$$V=\frac{1}{2}m\omega^2x^2,\ \dot{q}_k=\frac{p_k}{m},\ \dot{p}_k=-\left.\frac{\partial V}{\partial x}\right|_{q_k}=-m\omega^2q_k$$

$$g_k=N\mathrm{exp}\left(\frac{1}{\hbar}\left(-\frac{1}{2}m\omega(x-q_k)^2+ip_k(x-q_k)\right)\right)=\exp\left(\frac{1}{\hbar}\left(-\frac{1}{2}m\omega x^2+\xi_kx+\eta_k\right)\right)$$

$$\xi_k=m\omega q_k+ip_k,\ \eta_k=\ln N-\frac{1}{2}m\omega q_k^2-iq_kp_k$$

$$\dot{\xi}_k=m\omega\dot{q}_k+i\dot{p}_k=\omega p_k-im\omega^2q_k=-i\omega(m\omega q_k+ip_k)=-i\omega\xi_k$$

$$\dot{\eta}_k=-m\omega q_k\dot{q}_k-i(\dot{q}_kp_k+q_k\dot{p}_k)=-\dot{q}_k(m\omega q_k+ip_k)-iq_k\dot{p}_k=-\frac{p_k\xi_k}{m}+im\omega^2q_k^2$$

$$\tau_{mk}=\langle g_m|\dot{g}_k\rangle=\frac{1}{\hbar}\mathbb{S}_{mk}\left(\dot{\eta}_k+\dot{\xi}_k\frac{\xi_k+\xi_m^*}{2m\omega}\right)=\frac{1}{\hbar}\mathbb{S}_{mk}\left(im\omega^2q_k^2-\frac{p_k\xi_k}{m}-i\frac{\xi_k^2+\xi_k\xi_m^*}{2m}\right)=$$

$$=\frac{1}{\hbar}\mathbb{S}_{mk}\left(im\omega^2q_k^2-\frac{i\xi_k\xi_m^*}{2m}-\frac{i\xi_k(\xi_k-2ip_k)}{2m}\right)=\frac{1}{\hbar}\mathbb{S}_{mk}\left(im\omega^2q_k^2-\frac{i\xi_k\xi_m^*}{2m}-\frac{i|\xi_k|^2}{2m}\right)=$$

$$=\frac{1}{\hbar}\mathbb{S}_{mk}\left(im\omega^2q_k^2-\frac{i\xi_k\xi_m^*}{2m}-\frac{i(m^2\omega^2q_k^2+p_k^2)}{2m}\right)=\frac{1}{\hbar}\mathbb{S}_{mk}\left(\frac{i}{2}m\omega^2q_k^2-\frac{ip_k^2}{2m}-\frac{i\xi_k\xi_m^*}{2m}\right)$$

$$\mathbb{H}_{mk}=\left\langle g_m\left|\hat{T}\right|g_k\right\rangle+\frac{1}{2}m\omega^2\left\langle g_m\left|x^2\right|g_k\right\rangle=-\frac{\hbar^2}{2m}\left\langle g_m\left|\frac{\partial^2}{\partial x^2}\right|g_k\right\rangle+\frac{1}{2}m\omega^2\left\langle g_k\left|x^2\right|g_m\right\rangle=$$

$$=-\frac{\hbar^2}{2m}\left\langle g_m\left|\frac{1}{\hbar}\frac{\partial}{\partial x}(-m\omega x+\xi_k)\right|g_k\right\rangle+\frac{1}{2}m\omega^2\left\langle g_k\left|x^2\right|g_m\right\rangle=$$

$$=\frac{\hbar\omega}{2}\langle g_m|g_k\rangle-\frac{1}{2m}\left\langle g_m\left|(-m\omega x+\xi_k)^2\right|g_k\right\rangle+\frac{1}{2}m\omega^2\left\langle g_k\left|x^2\right|g_m\right\rangle=$$

$$=\left(\frac{\hbar\omega}{2}-\frac{\xi_k^2}{2m}\right)\langle g_m|g_k\rangle+\omega\xi_k\left\langle g_m\left|x\right|g_k\right\rangle-\frac{1}{2}m\omega^2\left\langle g_m\left|x^2\right|g_k\right\rangle+\frac{1}{2}m\omega^2\left\langle g_k\left|x^2\right|g_m\right\rangle=$$

$$=\left(\left(\frac{\hbar\omega}{2}-\frac{\xi_k^2}{2m}\right)+\omega\xi_k\frac{(\xi_m^*+\xi_k)}{2m\omega}\right)\langle g_m|g_k\rangle=\left(\frac{\hbar\omega}{2}+\frac{\xi_m^*\xi_k}{2m}\right)\langle g_m|g_k\rangle$$

$$\mathbb{H}_{mk}-i\hbar\boldsymbol{\tau}_{mk}=\left(\frac{\hbar\omega}{2}+\frac{\xi_m^*\xi_k}{2m}+\frac{1}{2}m\omega^2q_k^2-\frac{p_k^2}{2m}-\frac{\xi_m^*\xi_k}{2m}\right)\langle g_m|g_k\rangle=$$

$$=\left(\frac{\hbar\omega}{2}+\frac{1}{2}m\omega^2q_k^2-\frac{p_k^2}{2m}\right)\mathbb{S}_{mk}$$

$$(-i\mathbb{S}^{-1}(\mathbb{H}-i\hbar\boldsymbol{\tau}))_{nk}=-i\sum_m\mathbb{S}_{nm}^{-1}(\mathbb{H}_{mk}-i\hbar\boldsymbol{\tau}_{mk})=$$

$$=\left(\frac{\hbar\omega}{2}+\frac{1}{2}m\omega^2q_k^2-\frac{p_k^2}{2m}\right)\sum_m\mathbb{S}_{nm}^{-1}\mathbb{S}_{mk}=\left(\frac{\hbar\omega}{2}+\frac{1}{2}m\omega^2q_k^2-\frac{p_k^2}{2m}\right)\delta_{nk}$$