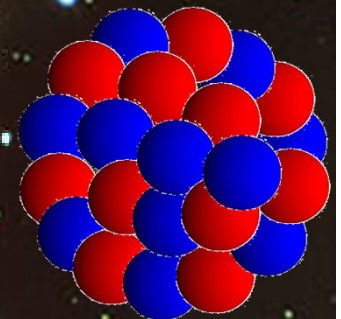
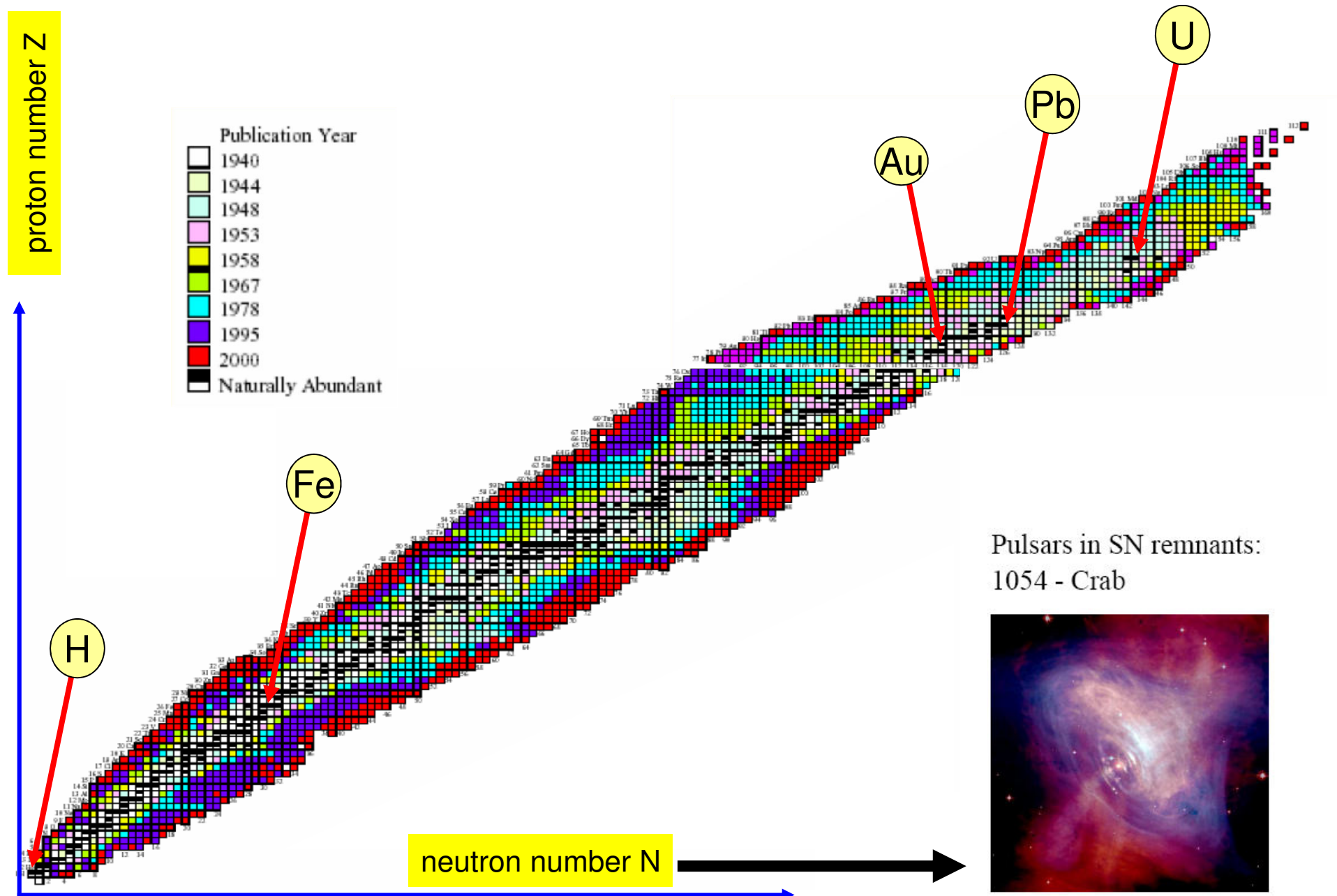


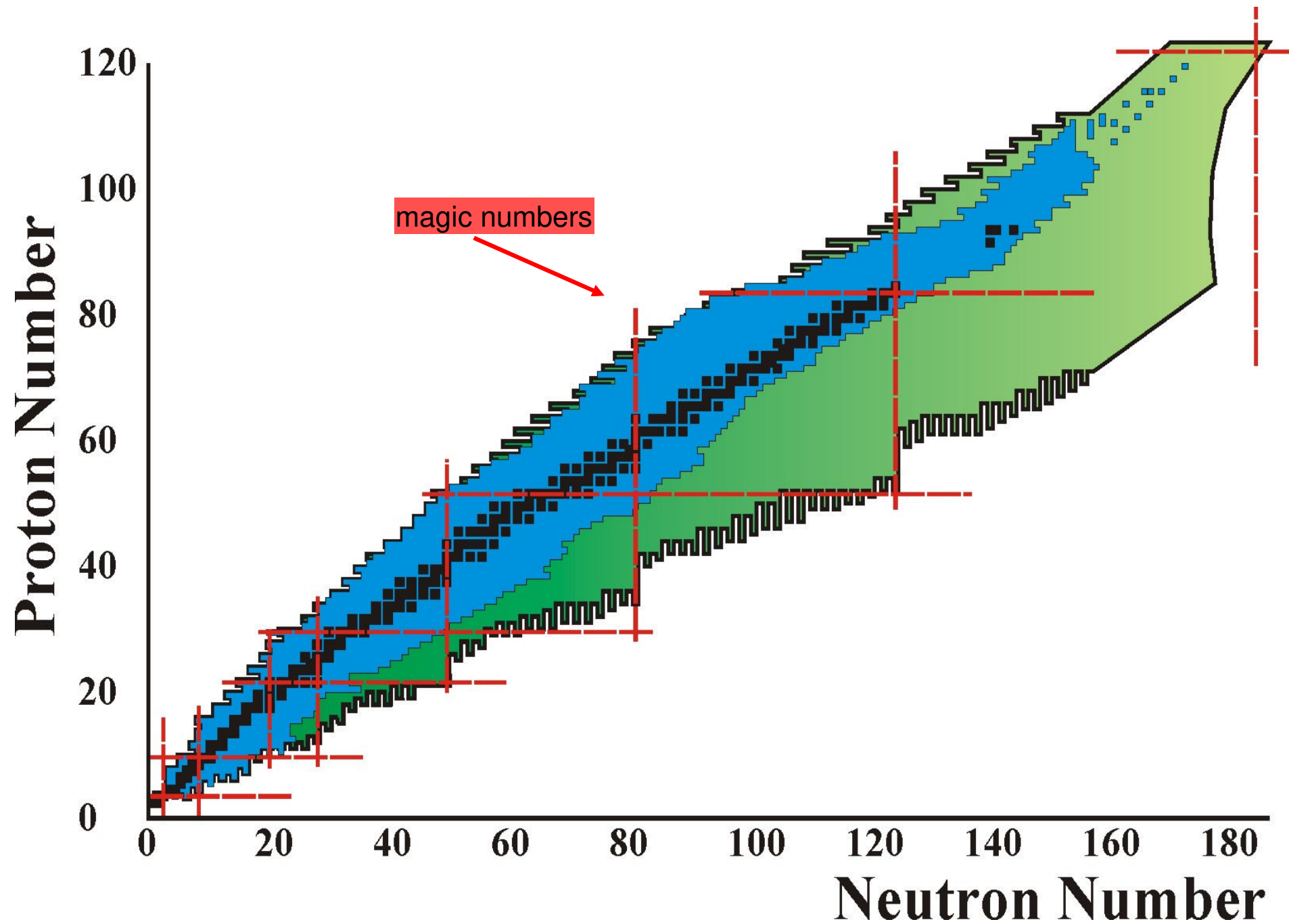
Nuclear Density Functional Theory Application to Nuclear Astrophysics

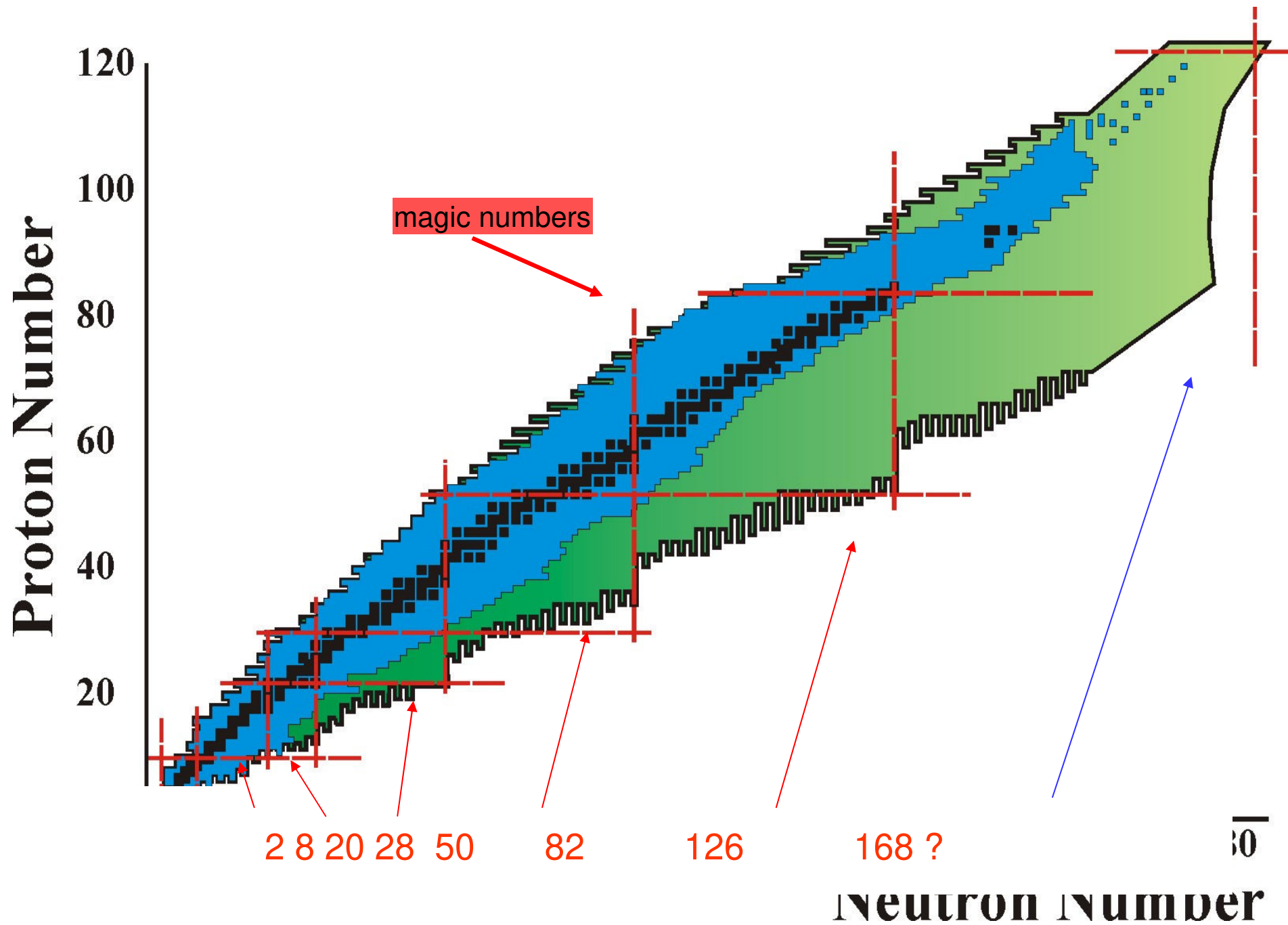
**3rd HEL.A.S. and DAAD
School Thessaloniki 2018**

**Georgios Lalazissis
Aristotle University of Thessaloniki**

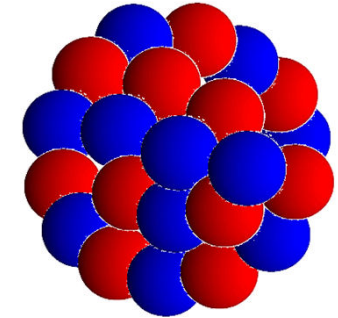








Forces acting in the nucleus:

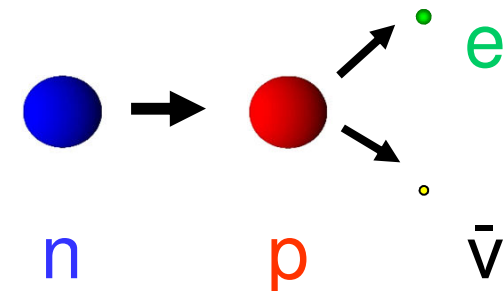


the **Coulomb force** repels the protons

the **strong interaction** ("nuclear force") causes binding
is stronger for pn-systems than nn-systems

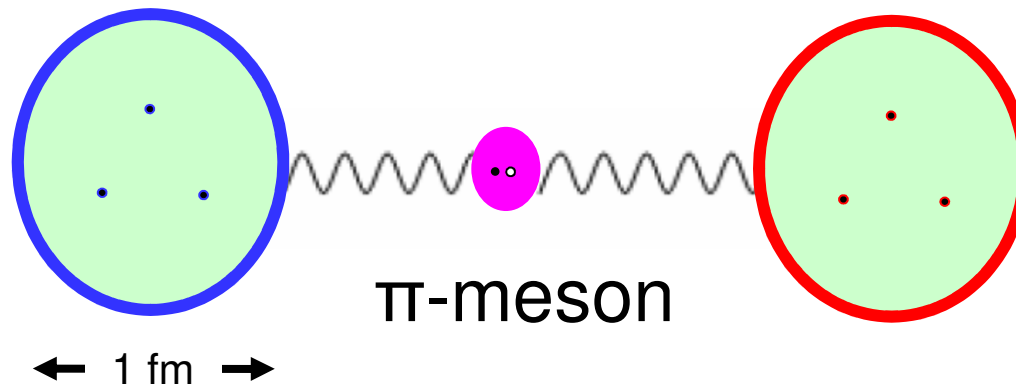
neutrons alone form no bound states
exception: neutron stars (**gravitation!**)

the **weak interaction** causes β -decay:



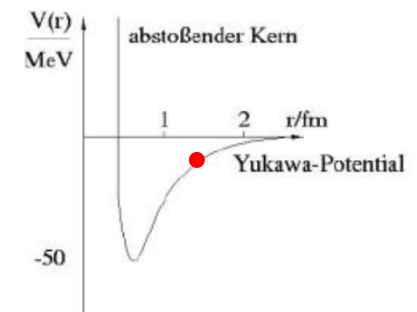
the nucleon-nucleon interaction:

distance > 1 fm

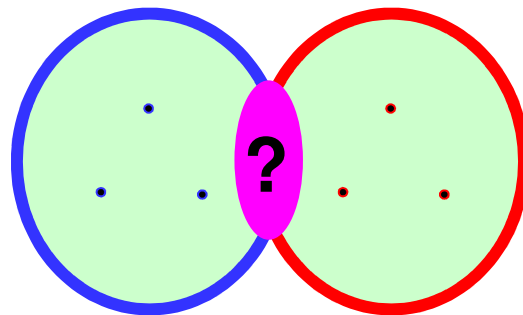


π -meson

attractive

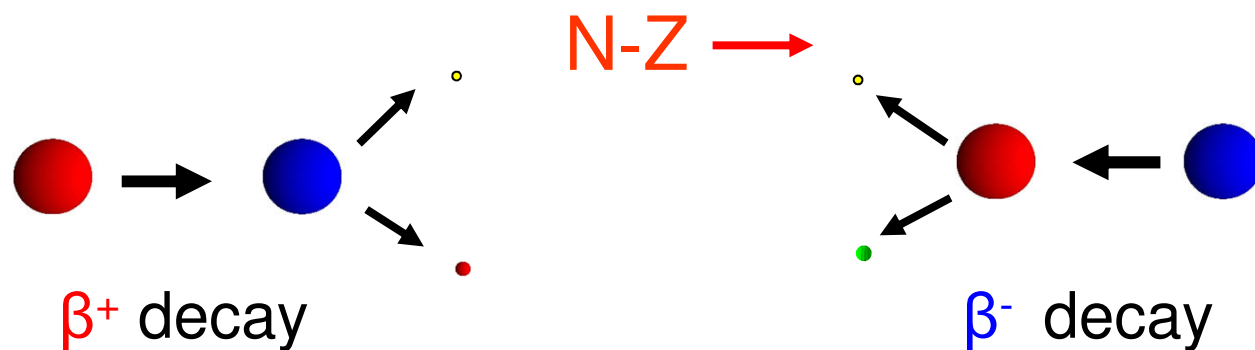
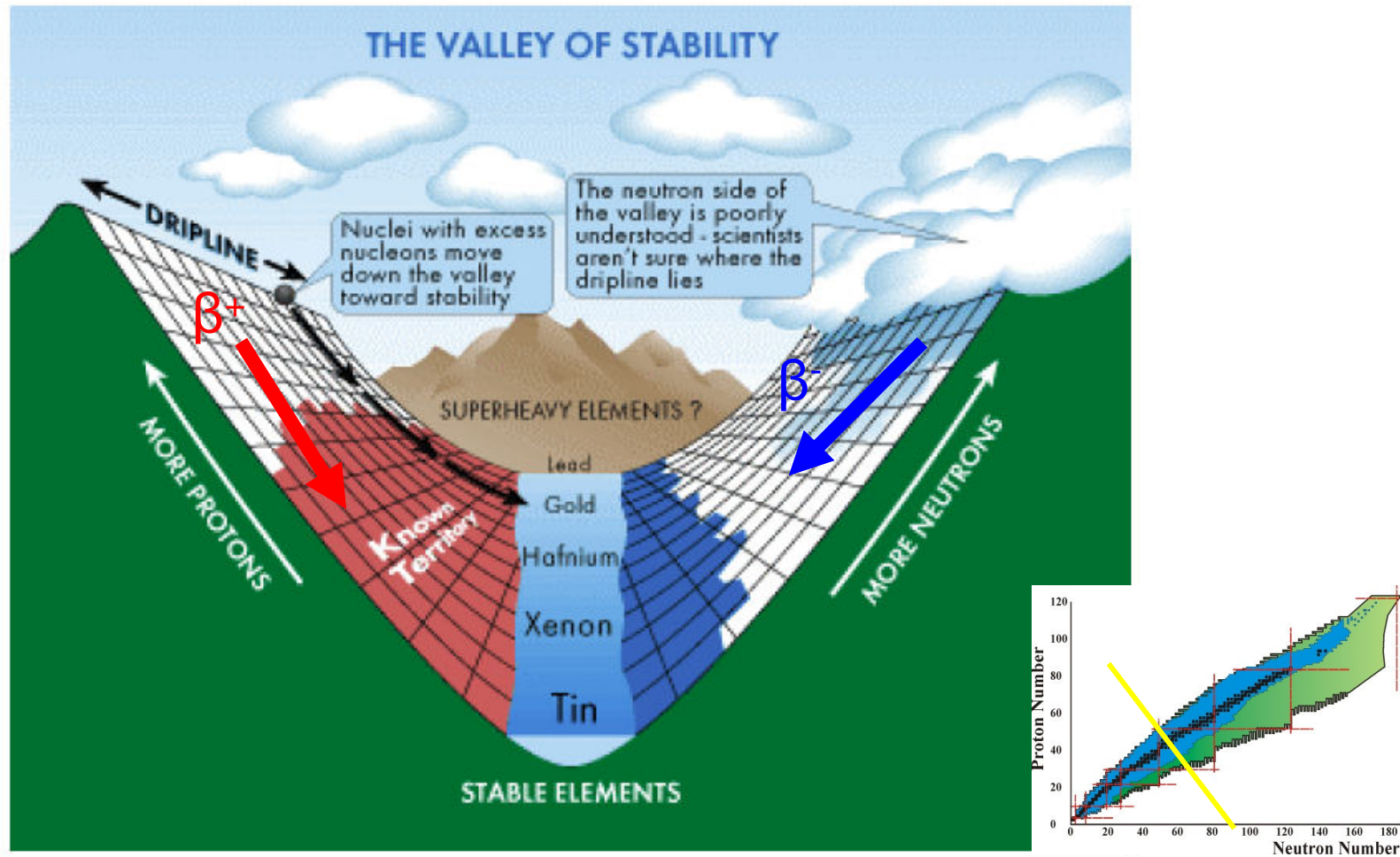


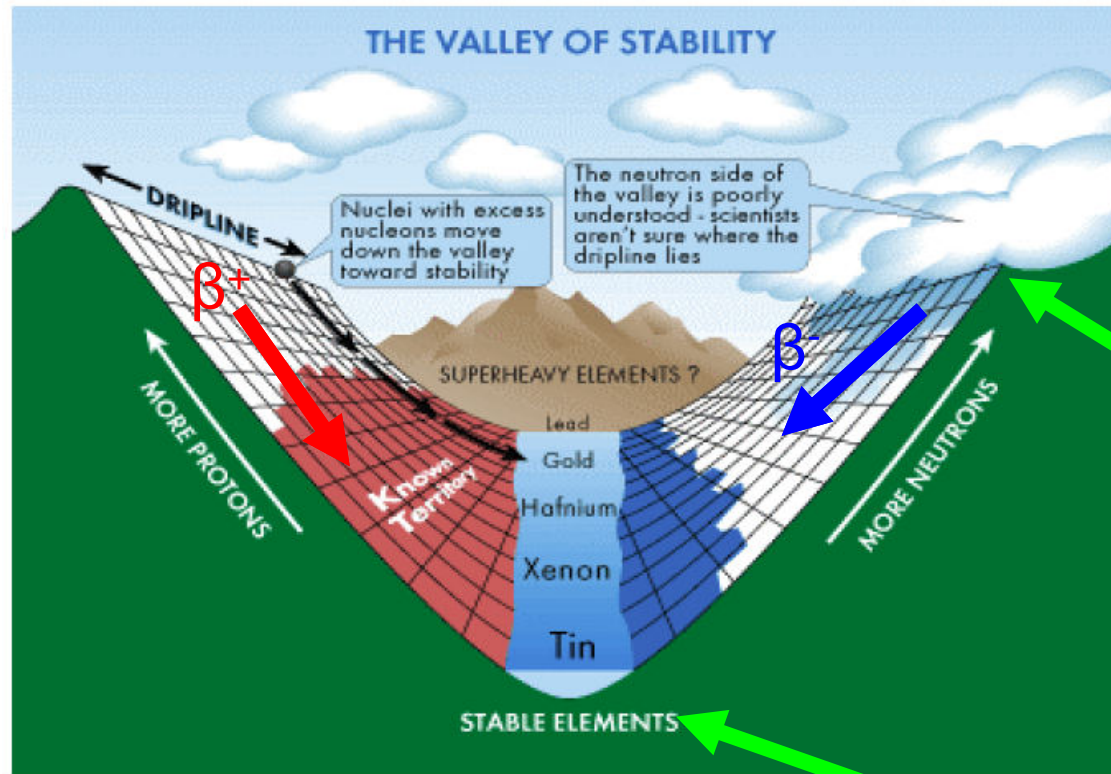
distance < 0.5 fm



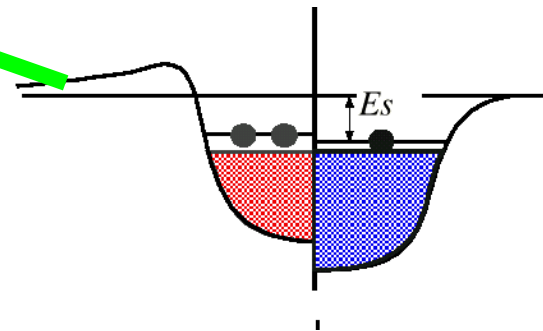
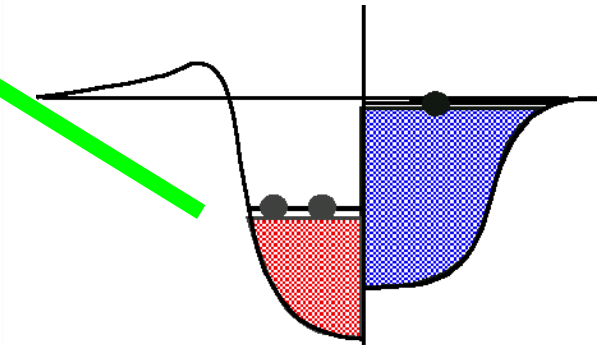
repulsive

three-body forces ?

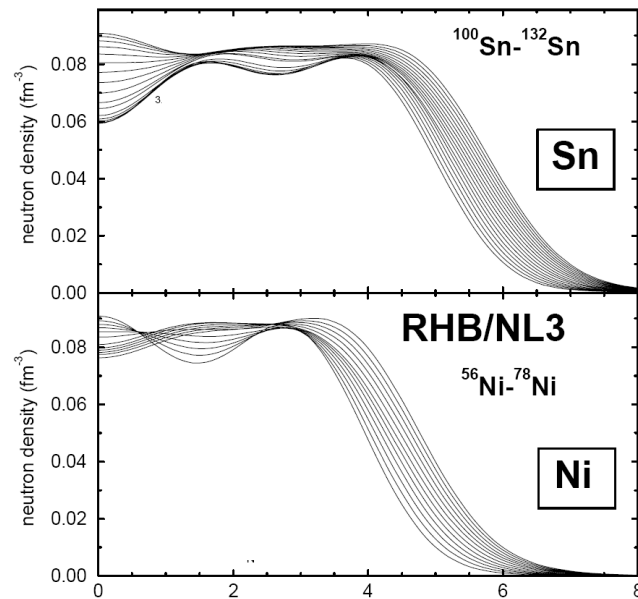
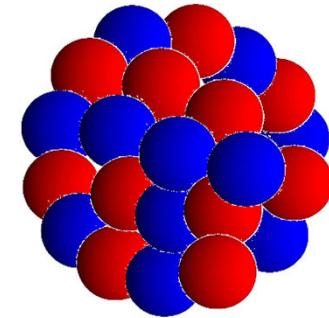




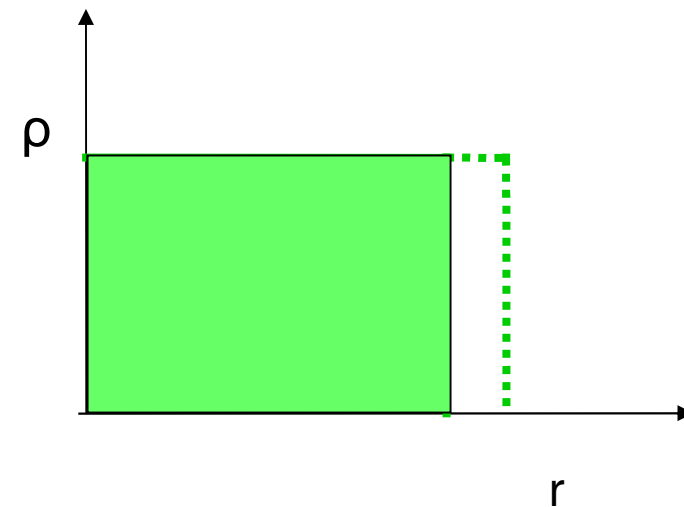
N-Z →



the nuclear density: $\rho(r)$

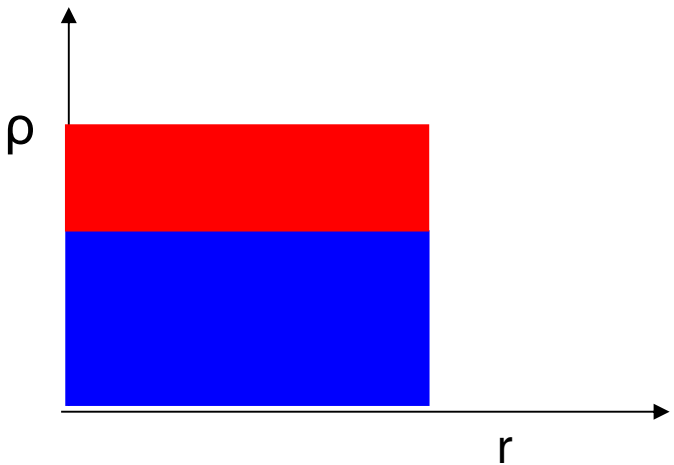
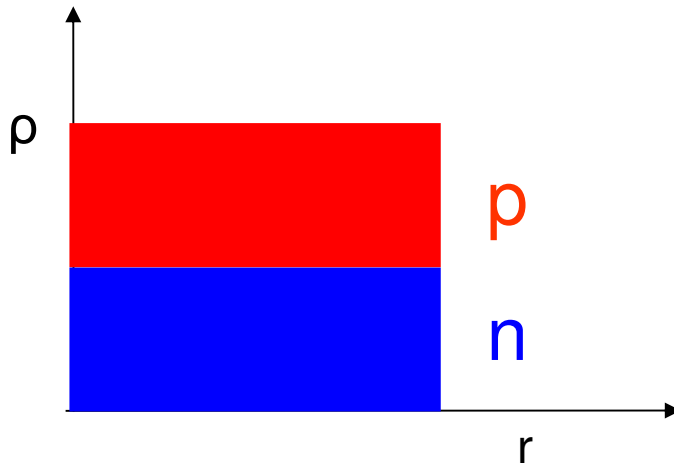


simplified representation:



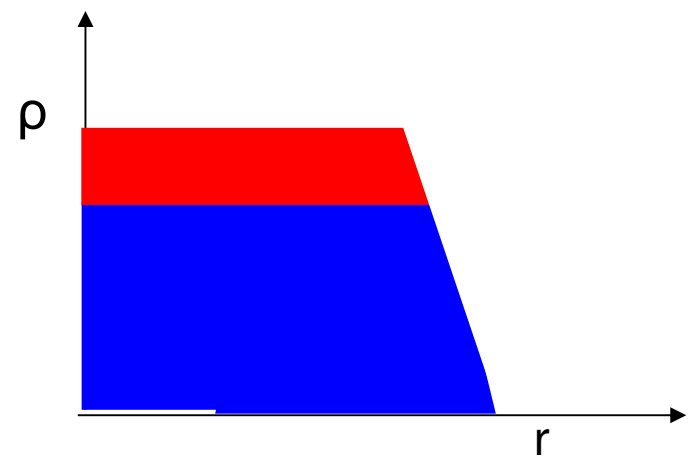
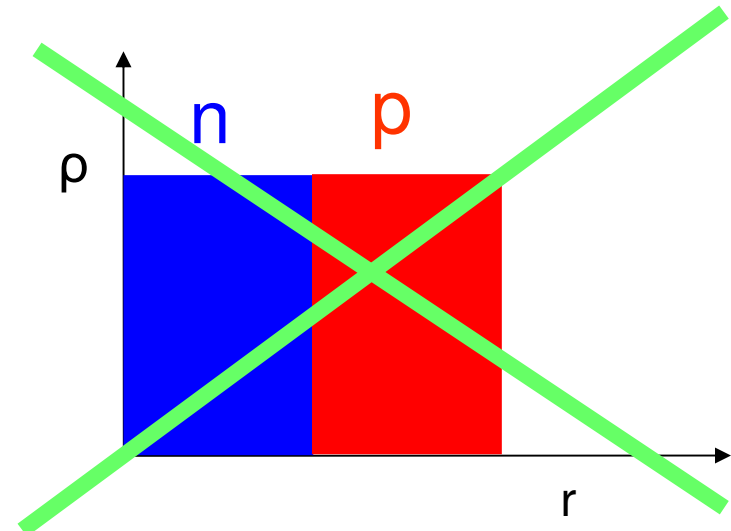
$$\rho = 1.6 \text{ nucleons/fm}^3$$

proton and neutron densities



small neutron excess

or?

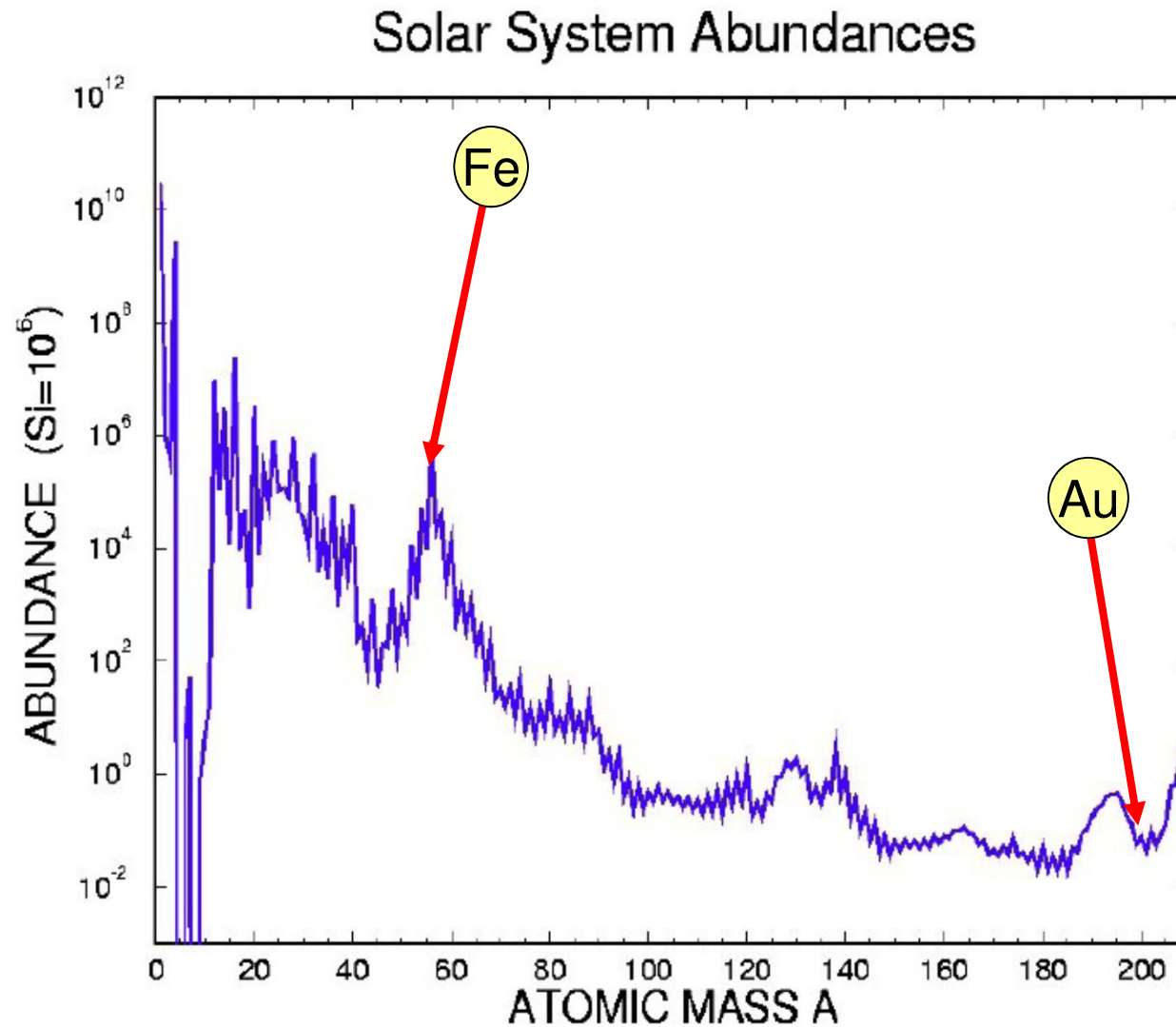


large neutron excess

Nuclei far from stability: what can we learn?

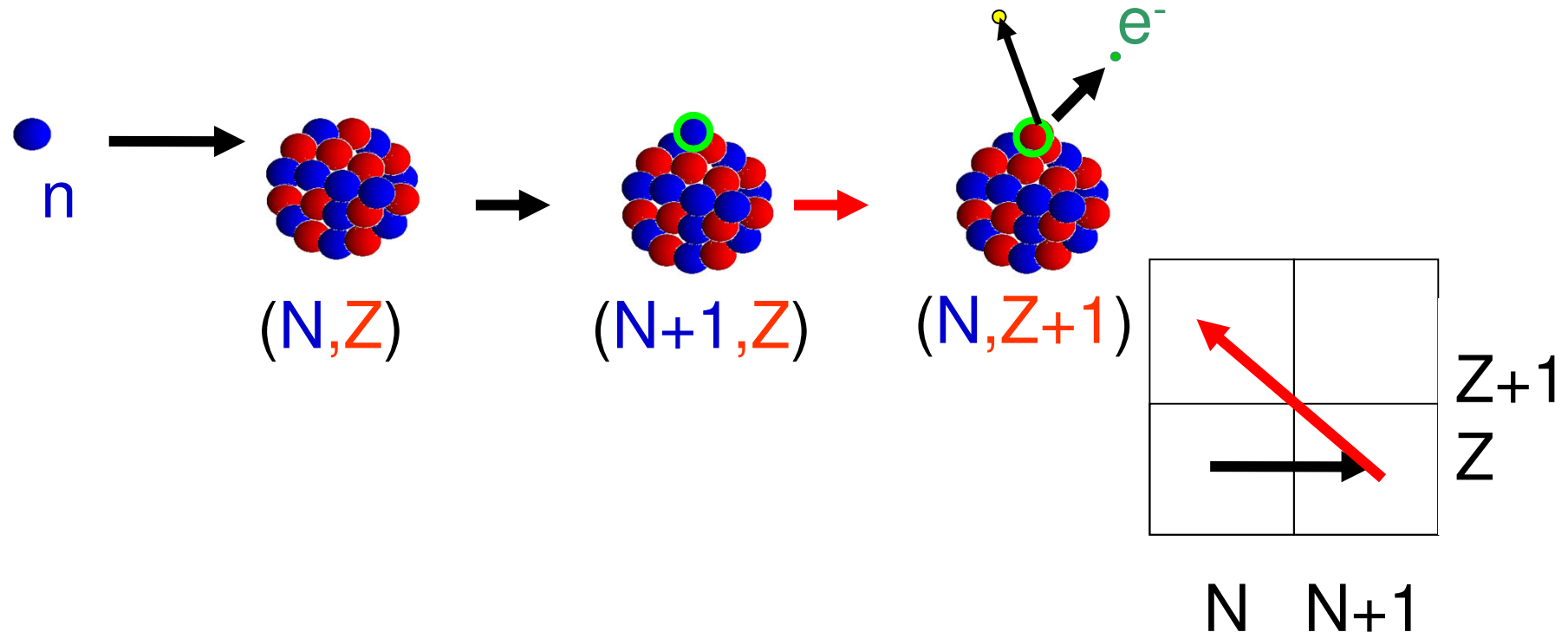
- the origin of more than half of the elements with $Z > 30$
- constraints on effective nuclear interactions
- evolution of shell structure
- reduction of the spin-orbit interaction
- properties of weakly-bound and open quantum systems
- exotic modes of collective excitations
(pygmy, toroidal resonances)
- possible new forms of nuclei (molecular states,
bubble nuclei, neutron droplets...)
- asymmetric nuclear matter equation of state and
the link to neutron stars
- applications in astrophysics

Abundancies of elements in the solar system

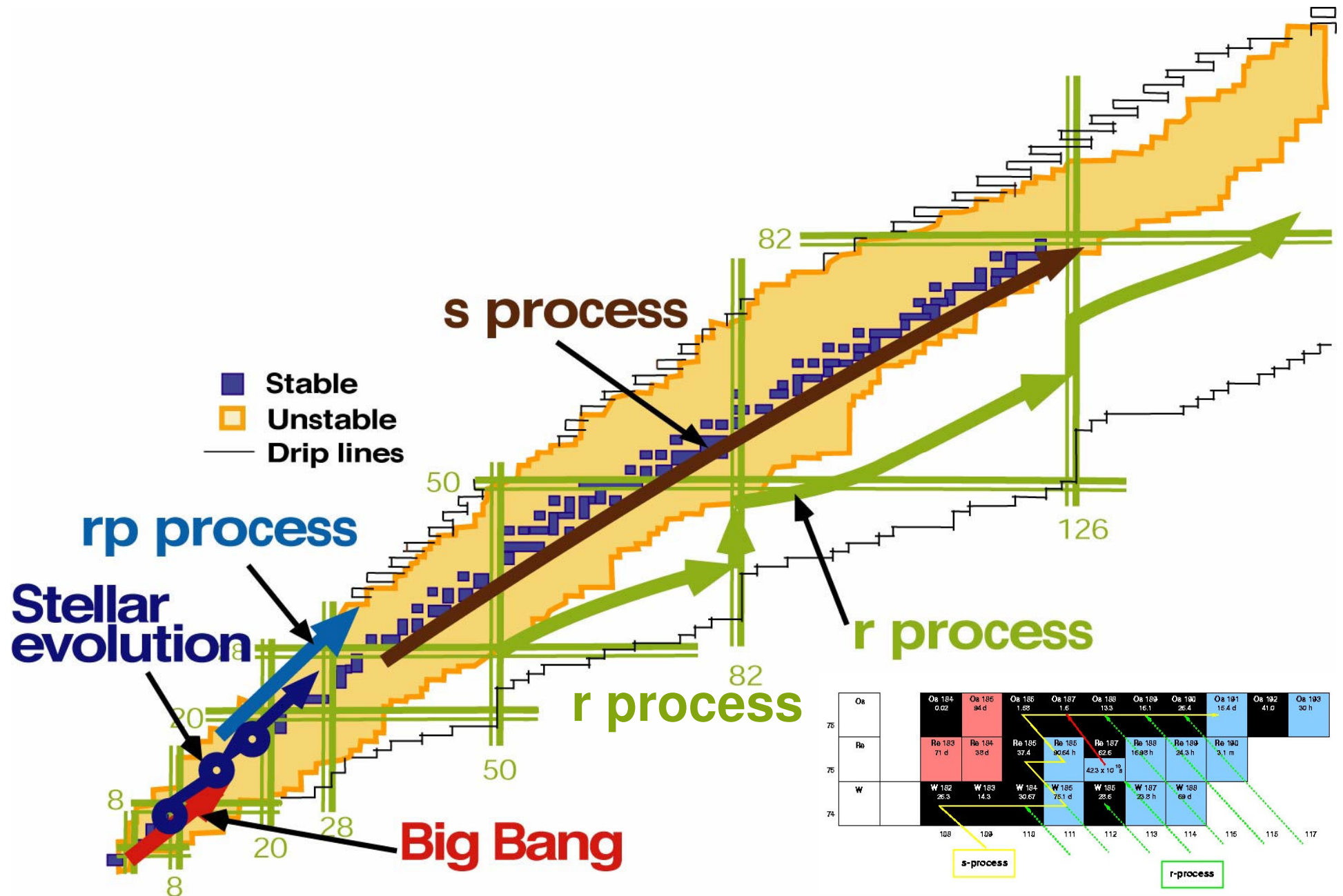


synthesis of heavy elements beyond Fe

neutron capture and successive β^- -decay:



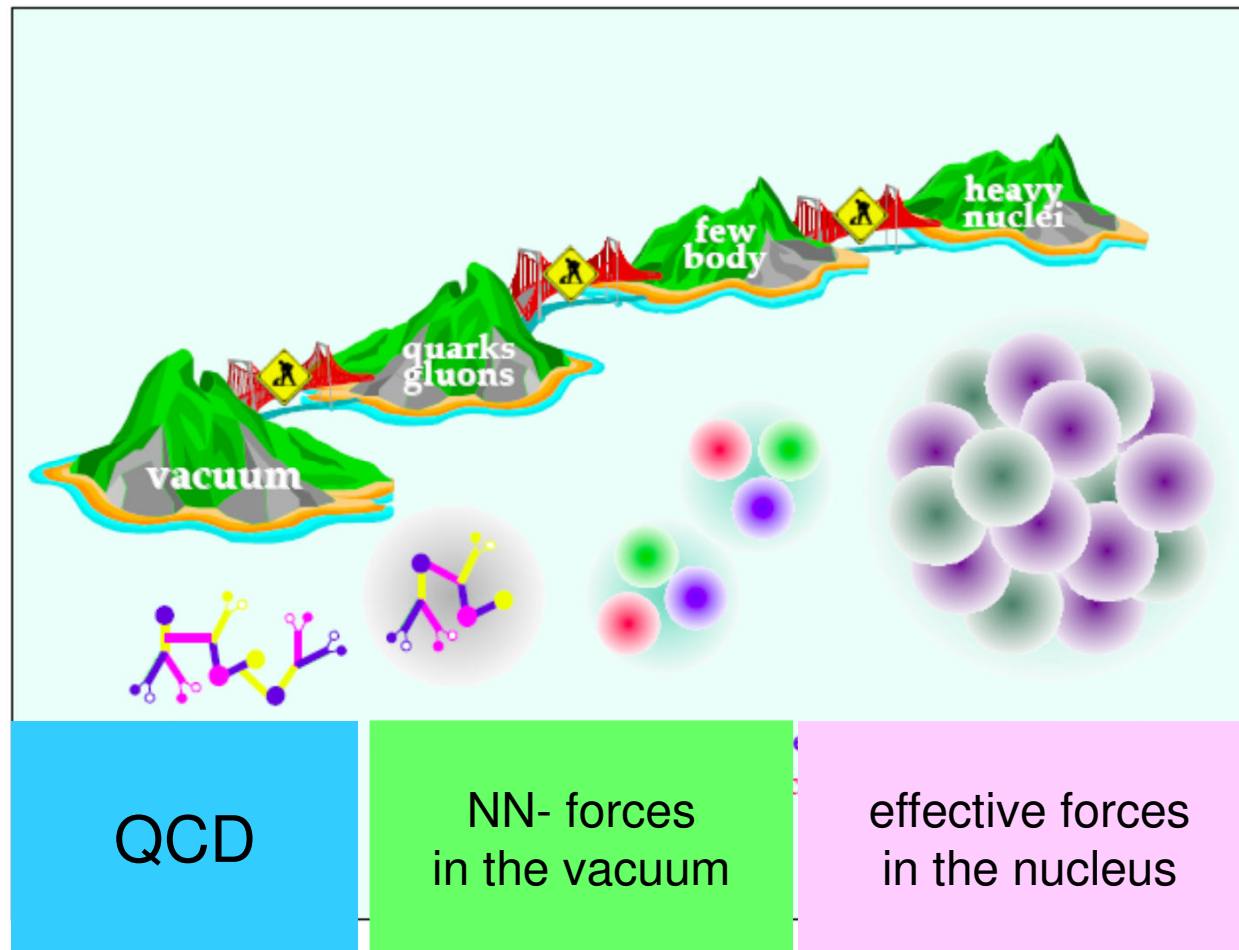
Study of Nucleosynthesis



What do the astrophysicists need ?

- nuclear masses (binding energies – **Q-values**)
- equation of state (EOS) of nuclear matter: **$E(\rho)$**
- isospin dependence **$E(\rho_p, \rho_n)$**
- nuclear matrix elements (**life times of β -decay ..**)
- cross section for neutron or electron capture
- fission probabilities
- cross sections for **neutrino reactions**
-
-

nuclei and QCD?



Scales: 1 GeV

100 keV

density functional theory:

theorem of Hohenberg und Kohn:

The exact energy of a quantum mechanical many body system is a functional of the local density $\rho(\mathbf{r})$

$$E[\rho] = \langle \Psi | H | \Psi \rangle$$

This functional is universal. It does not depend on the system, only on the interaction.

One obtains the exact density $\rho(\mathbf{r})$ by a variation of the functional with respect to the density

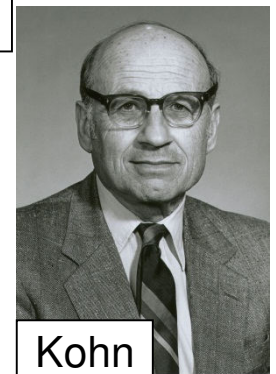
note:

$\rho(\mathbf{r})$ is a function of 3 variables.

$\Psi(\mathbf{r}_1 \dots \mathbf{r}_N)$ is a function of $3N$ variables.



Hohenberg



Kohn

Kohn-Sham theory:

In order to reproduce shell structure Kohn and Sham introduced a single particle potential $V_{\text{eff}}(r)$, which is defined by the condition, that after the solution of the single particle eigenvalue problem

$$\left\{ -\frac{\hbar^2}{2m} \Delta + V_{\text{eff}}(r) \right\} \phi_k(r) = \varepsilon_k \phi_k(r)$$

the density obtained as $\rho(r) = \sum_{i=1}^A |\phi_i(r)|^2$ is the exact density

Obviously to each density $\rho(r)$ there exist such a potential $V_{\text{eff}}(r)$.

The non interacting part of the energy functional is given by:

$$E_{\text{ni}}[\rho] = \int \frac{\hbar^2}{2m} \tau(r) d^3r = \int \frac{\hbar^2}{2m} \sum_{i=1}^A |\nabla \phi_i(r)|^2 d^3r = \sum_{i=1}^A \varepsilon_i - \int \rho(r) V_{\text{eff}}(r) d^3r$$

and obviously we have:

$$V_{\text{eff}}(r) = -\frac{\delta}{\delta \rho} E_{\text{ni}}[\rho] = -\frac{\delta}{\delta \rho} (E_{\text{HK}} - E_{\text{H}} - E_{\text{xc}})$$

limitations of exact density functionals:

formally exact

in practice

Hohenberg-Kohn: $E[\rho(\mathbf{r})]$

Kohn-Sham: $E[\rho(\mathbf{r}), \tau(\mathbf{r})]$

Skyrme: $E[\rho(\mathbf{r}), \tau(\mathbf{r}), J(\mathbf{r})]$

Gogny: $E[\rho(\mathbf{r}), \tau(\mathbf{r}), J(\mathbf{r}), \kappa(\mathbf{r})]$

no shell effects

no l·s,

no pairing

no config.mixing

generalized mean field: no configuration mixing,
no two-body correlations

local density: $\rho(\mathbf{r}) = \langle a^\dagger(\mathbf{r})a(\mathbf{r}) \rangle = \sum_i^A |\varphi_i(\mathbf{r})\rangle \langle \varphi_i(\mathbf{r})|$

kinetic energy density: $\tau(\mathbf{r}) = \sum_i^A |\nabla \varphi_i(\mathbf{r})\rangle \langle \nabla \varphi_i(\mathbf{r})|$

pairing density: $\kappa(\mathbf{r}) = \langle a^\dagger(\mathbf{r}, s) a^\dagger(\mathbf{r}, -s) \rangle$

twobody density: $\rho(\mathbf{r}, \mathbf{r}') = \langle a^\dagger(\mathbf{r}) a(\mathbf{r}) a^\dagger(\mathbf{r}') a(\mathbf{r}') \rangle$

Density functional theory in nuclei

- In nuclei DFT has been introduced by **effective Hamiltonians**:
by Vautherin and Brink (1972)

$$E = \langle \Psi | H | \Psi \rangle \approx \langle \Phi | \hat{H}_{eff}(\hat{\rho}) | \Phi \rangle \stackrel{?}{=} E[\hat{\rho}]$$

Skyrme Gogny Rel. MF

- Nuclei are **self-bound systems**.
The exact density is a constant. $\rho(r) = \text{const}$
Hohenberg-Kohn theorem is true, but useless
 $\rho(r)$ has to be replaced by the **intrinsic density**:

$$\rho_I(\vec{r}) = \rho(\vec{r} + \vec{R}_{CM}) \quad \text{with} \quad \vec{R}_{CM} = \frac{1}{A} \sum_i \vec{r}_i$$

- Density functional theory in nuclei is probably **not exact**,
but it is a very good approximation.

General properties of self-consistent mean field theories:

- the nuclear energy functional is so far **phenomenological** and not connected to any NN-interaction.
- it is expressed in terms of powers and gradients of the nuclear ground state density using the principles of **symmetry** and **simplicity**
- The remaining parameters are adjusted to characteristic properties of **nuclear matter** and **finite nuclei**

Virtues:

(i) the intuitive interpretation of mean fields results in terms of **intrinsic shapes** and of shells with **single particle states**

(ii) the **full model space** is used: no distinction between core and valence nucleons, **no need for effective charges**

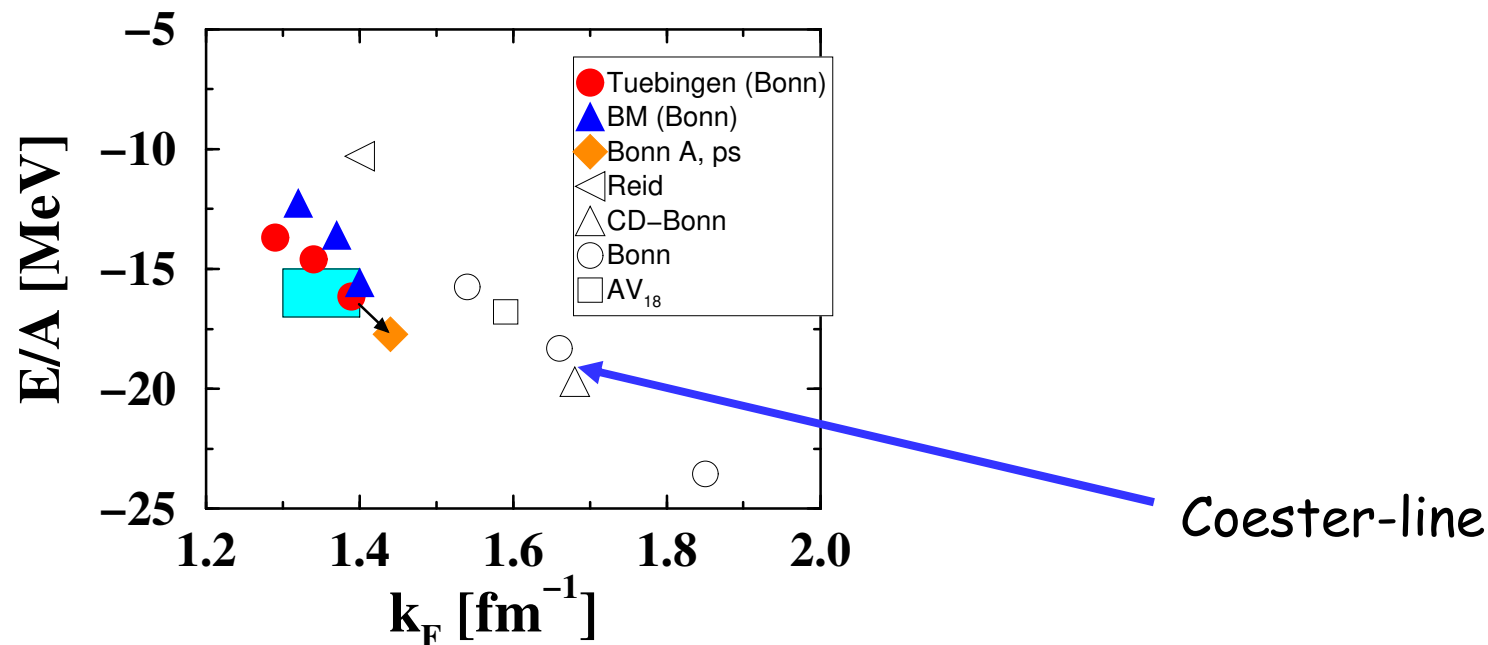
(iii) the functional is **universal**: it can be applied to all nuclei throughout the periodic chart, light and heavy, spherical and deformed

Why covariant ?

- no relativistic kinematic necessary: $\sqrt{p_F^2 + m_N^2} = m_N \sqrt{1 + 0.075}$
- non-relativistic DFT works well
- technical problems:
 - no harmonic oscillator
 - no exact soluble models
 - double dimension
 - huge cancellations V-S
 - no variational method
- conceptual problems:
 - treatment of Dirac sea
 - no well defined many-body theory

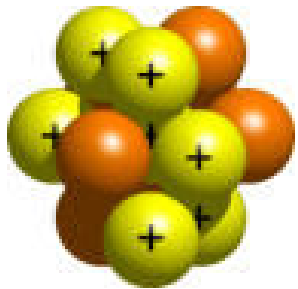
Why covariant?

- 1) Large spin-orbit splitting in nuclei
- 2) Large fields $V \approx 350$ MeV, $S \approx -400$ MeV
- 3) Success of Relativistic Brueckner
- 4) Success of intermediate energy proton scatt.
- 5) relativistic saturation mechanism
- 6) consistent treatment of time-odd fields
- 7) Pseudo-spin Symmetry
- 8) Connection to underlying theories ?
- 9) As many symmetries as possible

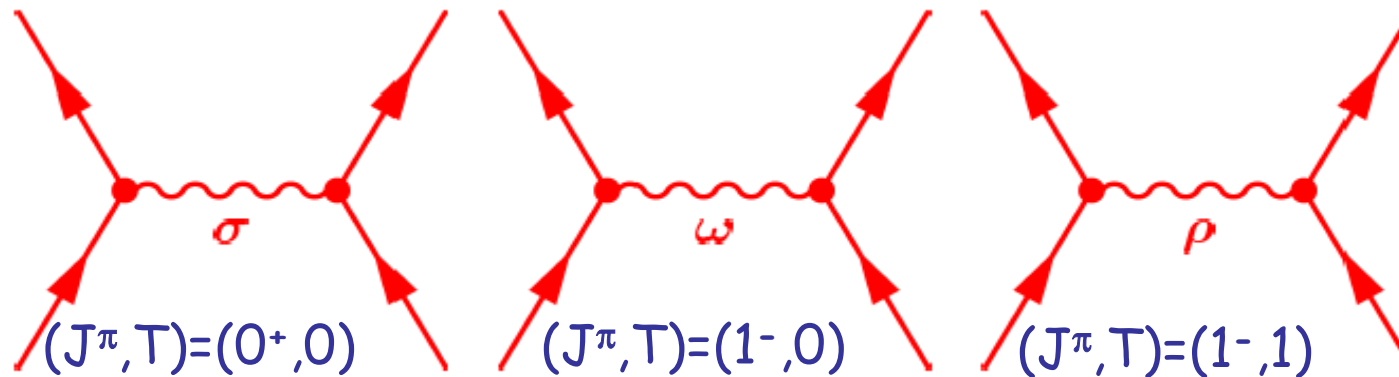


Walecka model

$$E[\hat{\rho}]$$



Nucleons are coupled by exchange of mesons through an **effective Lagrangian** (EFT)



$$S(r) = g_\sigma \sigma(r)$$

Sigma-meson:
attractive scalar field

$$V(r) = g_\omega \omega(r) + g_\rho \vec{\tau} \vec{\rho}(r) + eA(r)$$

Omega-meson:
short-range repulsive

Rho-meson:
isovector field

Lagrangian density

free Dirac particle

free meson fields

free photon field

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma \cdot \partial - m)\psi + \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 \\ & - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega^2 - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & - g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\gamma \cdot \omega\psi - g_\rho\bar{\psi}\gamma \cdot \vec{\rho}\vec{\tau}\psi - e\bar{\psi}\gamma \cdot A\frac{(1-\tau_3)}{2}\psi \end{aligned}$$

Parameter:

meson masses: $m_\sigma, m_\omega, m_\rho$

meson couplings: $g_\sigma, g_\omega, g_\rho$

interaction terms

$$\mathcal{L} = \mathcal{L}(\psi, \sigma, \omega, \rho, A)$$

Equations of motion

$$\partial_\mu \frac{\partial L}{\partial (\partial_\mu q_k)} - \frac{\partial L}{\partial q_k} = 0.$$

for the nucleons we find the Dirac equation

$$\left(\gamma^\mu (i \partial_\mu - V_\mu) - m + S \right) \psi_i = 0.$$

No-sea approxim. !

for the mesons we find the Klein-Gordon equation

$$\begin{aligned} (\partial^\nu \partial_\nu + m_\sigma^2) \sigma &= -g_\sigma \rho_s \\ (\partial^\nu \partial_\nu + m_\omega^2) \omega_\mu &= g_\omega j_\mu \\ (\partial^\mu \partial_\mu + m_\rho^2) \vec{\rho}_\mu &= g_\rho \vec{j}_\mu \\ \partial^\nu \partial_\nu A_\mu &= e j_\mu^{(em)} \end{aligned}$$

$$\rho_s(x) = \sum_{i=1}^A \bar{\psi}_i(x) \psi_i(x)$$

$$j_\mu(x) = \sum_{i=1}^A \bar{\psi}_i(x) \gamma_\mu \psi_i(x)$$

$$\vec{j}_\mu(x) = \sum_{i=1}^A \bar{\psi}_i(x) \vec{\tau} \gamma_\mu \psi_i(x)$$

$$j_\mu^{(em)}(x) = \sum_{i=1}^A \bar{\psi}_i(x) \frac{1}{2} (1 - \tau_3) \gamma_\mu \psi_i(x)$$

Static limit (with time reversal invariance)

for the nucleons we find the **static Dirac equation**

$$(\vec{\alpha} \vec{p} + V + \beta(m - S))\psi_i = \varepsilon_i \psi_i.$$

$$S = -g_s \sigma, \quad V = g_\omega \omega_0 + g_\rho \rho_0 + eA_0$$

for the mesons we find the **Helmholtz equations**

No-sea approxim. !

$$(-\Delta + m_\sigma^2)\sigma = -g_\sigma \rho_s$$

$$(-\Delta + m_\omega^2)\omega_0 = g_\omega \rho_B$$

$$(-\Delta + m_\rho^2)\rho_0^3 = g_\rho \rho^3$$

$$-\Delta A_0 = e\rho^{(em)}$$

$$\rho_s = \sum_{i=1}^A \bar{\psi}_i \psi_i$$

$$\rho_B = \sum_{i=1}^A \psi_i^+ \psi_i$$

$$\rho^3 = \sum_{i=1}^A \psi_i^+ \tau_3 \psi_i$$

$$\rho^{(em)} = \sum_{i=1}^A \psi_i^+ \frac{1}{2} (1 - \tau_3) \psi_i$$

Relativistic saturation mechanism:

We consider only the σ -field, the origin of attraction its source is the scalar density

$$m_\sigma^2 \sigma = -g_\sigma \sum_{i=1}^A \bar{\psi}_i \psi_i = -g_\sigma \sum_{i=1}^A (g_i^+ g_i - f_i^+ f_i)$$

for **high densities**, when the collapse is close, the Dirac gap $\approx 2m^*$ decreases, the small components **f_i** of the wave functions increase and reduce the scalar density, i.e. the source of the σ -field, and therefore also scalar attraction.

$$f_i(r) = \frac{1}{\varepsilon_i + 2\tilde{m}} \vec{\sigma} \cdot \vec{k} g_i(r)$$

$$m_\sigma^2 \sigma \approx -g_\sigma \rho_B - 2 \sum_{i=1}^A f_i^+ f_i = -g_\sigma \rho_B + \frac{1}{\tilde{m}} \sum_{i=1}^A \nabla g_i^+ \nabla g_i$$

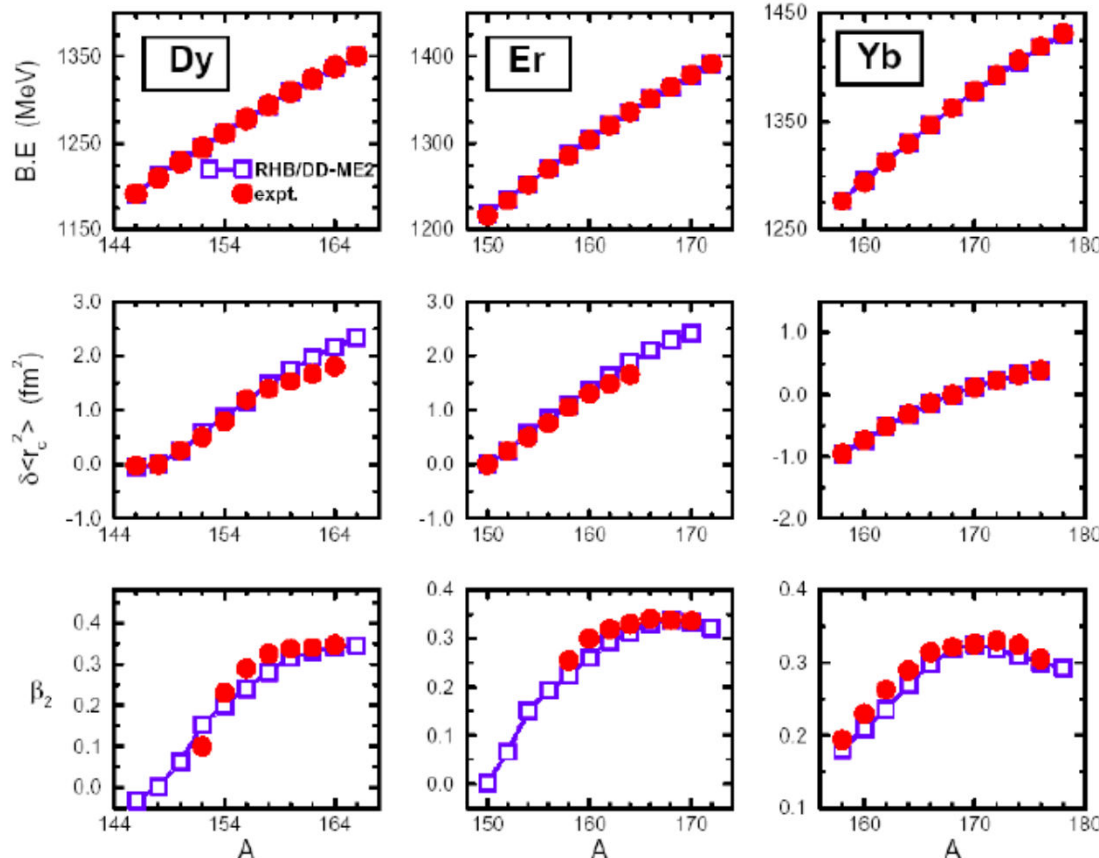
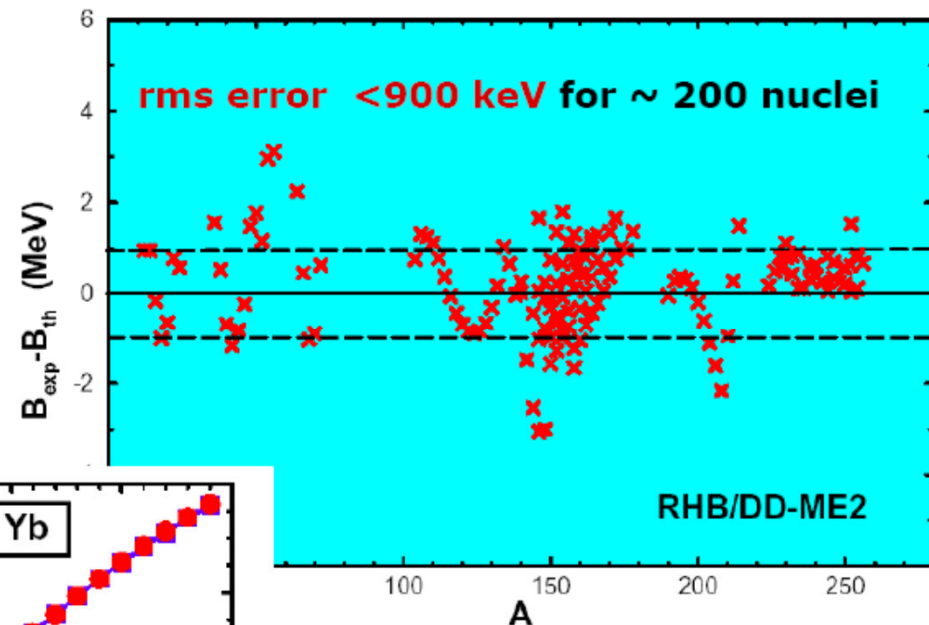
In the non-relativistic case, Hartree with Yukawa forces would lead to collapse

Successes of relativistic investigations:

- Masses and radii
- Isotope shifts
- Neutron halo's
- Proton emitters
- Collective vibrations
- Pygmy modes
- Beyond mean field: transitional nuclei
- Beyond mean field: complex configurations

Relativistic Hartree-Bogoliubov
calculations: DD-ME2 + Gogny
DIS pairing

Absolute deviations of the
calculated binding energies
from experimental values:



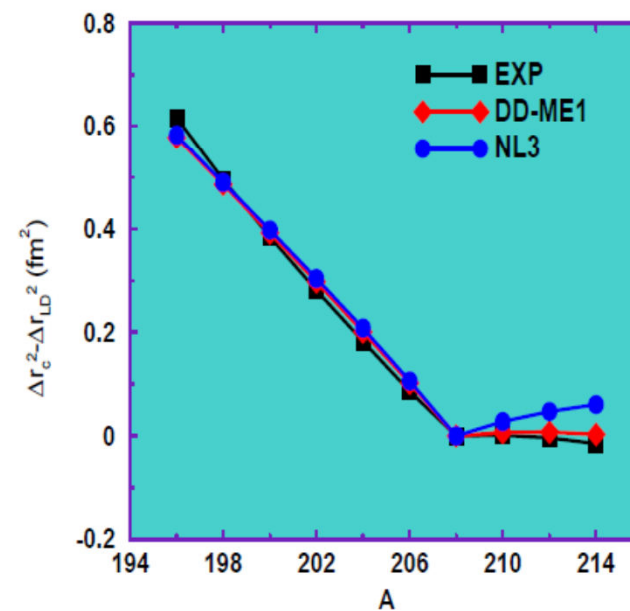
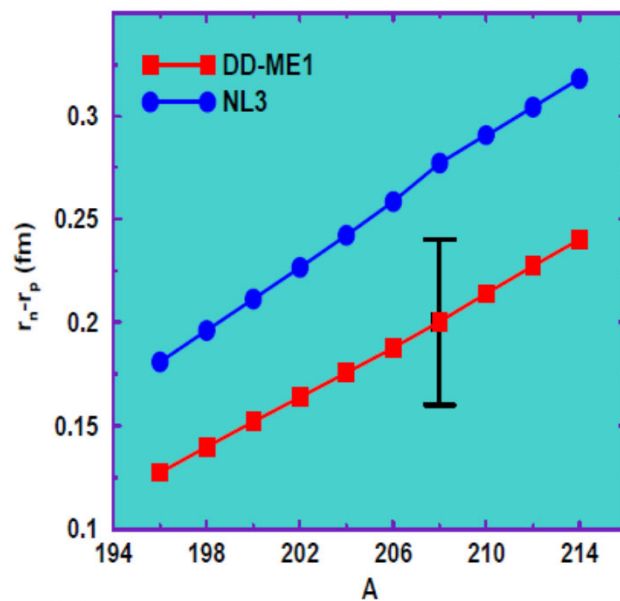
Binding energies, charge
isotope shifts and
quadrupole deformations
of isotopic chains in the
rare-earth region.

Phys. Rev. C **71**, 024312 (2005)

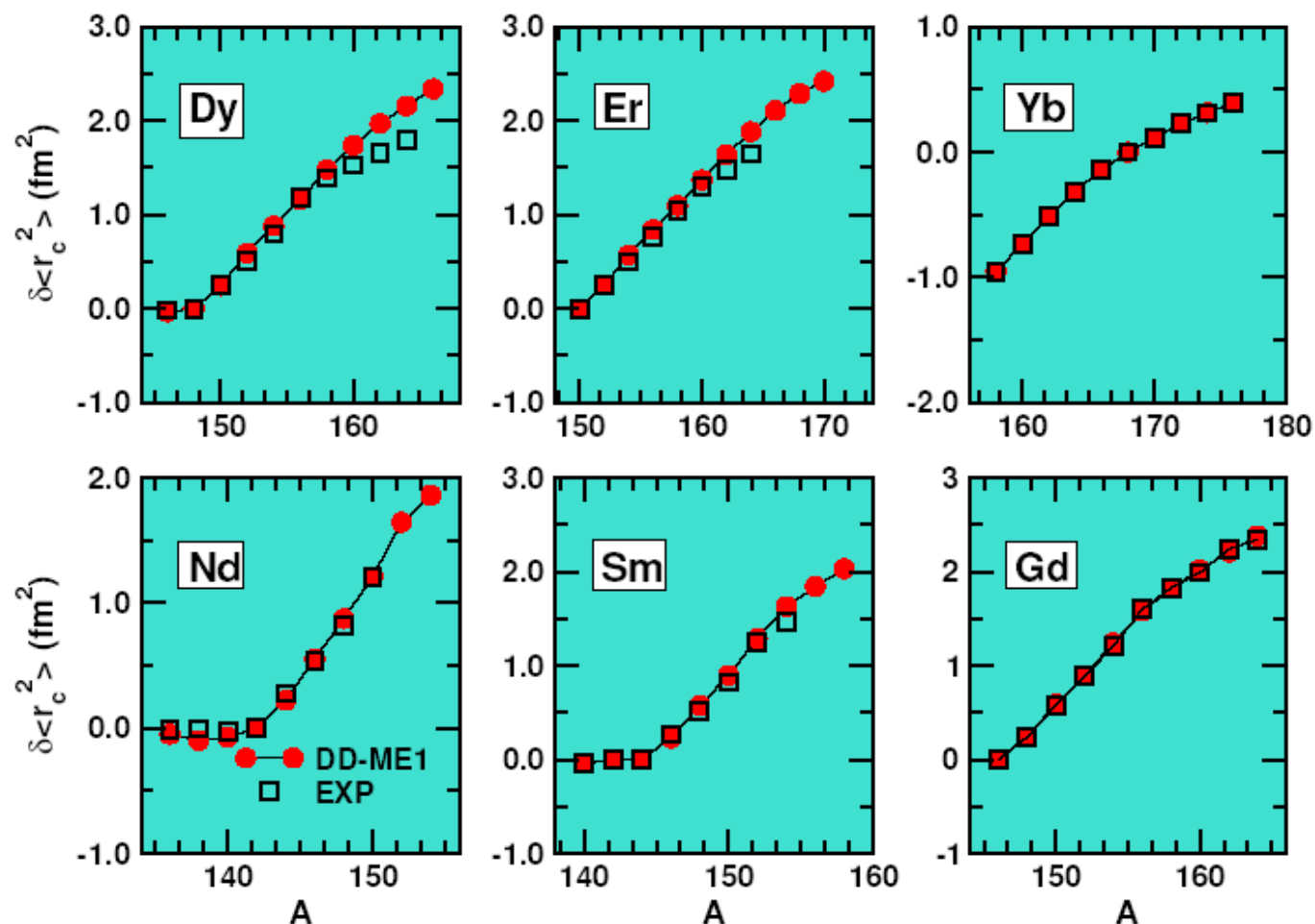
Kink in the isotopic shifts of radii: relativistic

Pb isotopes

- differences between neutron and proton radii
- charge isotope shifts



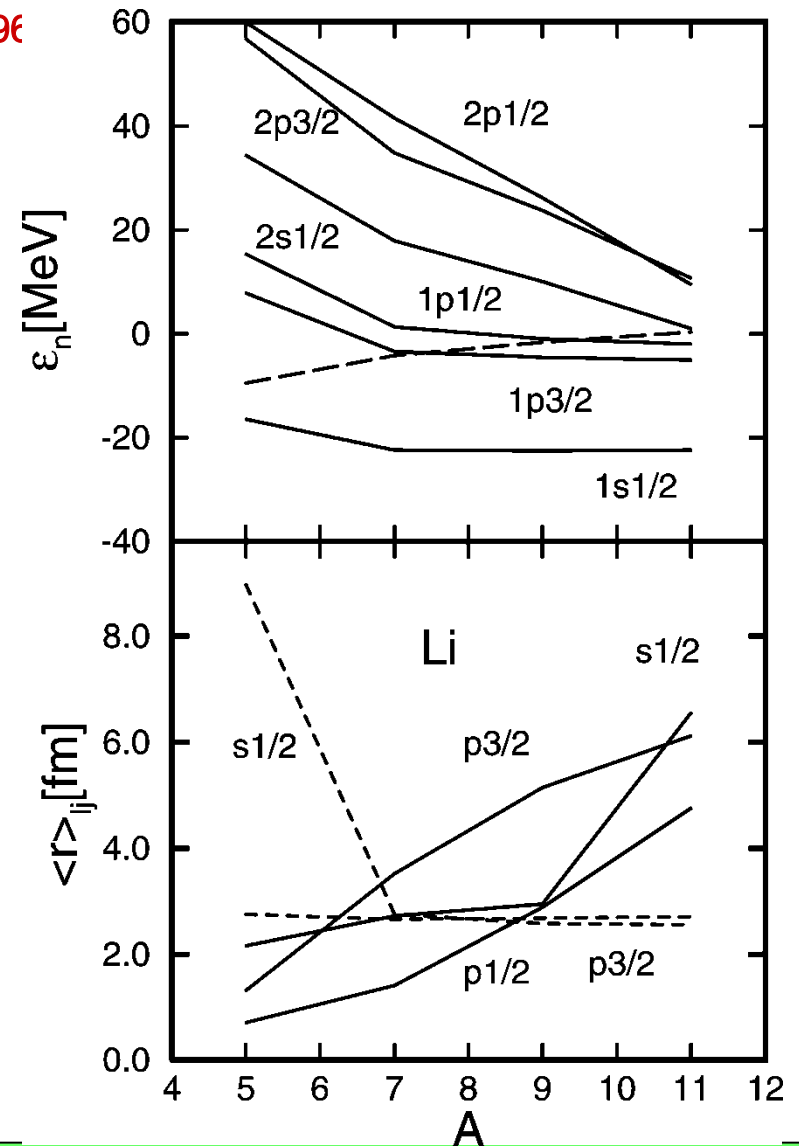
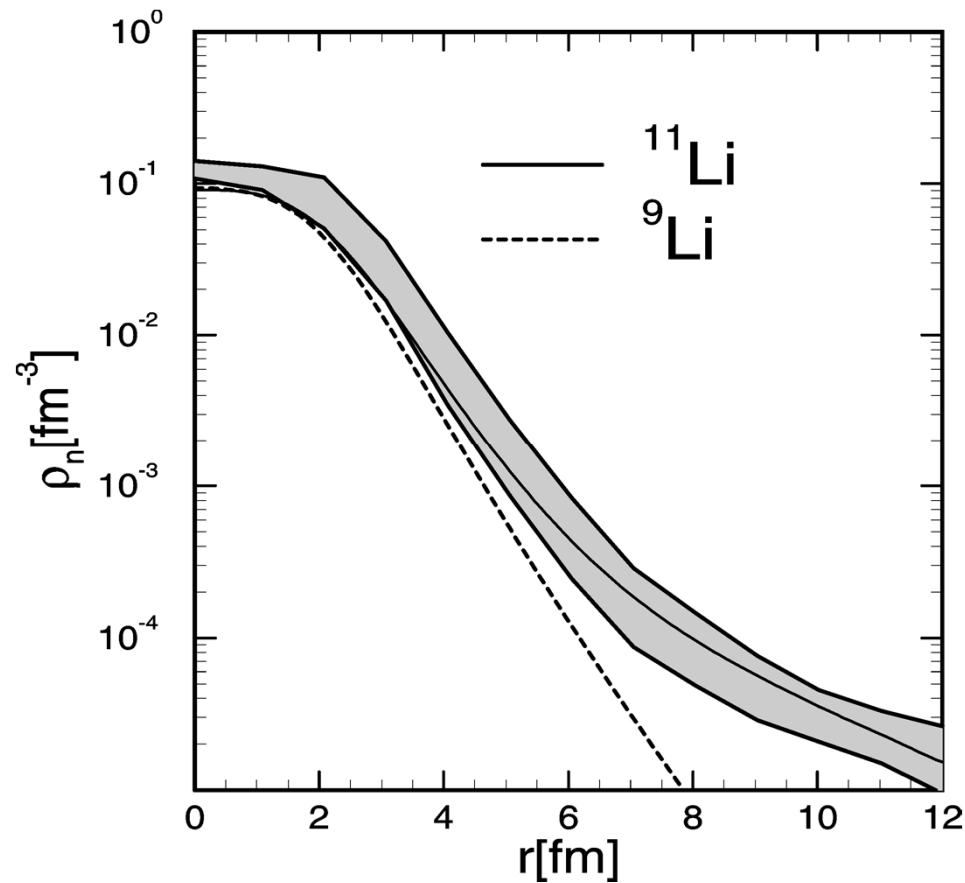
NL3: Isotope shifts in deformed nuclei:



Empirical data: E.G. Nadjakov et al., At. Data Nucl. Data Tables 56, 133(1994)

Density distribution in Li-nuclei

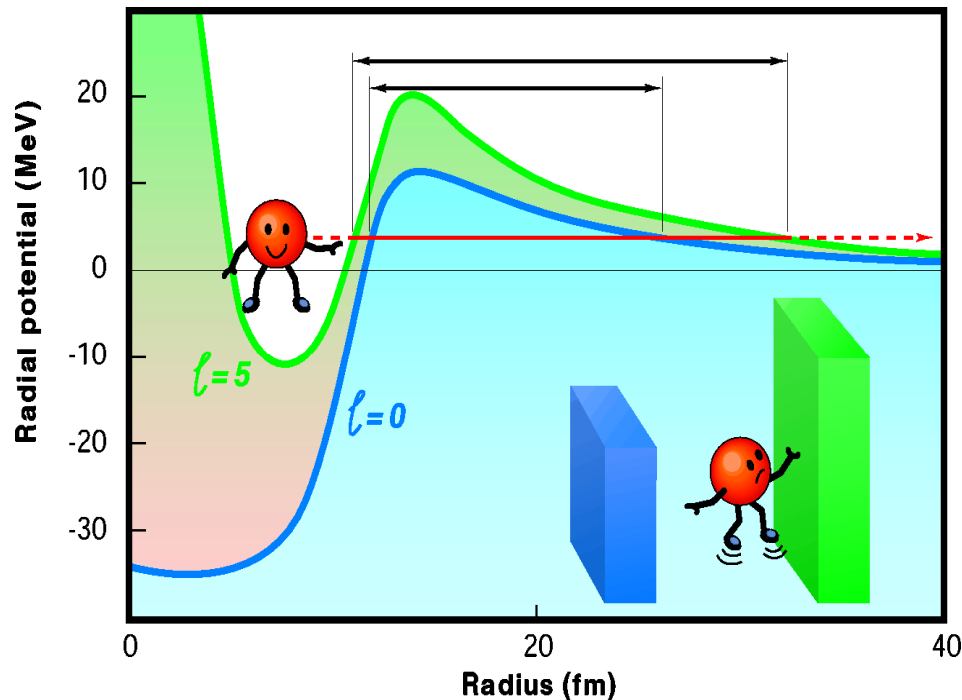
J. Meng and P. Ring, PRL 77, 3963 (1996)



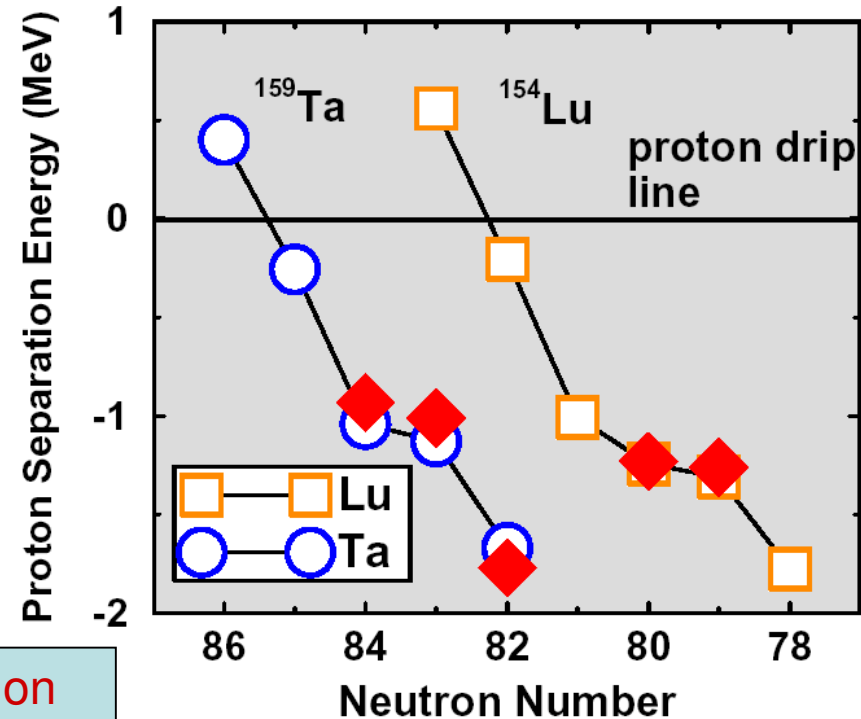
Proton emitters at the proton dripline

Vretenar, Lalazissis, Ring, Phys.Rev.Lett. 82, 4595 (1999)

characterized by exotic ground-state decay modes such as the direct emission of charged particles and β -decays with large Q-values.



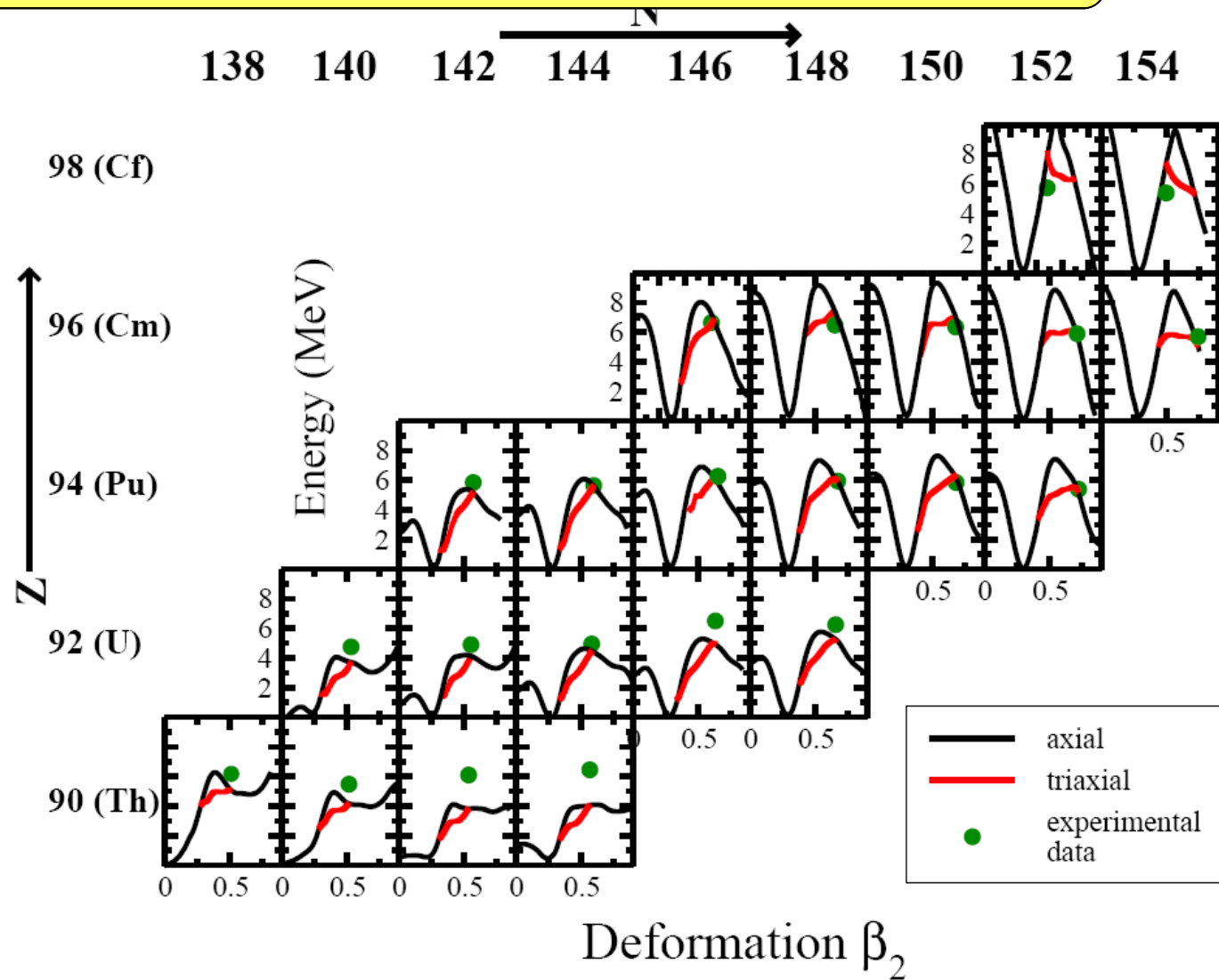
Ground-state proton emitters



Self-consistent RHB calculations -> separation energies, quadrupole deformations, odd-proton orbitals, spectroscopic factors

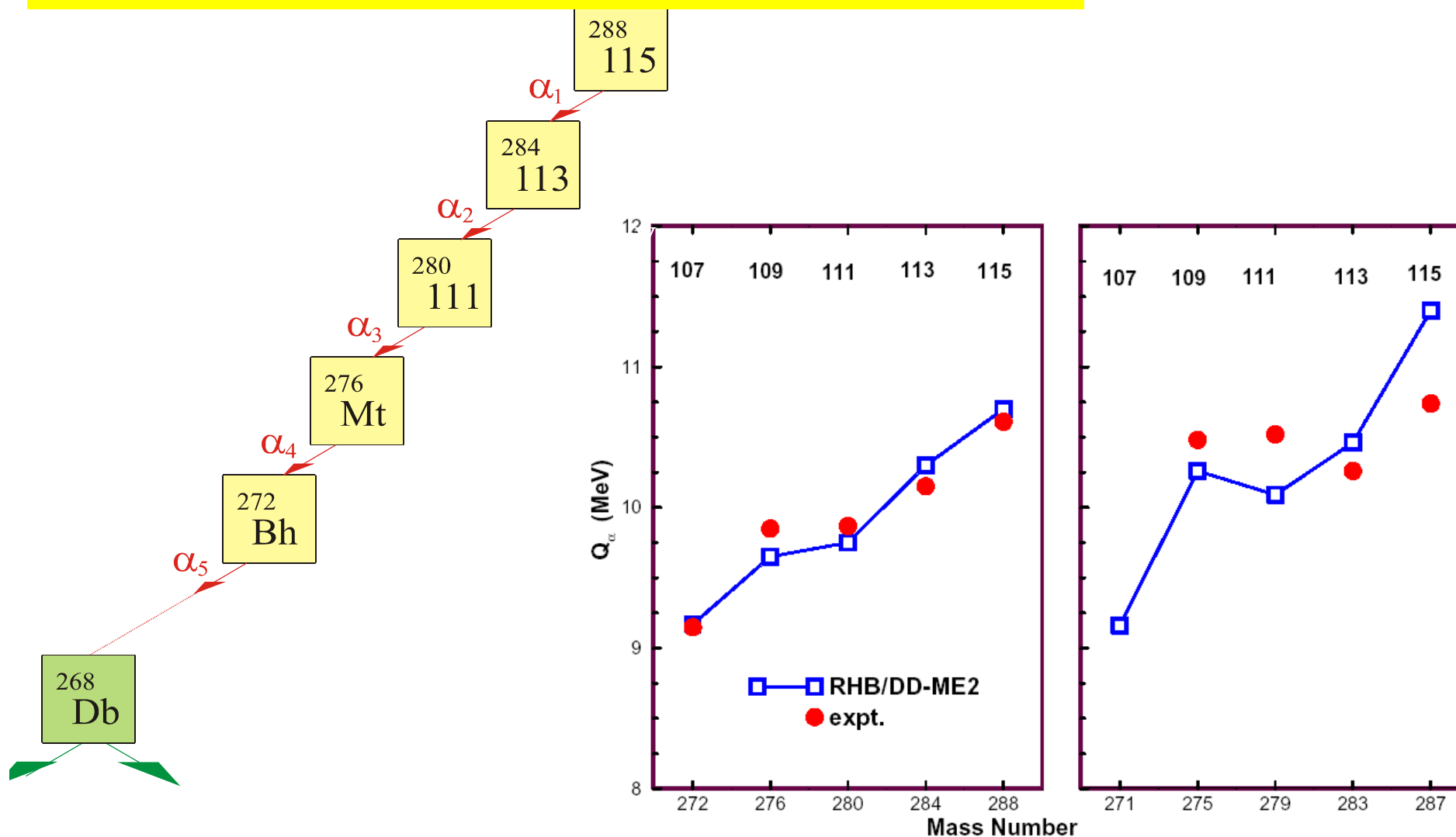
Lalazissis, Vretenar, Ring
Phys.Rev. C60, 051302 (1999)

Fission barriers for triaxially deformed shapes:



Nucleus	B.E (MeV)	r_C (fm)	r_n (fm)	Q_p (b)	H_p (b^2)
^{224}Ra	1720.47 (1720.31)	5.71	5.85	4.93 (6.33)	0.45
^{226}Ra	1731.13 (1731.61)	5.74	5.88	6.22 (7.19)	0.65
^{228}Ra	1741.67 (1742.49)	5.76	5.92	7.44 (7.76)	0.79
^{230}Ra	1751.94 (1753.05)	5.79	5.95	8.39	0.86
^{228}Th	1743.04 (1742.49)	5.78	5.90	7.64 (8.42)	0.88
^{230}Th	1751.94 (1753.05)	5.80	5.93	8.57 (8.99)	0.97 (1.09)
^{232}Th	1766.10 (1766.92)	5.82	5.96	9.28 (9.66)	1.00 (1.22)
^{234}Th	1776.80 (1777.68)	5.84	5.99	9.78 (8.96)	0.96
^{232}U	1766.39 (1765.97)	5.83	5.94	9.57 (10.00)	1.10
^{234}U	1778.66 (1778.57)	5.85	5.97	10.10 (10.35)	1.10 (1.40)
^{236}U	1790.29 (1790.42)	5.87	6.00	10.46 (10.80)	1.03 (1.30)
^{238}U	1801.38 (1801.69)	5.88	6.02	10.74 (11.02)	0.94 (0.83)
^{240}U	1811.82 (1812.44)	5.90	6.05	11.03	0.86
^{238}Pu	1801.85 (1801.27)	5.89	6.01	11.09 (11.26)	1.00 (1.38)
^{240}Pu	1813.84 (1813.46)	5.91	6.03	11.32 (11.44)	1.00 (1.15)
^{242}Pu	1825.26 (1825.01)	5.92	6.05	11.55 (11.61)	0.90
^{244}Pu	1836.00 (1836.06)	5.94	6.08	11.61 (11.73)	0.79
^{246}Pu	1845.97 (1846.66)	5.95	6.10	11.52 (11.52)	0.66

Superheavy Elements: Q_α -values



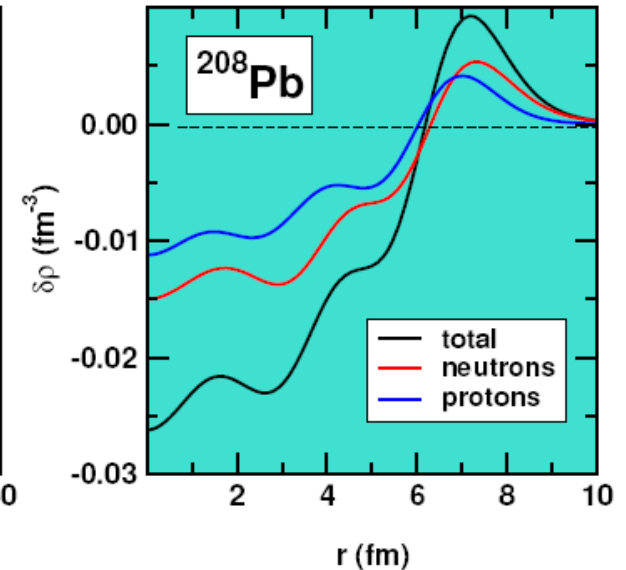
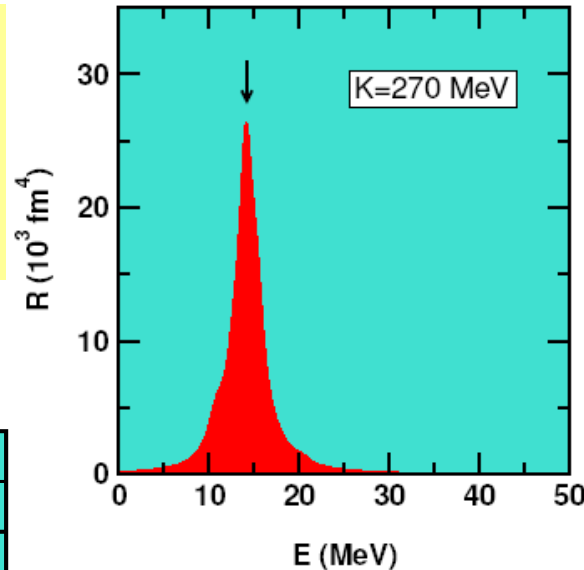
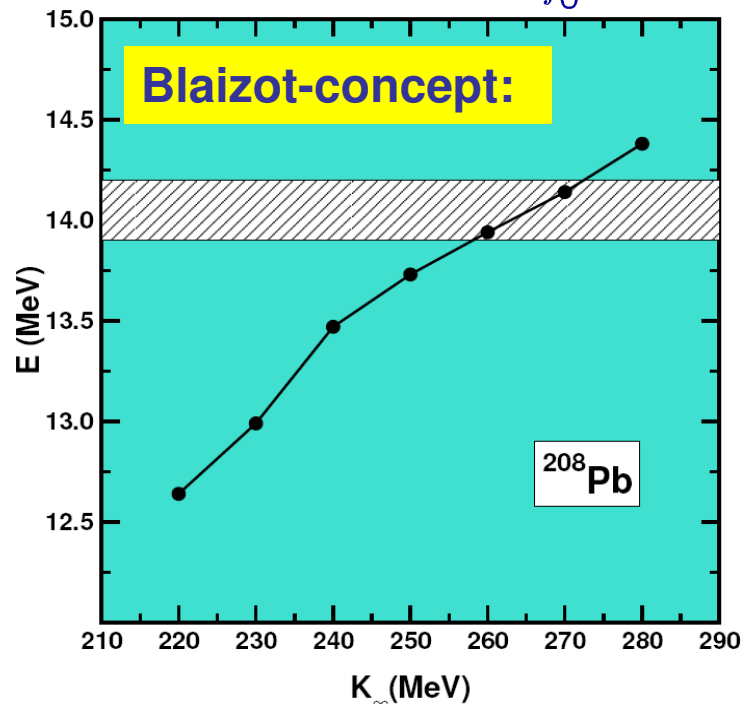
● Exp: Yu.Ts.Oganessian *et al*, PRC 69, 021601(R) (2004)

□ Lalzissis

Isoscalar Giant Monopole: IS-GMR

The ISGMR represents the essential source of experimental information on the nuclear incompressibility

$$K_0 = p_f^2 \left. \frac{d^2 E/A}{dp_f^2} \right|_{p_{f0}}$$



constraining the nuclear matter compressibility

$$\rho(t) = \rho_0 + \delta\rho(t)$$

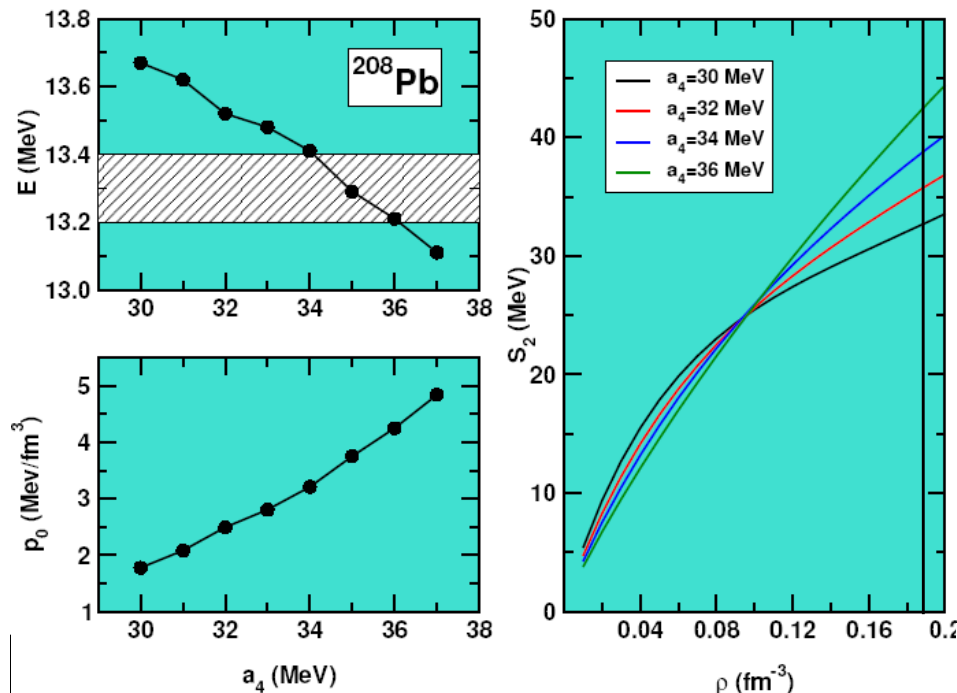
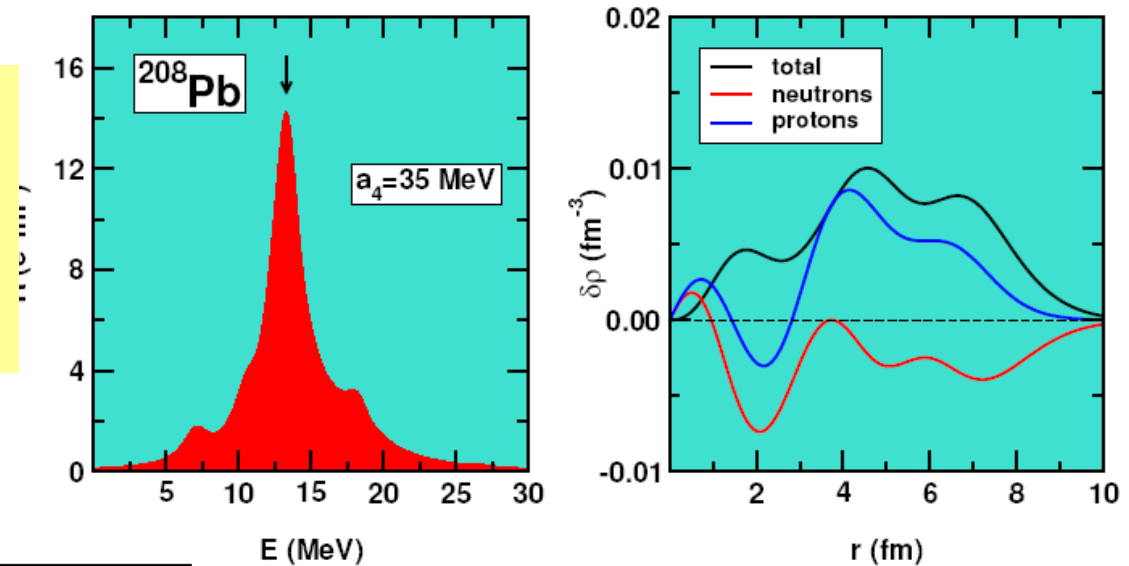
RMF models reproduce the experimental data only if

$$250 \text{ MeV} \leq K_0 \leq 270 \text{ MeV}$$

T. Niksic et al., PRC 66 (2002) 024306

Isovector Giant Dipole: IV-GDR

the IV-GDR represents one of the sources of experimental informations on the nuclear matter symmetry energy



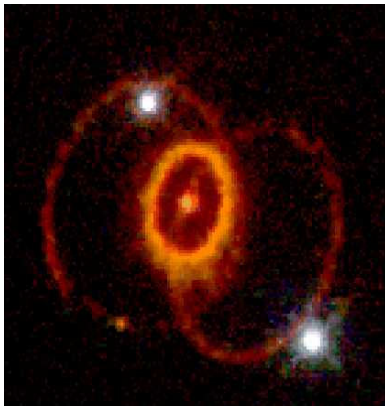
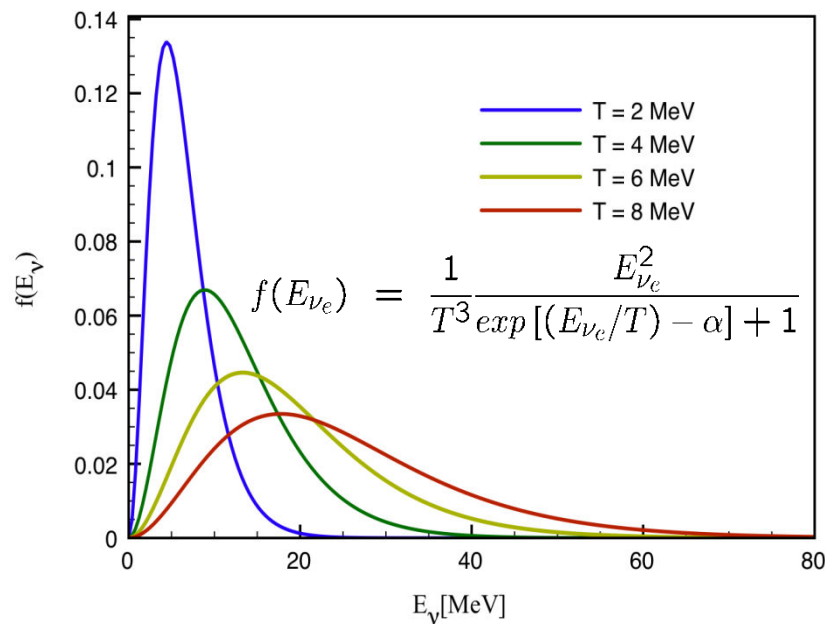
constraining the nuclear matter symmetry energy

the position of IV-GDR is reproduced if

$$32 \text{ MeV} \leq a_4 \leq 36 \text{ MeV}$$

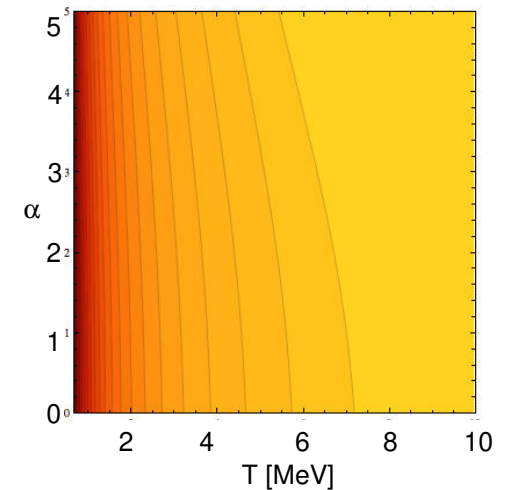
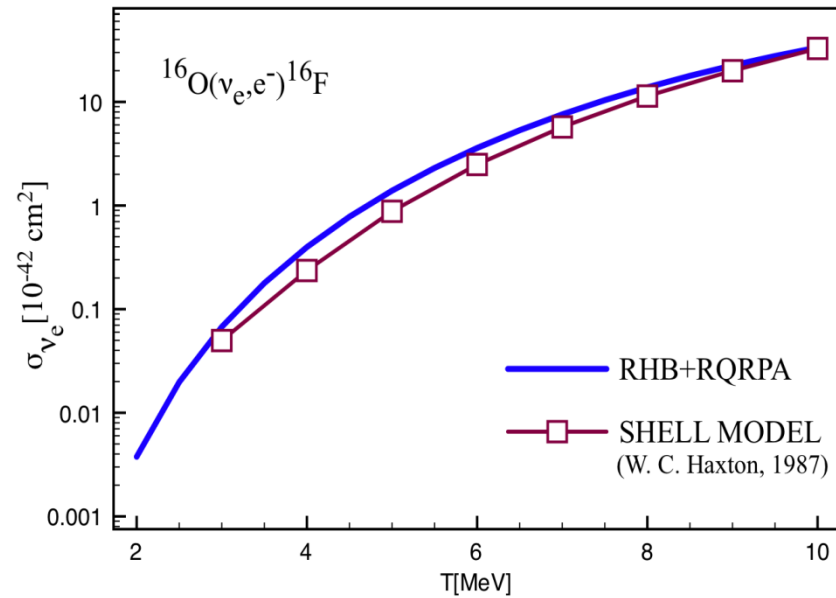
Cross section averaged over supernova neutrino flux

Supernova neutrino flux is given by Fermi-Dirac spectrum



Cross section averaged over Supernova neutrino flux

$$\langle \sigma_{\nu} \rangle = \frac{\int dE_{\nu} \sigma_{\nu}(E_{\nu}) f(E_{\nu})}{\int dE'_{\nu} f(E'_{\nu})}$$



How many parameters ?

4 + 3 parameters

symmetric nuclear matter: E/A , ρ_0 \longrightarrow G_σ G_ω

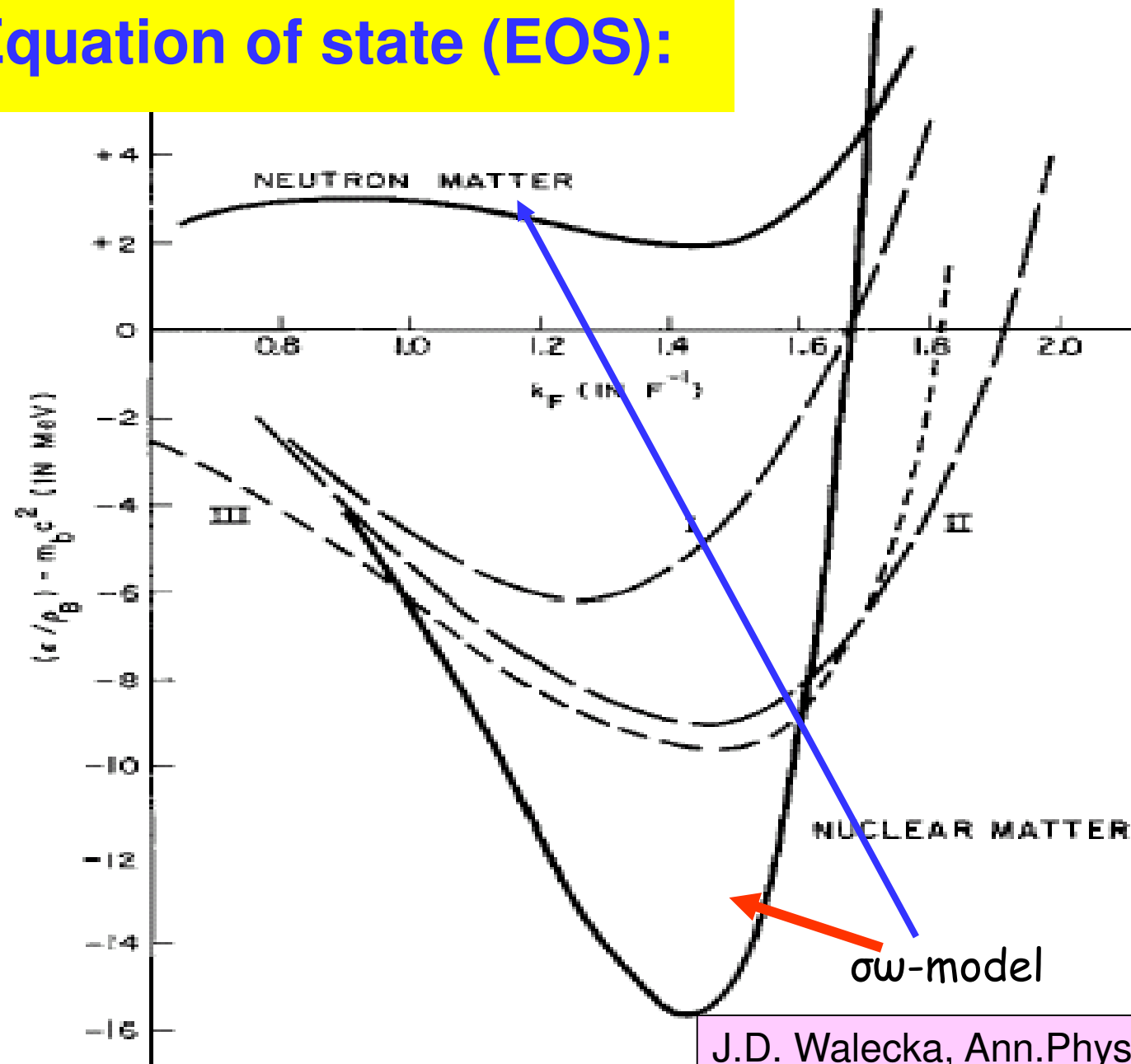
finite nuclei ($N=Z$): E/A , radii
spinorbit for free \longrightarrow m_σ

Coulomb ($N \neq Z$): a_4 \longrightarrow G_ρ

density dependence: $T=0$ K_∞ \longrightarrow g_2 g_3

$T=1$ $r_n - r_p$ \longrightarrow a_ρ

Equation of state (EOS):



Effective density dependence:

non-linear potential:

Boguta and Bodmer, NPA 431, 3408 (1977)

NL1, NL3..

$$\frac{1}{2} m_{\sigma}^2 \sigma^2 \Rightarrow U(\sigma) = \frac{1}{2} m_{\sigma}^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4$$

density dependent coupling constants:

R. Brockmann and H. Toki, PRL 68, 3408 (1992)

S. Typel and H.H. Wolter, NPA 656, 331 (1999)

T. Niksic, D. Vretenar, P. Finelli, and P. Ring, PRC 56 (2002) 024306

$$g_{\sigma}, g_{\omega}, g_{\rho} \Rightarrow g_{\sigma}(\rho), g_{\omega}(\rho), g_{\rho}(\rho)$$

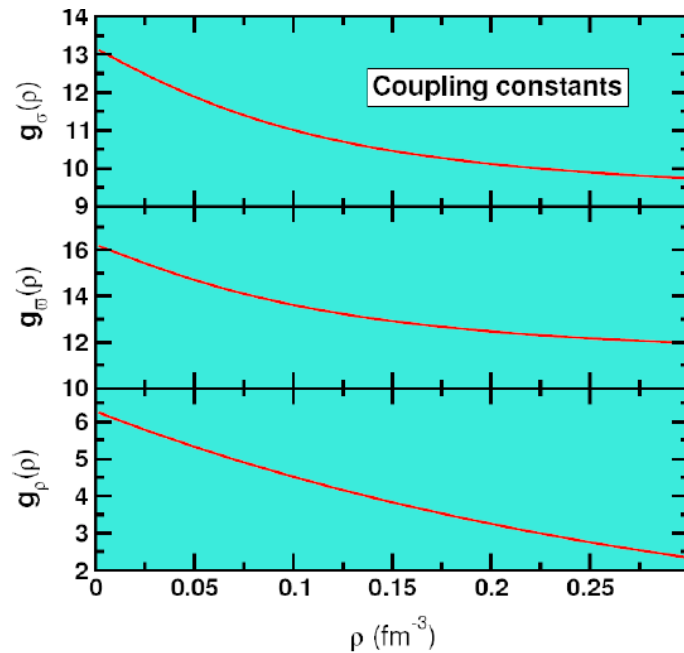
$$\mathbf{g} \rightarrow \mathbf{g}(\rho(r))$$

DD-ME1, DD-ME2

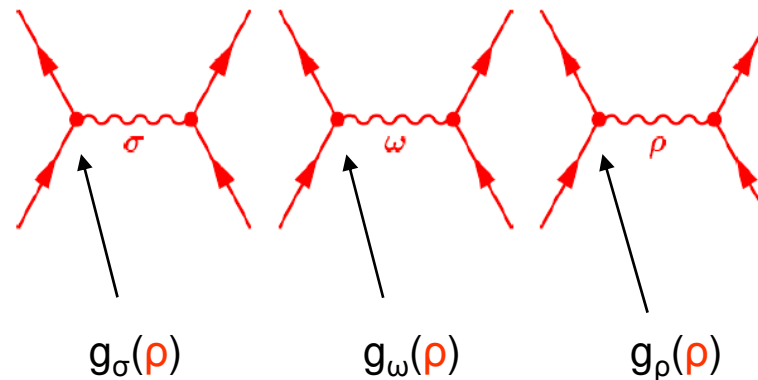
Effective density dependence:

The basic idea comes from **ab initio calculations**
density dependent coupling constants include **Brueckner correlations**
and **threebody forces**

non-linear meson coupling: **NL3**



Effective interactions with medium-dependent couplings:

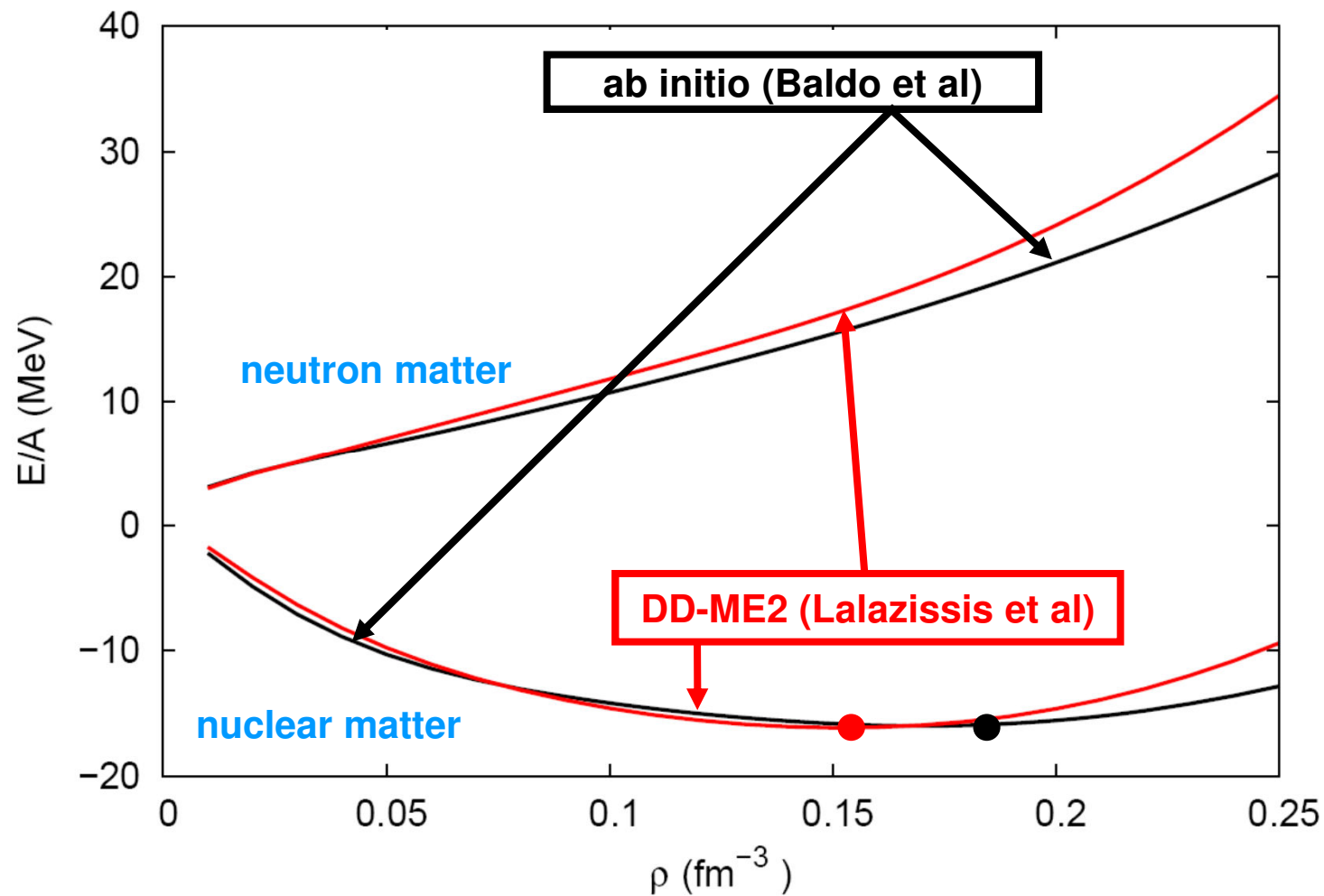


adjusted to ground state properties of finite nuclei

Typel, Wolter, NPA **656**, 331 (1999)
Niksic, Vretenar, Finelli, P.R., PRC **66**, 024306 (2002):
Lalazissis, Niksic, Vretenar, P.R., PRC **78**, 034318 (2008):

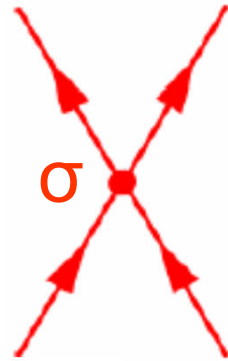
DD-ME1
DD-ME2

Comparison with ab initio calculations:



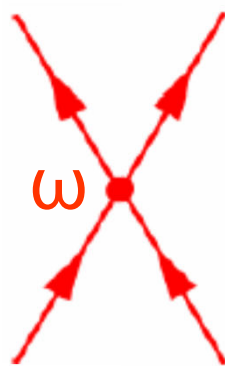
we find excellent agreement with ab initio calculations of Baldo et al.

Point-Coupling Models



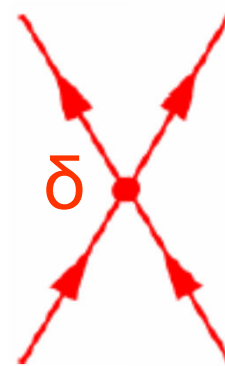
$J=0, T=0$

$$G_{\sigma} = \frac{g_{\sigma}^2}{m_{\sigma}^2}$$



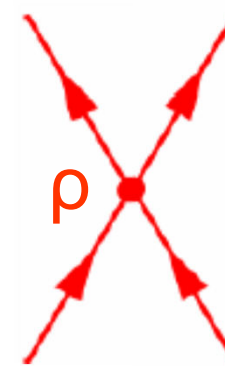
$J=1, T=0$

$$G_{\omega} = \frac{g_{\omega}^2}{m_{\omega}^2}$$



$J=0, T=1$

$$G_{\delta} = \frac{g_{\delta}^2}{m_{\delta}^2}$$



$J=1, T=1$

$$G_{\rho} = \frac{g_{\rho}^2}{m_{\rho}^2}$$

Manakos and Mannel, Z.Phys. **330**, 223 (1988)

Bürvenich, Madland, Maruhn, Reinhard, PRC **65**, 044308 (2002)

Lagrangian density for point coupling

free Dirac particle

$$\begin{aligned}
 \mathcal{L} = & \bar{\psi} (i\gamma \cdot \partial - m) \psi \\
 & + G_{\sigma}(\bar{\psi}\psi)(\bar{\psi}\psi) \quad + G_{\omega}(\bar{\psi}\gamma^{\mu}\psi)(\bar{\psi}\gamma_{\mu}\psi) \\
 & + G_{\delta}(\bar{\psi}\vec{\tau}\psi)(\bar{\psi}\vec{\tau}\psi) \quad + G_{\rho}(\bar{\psi}\gamma^{\mu}\vec{\tau}\psi)(\bar{\psi}\gamma_{\mu}\vec{\tau}\psi) \\
 & + D_{\sigma}(\bar{\psi}\partial^{\mu}\psi)(\bar{\psi}\partial_{\mu}\psi) \\
 & - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad + e^2\bar{\psi}\gamma^{\mu}A_{\mu}\frac{(1-\tau_3)}{2}\psi \quad (1)
 \end{aligned}$$

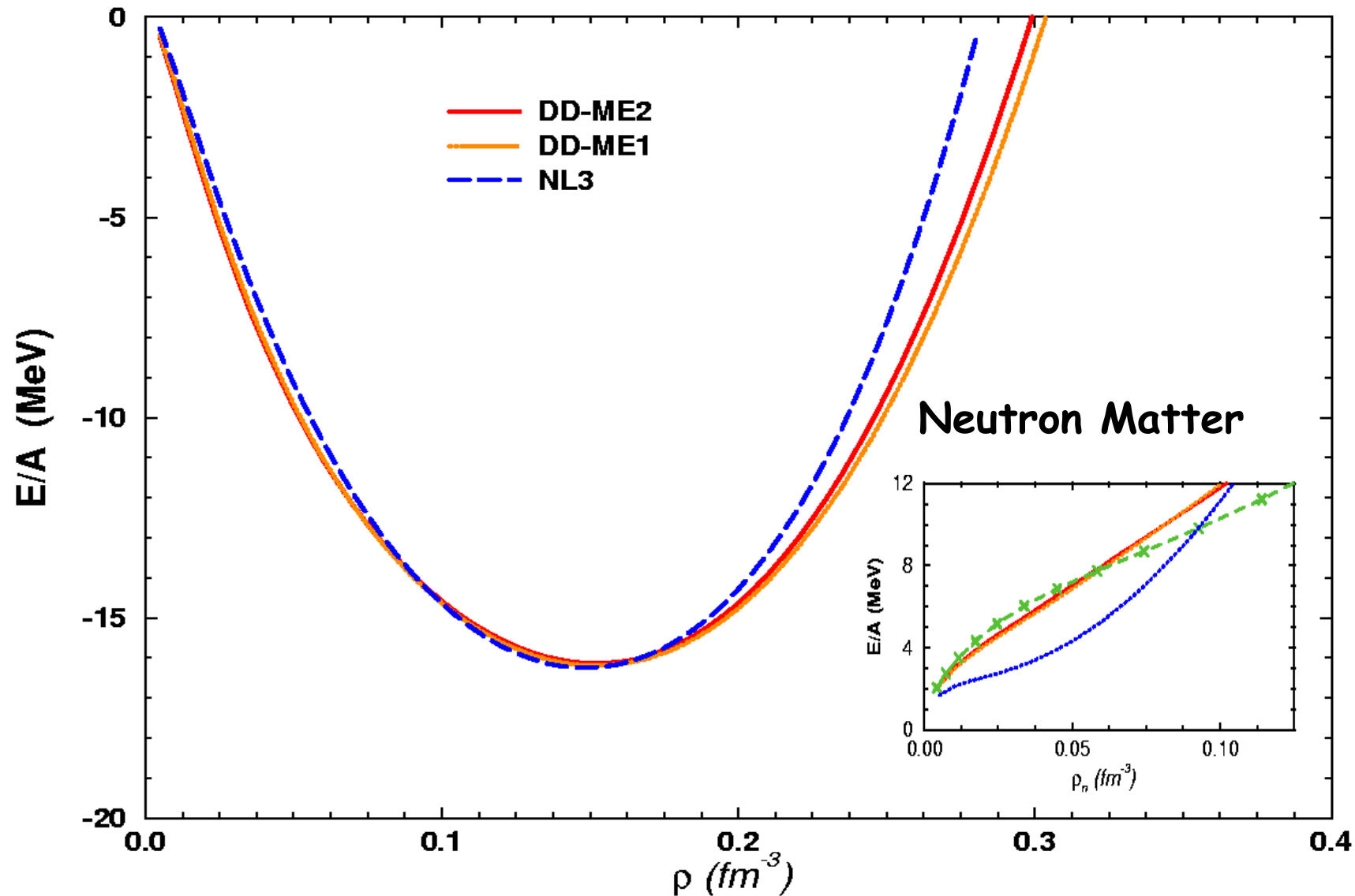
Parameter:

photon field

point couplings: $G_{\sigma}, G_{\omega}, G_{\delta}, G_{\rho}, \quad G_i = \left(\frac{g_i}{m_i}\right)^2$

derivative terms: D_{σ}

Nuclear matter equation of state



Symmetry energy

$$E(\rho, \alpha) = E(\rho, 0) + S_2(\rho)\alpha^2 + S_4(\rho)\alpha^4 + \dots$$

$$S_2(\rho) = a_4 + \frac{p_0}{\rho_{\text{sat}}^2} (\rho - \rho_{\text{sat}}) + \frac{\Delta K_0}{18\rho_{\text{sat}}^2} (\rho - \rho_{\text{sat}})^2 + \dots$$

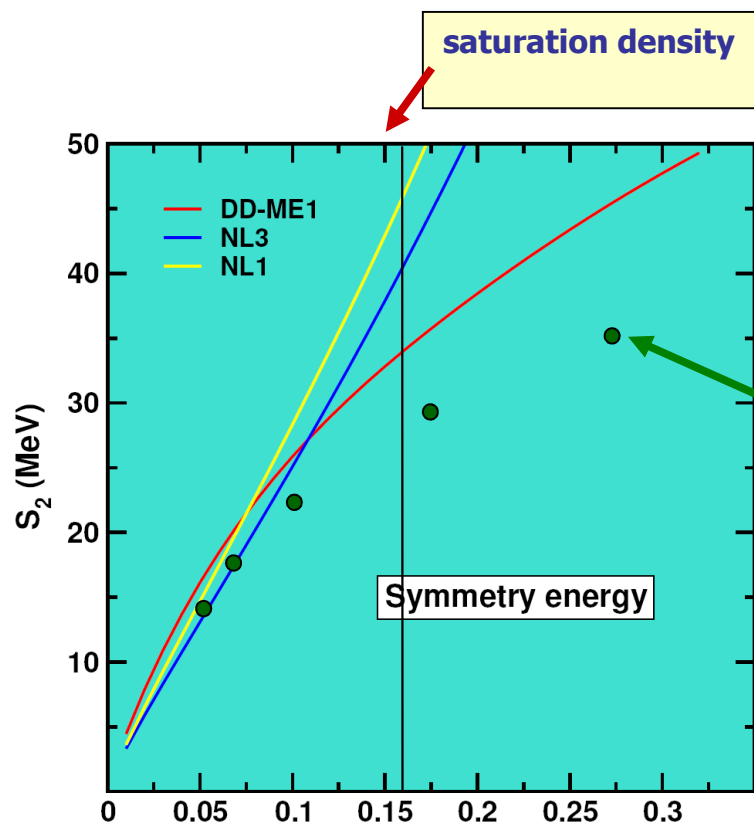
$$\alpha \equiv \frac{N-Z}{N+Z}$$

empirical values:

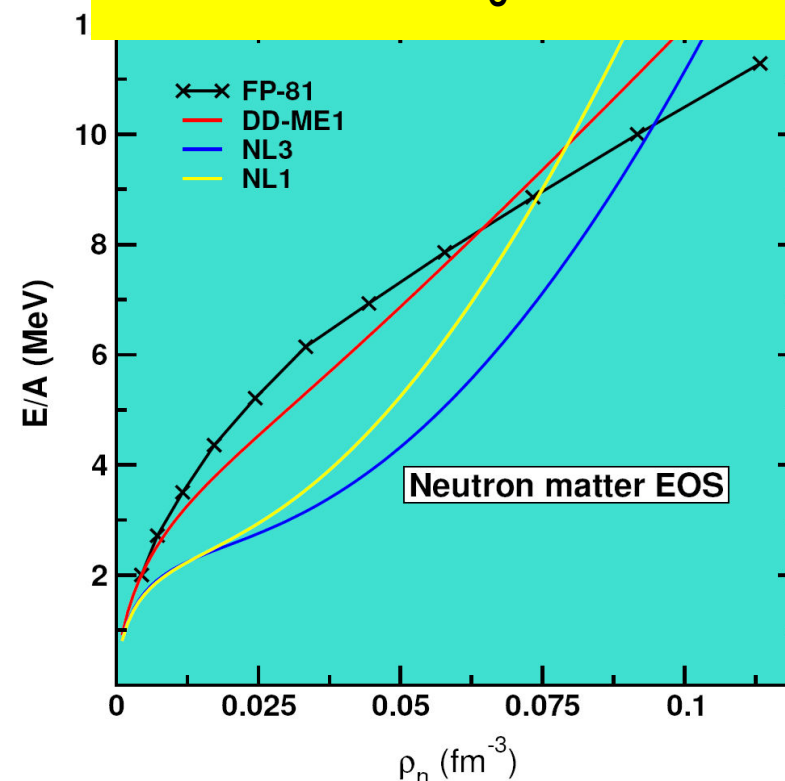
$$30 \text{ MeV} \leq a_4 \leq 34 \text{ MeV}$$

$$2 \text{ MeV/fm}^3 < p_0 < 4 \text{ MeV/fm}^3$$

$$-200 \text{ MeV} < \Delta K_0 < -50 \text{ MeV}$$

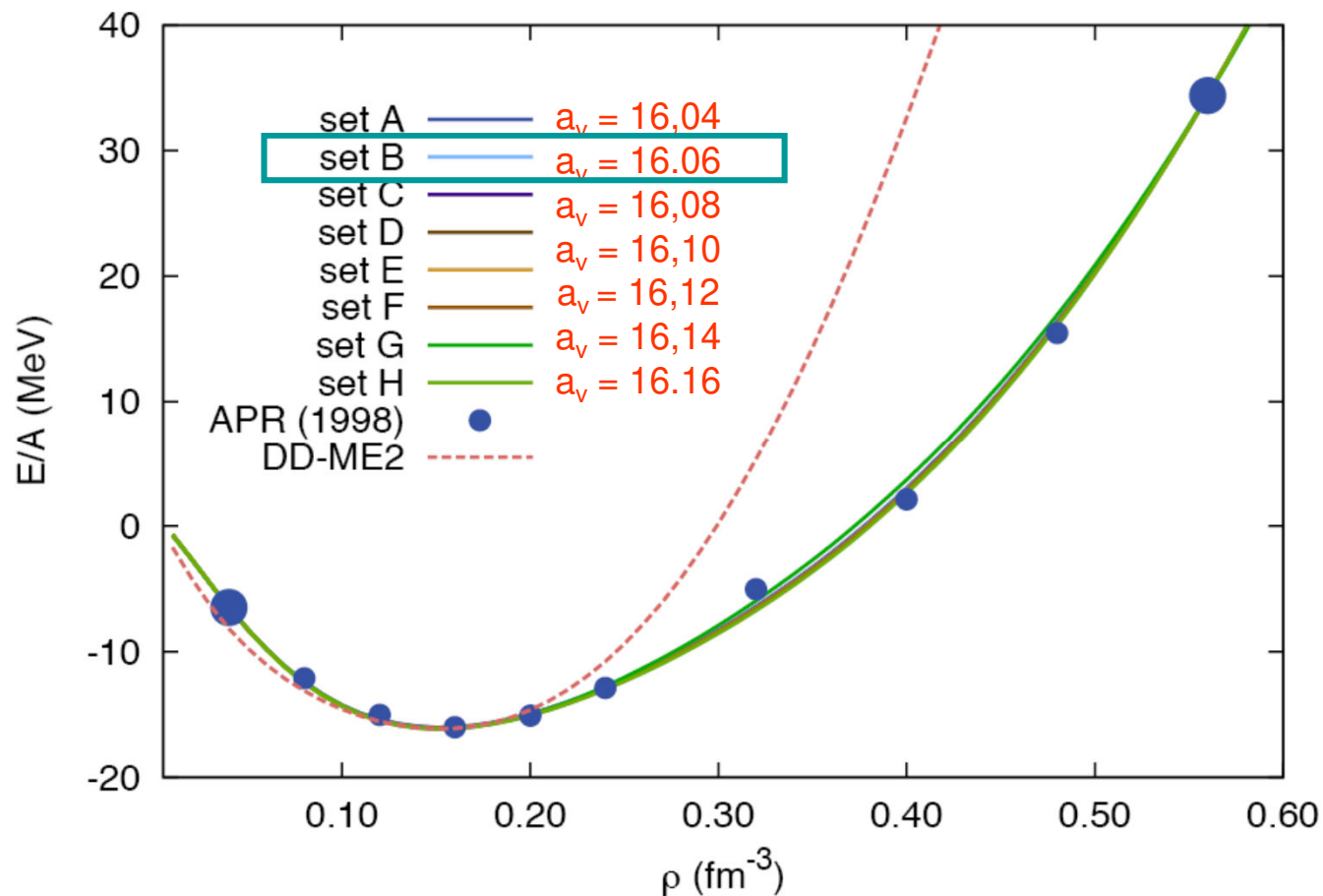


	DD-ME1	NL3	NL1
$a_4(\text{MeV})$	33.1	37.9	43.7
$p_0(\text{MeV/fm}^3)$	3.26	5.92	7.0
$\Delta K_0(\text{MeV})$	-128.5	52.1	67.3



Using ab initio data for the fit

point coupling model is fitted to microscopic nuclear matter
and to masses of 64 deformed nuclei:



$$\begin{aligned}\rho_{\text{sat}} &= 0.152 \text{ fm}^{-3} \\ m^* &= 0.58m \\ K_{\text{nm}} &= 230 \text{ MeV} \\ a_4 &= 33 \text{ MeV}\end{aligned}$$

DD-PC1

● A. Akmal, V.R. Pandharipande, and D.G. Ravenhall, PRC. 58, 1804 (1998).

construction areas

Is density functional theory exact
in self-bound systems as nuclei?

beyond mean field

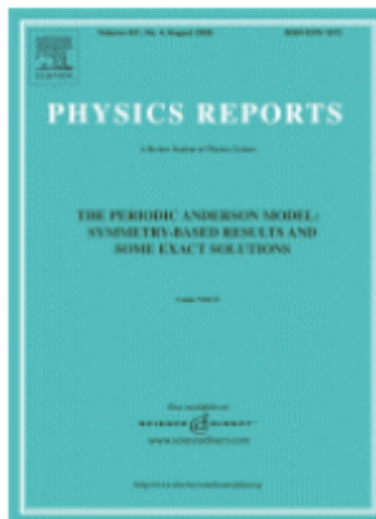
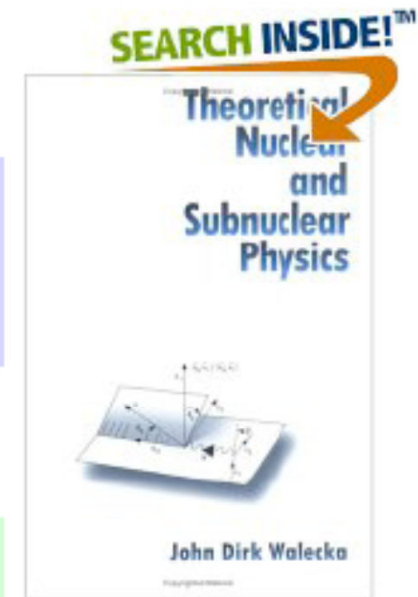
tensor-forces and single particle structure?

improvement of the functional

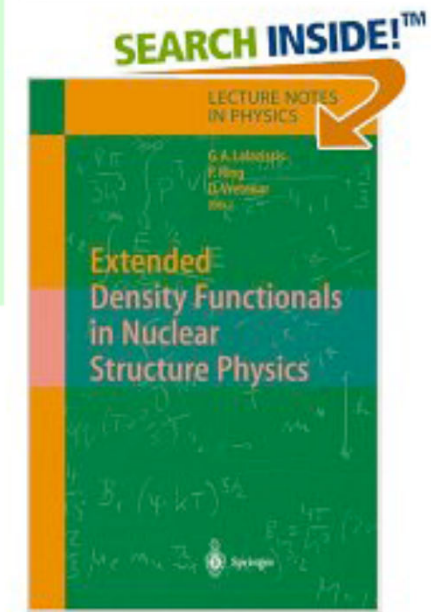
derivation of the functional from the NN-force ?

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D. Vretenar, A. V. Afanasjev, G. A. Lalazissis, P. Ring, Phys. Rep. 409, 101 (2005).

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- P. Ring, Y.K. Gambhir and G.A. Lalazissis, **105**, 77 (1997),
“Computer Program for the RMF Description of Ground State Properties of Even-Even Axially Deformed Nuclei .”