1. **Synaptic summation**

***General information***

Synaptic current for postsynaptic neuron that generated by j-th synapse is calculated according to [Ermentrout&Terman, 2010, Destexhe et al., 1994, Destexhe&Mainen, 1994, Destexhe et al., 1998]:

(1)

Where is maximal conductance; is gate variable that characterizes the transmitter release; is the factor that defines how effectively cells respond to neurotransmitters (=1 for the most synapses, except those the mechanism of synaptic plasticity is implemented); is reversal potential for *j*-th synapse.

Let suppose (for simplicity) that , are equal for N synapses (*j* = 1…N) and =1, then

(2)

According to (2) the synaptic current for postsynaptic neuron from all similar synapses (*j* = 1..N) is calculating as:

(3)

***Pulse model of synapse (fast synapse)***

The simplest model of transmitter release for *i*-th integration step [##ref]

(4)

where: - integration step; T - time constant; – rate of transmitter release; - weight of connection for *j*-th synapse between post- and pre- synaptic neurons; – Dirac function (1 then spike generated by presynaptic neuron; 0 otherwise); – the membrane potential of presynaptic neuron; =0; *i*=1…L.

Total synaptic current for postsynaptic neuron is:

(5)

where:

(6)

The proposed model can be used as rough approximation for the model of AMPA/GABA(a/b) synapses. The advantage of the proposed model of a synapse is that it is not necessary to store intermediate results of synaptic summation () into the local memory which improve the performance of synaptic computing.

**Implementation**

Rewrite the equation (6) as follow:

(7)

where: ; *i*, *k* = 1..L, =0.

Then, the implementation of equations (6, 7) could be written as follow:

//….

// initialization

float M = 0, Expt = exp( -t/T);

//….

forloop( i ){ // for all integration steps

//…

float W = 0;

forloop( j ){ // for all the-same-type-synapses

float w = w[j]\*Delta(Vpre[j]);

W = W+w;

}

M = Expt\*M+Alpha\*W; // M for next step of integration

Isyn = Gmax\*(V-Esyn)\*M; // Isyn for next step of integration

//…

}

//…

***Model of AMPA/GABA(a) synapse.***

The more precise model of synapses is based on [Destexhe et al., 1994, Destexhe&Mainen, 1994, Destexhe et al., 1998, Wang et al, 2004]. The transmitter release () for this j-th synapse is calculating as follow:

(8)

(9)

Let solve the differential equation (9) by one-step Euler method:

) (10)

where: - integration step; T - time constant; - weight of connection for *j*-th synapse between post- and pre- synaptic neurons; – rate of transmitter release; - the normalized concentration of neurotransmitter in the synaptic cleft; – the membrane potential of presynaptic neuron; =0; *i*=1…L.

Total synaptic current for postsynaptic neuron is calculating as follow:

(11)

where:

(12)

**Implementation**

Rewrite the equation (12) as follow:

(13)

where: ; *i*, *k* = 1..L, =0. (14)

Then, the implementation of equations (11, 13 and 14) could be written as follow:

//….

// initialization

float M = 0, dt = t/T;

forloop( j ){ // for all the-same-type-synapses

m[j] = 0;

}

//….

forloop( i ){ // for all integration steps

//…

float W = 0;

forloop( j ){ // for all the-same-type-synapses

float w = w[j]\*F(Vpre[j])(1-m[j]);

m[j] = m[j]\*(1-dt)+Alpha\*dt\*w; // m[j] for next step of integration

W = W+w;

}

M = M\*(1-dt)+Alpha\*dt\*W; // M for next step of integration

Isyn = Gmax\*(V-Esyn)\*M; // Isyn for next step of integration

//…

}

//…

***Model of GABA(b) synapse.***

The kinetic equations for GABA(b) synapses is calculating as follow [Destexhe & Sejnowski, 1995, Destexhe et al, 1996]:

(15)

(16)

(17)

(18)

Let solve the system of differential equations (15, 16) by one-step Euler method:

(19)

(20)

where: - integration step; - time constants; - weight of connection for *j*-th synapse between post- and pre- synaptic neurons; – rates of transmitter release; - the normalized concentration of neurotransmitter in the synaptic cleft; – the membrane potential of presynaptic neuron. Then according to (17):

(21)

The total synaptic current for postsynaptic neuron is calculating as follow:

(22)

where:

; =0; *i*=1…L. (23)

**Implementation**

//….

// initialization

float M = 0, G = 0, R = 0, dtr = t/Tr, dtg = t/Tg;

forloop( j ){ // for all the-same-type-synapses

m[j] = g[j] = r[j] = 0;

}

//….

forloop( i ){ // for all integration steps

//…

float W = 0;

forloop( j ){ // for all the-same-type-synapses

float r\_curr = r[j];

float w = w[j]\*F(Vpre[j])(1-r\_curr);

r[j] = r\_curr\*(1-dtr)+AlphaR\*dtr\*w; // r[j] for next step of integration

g[j] = g[j]\*(1-dtg)+AlphaG\*dtg\*r\_curr; // g[j] for next step of integration

W = W+w;

}

M = M\*(1-dt)+Alpha\*dt\*W; // M for next step of integration

Isyn = Gmax\*(V-Esyn)\*M; // Isyn for next step of integration

//…

}

//…

***Model of NMDA synapse.***

***General information.***

The synaptic current for postsynaptic neuron that generated by j-th NMDA synapse is calculated similar to equation (1) [Destexhe&Mainen, 1994, Ermentrout&Terman, 2010]

(24)

where is maximal conductance; is gate variable that characterizes the transmitter release, is reversal potential for *j*-th synapse; z(V) represents the magnesium block and is calculating as:

(25)

The model of transmitter release [Destexhe et al., 1994, Destexhe&Mainen, 1994, Destexhe et al., 1998] described similar to the model of transmitter release for AMPA/GABA(a) synapses (see eq. 9). Sometimes it is desirable to implement the NMDA channel so that there is greater flexibility in the rise time. In this case, the kinetic of transmitter release is modeled by two variables [Ermentrout&Terman, 2010, Wang et al, 2004]:

(26)

(27)

where (normalize concentration of transmitter) is calculating according to eq. 8

**Implementation**

The implementation of simplified model of synaptic current of NMDA synapse is similar to the model of AMPA/GABA(a) synapses (see eq. 13, 14). The magnesium block is calculating according to (eq. 25).

The more complicated model of the synapse (see 26, 27) might be implemented similar to implementation of GABA(b) synapses.

1. **Presynaptic inhibition**

***General information***

The presynaptic inhibition affects to the rate of transmitter release in synaptic vesicles (parameter in all equation for calculating of the dynamics of transmitter release, see eq(s) 4, 9, 17 etc). Then the simplest model of presynaptic inhibition can be written as follow:

(28)

where: is maximal rate of transmitter release; is presynaptic inhibition.

**Implementation**

The model of modulation will be implemented if necessary

1. **Synaptic plasticity**

***General information***

**Implementation**

n/a

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