“Beyond” General Relativity

with Neutron Stars

Kostas Kokkotas

October 12, 2018

Eberhard Karls Univerity of Tübingen

From Newton to Einstein

- **NEWTON**

$$\nabla^2 U(\vec{x}) = 4\pi G\rho(\vec{x}) \quad \text{Poisson equation} \tag{1}$$

$$U(\vec{x}) = -G\int \frac{\rho(\vec{x})}{|\vec{x}-\vec{x'}|} d^3\vec{x'} \tag{2}$$

$$\frac{d\vec{u}}{dt} + \nabla U = 0 \tag{3}$$

- **EINSTEIN**

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + \Lambda g^{\mu\nu} = \kappa T^{\mu\nu}\,, \tag{4}$$

$$\frac{du^\alpha}{ds} + \Gamma^\alpha_{\mu\nu} u^\mu u^\nu = 0 \tag{5}$$

where $\Lambda = \frac{8\pi G}{c^2}\rho_{\rm vac}$ is the so called cosmological constant.

The Scalar Theory of Gravity (Nordström 1912-13)

It is the simplest generalization of Newton's gravity.

Matter relativistically is described by the energy-momentum tensor $T_{\mu\nu}$, the only scalar with the dimensions of mass density is $T^{\mu}_{\mu} = T$.

Thus a "consistent" scalar relativistic theory of gravity will be given by the field equation

$$\Box U = -\frac{4\pi G}{c^2} T \quad \rightarrow \quad U \Box U = -\frac{4\pi G}{c^2} T \quad \rightarrow \quad R = \frac{24\pi G}{c^2} T \qquad (6)$$

This theory cannot be accepted because:

- Predicts a **retardation** of the perihelion of Mercury in contradiction to observations
- It does not allow one to **couple gravity to electromagnetism** since ${}^{(EM)}T^{\mu}_{\mu} = 0$.
- Does not predict **gravitational redshift**.
- Does not predict **deflection of light by matter** .

NOTE: A gravitational theory based on a **vector field** cannot be accepted since such a theory predicts that two massive particles would repel and not attract one another.

The Scalar-Tensor Theory of Gravity (Jordan-Brans-Dicke) i

It is possible to construct theories which combine **tensor**, **vector** *and* scalar *fields.*

Brans-Dicke assumed equivalence principle and also that gravity is described as spacetime curvature.
BUT they introduced a scalar field ϕ that determines the strength of gravitational "constant" G.
The key ingredients of the theory are:

- **matter** is represented by the energy-momentum tensor ${}^{(M)}T_{\mu\nu}$, and a coupling constant λ fix the scalar field ϕ
- The scalar field ϕ fixes the value of G.
- The gravitational field equations relate the curvature to the energy momentum tensors of the **scalar field** and **matter**.

The Scalar-Tensor Theory of Gravity (Jordan-Brans-Dicke) ii

The field equations are:

$$\begin{aligned} \Box\phi &= -4\pi\lambda\, {}^{(M)}T^{\mu}_{\mu} && (7) \\ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= -\frac{8\pi}{c^4\phi}\left[{}^{(M)}T_{\mu\nu} + {}^{(\phi)}T_{\mu\nu}\right] && (8) \end{aligned}$$

$$ {}^{(\phi)}T_{\alpha\beta} = \frac{\omega}{\phi^2}\left(\phi_{,\alpha}\phi_{,\beta} - \frac{1}{2}g_{\alpha\beta}\phi_{,\gamma}\phi^{,\gamma}\right) + \frac{1}{\phi}\left(\phi_{;\alpha\beta} - g_{\alpha\beta}\Box\phi\right) \qquad (9)$$

- Typically the coupling constant is written as:

$$\lambda = \frac{2}{3+2\omega} \qquad (10)$$

- In the limit $\omega \to \infty$, $\lambda \to 0$ so the scalar field is not affected by the matter distribution and can be set equal to a constant $\phi = 1/G$ (!).

In this case ${}^{(\phi)}T\mu\nu = 0$ and Brans-Dicke theory reduces to Einstein's in the limit $\omega \to \infty$.

The Scalar-Tensor Theory of Gravity (Jordan-Brans-Dicke) iii

- One of the key features of Brans-Dicke theory is that the effective gravitational "constant" G varies with time and is determined by the scalar field ϕ.
- Brans-Dicke theory predicts **light deflection** and the **precession of perihelia** of planets orbiting the Sun, BUT the formulae depend on ω.
- A variation of the G affects the orbits of planets e.g. altering the dates of eclipses.
- The action in this case is:

$$S_{\mathrm{BD}} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[\phi R - \frac{\omega}{\phi} \left(\phi_{,\alpha} \phi^{,\alpha} \right) + \mathcal{L}_M \right] \tag{11}$$

where $\mathcal{L}_M$ is the matter Lagrangian.

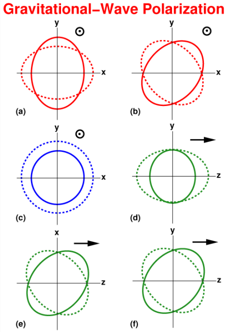
The Equivalence Principle is satisfied:

- The laws of SR are valid in the local Lorentz frame of the metric $g_{\mu\nu}$.

The Scalar-Tensor Theory of Gravity (Jordan-Brans-Dicke) iv

Consequence: the scalar field does not exert any direct influence on matter its only role is in participating in the field equations that determine the geometry of the space-time.

- The experiments as for today predict that $\omega \geq \mathbf{40,000}$ (Cassini-Huygens experiment, 2003). In 1973 $\omega \geq 5$, in 1981 $\omega \geq 30$!



How to set up a PPN theory (MTW Chapter 39.8) i

The PPN equations and post-Newtonian corrections to the metric coefficients $g_{\alpha\beta}$ could be found without writing down the field equations of the theory.

$$g_{ij} = \delta_{ij} + k_{ij} = \delta_{ij}(1 + 2\gamma U) + O(c^{-4}) \tag{12}$$

$$g_{0j} = k_{0j} = -\frac{7}{2}\Delta_1 U_j - \frac{1}{2}\Delta_2 W_j + O(c^{-5}) \tag{13}$$

$$g_{00} = -1 + 2U - 2\beta U^2 + 4\Psi - \zeta \mathcal{Q} - \eta \mathcal{D} \tag{14}$$

where

$$U_j(\vec{x}, t) = \int \frac{\rho_0(\vec{x}', t) v_j(\vec{x}', t)}{|\vec{x} - \vec{x}'|} d^3x' \tag{15}$$

$$W_j(\vec{x}, t) = \int \frac{\rho_0(\vec{x}', t)[(\vec{x} - \vec{x}') \cdot \vec{v}(\vec{x}', t)](x_j - x_j')}{|\vec{x} - \vec{x}'|^3} d^3x' \tag{16}$$

$$\Psi(\vec{x}, t) = \int \frac{\rho_0(\vec{x}', t)\, \psi(\vec{x}', t)}{|\vec{x} - \vec{x}'|} d^3x' \tag{17}$$

$$\psi = \beta_1 \vec{v}^2 + \beta_2 U + \frac{1}{2}\beta_3 \Pi + \frac{3}{2}\beta_4 p/\rho_0 \tag{18}$$

How to set up a PPN theory (MTW Chapter 39.8) ii

$$\mathcal{Q}(\vec{x}, t) = \int \frac{\rho_0(\vec{x}', t)[(\vec{x} - \vec{x}') \cdot v(\vec{x}', t)]^2}{|\vec{x} - \vec{x}'|^3} d^3x' \tag{19}$$

$$\mathcal{D}(\vec{x}, t) = \int \frac{\left[t_{\hat{j}\hat{k}}(\vec{x}', t) - \frac{1}{3}\delta_{jk} t_{\hat{m}\hat{m}}(\vec{x}', t)\right](x_j - x_j')(x_k - x_k')}{|\vec{x} - \vec{x}'|^3} d^3x' \tag{20}$$

The unknown constants β, β_1, β_2, β_3, β_4, ζ, η are PPN parameters.

SUMMARY

- For every metric theory of gravity, accurate through the post-Newtonian order the metric coefficients have the form (12) - (20).
- One theory is distinguished from another by the value of its **10** post-Newtonian parameters β, β_1, β_2, β_3, β_4, γ, ζ, η, Δ_1 and Δ_2
- These parameters are determined by comparing the field equations of the given theory with the form (12) - (20) of the post-Newtonian metric.

The PPN parameters i

Parameter	What measures relative to GR	value in GR
γ	How much space-curvature produced by unit rest mass ?	1
β	How much “nonlinearity” is there in the superposition law for gravity ?	1
ξ	Preferred-location effects ?	0
α_1, α_2, α_3	are there any preferred-frame effects ?	0
ζ_1, ζ_2, ζ_3, ζ_4	is there violation of conservation of total momentum ?	0

where

$$\zeta_1 = \zeta\,, \qquad \zeta_2 = 2\beta + 2\beta_3 - 3\gamma - 1 \tag{21}$$

$$\zeta_3 = \beta_3 - 1\,, \quad \zeta_4 = \beta_4 - \gamma \tag{22}$$

Current limits on the PPN parameters

parameter	Effect	Limit	Remarks
γ - 1	time delay	2.3×10^{-5}	Cassini tracking
	light deflection	4×10^{-4}	VLBI
β -1	perihelion shift	3×10^{-3}	$J_2 = 10^{-7}$ from helioseismology
	Nordtvedt effect	2.3×10^{-4}	$\eta_N = 4\beta - \gamma - 3$ assumed
ξ	Earth tides	10^{-3}	gravimeter data
α_1	orbital polarization	10^{-4}	Lunar laser ranging
		2×10^{-4}	PSR J2317+1439
α_2	spin precession	4×10^{-7}	solar alignment with ecliptic
α_3	pulsar acceleration	4×10^{-20}	pulsar $\dot{P}$ statistics
η_N	Nordtvedt effect	9×10^{-4}	Lunar laser ranging
ζ_1	–	2×10^{-2}	conbined PPN bounds
ζ_2	binary acceleration	4×10^{-5}	$\ddot{P}_p$ for PSR 1913+16
ζ_3	Newton's 3rd law	10^{-8}	lunar acceleration
ζ_4	–	6×10^{-3}	$6\zeta_4 = 3\alpha_3 + 2\zeta_1 - 3\zeta_3$ assumed

The PPN parameters in STT

In Brans-Dicke-Jordan theory we expect:

$$\begin{aligned}
\gamma &= \frac{1+\omega}{2+\omega} && (23)\\
\beta &= 1 && (24)\\
\beta_1 &= \frac{3+2\omega}{4+2\omega} && (25)\\
\beta_2 &= \frac{1+2\omega}{4+2\omega} && (26)\\
\beta_3 &= 1 && (27)\\
\beta_4 &= \frac{1+\omega}{2+\omega} && (28)\\
\zeta &= 0 && (29)\\
\eta &= 0 && (30)\\
\Delta_1 &= \frac{10+7\omega}{14+7\omega} && (31)\\
\Delta_2 &= 1 && (32)
\end{aligned}$$

Famous Men Words

- **EINSTEIN:**
  I would feel sorry for the good Lord. The theory is correct !
- **CHANDRASEKHAR (to C.M. Will)**
  Why do you spend so much time and energy testing GR? We know that the theory is right.

However, there is growing theoretical and experimental evidence that modifications of GR at small and large energies are somehow inevitable.

Alternative theories of gravity: Motivation

- **Theory**
  - Theories trying to unify all interactions: Kaluza-Klein theories, higher dimensional gravity, etc.
  - Quantum corrections in the strong field regime
- **Observations**
  - Dark energy and dark matter does not fit well in the standard GR framework
  - The strong field regime of gravity is essentially unconstrained

Studying alternative theories of gravity can give us a deeper insight in GR itself.

• Einstein's theory is the unique interacting theory of a Lorentz-invariant massless helicity-2 particle, and therefore new physics in the gravitational sector must introduce additional degrees of freedom.

Any additional degrees of freedom must modify the theory at low and/or high energies while being consistent with GR in the intermediate-energy regime, i.e. at length scales $1\mu m < L < 10^{11} m$, where the theory is extremely well tested.

Lovelock's theorem

GR emerges as the unique theory of gravity under specific assumptions.

- In 4D spacetimes the only divergence-free symmetric rank-2 tensor constructed solely from the metric $g_{\mu\nu}$ and its derivatives up to second differential order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological term.
  Lovelock's theorem suggests a natural route to Einstein's equations
  $$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \tag{33}$$
- The divergence-free nature of the Einstein tensor (that follows from the Bianchi identities) implies that $T_{\mu\nu}$ is also divergence free,
  $$\nabla_\mu T^{\mu\nu} = 0 \tag{34}$$
  This property is necessary for geodesic motion and it guarantees the validity of the weak equivalence principle, i.e. the universality of free fall.
- If we assume that the equations of motion for the gravitational field and the matter fields follow from a Lagrangian, the arguments above single out the Einstein-Hilbert action
  $$S = \frac{1}{16\pi} \int d^4x \sqrt{-g}\, R + S_M[\Psi, g_{\mu\nu}] \tag{35}$$

Circumventing Lovelock's theorem

1. Additional Fields (Dynamical, Nondynamical)
2. Violating of diffeomorfism invariance
   - Lorentz invariance
   - Massive gravity
3. Higher Dimensions
4. WEP violations

The main alternative theories (ATG) i

There is a very wide range of alternative theories of gravity constructed from different generalizations/modifications of Einstein's theory.

- **Scalar-tensor theories and their generalizations**. Including multiscalar and Horndeski theories
- **F(R) theories**
- **Theories whose action contains terms quadratic in curvature.** Including Einstein-dilaton-Gauss-Bonnet (EdGB) and dynamical Chern-Simons (dCS) theories
- **Lorentz-violating theories**. Including Einstein-Aether, Hořava and n-Dirac-Born-Infeld (n-DBI) gravity.
- **Massive gravity theories**.
- **Theories involving non-dynamical fields**. Including the Palatini formulation of F(R) gravity and Eddington-inspired Born-Infeld (EiBI) gravity.

The main alternative theories (ATG) ii

- We will concentrate on the most natural and widely used generalizations:
  - **Scalar-tensor theories of gravity**
  - $F(R)$ **theories of gravity**
  - **Massive theories**
- They are in agreement with all the observations and do not posses any intrinsic problems.
- Widely used as an alternative explanation of the dark energy phenomena.
- Scalar-tensor theories can be consider as an Einstein theory of gravity but with variable gravitational constant.

Compact objects such as black holes (BHs) and neutron stars (NSs) are our best natural laboratories to constrain strong gravity. In these celestial bodies gravity prevails over all other interactions, and collapse leads to large-curvature, strong-gravity environments.

Scalar Tensor Theories: derivation i

Essence: one or several scalar field that can be viewed as mediators of the gravitational interaction in addition to the spacetime metric

Physical (Jordan) frame action:

$$S_J = \frac{1}{16\pi}\int d^4x\sqrt{-\tilde{g}}\left[\Phi\tilde{R} - \frac{\omega(\Phi)}{\Phi}\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - U(\Phi)\right] + S_m\left[\Psi_m;\tilde{g}_{\mu\nu}\right] \tag{36}$$

The theory has only 2 free functions of the scaler field Φ: the potential $U(\Phi)$ and a coupling function $\omega(\Phi)$.

If $U(\Phi)$ is a slowly varying function of Φ, as expected on cosmological scenarios, it is of negligible importance in the propagation of Φ on smaller (stellar) scales. In Brans-Dicke theory $\omega(\Phi) = \omega_{\mathrm{BD}}$=const and $U(\Phi) = 0$.

Conformal transformation

$$g_{\mu\nu} = \mathcal{A}^{-2}(\phi)\tilde{g}_{\mu\nu} + \quad \text{redefine the scalar field} \quad \Phi = \Phi(\phi) \tag{37}$$

Scalar Tensor Theories: derivation ii

Einstein frame action (much simpler):

$$S_E = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - 2g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] + S_m \left[\Psi_m ; \mathcal{A}^2(\phi) g_{\mu\nu} \right] \quad (38)$$

where $V(\phi) = \mathcal{A}^4(\phi) U[\Phi(\phi)]$ while the new term $\mathcal{A}^2(\phi) g_{\mu\nu}$ is the price we pay for simplicity: Explicit coupling between the matter and the scalar field.

The relations between the two frames are:

$$\Phi = \mathcal{A}^2(\phi) \quad (39)$$

$$3 + 2\omega(\Phi) = \alpha^{-2}(\phi) \quad (40)$$

$$\alpha \equiv \frac{d \ln \mathcal{A}(\phi)}{d\phi} \quad (41)$$

Thus, the theory is fixed once $\omega(\Phi)$ or, equivalently, $\alpha(\phi)$ is fixed and the scalar potential is chosen.

The advantage of this formulation is that the scalar field is now minimally coupled to the Einstein metric, as can be seen from action (38).

Field equations in STT (Einstein frame)

$$G_{\mu\nu} = 8\pi T_{\mu\nu} + 2\partial_\mu\partial_\nu\phi - g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - \frac{1}{2}g_{\mu\nu}V(\phi) \tag{42}$$

$$\Box\phi = -4\pi\alpha(\phi)T + \frac{1}{4}\frac{dV(\phi)}{d\phi} \tag{43}$$

where

$$\alpha(\phi) = \frac{d\ln\left(\mathcal{A}(\phi)\right)}{d\phi} \tag{44}$$

These equations have to be supplemented with:

- Equations for hydrostatic equilibrium
- Equation of state for the nuclear matter

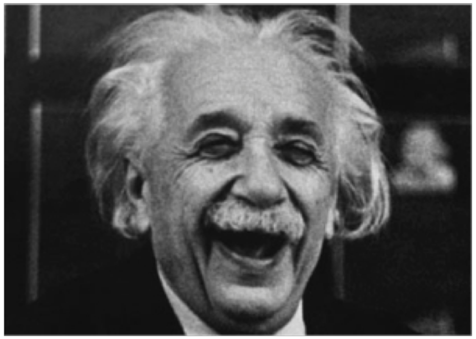
If $V(\phi)$ is a slowly varying function of ϕ, as expected on cosmological scenarios, it is of negligible importance in the propagation of ϕ on smaller (stellar) scales.

The TOV equations in STT (Einstein frame)

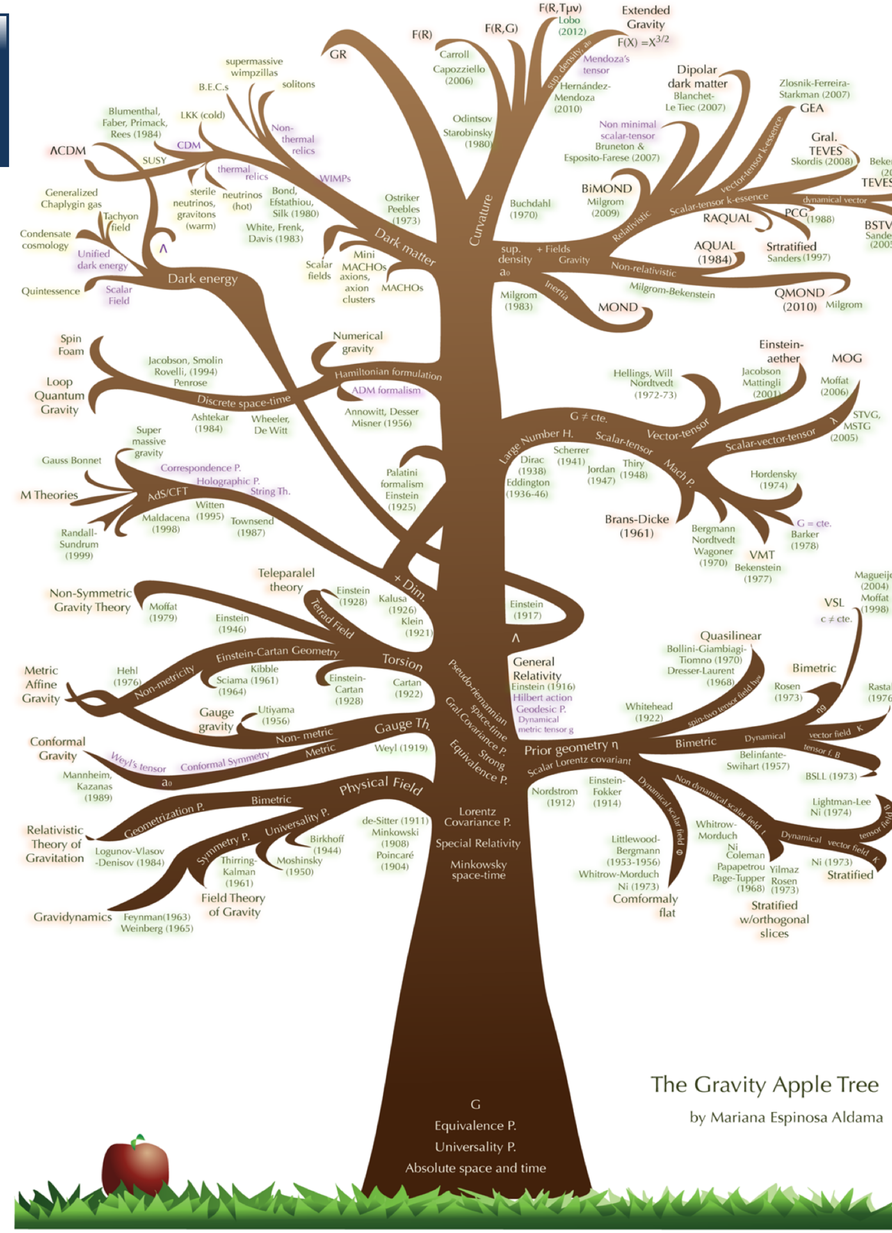
$$
\begin{aligned}
\frac{d\mu}{dr} &= 4\pi G_* r^2 A^4 \tilde{\rho} + \frac{1}{2} r(r-2\mu)\Psi^2, && (45)\\
\frac{d\Phi}{dr} &= 4\pi G_* \frac{r^2 A^4 \tilde{P}}{r-2\mu} + \frac{1}{2} r\Psi^2 + \frac{\mu}{r(r-2\mu)}, && (46)\\
\frac{d\varphi}{dr} &= \Psi, && (47)\\
\frac{d\Psi}{dr} &= 4\pi G_* \frac{rA^4}{r-2\mu}\left[\alpha(\tilde{\rho}-3\tilde{P}) + r(\tilde{\rho}-\tilde{P})\Psi\right] - \frac{2(r-\mu)}{r(r-2\mu)}\Psi, && (48)\\
\frac{d\tilde{P}}{dr} &= -\left(\tilde{\rho}+\tilde{P}\right)\left[\frac{d\Phi}{dr} + \alpha\Psi\right]\\
&= -\left(\tilde{\rho}+\tilde{P}\right)\left[4\pi G_* \frac{r^2 A^4 \tilde{P}}{r-2\mu} + \frac{1}{2} r\Psi^2 + \frac{\mu}{r(r-2\mu)} + \alpha\Psi\right]. && (49)
\end{aligned}
$$

- Berti et al
  Testing general relativity with present and future astrophysical observations
  Class. Quantum Grav. 32 (2015) 243001 (179pp)
- Will, Clifford C.M.
  The Confrontation between General Relativity and Experiment
  Living Reviews in Relativity, Volume 9, article id. 3, 100 pp
- Yagi, Kent; Yunes, Nicolas
  I-Love-Q relations: from compact stars to black holes
  Classical and Quantum Gravity, Volume 33, 095005 (2016).
- Doneva, Daniela D.; Pappas, George
  Universal Relations and Alternative Gravity Theories
  eprint arXiv:1709.08046

GTR Prevails !!



[gremlin](), [goblin]() (creature which according to Greek folklore lives underground and comes up on Christmas Day to cause minor mischief, disappearing back into the earth on the feast of the [Epiphany]())



Genealogical tree of the gravitation theories developed between 1904 and 2014, structured with the aid of formal concept analysis method, and a diachronic perspective. Authors and dates are presented in green. Common theory names are shadowed in red, while models for dark matter and dark energy are presented in orange. Principles followed are in violet. Some selected principles are written in the middle of branches as they allow to distinguish groups of theories. The timeline grows upwards and radially.

Catalog of ATG vs Lovelock

Table 1. Catalog of several theories of gravity and their relation with the assumptions of Lovelock's theorem. Each theory violates at least one assumption (see also figure 1), and can be seen as a proxy for testing a specific principle underlying GR.

Theory	Field content	Strong EP	Massless graviton	Lorentz symmetry	Linear $T_{\mu\nu}$	Weak EP	Well- posed?	Weak-field constraints
Extra scalar field								
Scalar–tensor	S	×	✓	✓	✓	✓	✓[34]	[35–37]
Multiscalar	S	×	✓	✓	✓	✓	✓[38]	[39]
Metric $f(R)$	S	×	✓	✓	✓	✓	✓[40, 41]	[42]
Quadratic gravity								
Gauss–Bonnet	S	×	✓	✓	✓	✓	✓?	[43]
Chern–Simons	P	×	✓	✓	✓	✓	×✓? [44]	[45]
Generic	S/P	×	✓	✓	✓	✓	?	
Horndeski	S	×	✓	✓	✓	✓	✓?	
Lorentz-violating								
Æ-gravity	SV	×	✓	×	✓	✓	✓?	[46–49]
Khronometric/ Hořava–Lifshitz	S	×	✓	×	✓	✓	✓?	[48–51]
n-DBI	S	×	✓	×	✓	✓	?	none ([52])
Massive gravity								
dRGT/Bimetric	SVT	×	×	✓	✓	✓	?	[17]
Galileon	S	×	✓	✓	✓	✓	✓?	[17, 53]
Nondynamical fields								
Palatini $f(R)$	—	✓	✓	✓	×	✓	✓	none
Eddington–Born–Infeld	—	✓	✓	✓	×	✓	?	none
Others, not covered here								
TeVeS	SVT	×	✓	✓	✓	✓	?	[37]
$f(R)\mathcal{L}_m$	?	×	✓	✓	✓	×	?	
$f(T)$	?	×	✓	×	✓	✓	?	[54]

Note. See text for details of the entries. Key to abbreviations: S: scalar; P: pseudoscalar; V: vector; T: tensor; ?: unknown; ✓?: not explored in detail or not rigorously proven, but there exist arguments to expect ✓. The occurrence of ×✓? means that there exist arguments in favor of well-posedness within the EFT formulation, and against well-posedness for the full theory. Weak-field constraints (as opposed to strong-field constraints, which are the main topic of this review) refer to Solar System and binary pulsar tests. Entries below "Others, not covered here" are not covered in this review.

Berti etal (2015)

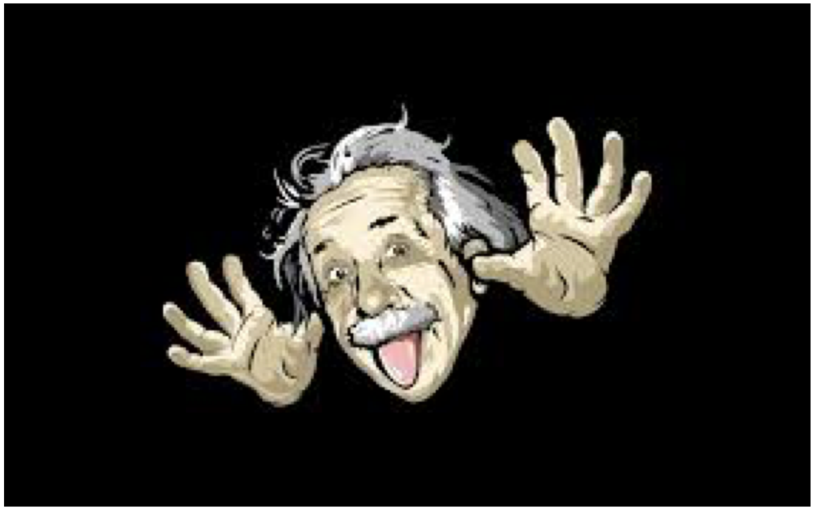
Catalog of NS properties in ATG

Table 3. Catalog of NS properties in several theories of gravity. Symbols and abbreviations are the same as in table 2.

Theory	Structure			Collapse	Sensitivities	Stability	Geodesics
	NR	SR	FR				
Extra scalar field							
Scalar–tensor	[26, 114–118]	[116, 119, 120]	[121–123]	[124–131]	[132]	[133–143]	[122, 144]
Multiscalar	?	?	?	?	?	?	?
Metric $f(R)$	[145–157]	[158]	[159]	[160, 161]	?	[162, 163]	?
Quadratic gravity							
Gauss–Bonnet	[164]	[164]	[82]	?	?	?	?
Chern–Simons	≡GR	[27, 45, 165–167]	?	?	[166]	?	?
Horndeski	?	?	?	?	?	?	?
Lorentz-violating							
Æ-gravity	[168, 169]	?	?	[170]	[48, 49]	[162]	?
Khronometric/ Hořava–Lifshitz	[171]	?	?	?	[48, 49]	?	?
n-DBI	?	?	?	?	?	?	?
Massive gravity							
dRGT/Bimetric	[172, 173]	?	?	?	?	?	?
Galileon	[174]	[174]	?	[175, 176]	?	?	?
Nondynamical fields							
Palatini $f(R)$	[177–181]	?	?	?	—	?	?
Eddington–Born–Infeld	[182–188]	[182, 183]	?	[183]	—	[189, 190]	?

Berti etal (2015)

NEUTRON STARS &
ALTERNATIVE THEORIES OF GRAVITY



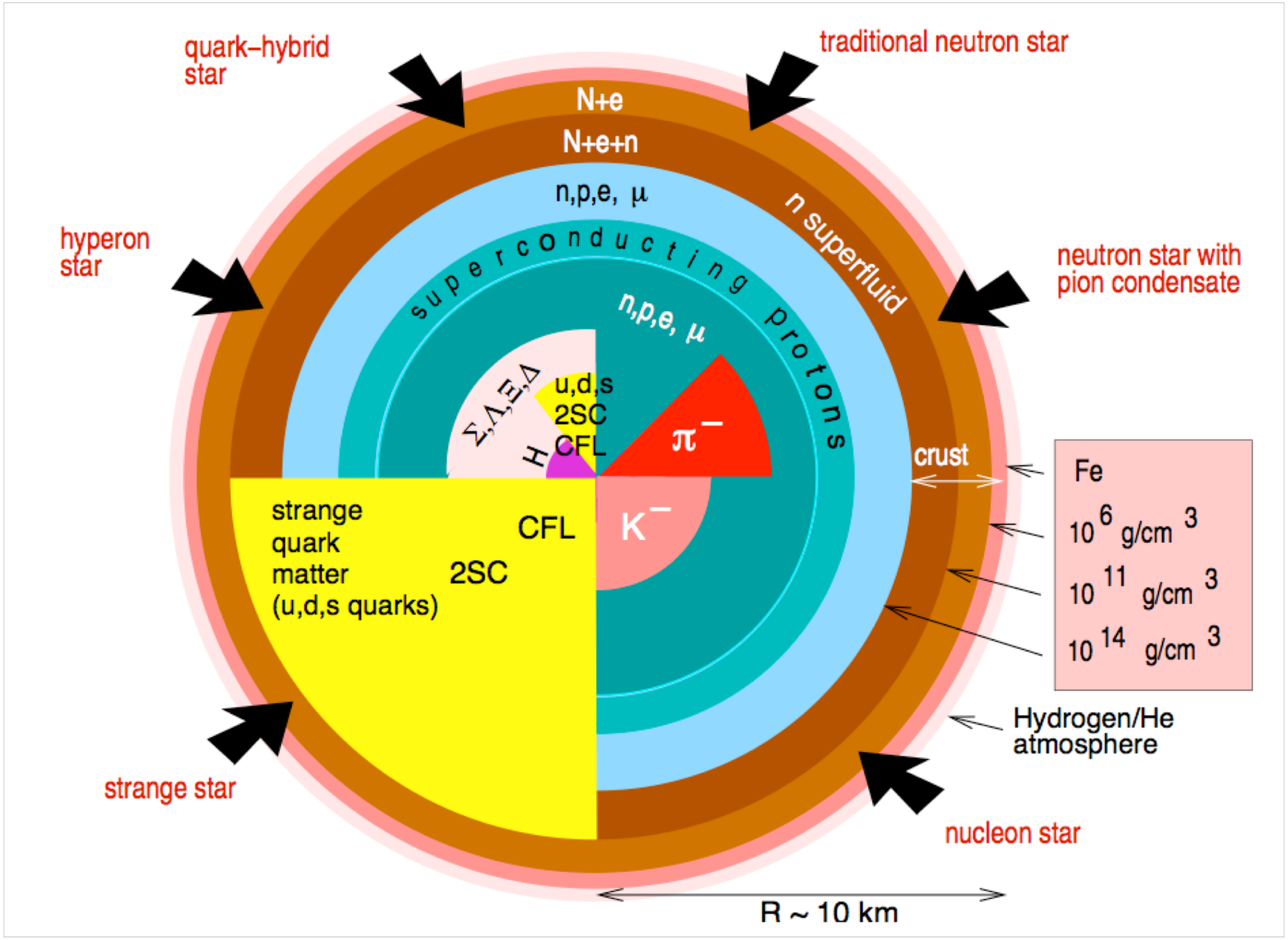
Expect (and Prepare for) The Unexpected

ATG and Neutron Stars

The **enormous gravitational field** of NSs, the **high density of matter** at their cores and the existence of pulsars with **fast spin** and **large magnetic fields** make them ideal laboratories to study all fundamental interactions

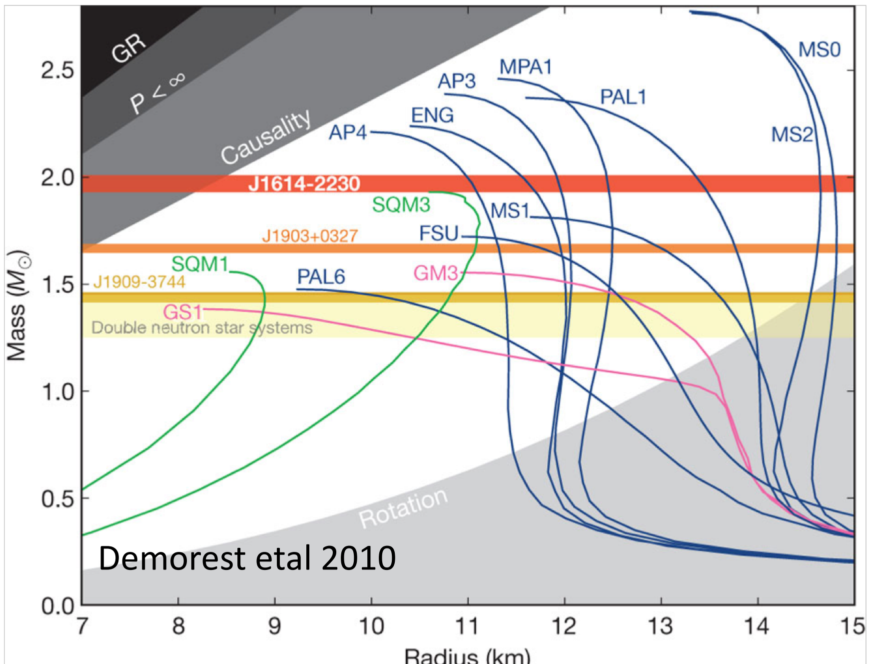
- The structure of compact stars depends on the **coupling of gravity with matter** in strong-field regions.
- NSs are a valuable alternative to BHs in tests of strong-field gravity, because **they can probe** (and possibly rule out) those theories that are close to GR in vacuum, but differ in the description of the coupling between matter and gravity.

Zooming into a Neutron Star



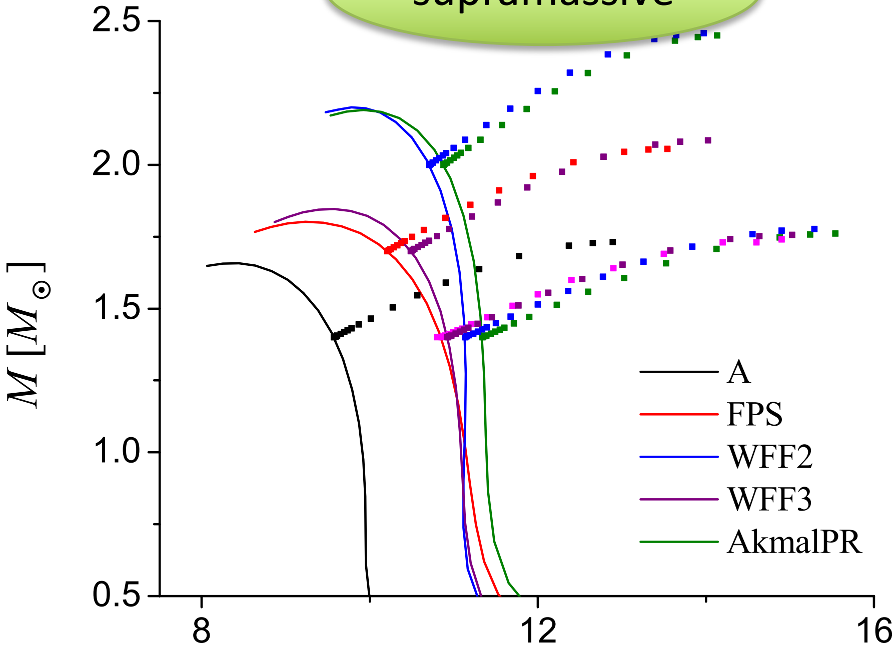
Neutron Stars: Mass vs Radius

Static Models



Rotating Models

supramassive



The holy grail of NS astrophysics… is the determination of the equation of state (EOS) of matter at supra-nuclear densities.
The most direct way of constraining the EOS is to measure simultaneously the neutron star mass and radius.

Constraints on NS Radius

Main methods in EM spectrum:

- Thermonuclear X-ray bursts (photospheric radius expansion)
- Burst oscillations (rotationally modulated waveform)
- Fits of thermal spectra to cooling neutron stars
- khZ QPOs in accretion disks around neutron stars
- Pericenter precession in relativistic binaries (double pulsar J0737)

Main methods in GW spectrum:

- Tidal effects on waveform during inspiral phase of NS-NS mergers
- Tidal disruption in BH-NS mergers
- Post-merger phase of NS-NS mergers and Oscillations

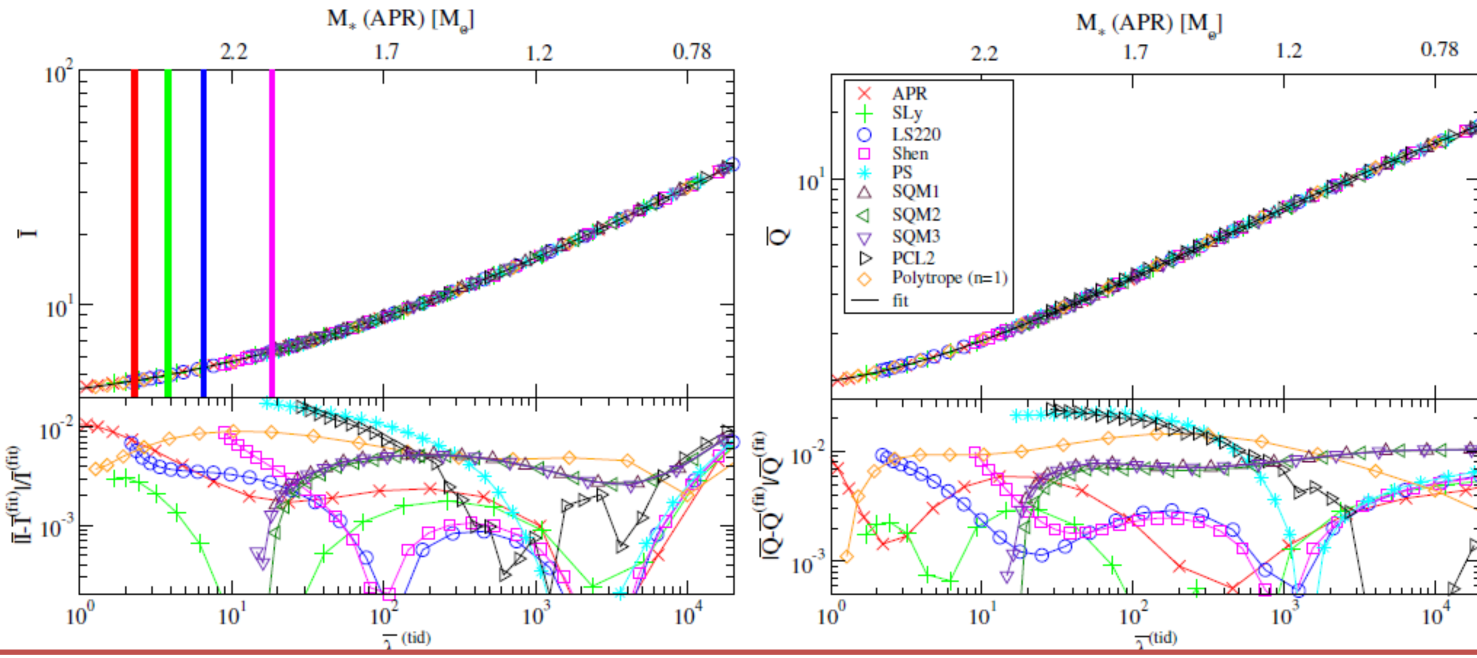
Neutron Stars & “universal relations”

Need for relations between the “observables” and the “fundamentals” of NS physics

Average Density	$\bar{\rho} \sim M / R^3$	
Compactness	$z \sim M / R$	$\eta = \sqrt{M^3 / I}$
Moment of Inertia	$I \sim MR^2$	$I \sim J / \Omega$
Quadrupole Moment	$Q \sim R^5 \Omega^2$	
Tidal Love Numbers	$\lambda \sim I^2 Q$	

I-Love-Q relations

EOS independent relations were derived by **Yagi & Yunes(2013**) for non-magnetized stars in the slow-rotation and small tidal deformation approximations.



… the relations proved to be valid (*with appropriate normalizations*) even for *fast rotating* and *magnetized* stars

✓ Yagi-Yunes Phys. Reports (arXiv:1608.02582)

Oscillations & Instabilities

p-modes: main restoring force is the pressure (f-mode) *(>1.5 kHz)*

$$\sigma \approx \sqrt{\frac{GM}{R^3}}$$

Inertial modes: (r-modes) main restoring force is the Coriolis force

$$\sigma \approx \Omega$$

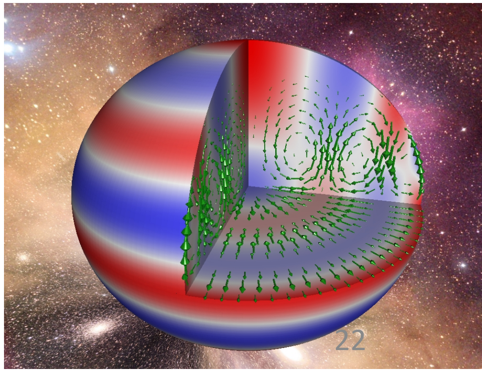
w-modes: pure space-time modes (only in GR) *(>5kHz)*

$$\sigma \approx \frac{1}{R}\left(\frac{GM}{Rc^2}\right)$$

Torsional modes (t-modes) *(>20 Hz)* shear deformations. Restoring force, the weak Coulomb force of the crystal ions.

$$\sigma \approx \frac{v_S}{R} \sim 16\ \ell\ \mathrm{Hz}$$

... and many more



shear, g-, Alfven, interface, … modes

Equilibrium neutron star solutions: Scalar-Tensor Theory

Scalar-tensor theories with massless scalar field

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\,(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \cancel{4V(\varphi)}) + S_m[\Psi_m; \boxed{A^2(\varphi)}g_{\mu\nu}]$$

Coupling function $\alpha(\varphi) = \frac{d\,lnA(\varphi)}{d\,\varphi}$

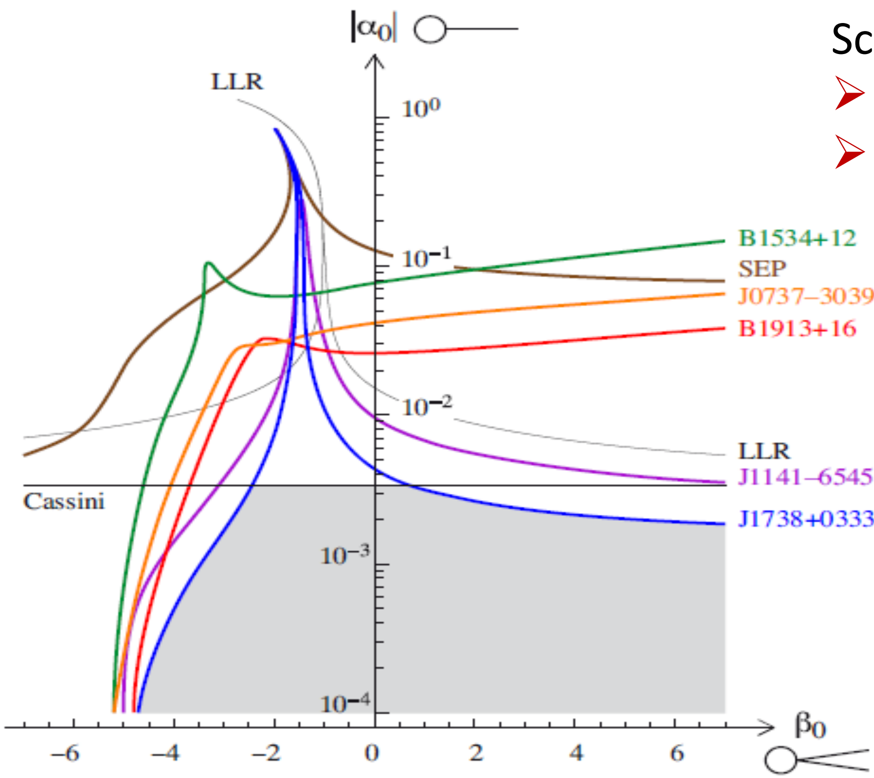
- The coupling function can be expanded as $\boldsymbol{\alpha(\varphi) = \alpha_0 + \beta\varphi +}$ **higher order terms**
  1. $\boldsymbol{\alpha(\varphi) = \alpha_0}$
     - Equivalent to the Brans-Dicke theory.
     - Differs from GR in the weak field regime.
     - Neutron stars have nontrivial scalar field for every $\alpha_0 \neq 0$
  2. $\boldsymbol{\alpha(\varphi) = \beta\varphi}$ **(**$\alpha_0 = 0$**)**
     - Equivalent to GR in the weak field regime.
     - Can differ significantly when strong fields are considered.
     - Nonuniqueness of the neutron star solutions can exist – one solution with trivial scalar field and one or several others with nontrivial scalar field.
- **Higher order terms** in $\alpha(\varphi)$ lead to qualitatively similar results

Equilibrium neutron star solutions: Scalar-Tensor Theory

Observational constraints

$\boldsymbol{\alpha_0 < 0.0035}$ $(Cassini)$ and $\boldsymbol{\beta > -4.5}$

(Damour & Esposito-Farese (1996,1998), Will (2006), Freire et al (2012), Antoniadis et al (2013))



Freire et al (2012)

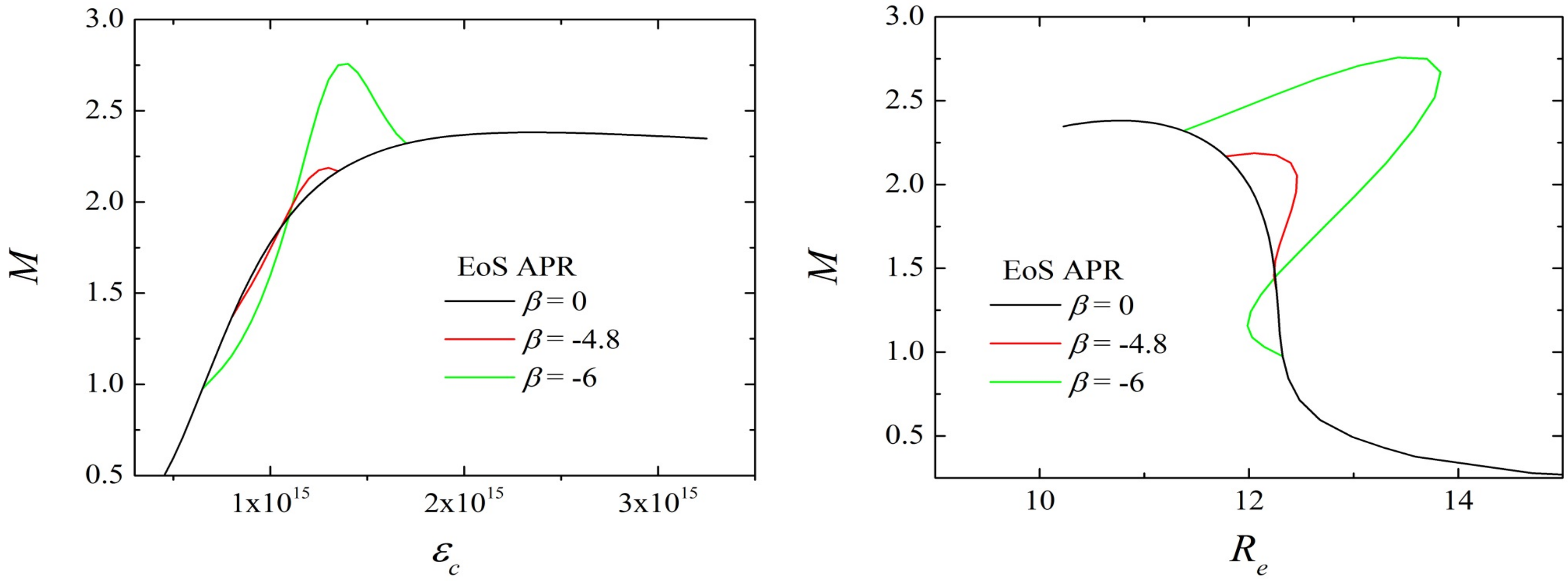
Scalarized solutions exist only for

- $\beta < -4.35$ in the static case and
- $\beta < -3.9$ in the rapidly rotating case.

Equilibrium neutron star solutions:
Scalar-Tensor Theory

Spontaneous Scalarizarion is possible for β<-4.35 **(Damour+Esposito-Farese 1993)** introducing macroscopically (and observationally) significant modifications to the structure of the star[52].

The solutions become non-unique: for certain ranges of the parameter space:
NS solutions in GR coexist with scalarized NSs.

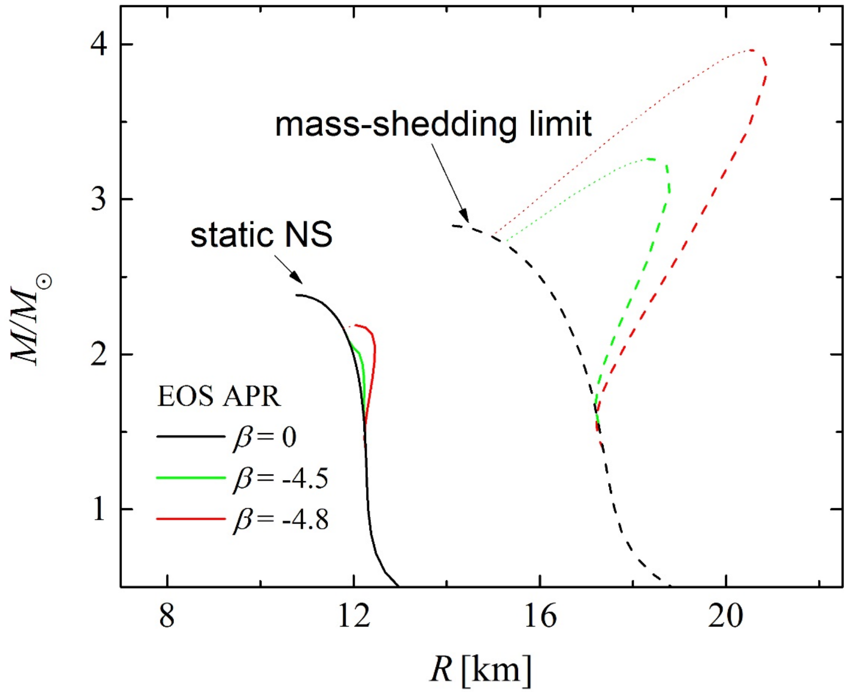


The solutions with nontrivial scalar field are *energetically more favorable* than their GR counterpart (Harada 1997, Harada 1998, Sotani+Kokkotas 2004)**.**

Equilibrium neutron star solutions:
Scalar-Tensor Theory

- **Slow rotation approximation** was also considered (Damour&Esposito-Farese (1996), Sotani (2012), Pani & Berti(2014)).
- **Rapid rotation** – changes the picture significantly (Doneva , Yazadjiev, Stergioulas, KK (2013)

Coupling function $\boldsymbol{\alpha(\varphi) = \beta\varphi}$

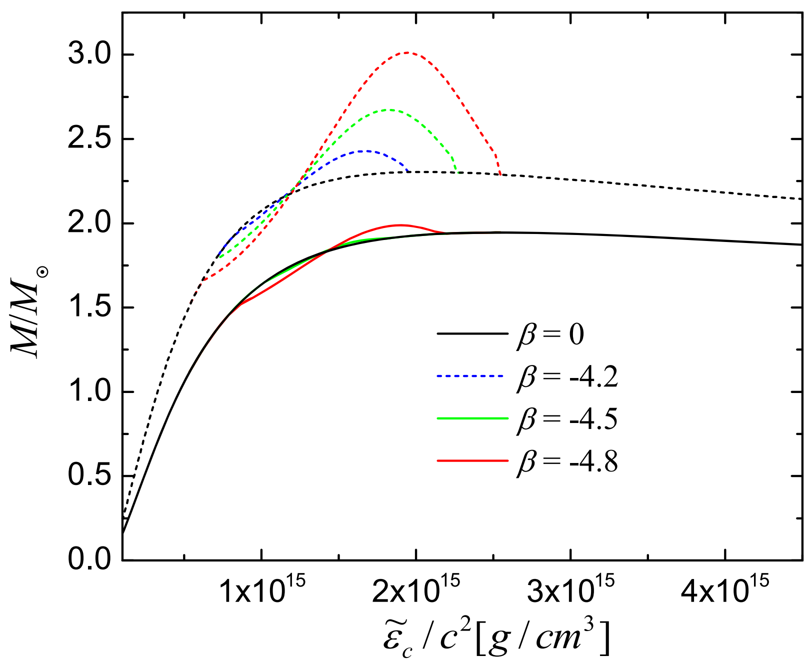


- Scalarization possible also for **positive $\boldsymbol{\beta}$** and negative trace of the energy momentum tensor. Possible for stiff EOS and very massive stars, not fully studied yet (Mendes (2015), Mendes & Ortiz (2016), Palenzuela & Liebling (2015)).
- **Tensor-multi-scalar theories** (Horbatsch et al (2015)) – new interesting phenomena, still in development.

Equilibrium neutron star solutions:
Scalar-Tensor Theory

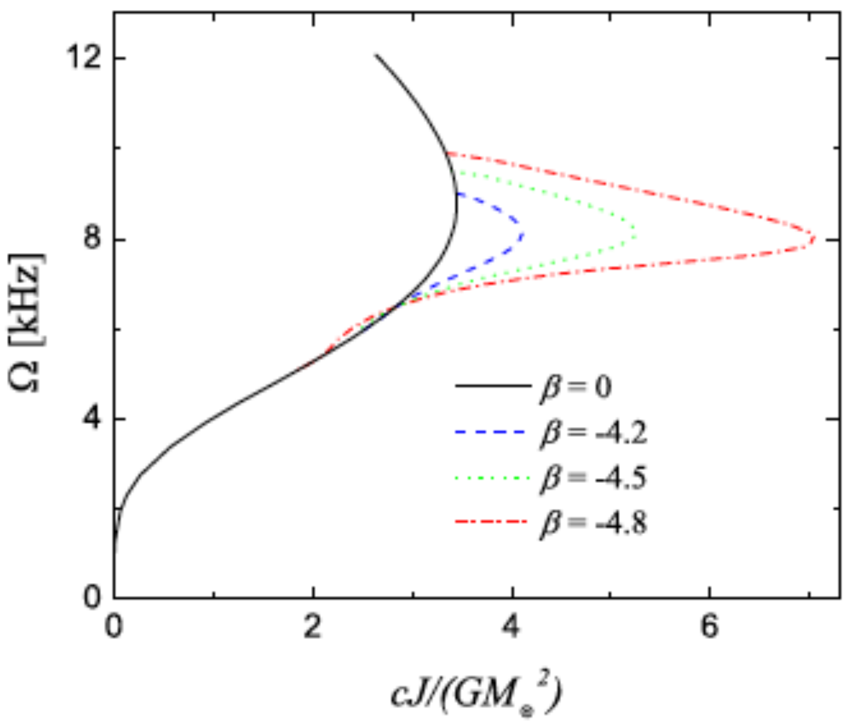
- **Slow rotation approximation** was also considered (Damour&Esposito-Farese (1996), Sotani (2012), Pani & Berti(2014)).
- **Rapid rotation** – changes the picture significantly (Doneva , Yazadjiev, Stergioulas, KK (2013)

Coupling function $\boldsymbol{\alpha(\varphi) = \beta\varphi}$

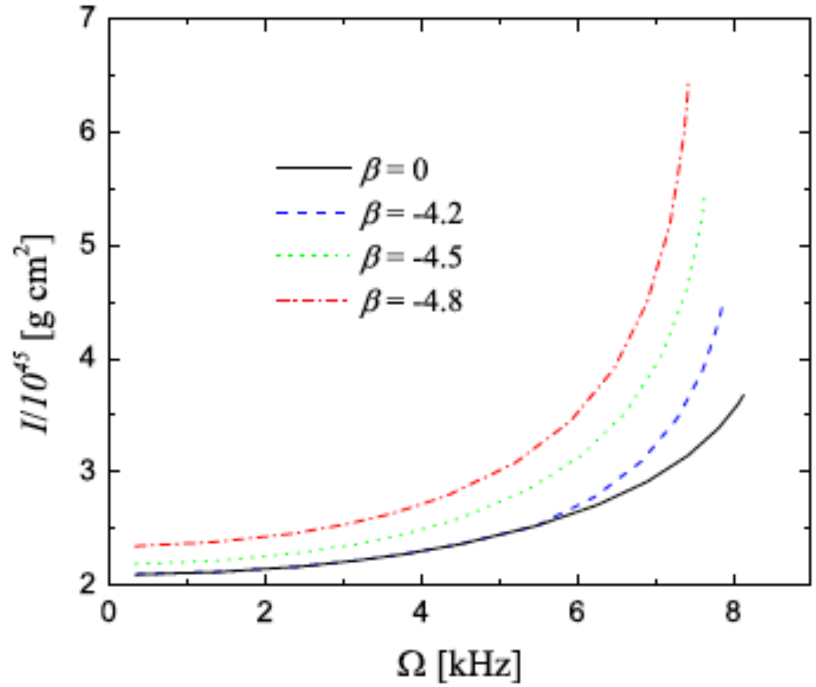


- Scalarization possible also for **positive $\boldsymbol{\beta}$** and negative trace of the energy momentum tensor. Possible for stiff EOS and very massive stars, not fully studied yet (Mendes (2015), Mendes & Ortiz (2016), Palenzuela & Liebling (2015)).
- **Tensor-multi-scalar theories** (Horbatsch et al (2015)) – new interesting phenomena, still in development.

Equilibrium neutron star solutions:
Scalar-Tensor Theory



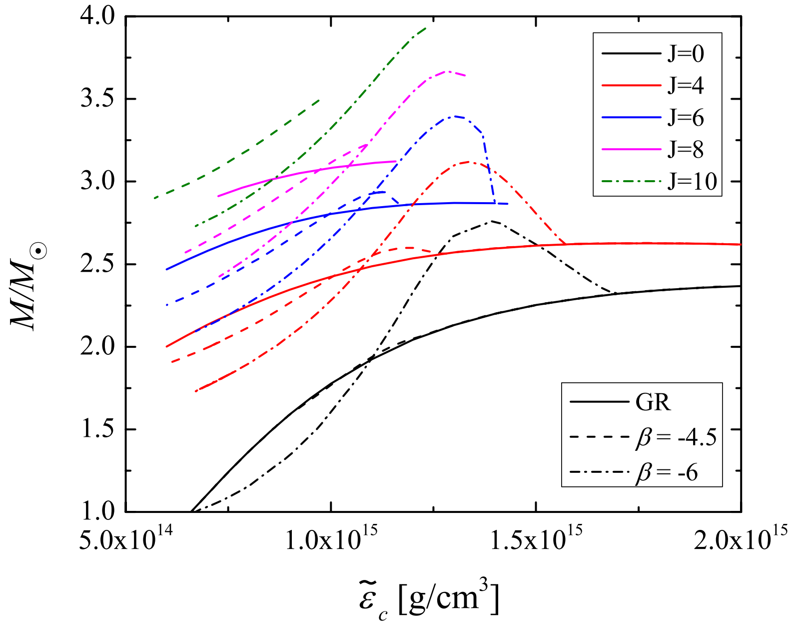
The **angular velocity** as a function of the **angular momentum** for sequences of stars rotating at the mass-shedding limit.

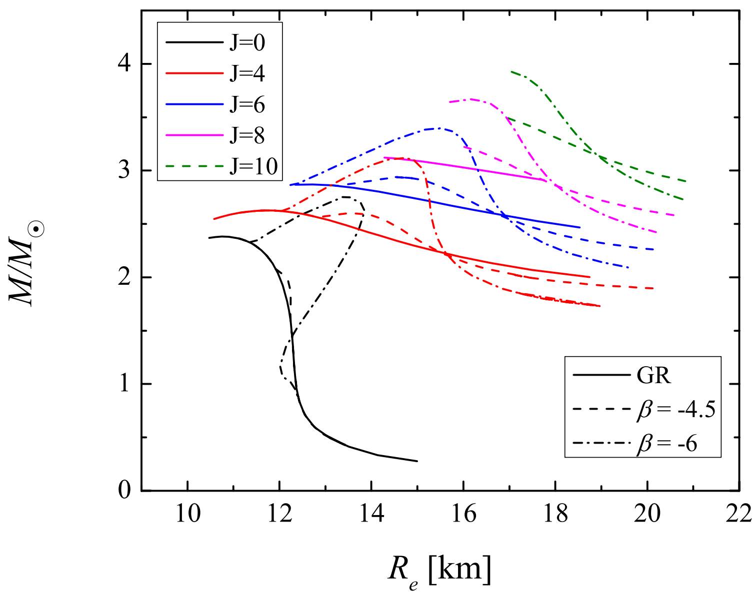


Moment of inertia as function of **angular velocity**

Differentially Rotating NSs:
Scalar-Tensor Theory

Doneva, Yazadjiev, Stergioulas, Kokkotas 2018





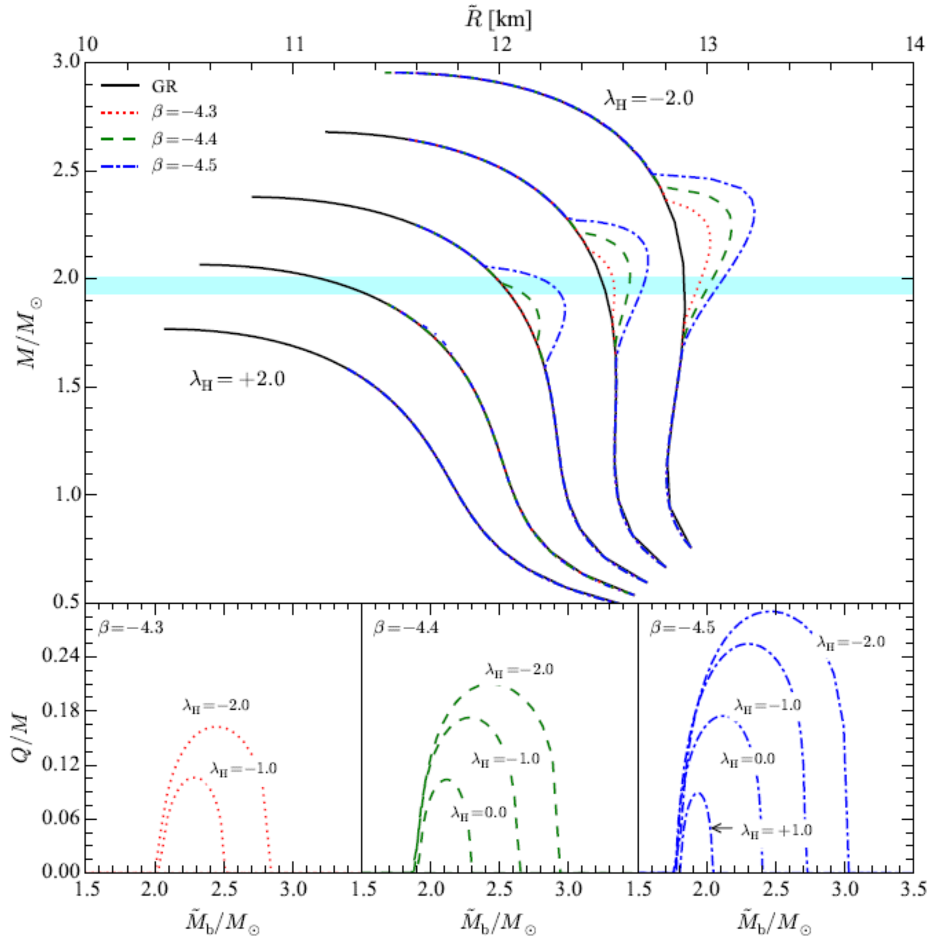
The mass as a function of the central energy density for differentially rotating sequences of neutron stars with constant angular momentum and as function of radius.

The maximum mass that a differentially rotating NS can sustain **increases significantly** for scalarized solutions and such stars can reach larger angular momenta.

The scalar field also makes rapidly rotating models less quasi-toroidal than their GR counterparts.

Equilibrium neutron star solutions:
Scalar-Tensor Theory

Anisotropic scalar-tensor neutron stars (Silva et al (2015)) – the deviations from GR are magnified significantly for strong degree of anisotropy



Scalar Tensor Theories with a massive scalar field

Neutron stars, with a massive scalar field could, in principle, have rather different structure and properties compared to their counterparts in the massless case.

Ramazanoglu, Pretorious *Spontaneous scalarization with massive field* (2016)

Yazadjiev, Doneva & Popchev *Slowly rotating neutron stars in scalar-tensor theories with a massive scalar field* (2016)

Doneva & Yazadjiev *Rapidly rotating neutron stars with a massive scalar field - structure and universal relations* (2016)

Scalar-Tensor Theory
with massive scalar field

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)\right) + S_m[\Psi_m; \boxed{\mathcal{A}^2(\varphi)} g_{\mu\nu}]$$

Coupling function $(k(\varphi) = \frac{d\, lnA(\varphi)}{d\ \varphi}$

Two types of coupling functions :

1) Brans-Dicke coupling $k(\varphi) = \alpha_0 \Leftrightarrow A(\varphi) = \exp(\alpha_0\varphi)$

2) Theory with spontaneous scalarization $k(\varphi) = \beta\varphi \Leftrightarrow A(\varphi) = \exp\left(\frac{\beta}{2}\varphi^2\right)$, where $\beta < 0$

✓ **Massive scalar field with a potential** $V(\varphi) = \frac{1}{2}m_\varphi^2\varphi^2$

Theoretical & observational bounds on the parameters

L. Perivolaropoulos, PRD ***81****, 047501 (2010); J. Alsing, E. Berti, C. M. Will, H. Zaglauer, PRD* ***85****, 064041 (2012); M. Hohmann, L. J¨arv, P. Kuusk, E. Randla, PRD* ***88****, 084054 (2013); A. Scharer, R. Ang´elil, R. Bondarescu, P. Jetzer, and A. Lundgren, PRD* ***90****, 123005 (2014); L. Jarv, P. Kuusk, M. Saal, and O. Vilson, PRD* ***91****, 024041 (2015)*

The recent astrophysical and cosmological observations have severely constrained the basic parameters of the scalar-tensor theories with a massless scalar field leaving a narrow window for new physics beyond general relativity.

The situation changes drastically if we consider a massive scalar field.

The scalar field mass m_φ leads to a finite range of the scalar field of the order of its Compton wavelength $\lambda_\varphi = 2\pi/m_\varphi$.

- The presence of the scalar field will be suppressed outside the compact objects at distances $D > \lambda_\varphi$.
- This means in turn that all observations of compact objects involving distances greater than λ_φ cannot put constraints, or at least stringent constraints, on the scalar tensor theories.

Theoretical & observational bounds on the parameters

Massive Brans-Dicke theory

- For massive Brans-Dicke theory with $m_{\varphi} \geq 2\times10^{-25} GeV$ $(or\ \lambda_{\varphi} \leq 10^{11} m)$ the Solar System observations cannot put constraints on the Brans-Dicke parameter α_0 and all values of α_0 $(\omega_{BD} > -3/2)$ are observationally allowed.
- **The massive gravitational scalar suppresses also the dipole radiation and the compact binaries** cannot constrain severely the Brans-Dicke parameter if their orbit radius is significantly greater than λ_{φ}.

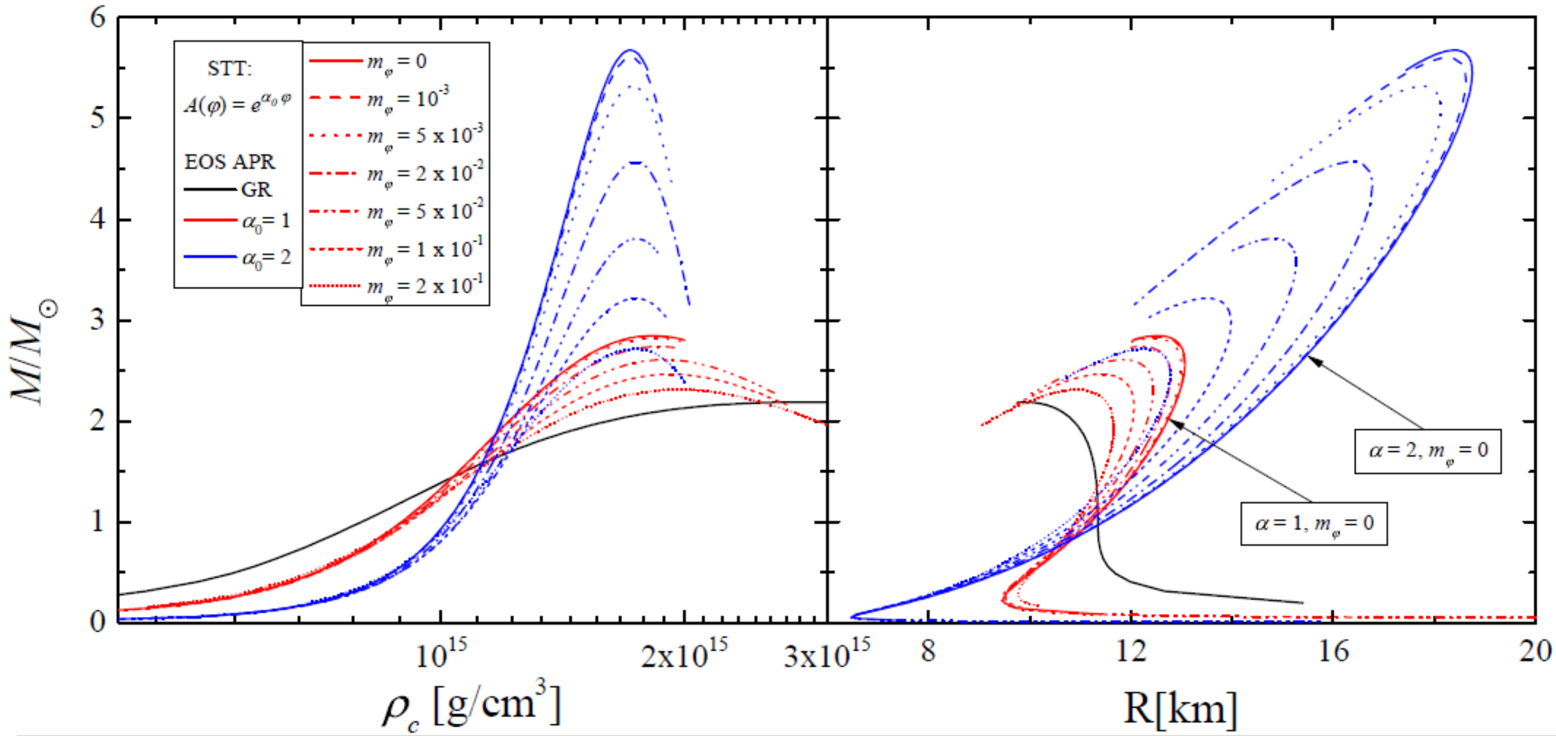
Scalar-tensor theory with $k(\varphi) = \beta\varphi$

- **The mass of the scalar field can effectively suppress the scalar gravitational waves** and reconcile the scalar-tensor theories with the binary neutron star observations for a much larger range of $\boldsymbol{\beta}$.
- **If the Compton wavelength of the scalar field $\boldsymbol{\lambda_{\varphi}}$ is much smaller than the separation of the two stars** in the binary system **the emitted scalar gravitational radiation will be negligible.**

Neutron stars in massive Brans-Dicke theory

$$m_\varphi \to m_\varphi R_0 = 2\pi\, R_0/\lambda_\varphi \qquad R_0 = 1.47664\, km$$

Brans-Dicke coupling $k(\varphi) = \alpha_0 \Leftrightarrow A(\varphi) = \exp(\alpha_0\varphi)$

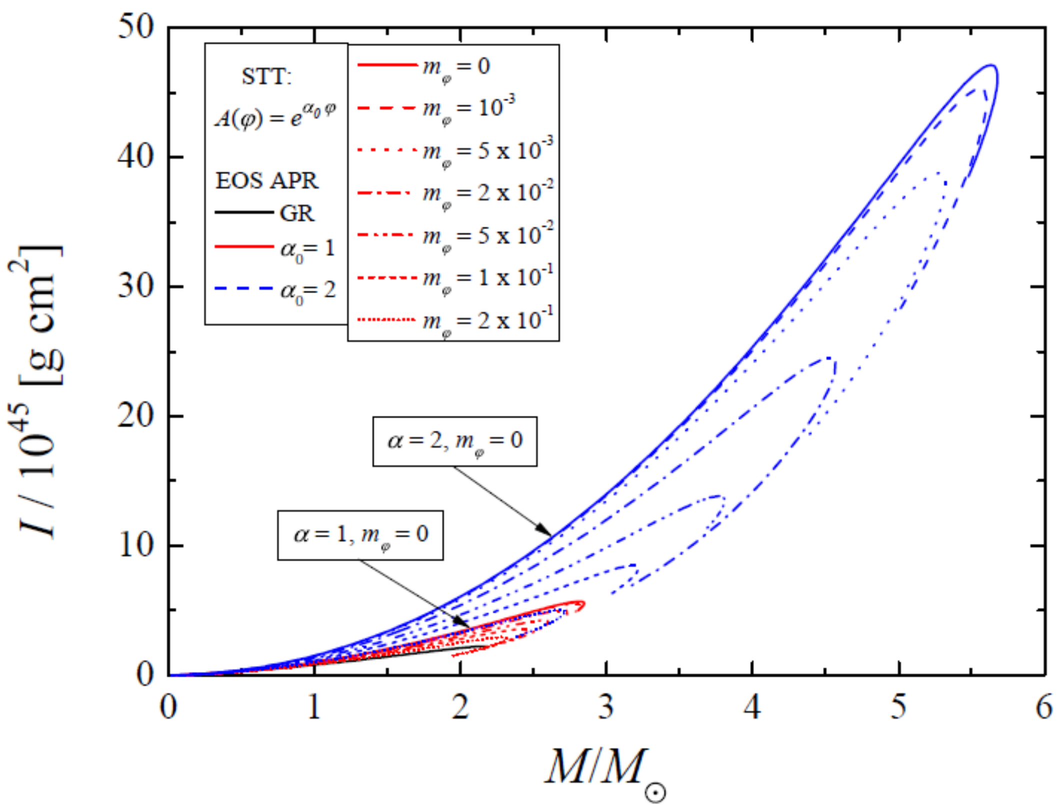


The allowed range for m_φ is $\mathbf{10^{-16}\, eV \leq m_\varphi \leq 10^{-9}\, eV}$

… normalized $\mathbf{7x10^{-7} \leq m_\varphi \leq 7}$

STT of gravity - observations

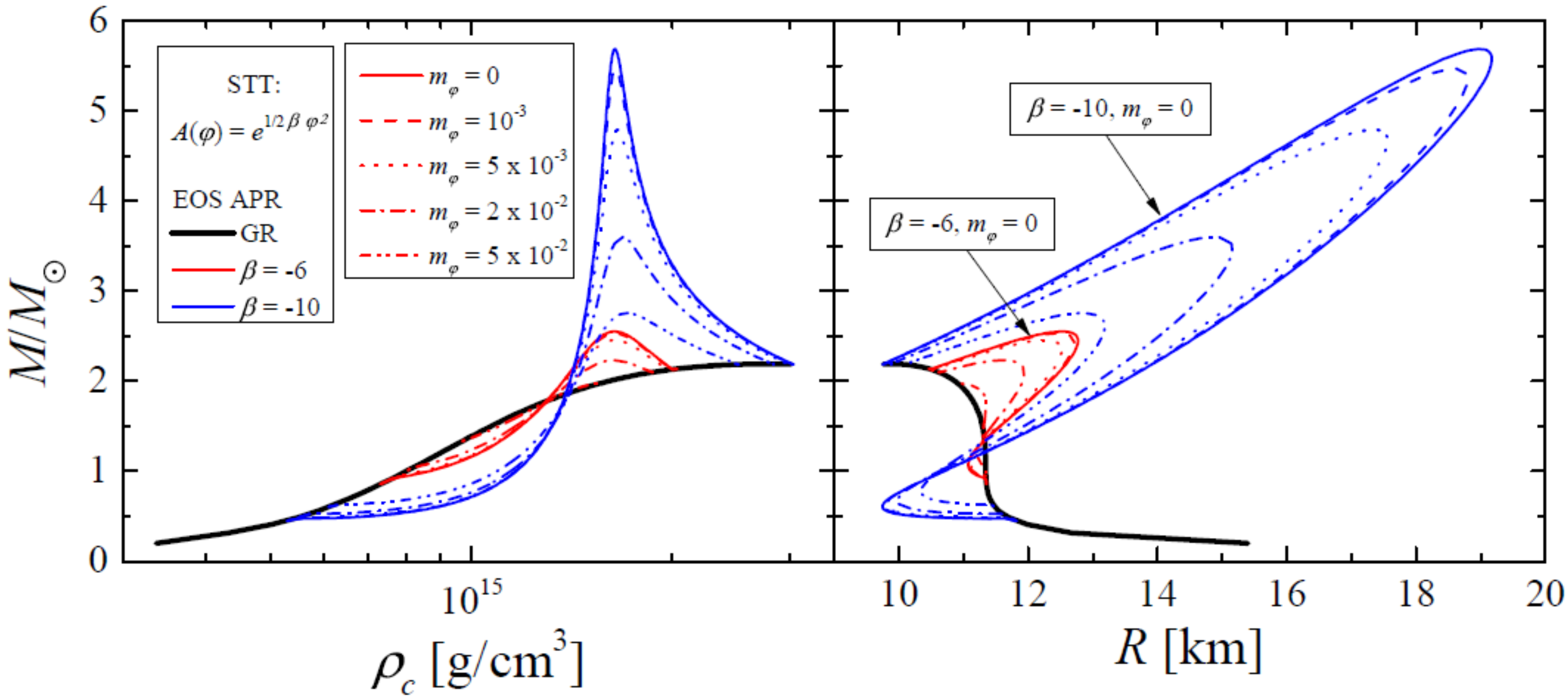
Brans-Dicke coupling $k(\varphi) = \alpha_0 \Leftrightarrow A(\varphi) = \exp(\alpha_0 \varphi)$



STT of gravity - observations

Theory with **spontaneous scalarization**

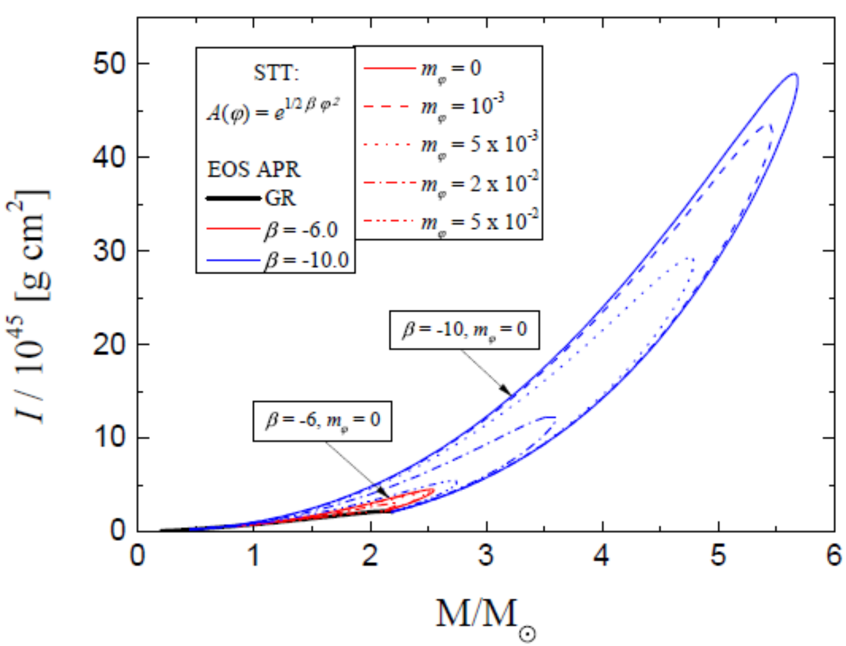
$$k(\varphi) = \beta\varphi \;\Leftrightarrow\; A(\varphi) = \exp\left(\frac{\beta}{2}\varphi^2\right), \beta < 0$$

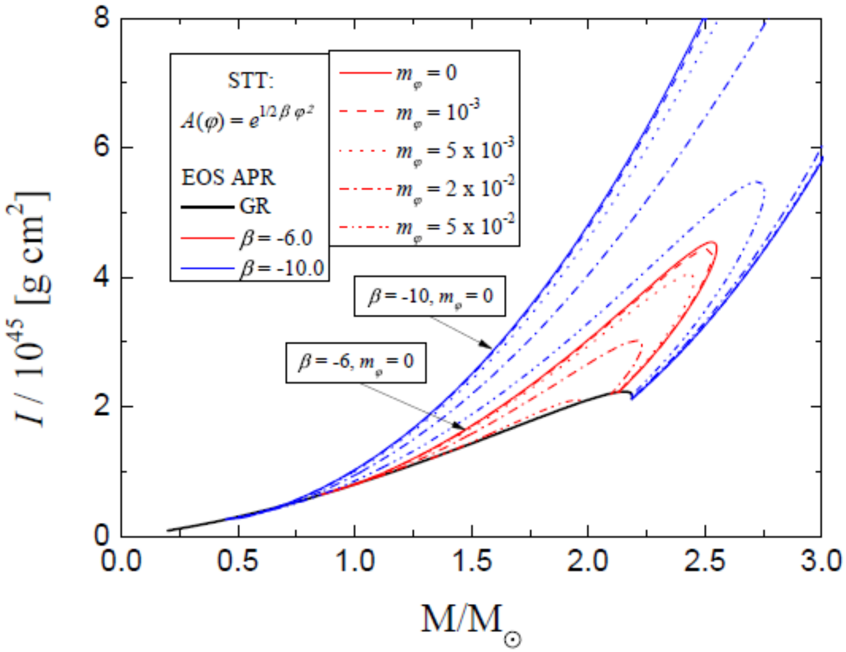


STT of gravity - observations

Theory with **spontaneous scalarization**

$$k(\varphi) = \beta\varphi \;\Leftrightarrow A(\varphi) = \exp\left(\frac{\beta}{2}\varphi^2\right), \beta < 0$$





Conclusions (massive field)

- ✓ In scalar-tensor theories with a **massless scalar field neutron stars differ almost marginally from GR** if one considers coupling parameters that are in agreement with the present observations.

- ✓ The inclusion of **scalar field mass changes the picture dramatically**. It suppresses the scalar field at length scale of the order of the Compton wavelength which helps us reconcile the theory with the observations for a much broader range of the coupling parameters.

- ✓ **The structure and the properties of the neutron stars in massive STT can differ drastically from the pure GR solutions if sufficiently large masses of the scalar field are considered.**

$f(R)$ theories of gravity

Alternative theories of gravity: $f(R)$ theories

- **Motivation:** widely used as an alternative explanation of the *accelerated expansion of the universe*

- **Studied mainly at cosmological scales**, but every theory of gravity should pass via the observations at astrophysical scale too

- **Action:** $$\boxed{S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,f(R) + S_{\mathrm{matter}}(g_{\mu\nu}, \chi)}$$

- Free of tachyonic instabilities and the appearance of ghosts when:

$$\boxed{\frac{d^2f}{dR^2} \geq 0, \qquad \frac{df}{dR} > 0}$$

- **Mathematical treatment** of the problem: $f(R)$ *theories are mathematically equivalent to a particular class of massive scalar-tensor theories.*

Alternative theories of gravity: $f(R)$ theories of gravity

Example: R^2 gravity ($f(R) = R + aR^2$)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_{\text{matter}}(g_{\mu\nu}, \chi),$$

This action is **mathematically equiva**lent to the following scalar-tensor theory action

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-\tilde{g}}\left[\boxed{F(\Phi)}\tilde{R} - \cancel{Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi} - \boxed{2U(\Phi)}\right] + S_m\left[\Psi_m; \tilde{g}_{\mu\nu}\right]$$

$$= \Phi = f'(R)$$

$$= \frac{1}{8a}(\Phi - 1)^2 \;\Rightarrow\; m_\Phi = \frac{1}{\sqrt{6a}}$$

Equilibrium neutron star solutions:
$f(R)$ theories of gravity

- We will concentrate on the $\boldsymbol{R^2}$ **gravity** ($f(R) = R + a\,R^2$) case, that is expected to give the dominant contribution at astrophysical scales.

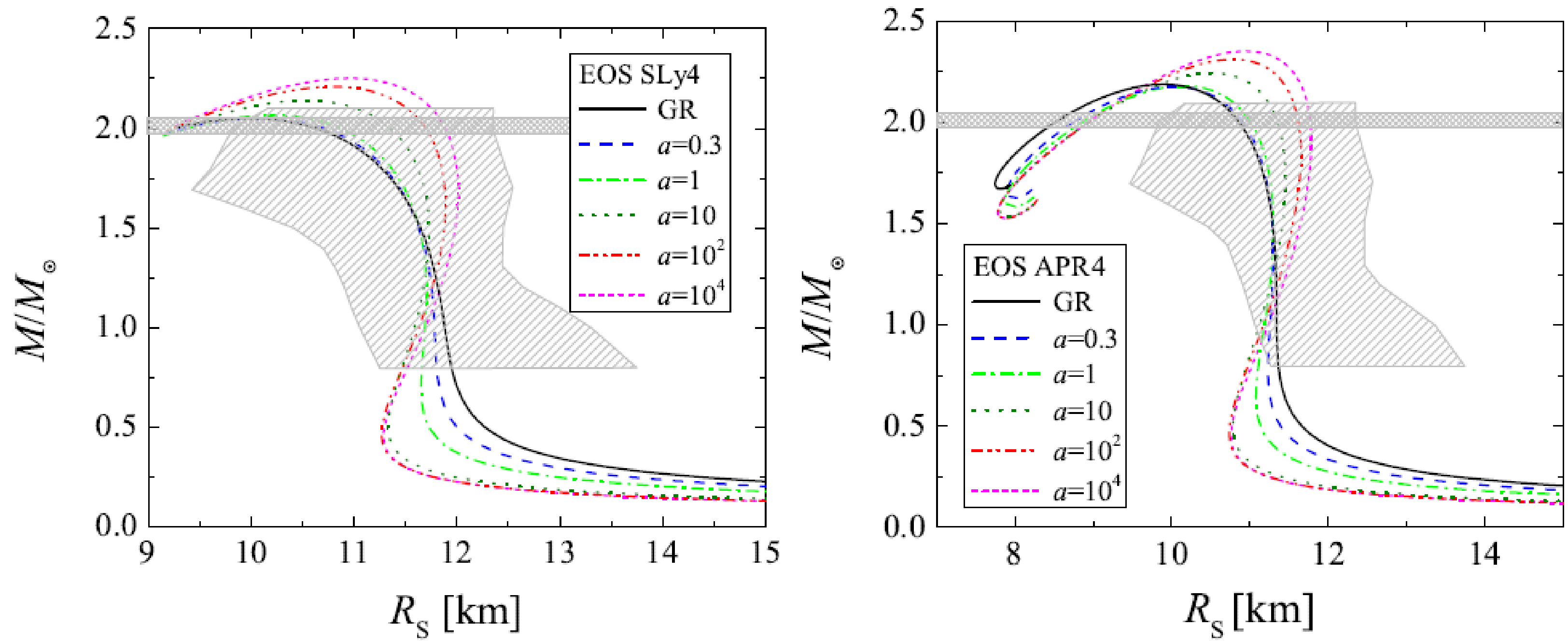
- **Pertubative approach**, assuming that $\boldsymbol{a}$ is a small number, (Cooney, DeDeo, Psaltis (2010)) widely used in the past, but it was shown to be "**misleading"** (Yazadjiev, Doneva, Kokkotas, Staykov (2014))

- **Observational constraints** – the most severe coming from the Gravity Probe B experiments $a < 2.5{\times}10^5$ (or $a < 5{\times}10^{11} m^2$ in physical units).

- The **scalar-tensor representation** of $f(R)$ theories is commonly employed.

- *The field equation for the Ricci scalar curvature (or equivalently the scalar field) is stiff which poses a computational difficulty.*

NSs in f(R)-gravity: Static Models



Yazadjiev, Doneva, Kokkotas, Staykov (2014)

$$f(R) = R + aR^2$$

- The differences between the R^2 and GR are comparable with the uncertainties in the nuclear matter equations of state.
- The current observations of the NS masses and radii alone can not put constraints on the value of the parameters a, **unless the EoS is better constrained in the future.**

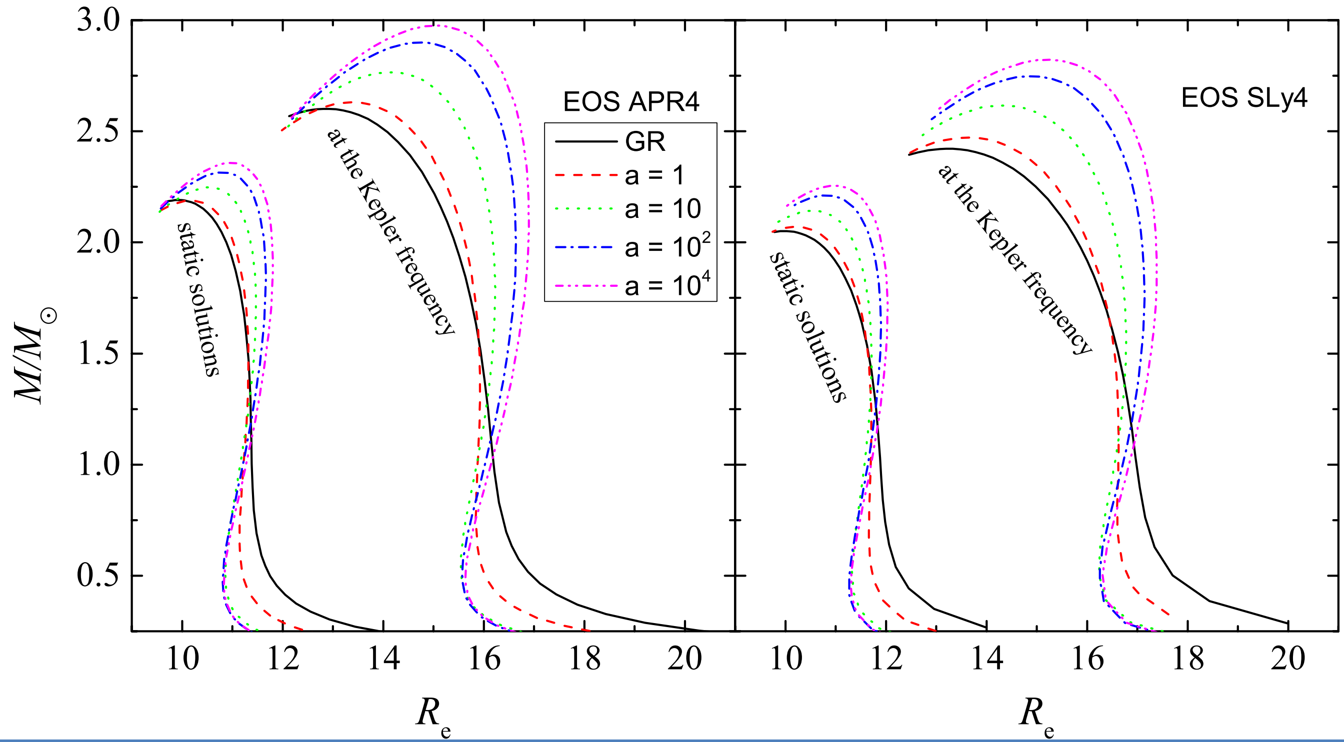
See also: Capozziello, De Laurentis, Farinelli, Odintsov (2015)

NSs in f(R)-gravity: Fast Rotation

$$f(R) = R + aR^2$$

Mass vs Radius diagrams for two realistic EOS

Yazadjiev, Doneva, Kokkotas, (2015)

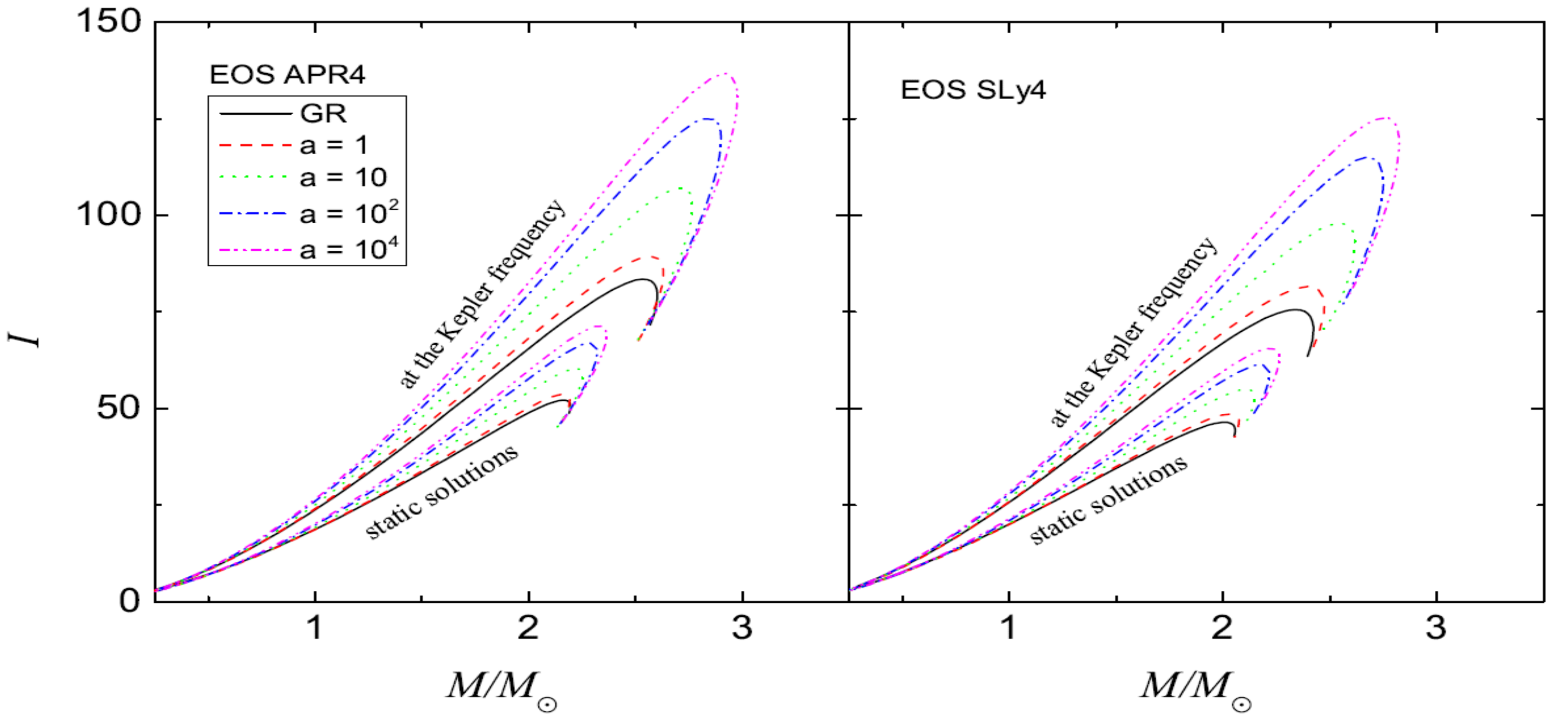


Difficult to set constraints on the *f* (*R*) theories using measurement of the neutron star **M** and **R** alone, until the EOS can be determined with smaller uncertainty.

NSs in f(R)-gravity: Fast Rotation

$$f(R) = R + aR^2$$

Yazadjiev, Doneva, Kokkotas (2015)

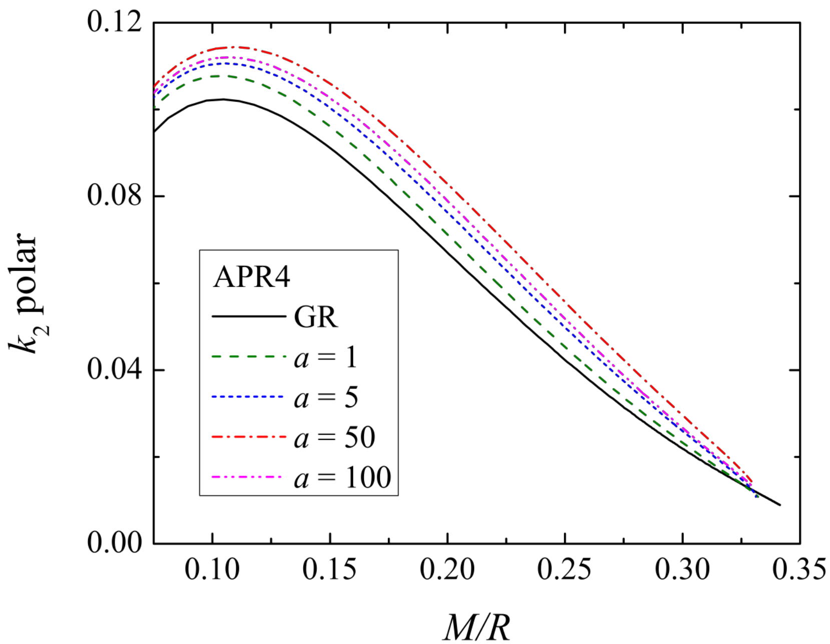


- ✓ The differences in the neutron star moment of inertia on the other hand can be much more dramatic. **BEYOND THE UNCERTAINTY DUE TO THE EOS**
- ✓ Large deviations can be potentially measured by the forthcoming observations of the NS moment of inertia [Lattimer-Schutz 2005, Kramer-Wex 2009] that can lead to a direct test of the R^2 gravity.

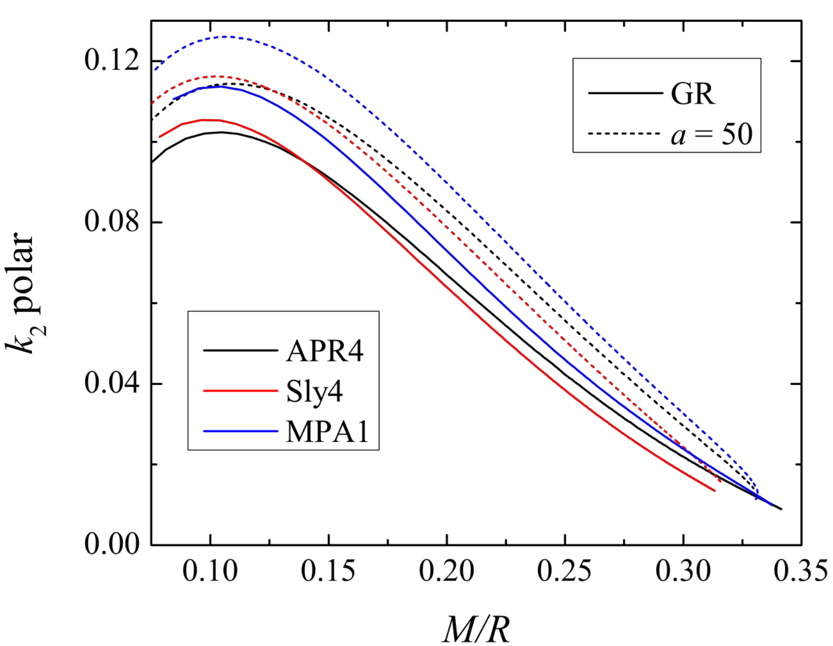
Tidal Love numbers in f(R)

$$f(R) = R + aR^2$$

Yazadjiev, Doneva, Kokkotas (2018)



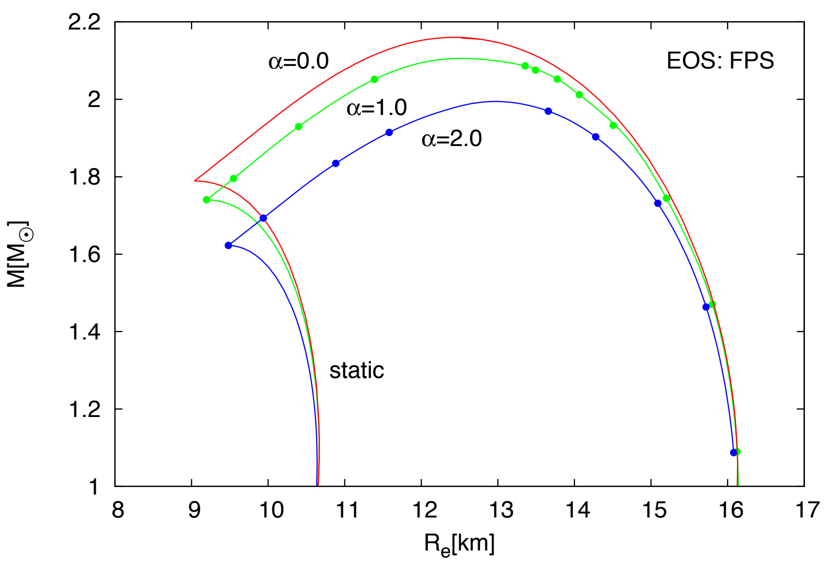
The polar tidal Love numbers as functions of the stellar compactness for the ARP4 EOS and several values of the parameters a.

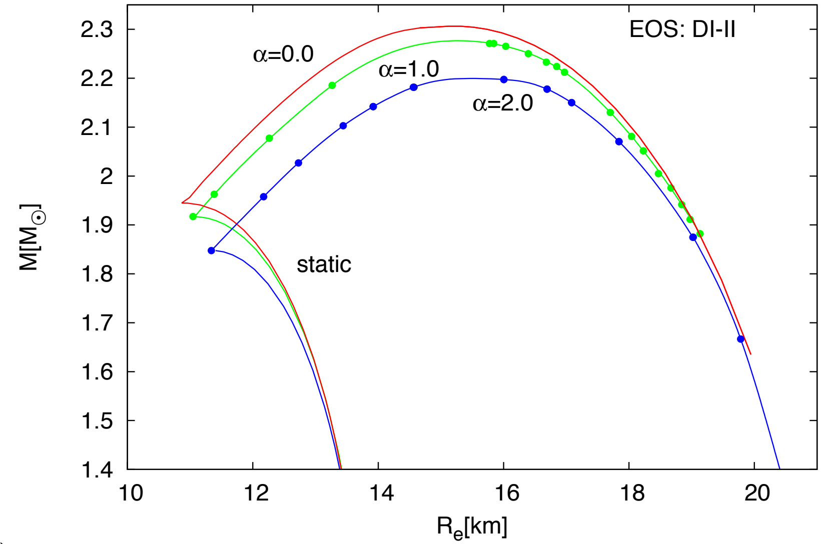


The polar tidal Love numbers as functions of the stellar compactness for several EOS, and for the pure GR case and a=50.

Dilatonic Einstein-Gauss-Bonnet Theory

- The theory is **motivated from string theory**.
- String theory predicts the presence of **higher curvature terms** in the action as well as further fields.
- In the low energy effective action obtained from heterotic string theory contains as **basic ingredients a Gauss-Bonnet** (GB) term and a dilaton field

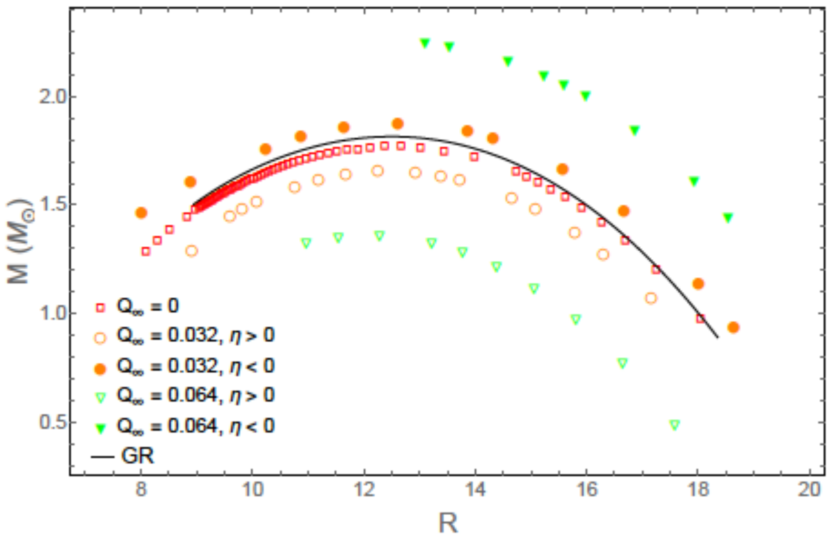


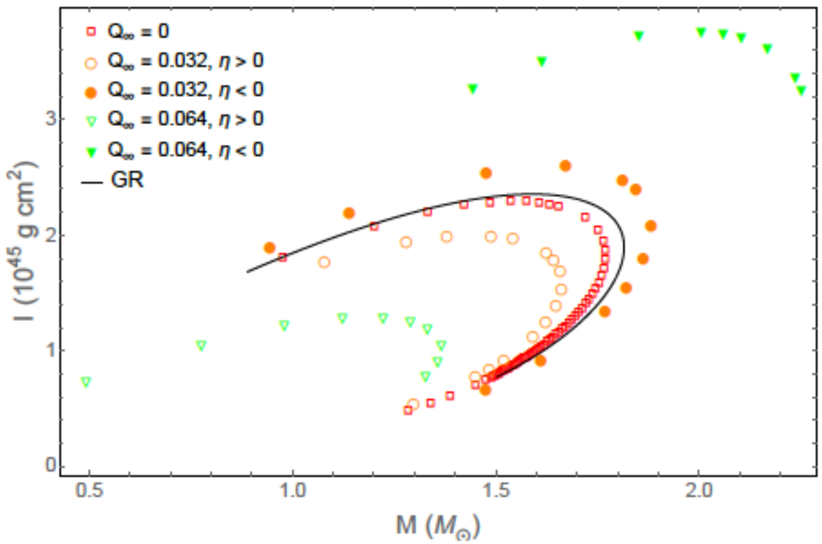


Kleihaus, Kunz, Mojica, Zagermann 2016

Hondersky gravity

The most **general extension of Einstein's theory** of general relativity with a **single scalar degree of freedom** and **second-order field equations**.



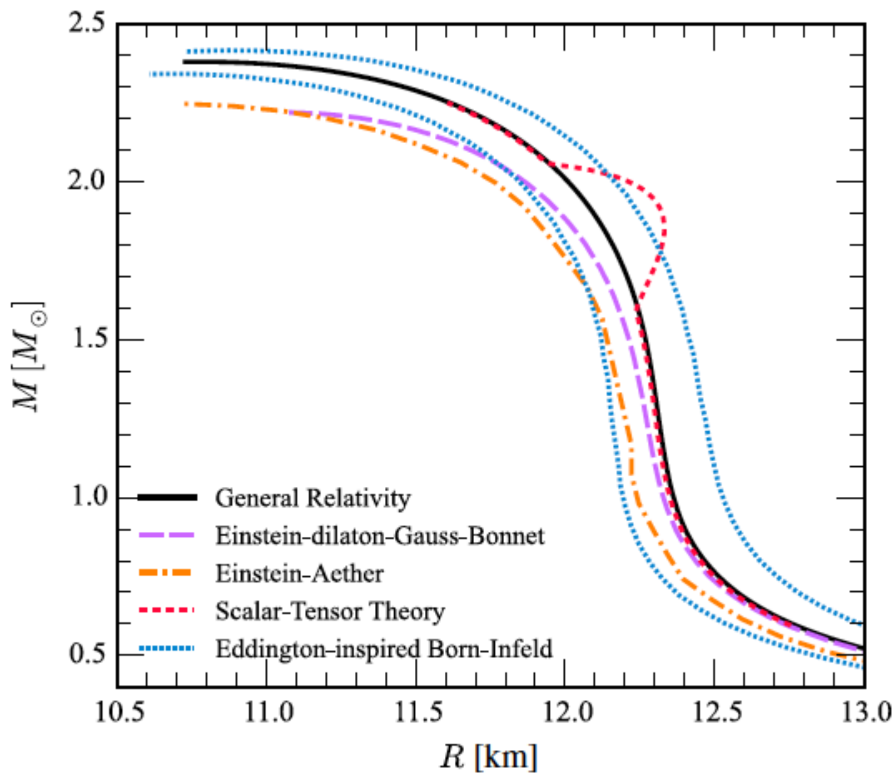


Babichev and Charmousis (2014)
Babichev, Charmousis, Lehebel (2016)
Babichev etal (2016)
Barausse and Yagi (2015)
Cisterna et al. (2015)
Maselli et al. (2016a, 2016b)
…

Post-TOV approximation

The gravity theory degeneracy problem:

Exists even if we do know the correct equation of state



The logic underpinning the formalism is that by parametrizing the deviation of the stellar structure equations from their GR counterparts, thus producing a set of post-TOV equations.

Glampedakis, Pappas, Silva, Berti (2015, 2016)

Post-TOV approximation

Post-TOV equations: *describe smooth modifications of he TOV equations, parametrized by the post-TOV parameters*

$$\frac{dp}{dr} = \left(\frac{dp}{dr}\right)_{\mathrm{GR}} - \frac{\rho m}{r^2}\mathcal{P} \qquad \frac{dm}{dr} = \left(\frac{dm}{dr}\right)_{\mathrm{GR}} + 4\pi r^2 \rho \mathcal{M}$$

where

$$\mathcal{P} \equiv \pi_1 \frac{m^3}{r^5\rho} + \pi_2 \frac{m^2}{r^2} + \pi_3 r^2 p + \pi_4 \frac{\Pi p}{\rho}$$

$$\mathcal{M} \equiv \mu_1 \frac{m^3}{r^5\rho} + \mu_2 \frac{m^2}{r^2} + \mu_3 r^2 p + \mu_4 \frac{\Pi p}{\rho} + \mu_5 \frac{\Pi^3 r}{m}$$

Glampedakis, Pappas, Silva, Berti (2015, 2016)

Astrophysical Implications

Astrophysical Implications

- **Final goal** – test the strong field regime of gravity via neutron star observations and impose constraints on the alternative theories

- **Obstacles**:
  - Accuracy of observations
  - Accurate models of the observed phenomena
  - EOS uncertainty

- **Ways out:**
  - Deviation from GR stronger than the EOS uncertainty for the allowed range of parameters
  - EOS independent relations

Astrophysical Implications

Possible approaches for testing alternative theories of gravity

- **Direct observation of the mass and radius.**

- **Observations of the moment of inertia**: applicable for example for $f(R)$ theories Staykov at al (2014) and Eddington inspired gravity Pani, Cardoso, Delsate (2011)

- **Quasiperiodic oscillations** DeDeo&Psaltis (2004), Doneva, Yazajdjiev,Stergioulas, Kokkotas, Athanasiadis (2014), Staykov, Doneva, Yazadjiev (2015)

- **The redshift of surface spectral lines** in X-rays and γ-rays DeDeo&Psaltis(2003)

- **Gravitational wave emission of oscillating neutron stars**

- **Neutron star mergers**

- **Universal relations**

Astrophysical Implications

Universal relations

- **EOS independent relations** between the properties, including the oscillation spectrum, of neutron stars. Normally a proper normalization of the quantities is required.

- Very convenient way to **circumvent the EOS uncertainty**.

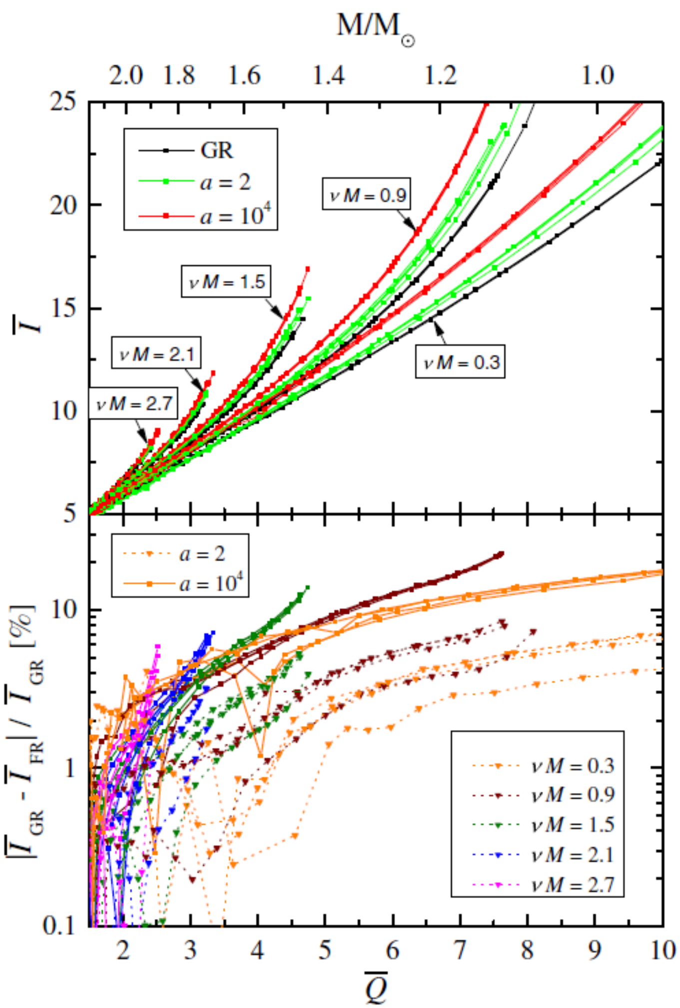
- Attracted particular attraction with the paper of *Yagi&Yunes (2013)*

- The focus is on the **I-Love-Q relations** but many other universal relations exist (Lattimer&Schutz(2005),Yagi et al (2014), AlGendy&Morsink(2014), Breu&Rezzolla(2016))

- **General idea** for **testing the strong field** regime of gravity: if the two parameters that enter in a universal relation are measured independently, then a possible deviation from the GR EOS independent relations can be measured.

Astrophysical Implications

Example R^2 theories:



- **I-Love-Q relations**: appreciable deviations from GR only for some alternative theories of gravity (**dynamical Chern-Simons gravity** Yagi&Yunes(2013), $f(R)$ gravity Doneva, Yazadjiev, Kokkotas (2015), **massive STT** Yazadjiev,Doneva (2016))

- **Most of the studied alternative theories of gravity give only marginal deviations from GR** (eg. Sham, Lin, Leung(2014); Kleinhaus, Kunz, Mojica (2014), Pani, Berti (2014), Pappas, Sotiriou (2015)).

- **Unnormalized relations** STILL differ significantly from GR. Solution:
  - Different normalization
  - Different universal relation

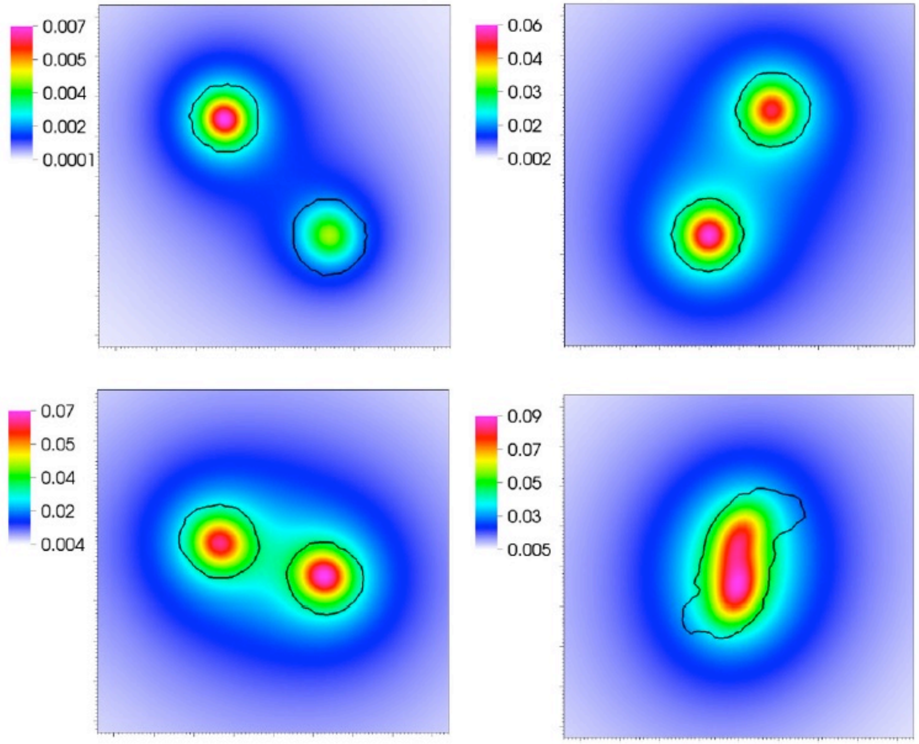
- **Strong point:** these relations are also theory independent up to a good extend that might have different application.

Astrophysical Implications

Dynamical scalarization – NS mergers

Even if the two NS are not scalarized when separated, in close binary system they **develop strong scalar field**.

Coupling function $\boldsymbol{\alpha(\varphi) = \beta\varphi}$



The observational signature of the scalarized merging neutron stars has been studied in Barause et al (2013), Palenzuela et al (2014), Shibata et al (2014), Sampson (2014), Taniguchi et al (2015).

Astrophysical Implications

Neutron star oscillations

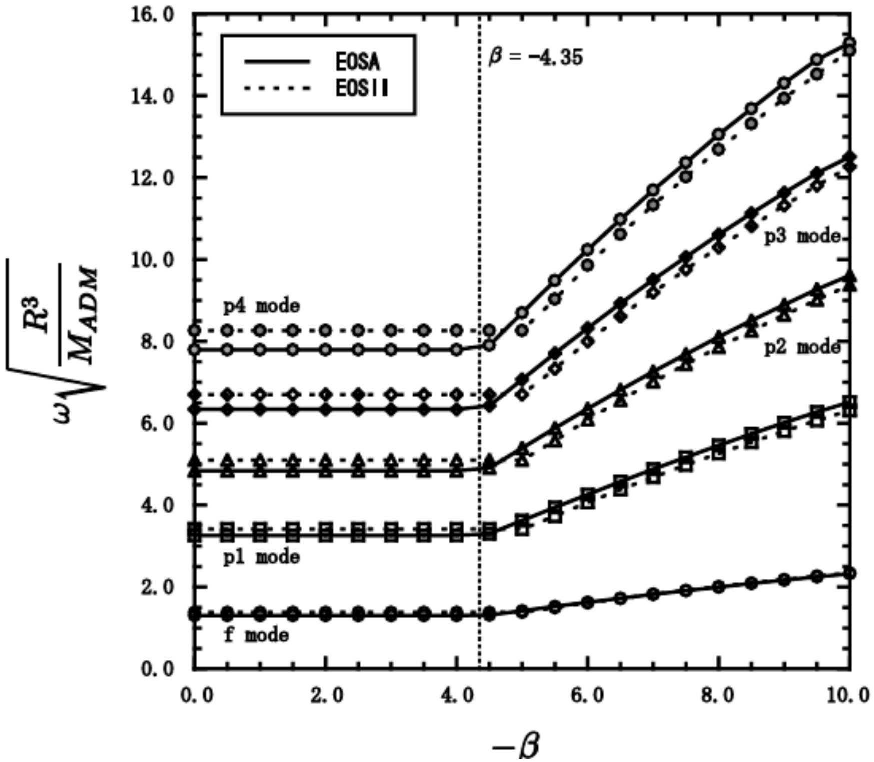
- The study was **initiated** with the work of Sotani & Kokkotas (2004,2005) for *f*-, *p*- and *w*-modes in STT.

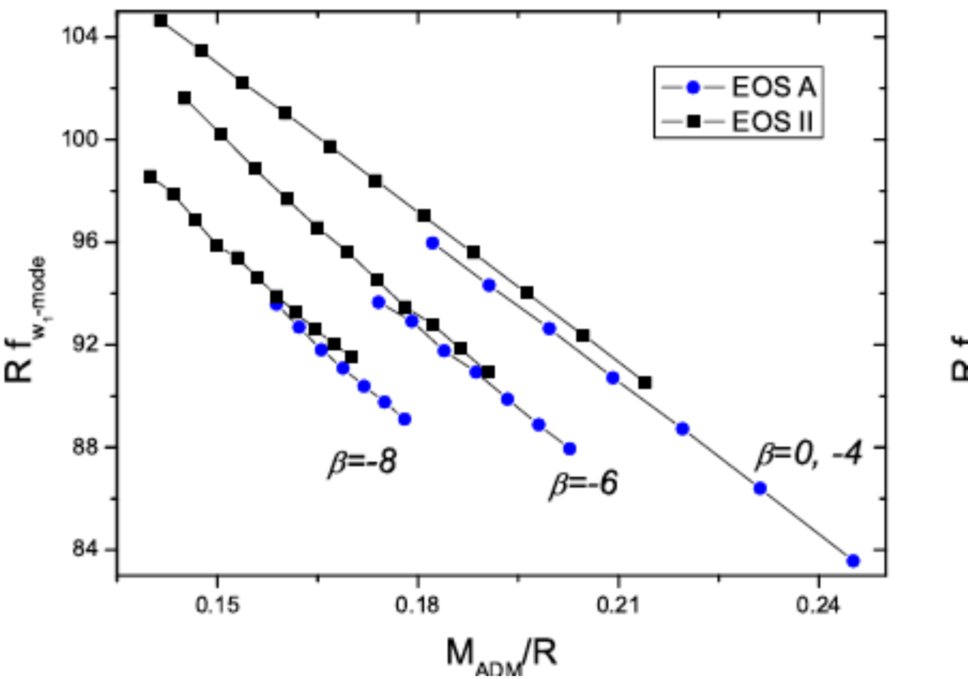
- The main idea is to constrain the deviations from GR using the emitted gravitational wave signal or in some cases electromagnetic signal, related to neutron star oscillations

- Several alternative theories studied until now – **STT** Sotani&Kokkotas (2004, 2005), Silva et al (2014), **TeVeS** Sotani (2010, 2011, 2009), $\boldsymbol{f(R)}$ Staykov et al (2015), **Einstein-Gauss-Bonnet-dilaton gravity** Blázquez-Salcedo et al (2016)

- Fundamental ***f*-modes**, **torsional** modes, ***w*-modes** and others are studied. In many cases the Cowling approximation is employed.

Stellar Oscillations in STT





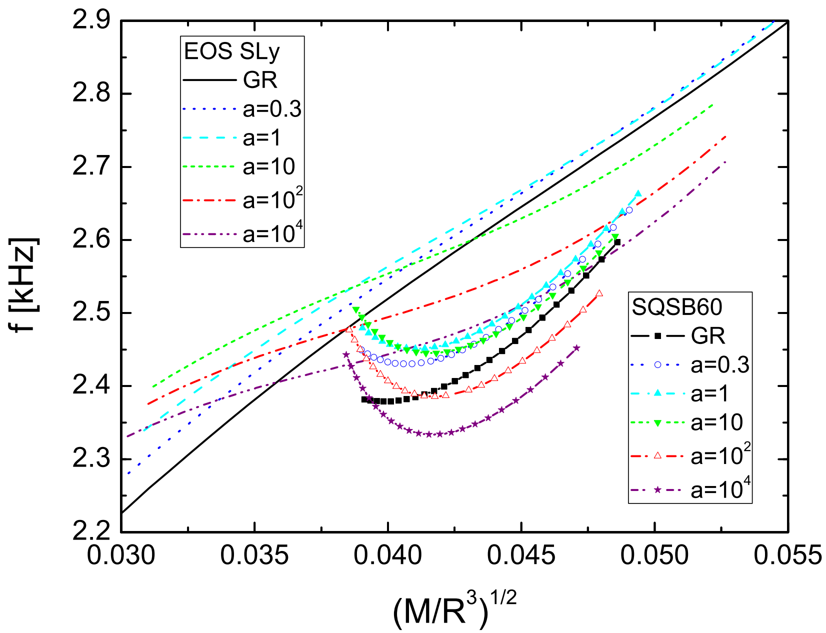
$$\frac{\partial \omega_n}{\partial(-\beta)} \approx \frac{n}{4}$$

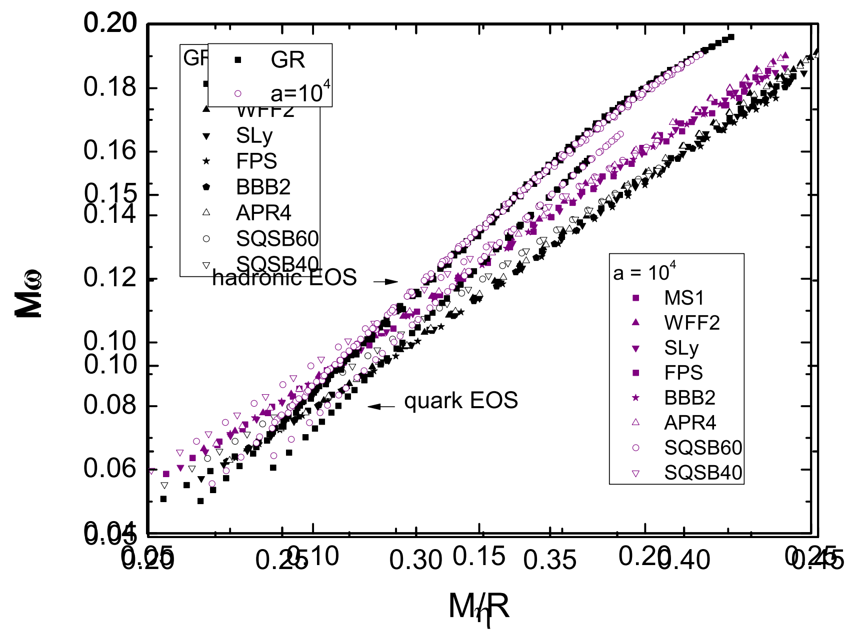
Sotani-Kokkotas (2004,2005)

Asteroseismology in ATG

Asteroseismology relations in R^2 theories

- ***f*-mode** oscillation frequencies, nonrotating case
- Quite **EOS independent** with suitable choice of normalization



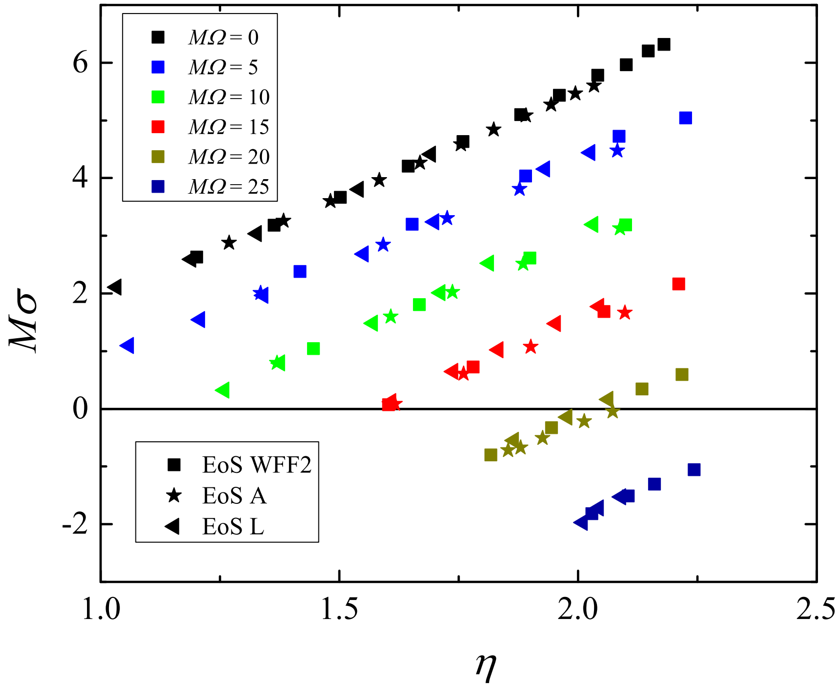


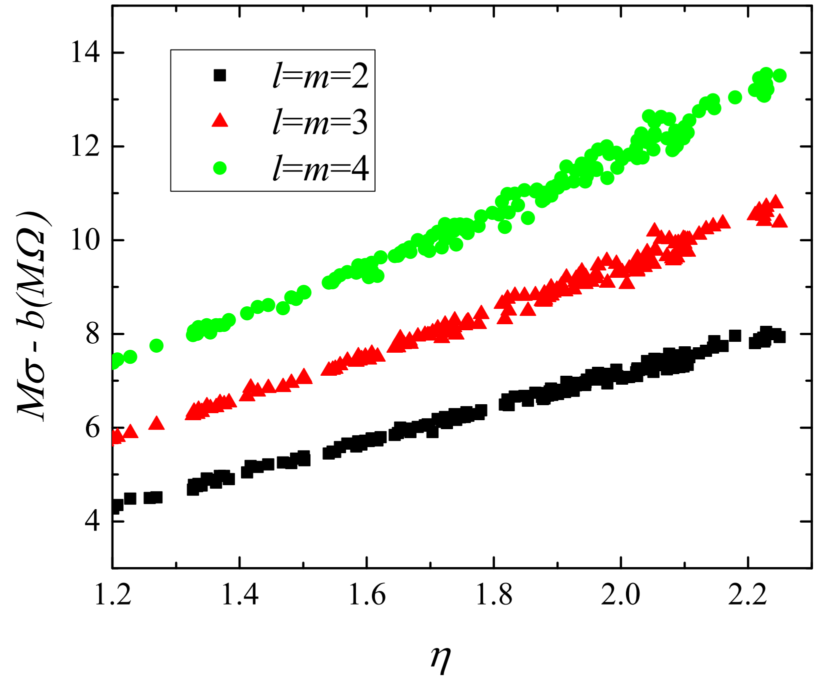
$$\eta = \sqrt{M^3 / I}$$

- The maximum deviation between the f-mode frequencies in GR and R^2 gravity is up to 10% and depends on the value of the R^2 gravity parameter a.
- Alternative normalizations show nicer relations

Asteroseismology: but in GR

$$M\sigma_i^{unst} = [(0.56 - 0.94\ell) + (0.08 - 0.19\ell)M\Omega + 1.2(\ell+1)\eta]$$



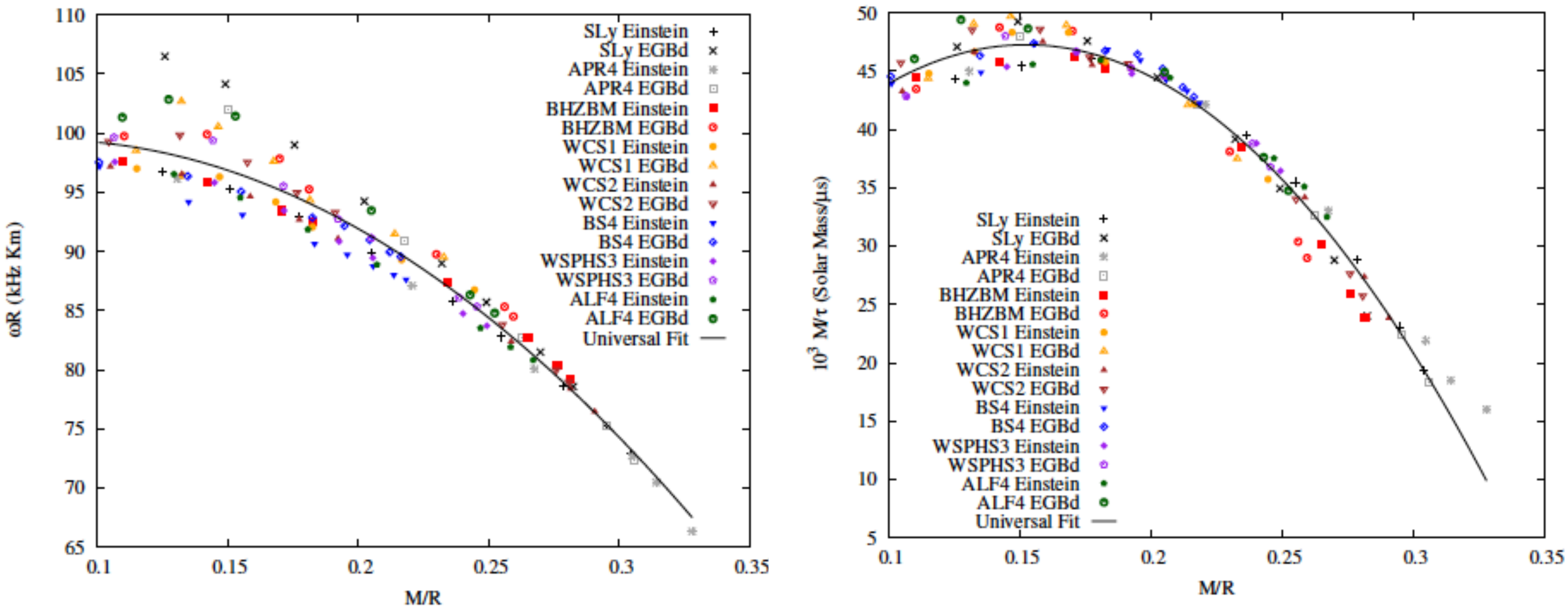


$$\eta = \sqrt{M^3/I}$$

The l = 2 f-mode oscillation frequencies as functions of the parameter η

Doneva-Kokkotas 2015

Dilatonic Einstein-Gauss-Bonnet Theory
Axial w-modes



Blazquez-Salcedo, Gonzalez-Romero, Kunz, Mojica, Navarro-Lerida (2016)

Conclusions

- ✓ **Neutron stars in alternative theories of gravity can have significantly different properties compared to their general relativistic counterparts.**
- ✓ **Rotation can magnify the deviations and lead to new observational consequences.**
- ✓ **A further study of the astrophysical implications is required in order to check what are the most promising astrophysical implications.**
- ✓ **Further info: Berti et al (2015), Yagi & Yunes (2016), Doneva & Pappas (2017)**