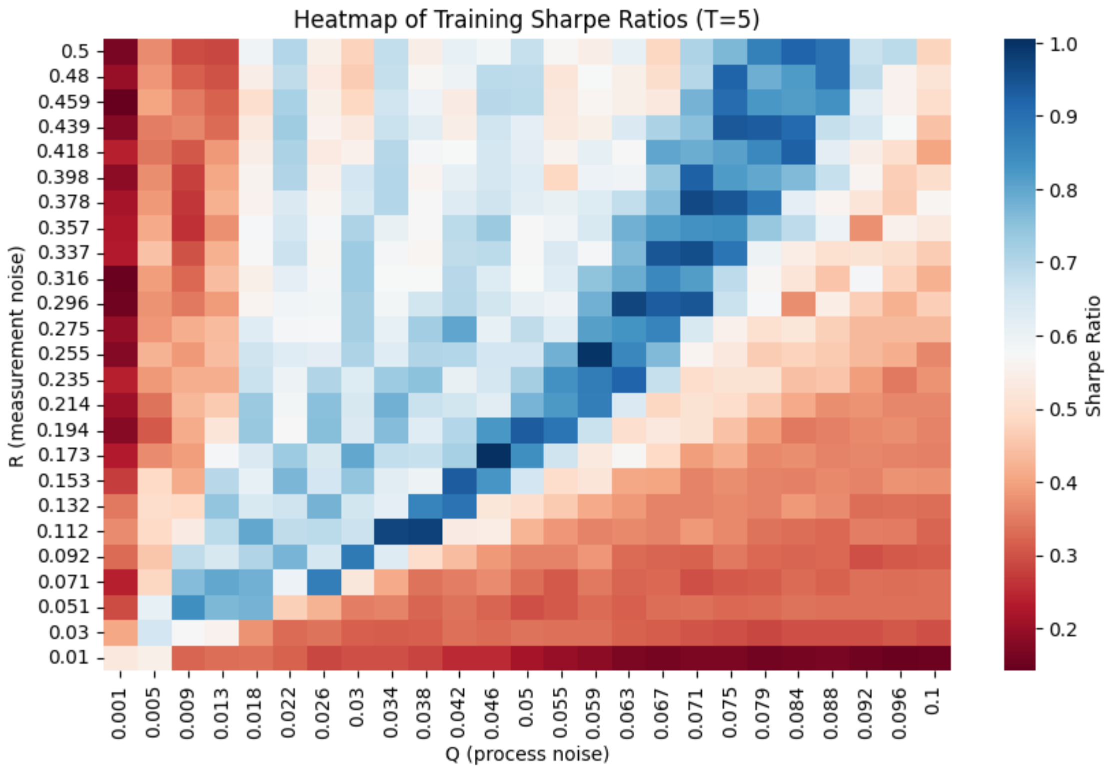

Figure 4: Heatmap for Kalman Filter Strategy (given $T^* = 5$)

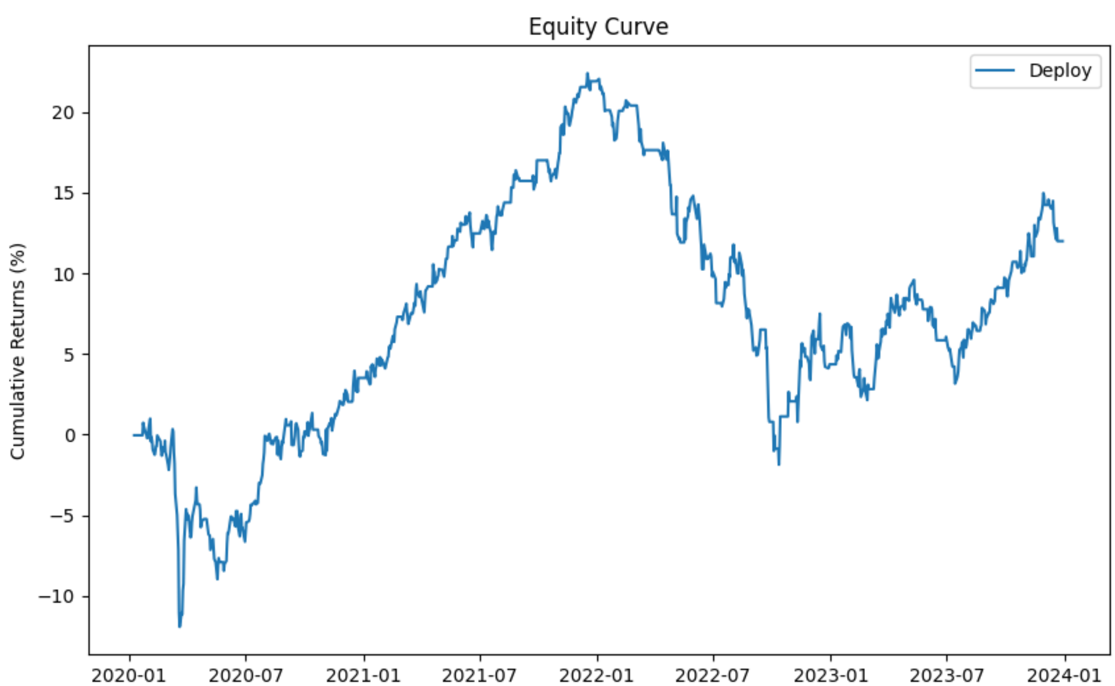

Figure 5: Equity Curve for Kalman Filter Strategy

5.2 Molodtsova-Papell Definition

We define our *Fisher Equation* regression as follows:

$$\Delta s_{t+1} = \beta_0 + \beta_1(\pi_t^{us} - \pi_t^{uk}) + \beta_2(\text{gap}_t^{us} - \text{gap}_t^{uk}) + \varepsilon_t$$

where $\Delta s_{t+1} \equiv s_{t+1} - s_t$ is the log difference in the GBP/USD exchange rate at time $t+1$, π_t^i is the PCE inflation index, and $\text{gap}_t^i \equiv y_t^i - \bar{y}_t^i$ is the output gap at time t for country i. Furthermore, y_t^i is the Industrial Production Index at time t for country i and

$$\bar{y}_t^i = h_i^{-1} \sum_{n=t-h_i+1}^{t} y_n^i$$

for $i \in \{us, uk\}$ such that $\bar{y}_t^i$ is the simple rolling mean over window length h_i (a hyper-paramter which is tuned in figure 2 for the U.S. and U.K. separately).

5.3 Kalman Filter Definition

We define our Kalman filters in two steps: Prediction and Update. In the **Prediction step** we have

$$\begin{aligned}\hat{x}_{t|t-1} &= \hat{x}_{t-1|t-1} \\ \hat{P}_{t|t-1} &= \hat{P}_{t-1|t-1} + Q\end{aligned}$$

where $\hat{x}_{t|t-1}$ is the predicted IRD given information up to time $t-1$ and $\hat{P}_{t|t-1}$ is the associated estimation error variance. Q is the process noise variance, i.e., the variance of our state variable (IRD). Using the results from the prediction step, we compute the Kalman gain as $K_t = \hat{P}_{t|t-1}^{-1}(\hat{P}_{t|t-1} + R)$ where R is the measurement variance. This then allows us to enter the **Update step**:

$$\begin{aligned}\hat{x}_{t|t} &= \hat{x}_{t|t-1} + K_t(z_t - \hat{x}_{t|t-1}) \\ \hat{P}_{t|t} &= (1 - K_t)\hat{P}_{t|t-1}\end{aligned}$$

where $\hat{x}_{t|t}$ is our updated prediction, z_t is the measurement of the IRD at time t, and $\hat{P}_{t|t}$ is the updated estimation error variance. These $\hat{x}_{t|t}$ and $\hat{P}_{t|t}$ are then used in subsequent estimations.

For our case, this approach differs from Pandas' exponentially weighted average in that we actually compute the optimal Kalman gain K_t for each time t rather than tuning it arbitrarily. Instead, our optimization occurs over the assumed values for Q and R which are not typically observable but assumed to come from a normal distribution. The beauty of this implementation is that it generalizes very nicely into high dimensions. We need only include a few matrices that allow us to consider a system of equations – allowing for multivariate estimation.

References

[1] Amir. Kalman filter with python code, January 2023. https://medium.com/@ab.jannatpour/kalman-filter-with-python-code-98641017a2bd.

[2] Tanya Molodtsova and David H. Papell. Taylor rule exchange rate forecasting during the financial crisis. *NBER International Seminar on Macroeconomics*, 9(1), 2012.