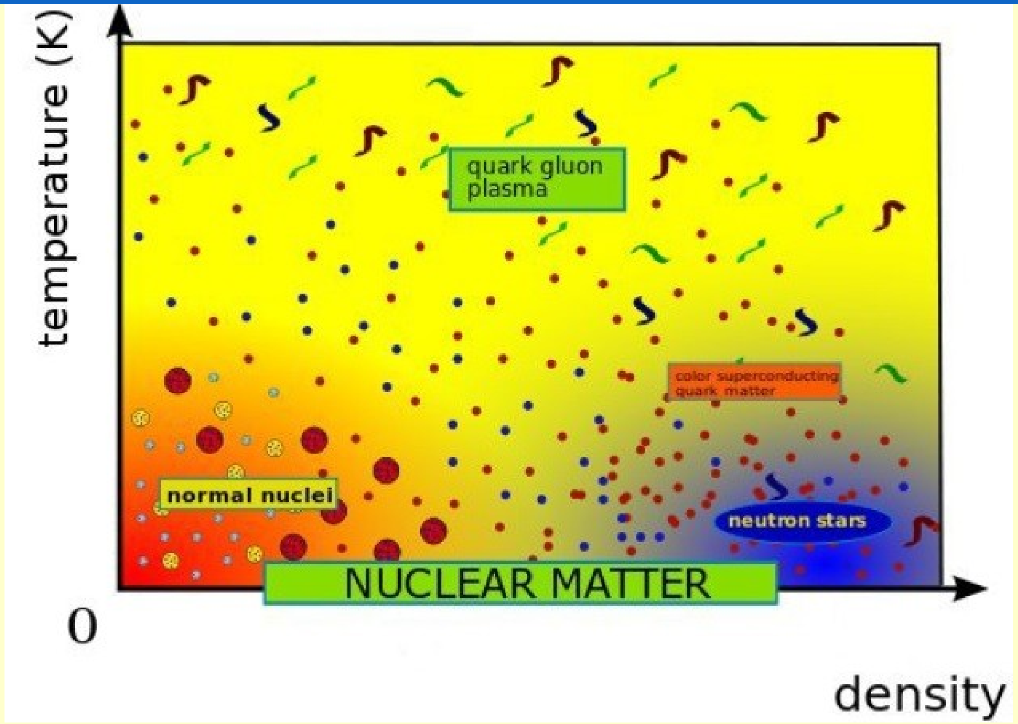
Relativistic Equation of States (EoS) for Neutron Stars

Theodoros Gaitanos

ΤΜΗΜΑ ΦΥΣΙΚΗΣ
ΑΡΙΣΤΟΤΕΛΕΙΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΘΕΣΣΑΛΟΝΙΚΗΣ



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Introduction

Neutron Stars (NS)

SN 1987A

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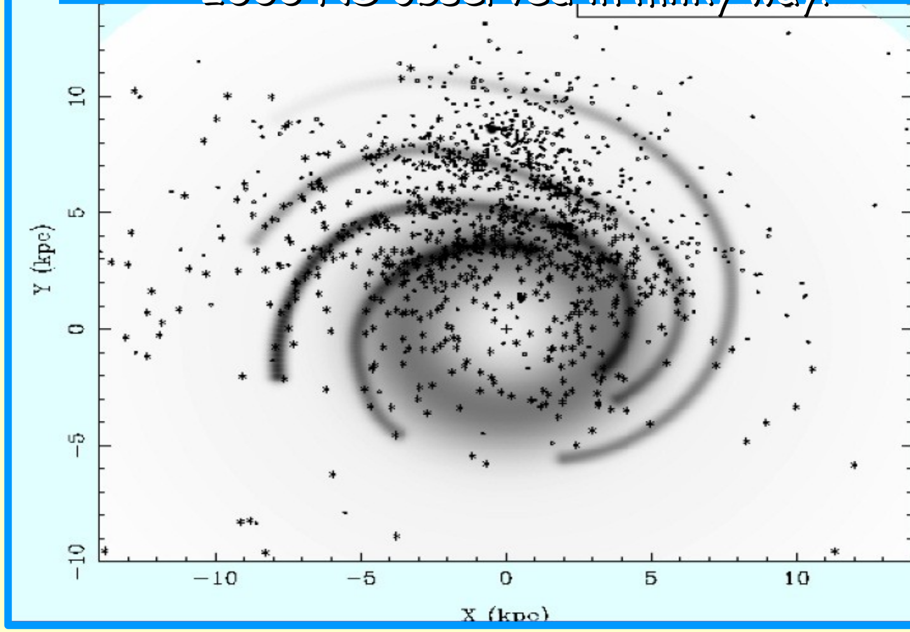
what we know from observations

SN 1987A

~2000 NS observed in milky way!



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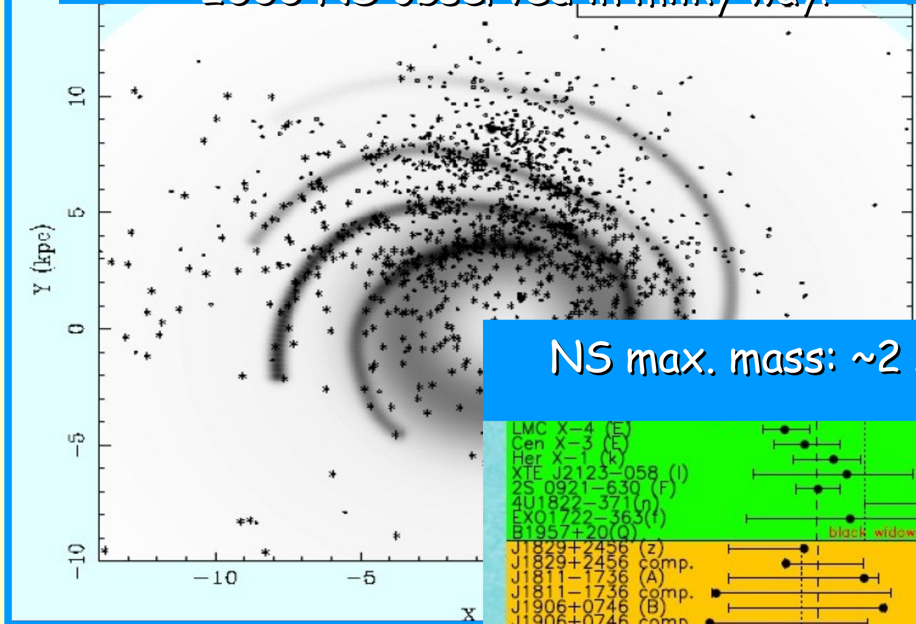
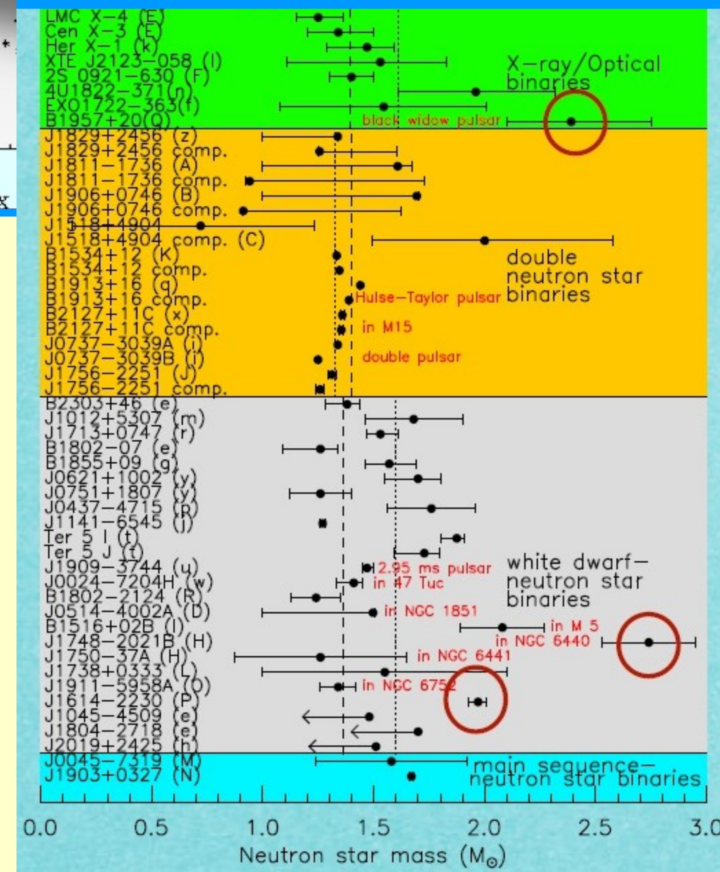
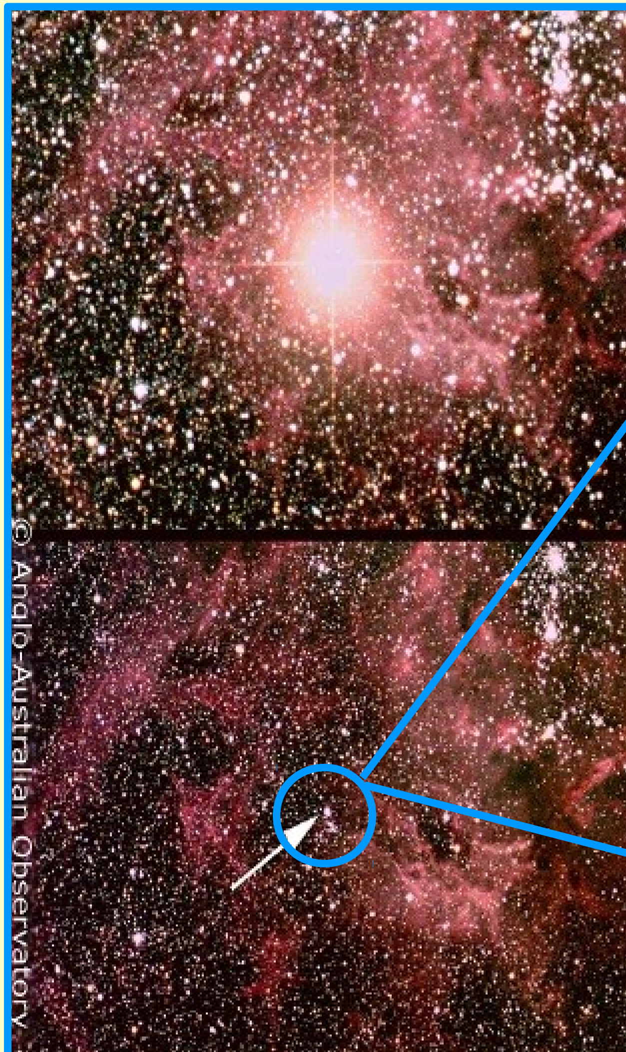


NS max. mass: ~2 x solar mass

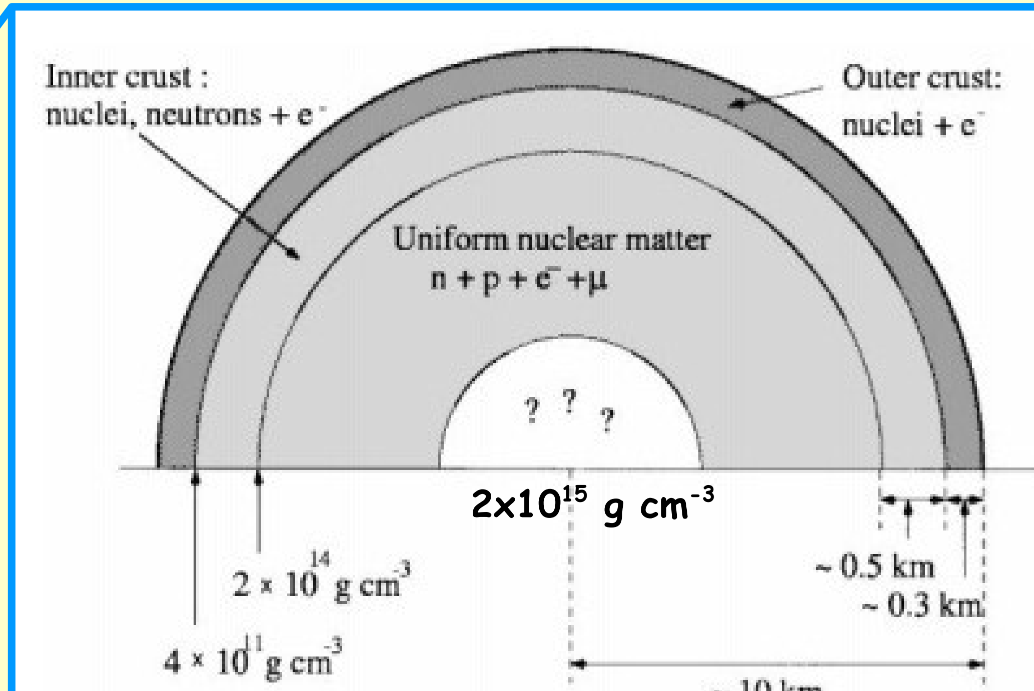


Neutron Stars (NS)

- stable "cosmic nucleus" made of p,n,e^-,...
- densest visible matter in the universe
  1.5-2 x solar mass inside 10km radius



SN 1987A



earth's average density~5.5 g cm^{-3}
earth's radius ~6.371km

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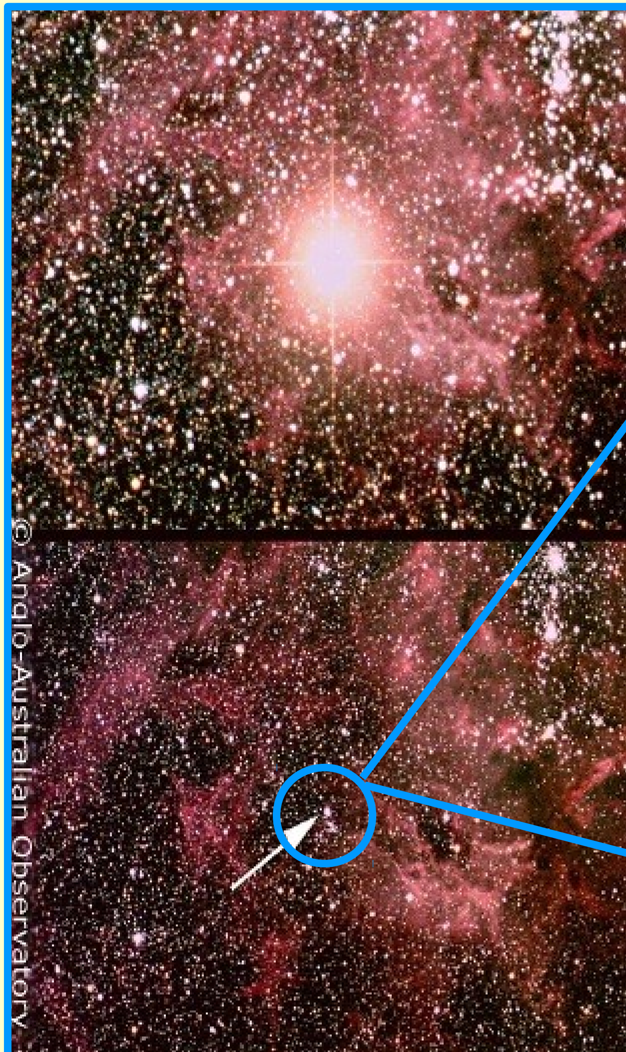
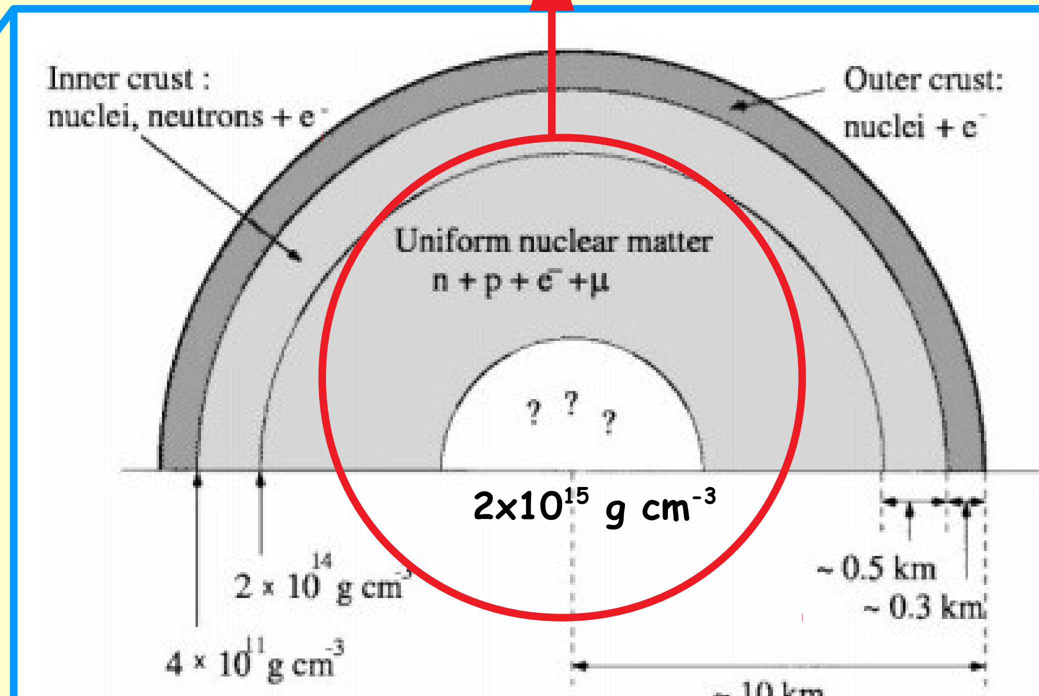


SN 1987A

Theoretical model of cold & compressed nuclear matter



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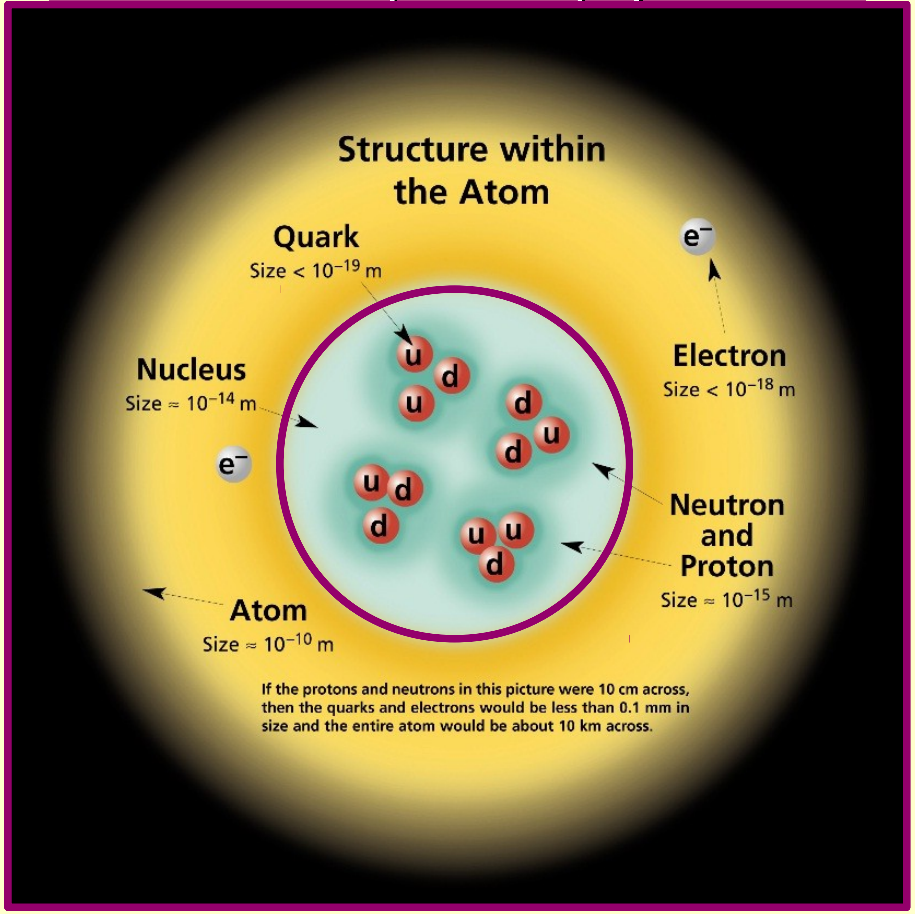
Units in particle physics



Units:
$\hbar = c = 1$
1 GeV=1.8 10^{-24} g
1 fm = 10^{-15} m

Temperature units:
[1MeV]~[10^{10}K]
nuclei:T~0MeV
(earth:T~180K-330K)

Density units:
nuclei:ρ_0 ~ 2.5 10^{14} g cm^{-3} = 0.165 fm^{-3} (saturation density ρ_0 or ρ_{sat})
(earth:ρ ~ g cm^{-3})

Basics: Uniform nuclear & neutron matter…

What is nuclear matter?

* Easy, an infinite system of nucleons (p,n) at constant density ρ

* Symmetric nuclear matter: p=n / neutron Matter: only n

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What is EoS?

* Equation of State = (total energy density/density – mass) as function of ρ (and T)

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What is EoS?

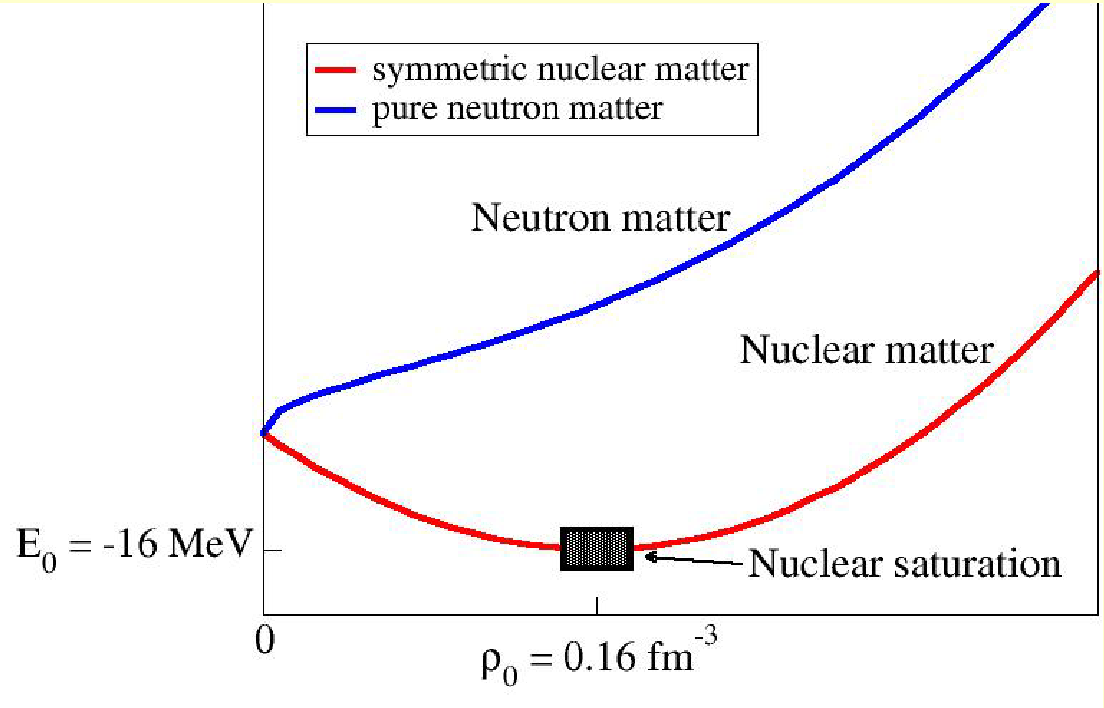
* Equation of State = (total energy density/density - mass) as function of ρ (and T)

* Saturation = min. of EoS → ground state of nuclear matter (stable nuclei)

* ρ_0 = saturation density

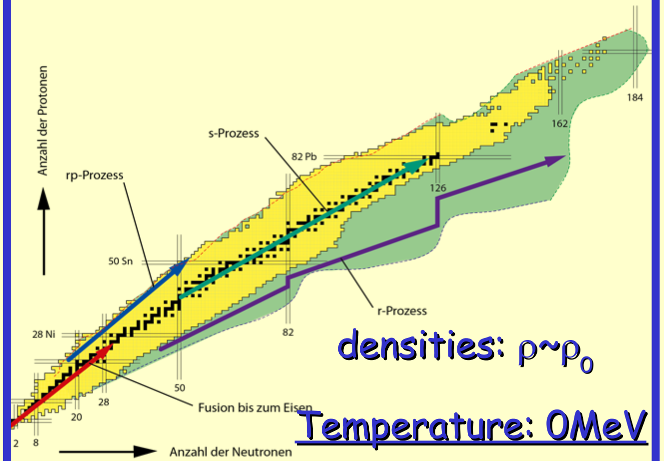
* Nuclear matter saturates

* Neutron matter positive pressure



Accessing EoS from observations

finite nuclei



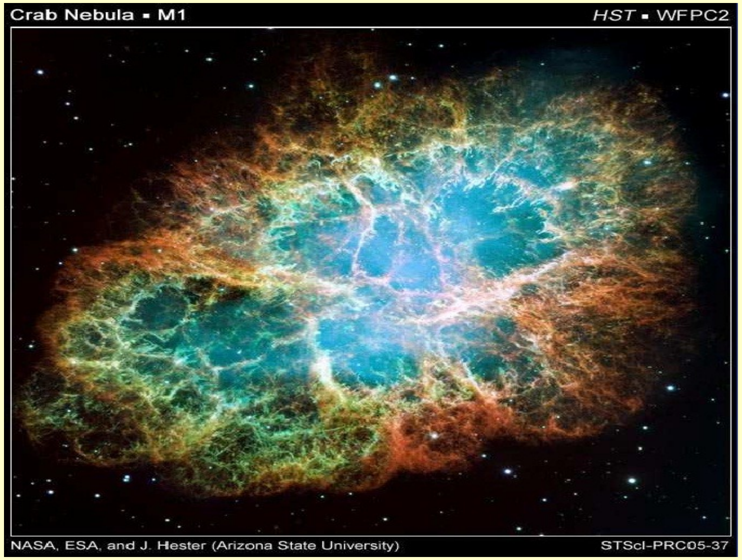
densities: $\rho \sim \rho_0$

Temperature: 0MeV

Accessing EoS from observations

Neutron stars

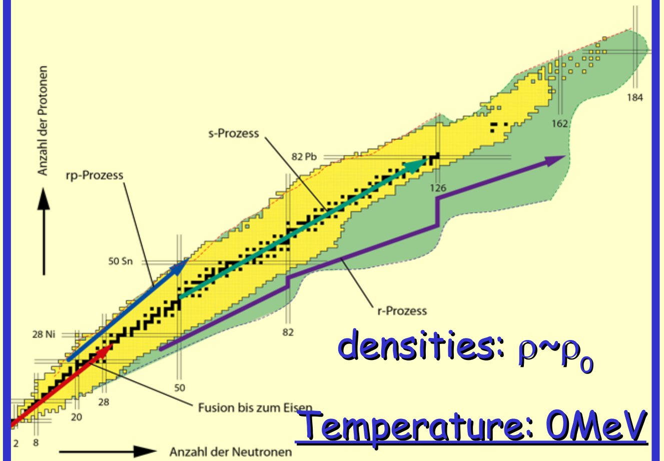
(mass & radius)



densities: $\rho \sim (8\text{-}10)\rho_0$

Temperature: ~0 MeV

finite nuclei



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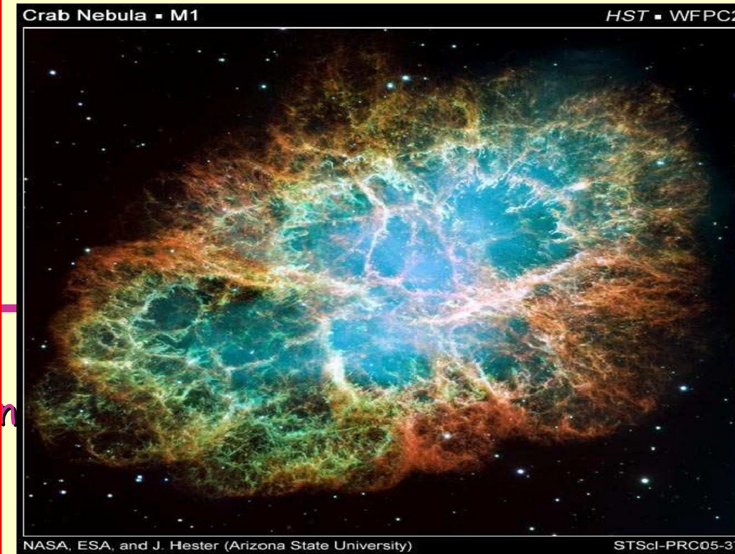
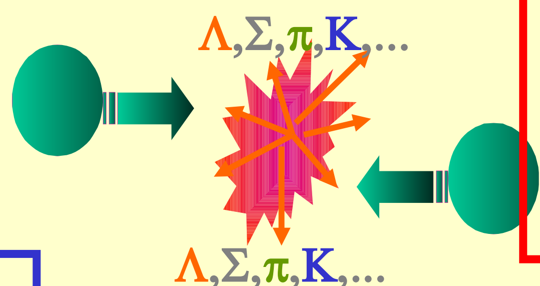


densities: $\rho \sim (8\text{-}10)\rho_0$

Temperature: ~0 MeV

Heavy-ion collisions

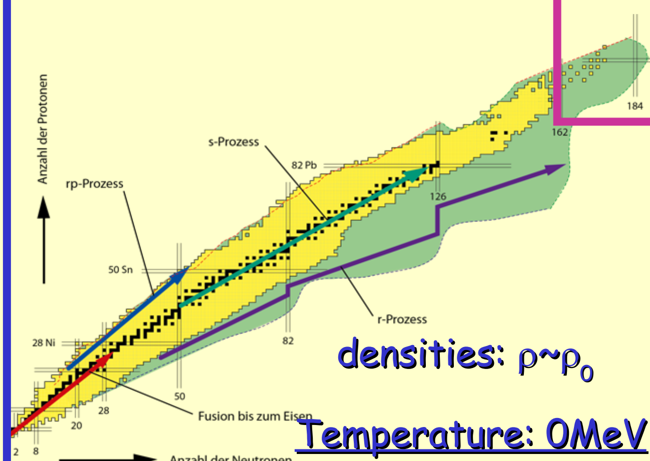
(collective flow, meson production)



densities: $\rho \sim 5\rho_0$

Temperture: 5-50 MeV

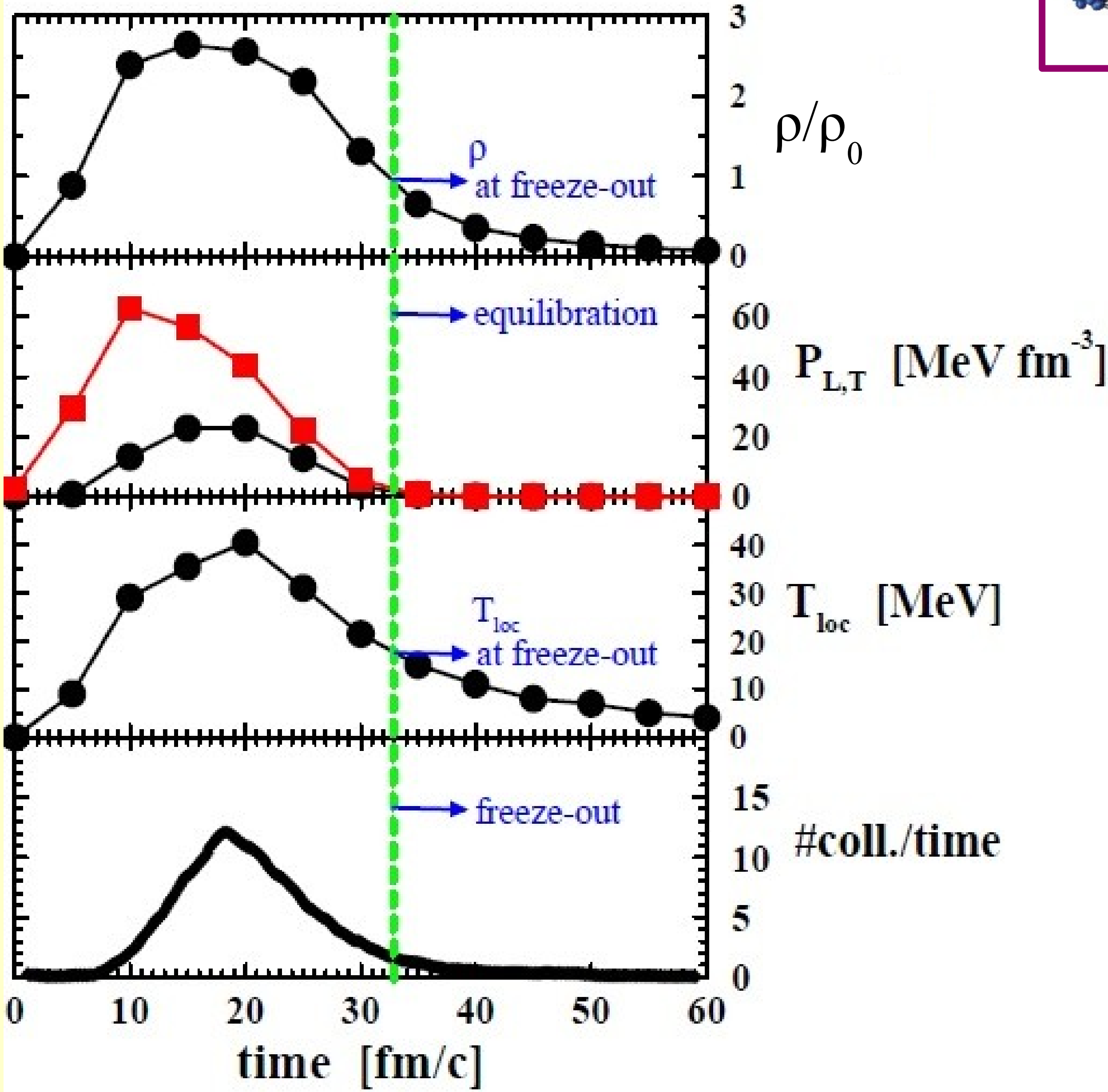
finite nuclei

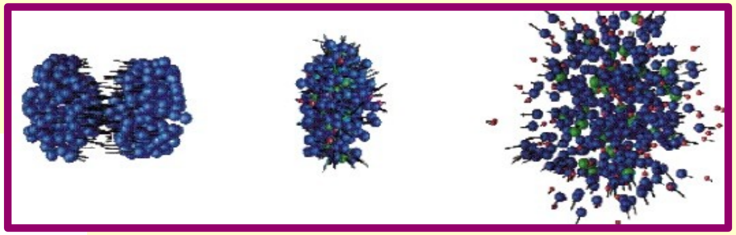


densities: $\rho \sim \rho_0$

Temperature: 0MeV

Densities & temperatures in fireball…





Au+Au@0.6AGeV

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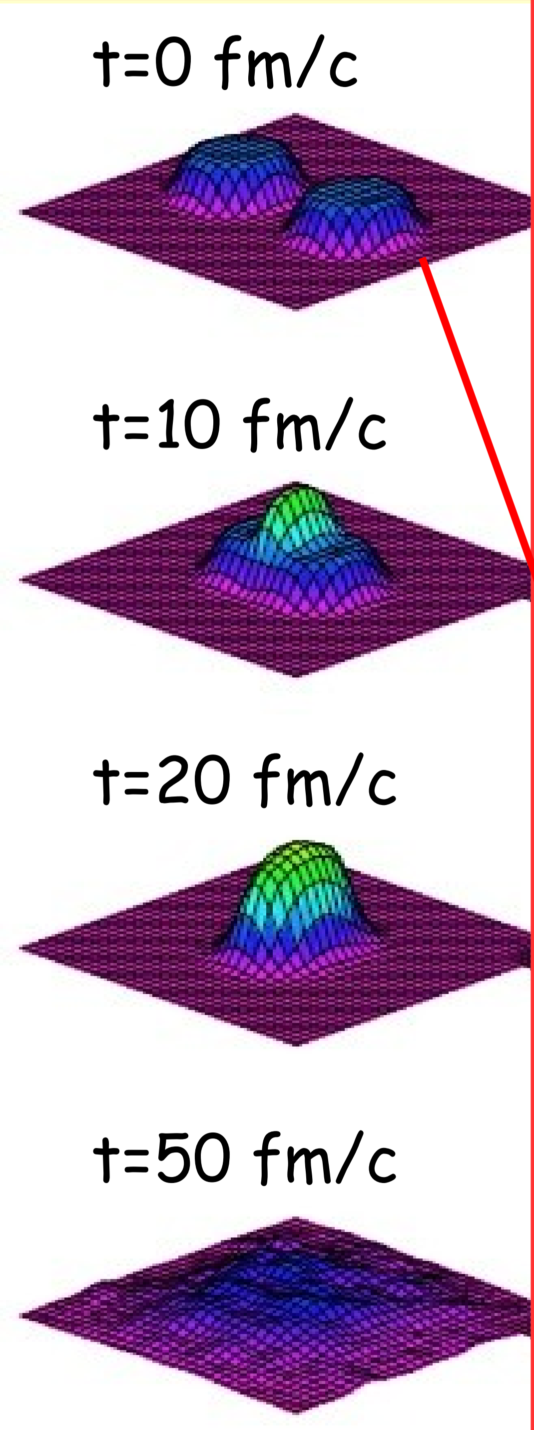

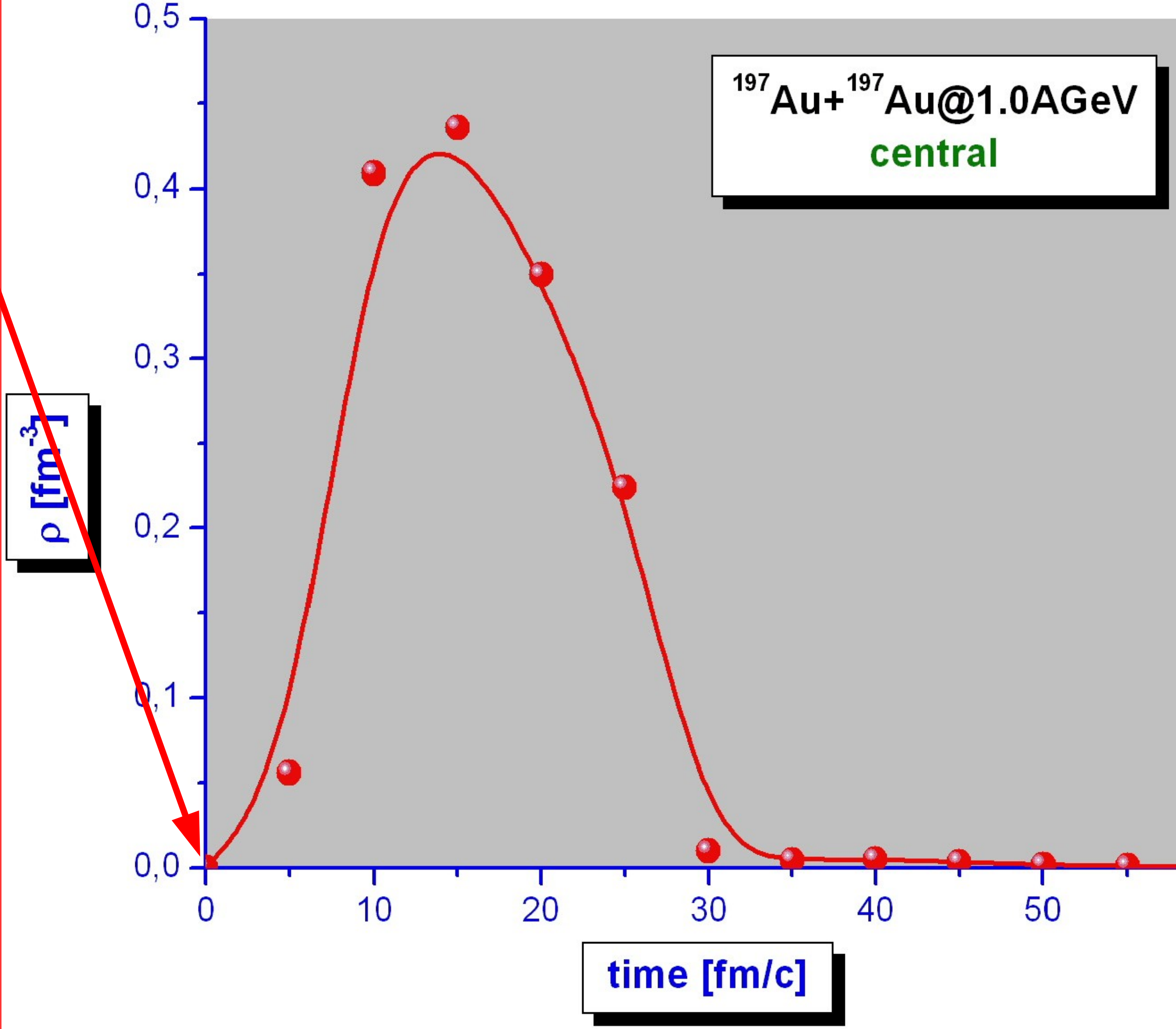

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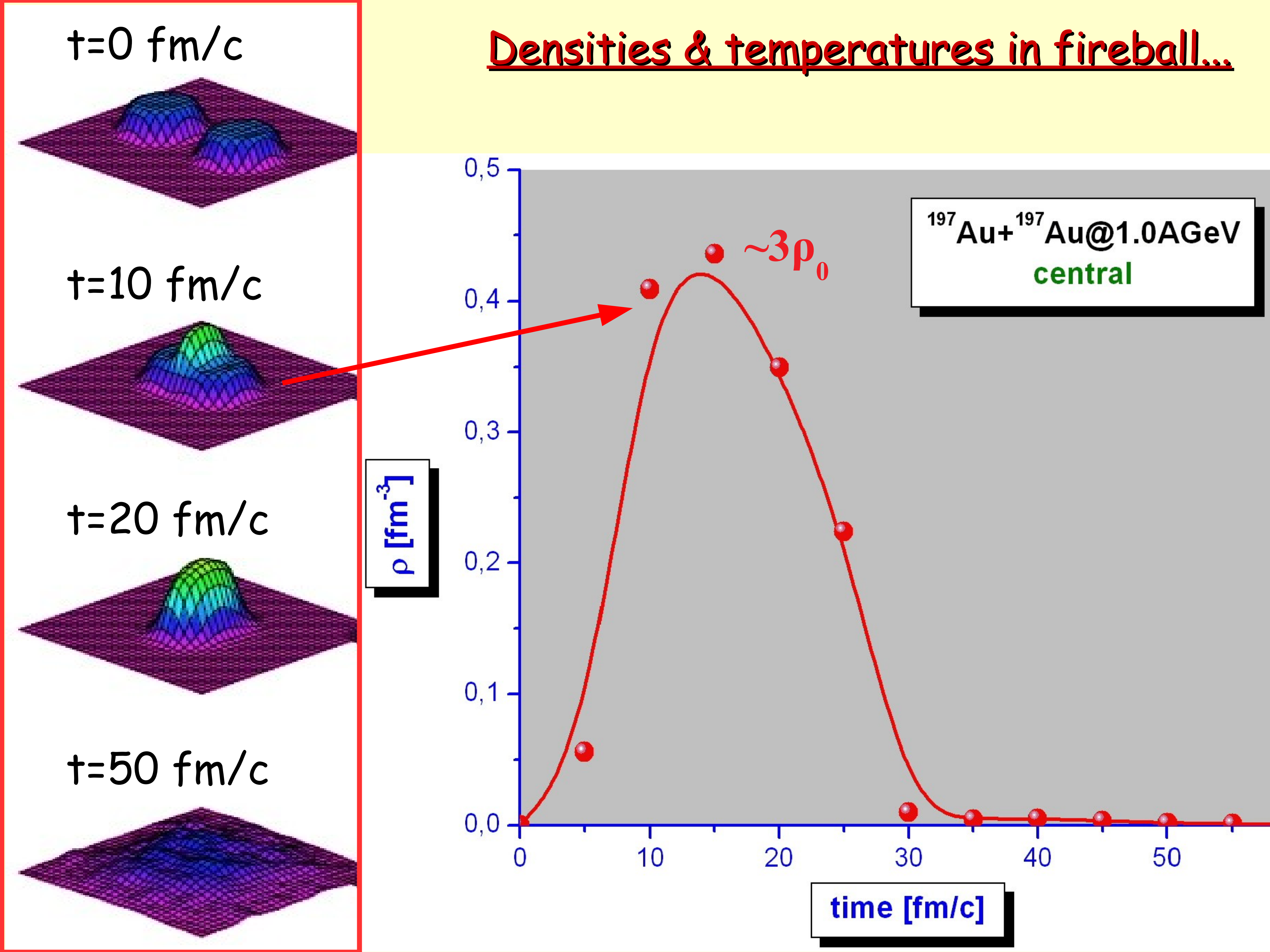

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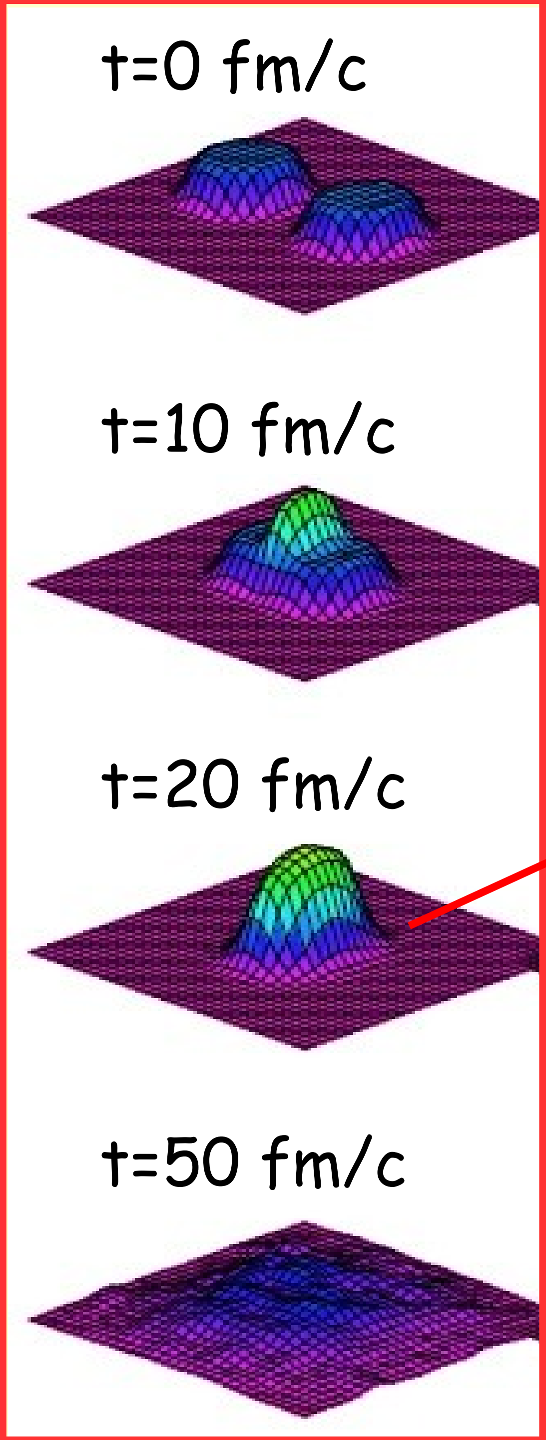

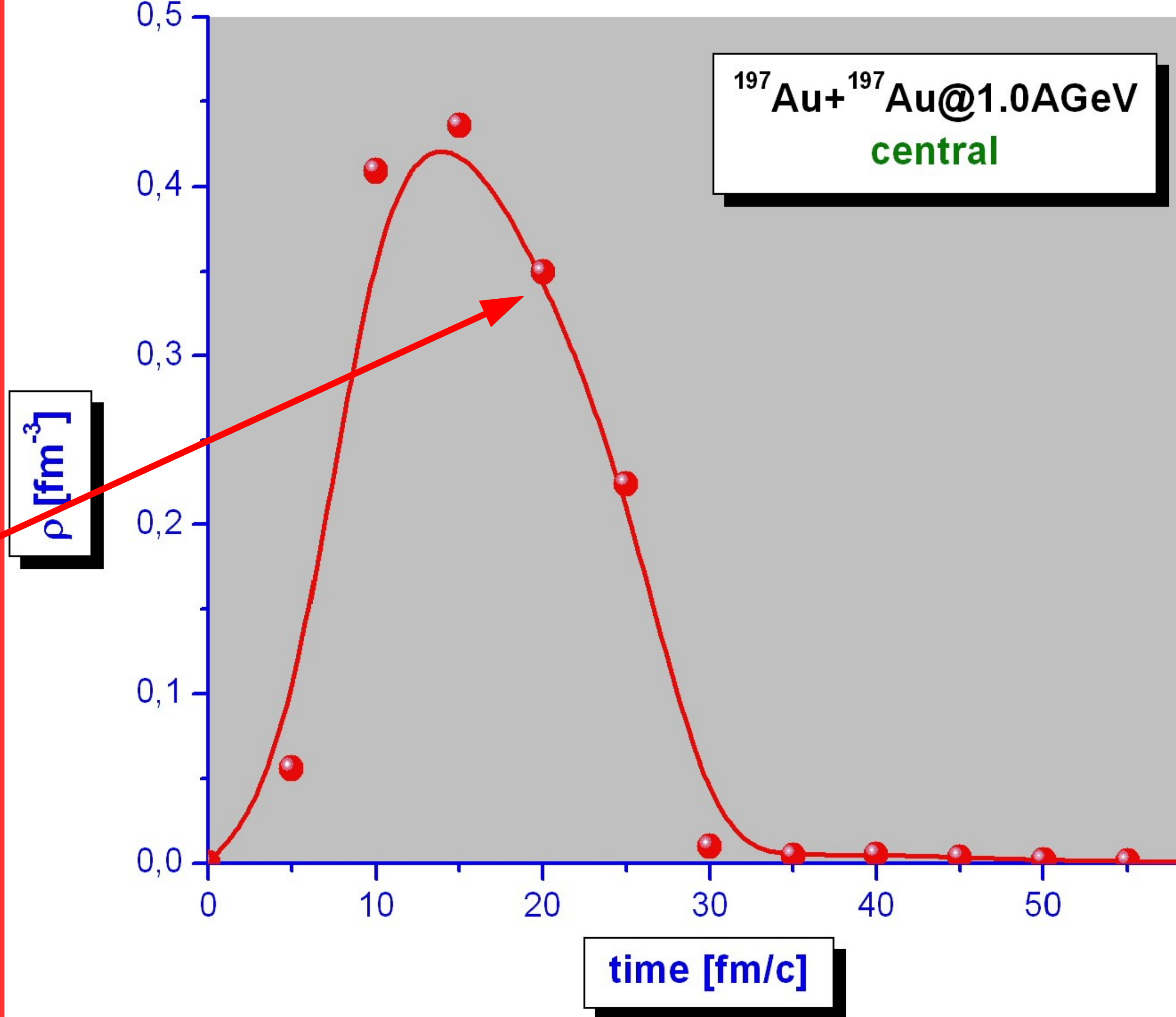

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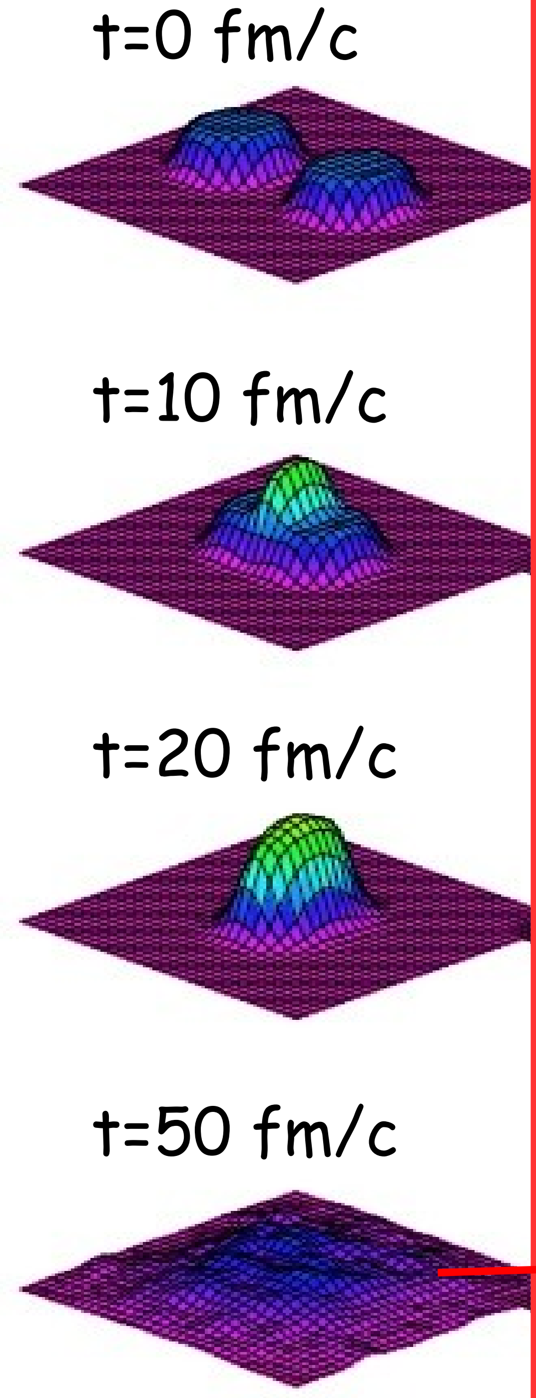

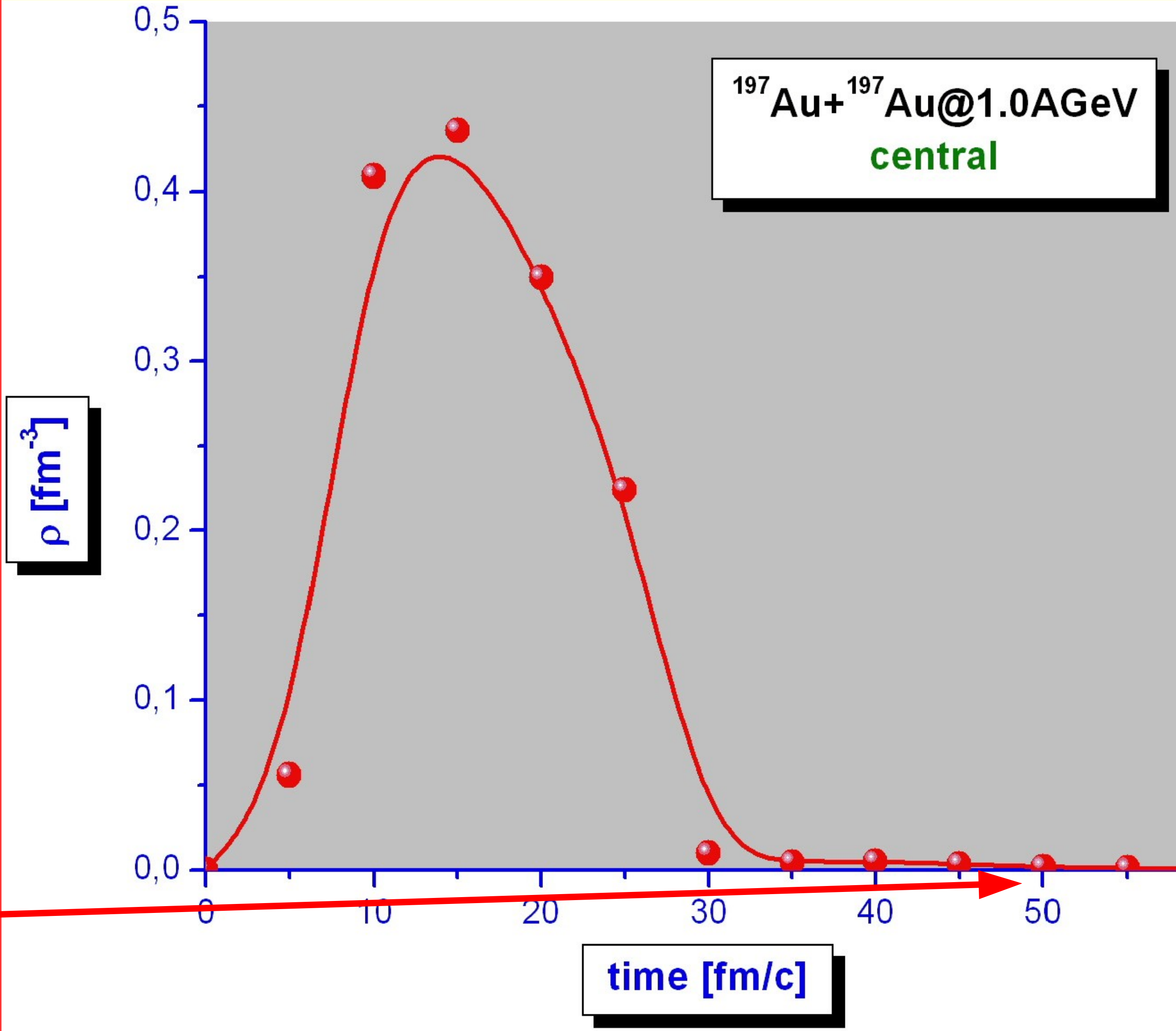

Relativistic Nuclear Models

Classical mechanics versus relativistic field theory

Point-like particles	relativistic fields
dof: coordinates q_i	$\varphi(t,\vec{r}) = \varphi(x)$
Velocities $\dot{q}_i$	$\partial^\mu \varphi(x)$
Lagrange function	Lagrangian density
$\mathcal{L} = \mathcal{L}(q_i, \dot{q}_i)$	$\mathcal{L} = \mathcal{L}[\varphi, \partial_\mu \varphi]$
Equations of motion	Equations of motion
$\frac{d\mathcal{L}}{dq_i} - \frac{d}{dt}\frac{d\mathcal{L}}{d\dot{q}_i} = 0$	$\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} = 0$

Classical mechanics versus relativistic field theory

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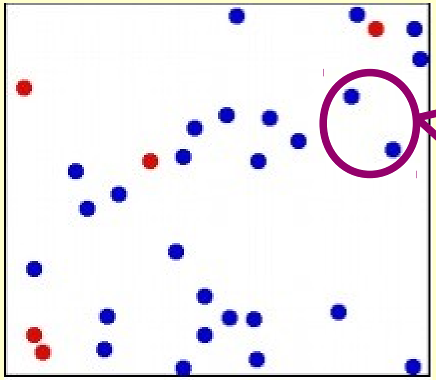
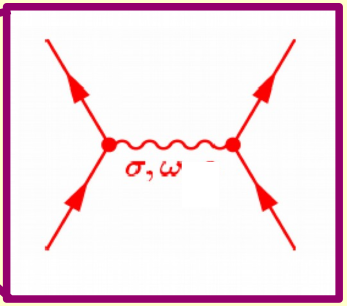
Relativistic Hadrodynamics: Lagrangian formalism for fields



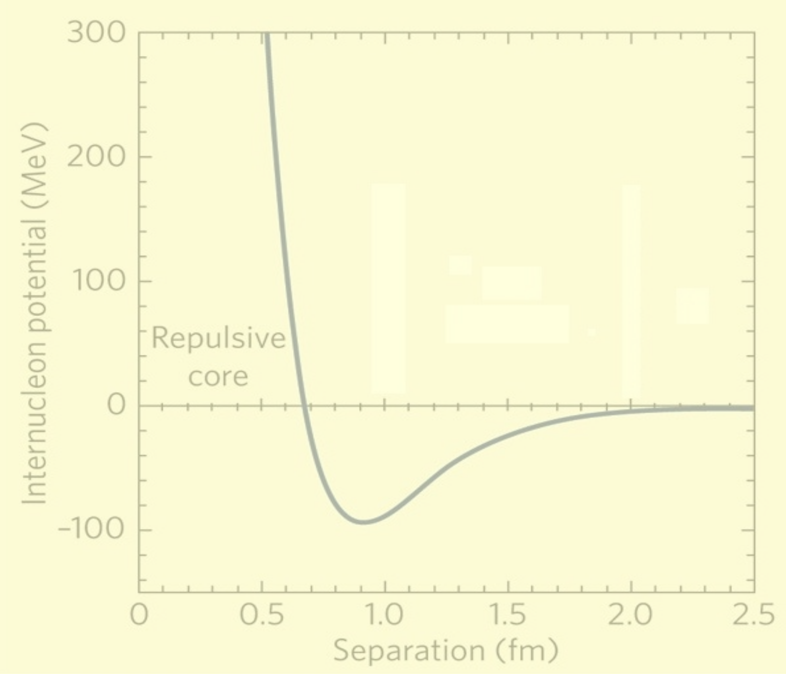


Degrees of freedom

Nucleons → Dirac Spinor Ψ

Interaction → σ, ω mesons

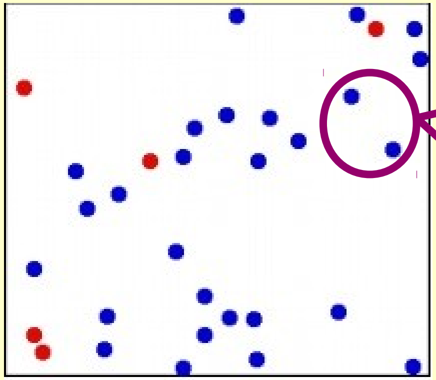
Interaction: short range attractive & repulsive for r<0.8 fm

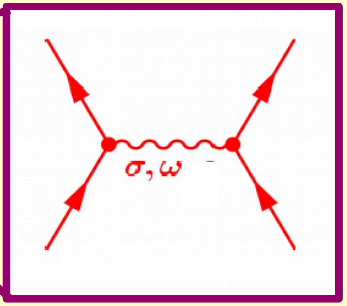


Yukawa's concept

screened Coulomb potential

Relativistic Hadrodynamics: Lagrangian formalism for fields





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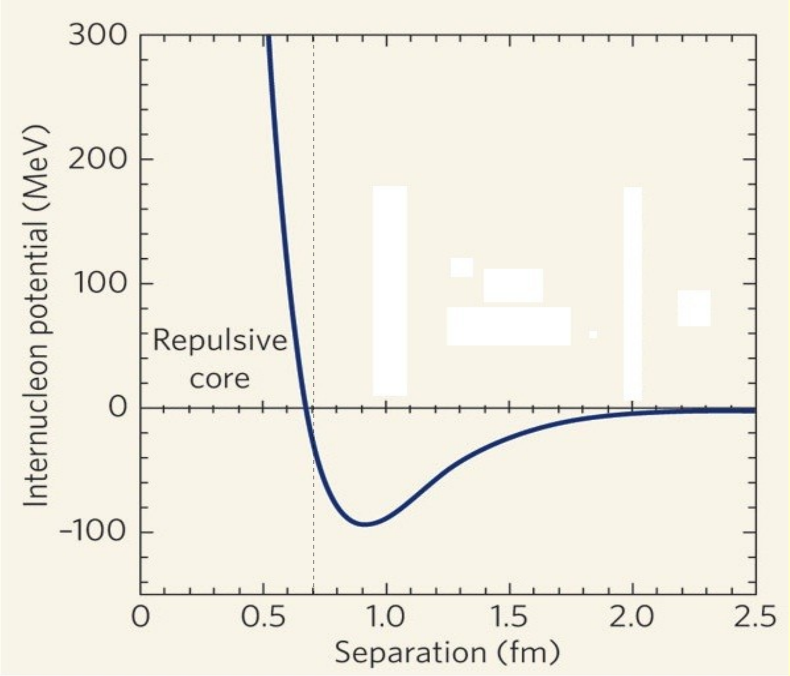

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$$V(r) = -g^2\,\frac{1}{r}e^{-mr}$$

coupling strength

particle mass

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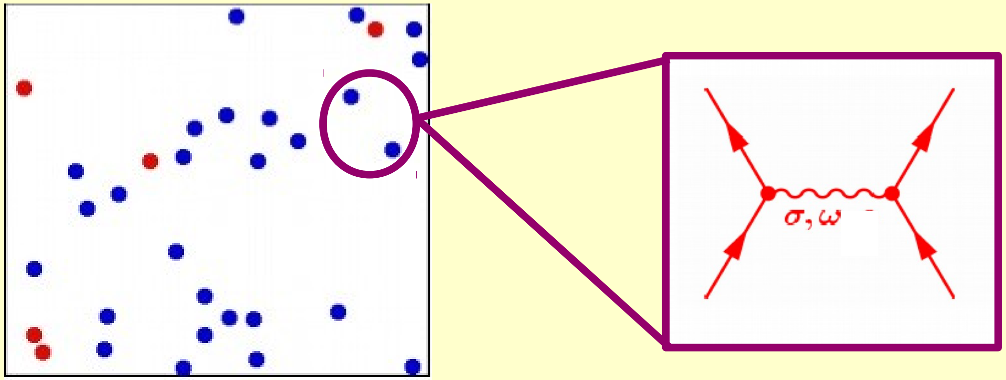

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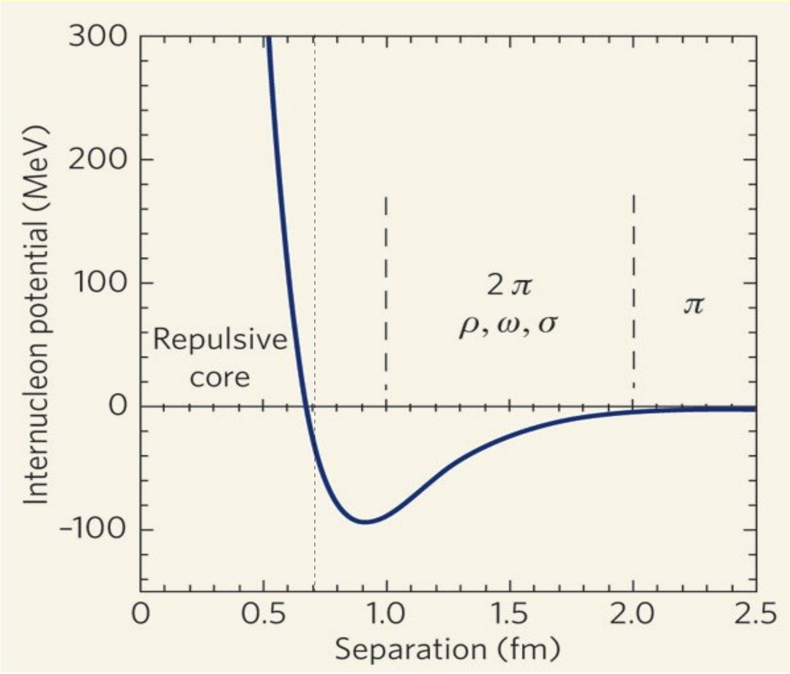

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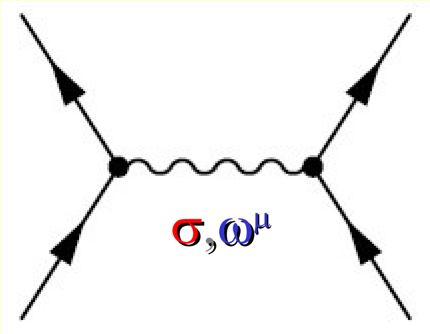
Relativistic Hadrodynamics:

Free Lagrangian for the Dirac field (spin ½ nucleons)

$$\mathcal{L}_D = \overline{\Psi}\gamma_\mu i\partial^\mu \Psi - m\overline{\Psi}\Psi$$

Interaction Lagrangian: minimal coupling (analogy to electrodynamics)



σ → scalar (spin 0) field
ω^μ → vector (spin 1) field
} isoscalar sector (T=0)

$$\mathcal{L}_{int} = g_\sigma \overline{\Psi}\Psi\sigma - g_\omega \overline{\Psi}\gamma_\mu\Psi\omega^\mu$$
$$+ g_\delta \overline{\Psi}\vec{\tau}\Psi\vec{\delta} - g_\rho \overline{\Psi}\gamma_\mu\vec{\tau}\Psi\vec{\rho}^\mu$$

J.D. Walecka, Ann. Phys. (N.Y.) 83 (1974) 497

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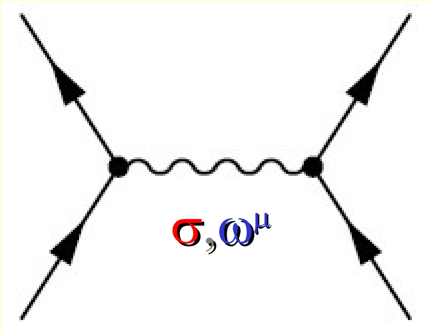

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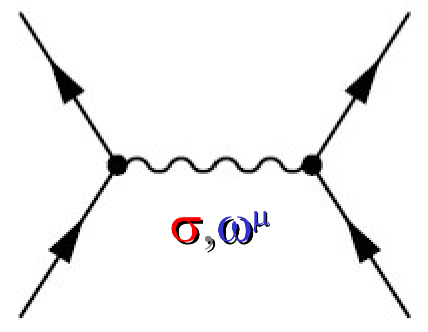

$\sigma \rightarrow$ scalar (spin 0) field
$\omega^\mu \rightarrow$ vector (spin 1) field
} isoscalar sector (T=0)

$$\mathcal{L}_{int} = g_\sigma \overline{\Psi}\Psi\sigma - g_\omega \overline{\Psi}\gamma_\mu\Psi\omega^\mu + g_\delta \overline{\Psi}\vec{\tau}\Psi\vec{\delta} - g_\rho \overline{\Psi}\gamma_\mu\vec{\tau}\Psi\vec{\rho}^\mu$$

$\delta \rightarrow$ scalar (spin 0) field
$\rho^\mu \rightarrow$ vector (spin 1) field
} isovector sector (T=1)

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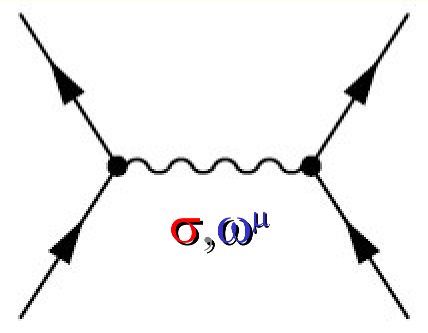

$\sigma \rightarrow$ scalar (spin 0) field $\delta \rightarrow$ scalar (spin 0) field

$\omega^\mu \rightarrow$ vector (spin 1) field $\rho^\mu \rightarrow$ vector (spin 1) field

$$\mathcal{L}_{int} = g_\sigma \overline{\Psi}\Psi\sigma - g_\omega \overline{\Psi}\gamma_\mu \Psi \omega^\mu + g_\delta \overline{\Psi}\vec{\tau}\Psi\vec{\delta} - g_\rho \overline{\Psi}\gamma_\mu \vec{\tau}\Psi\vec{\rho}^{\,\mu}$$

Full Lagrangian:

$$\mathcal{L} = \overline{\Psi}\gamma_\mu \left(i\partial^\mu - g_\omega\omega^\mu - g_\rho\vec{\tau}\,\vec{\rho}^{\,\mu}\right)\Psi - \left(m - g_\sigma\sigma - g_\delta\vec{\tau}\,\vec{\delta}\right)\overline{\Psi}\Psi + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\delta + \mathcal{L}_\rho$$

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Relativistic Hadrodynamics:

Euler-Lagrange equations of motion for Dirac field Ψ:

$$\frac{\partial \mathcal{L}}{\partial \overline{\Psi}} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \overline{\Psi})} = 0$$

$$\mathcal{L}_D = \overline{\Psi}\gamma_\mu \left(i\partial^\mu - g_\omega \omega^\mu\right) \Psi - \left(m - g_\sigma \sigma\right)\overline{\Psi}\Psi + \mathcal{L}_\sigma + \mathcal{L}_\omega$$

$$\frac{\partial \mathcal{L}}{\partial \overline{\Psi}} = \gamma_\mu \left(i\partial^\mu - g_\omega \omega^\mu\right) \Psi - \left(m - g_\sigma \sigma\right)\Psi \qquad \frac{\partial \mathcal{L}}{\partial(\partial_\mu \overline{\Psi})} = 0$$

→ **In-medium Dirac equation for spinor field Ψ:**

$$\boxed{\gamma_\mu \left(i\partial^\mu - g_\omega \omega^\mu\right) \Psi - \left(m - g_\sigma \sigma\right)\Psi = 0}$$

J.D. Walecka, Ann. Phys. (N.Y.) 83 (1974) 497

Relativistic Hadrodynamics:

$$\gamma_\mu \left(i\partial^\mu - g_\omega \omega^\mu\right)\Psi - \left(m - g_\sigma \sigma\right)\Psi = 0$$

$$m_\sigma^2 \sigma + \frac{\partial U}{\partial \sigma} = g_\sigma \sum_{i=p,n} \left\langle \overline{\Psi}_i \Psi_i \right\rangle = g_\sigma \rho_s$$

$$m_\omega^2 \omega = g_\omega \sum_{i=p,n} \left\langle \overline{\Psi}_i \gamma^0 \Psi_i \right\rangle = g_\omega \rho_0$$

$$m_\rho^2 \rho = g_\rho \sum_{i=p,n} \tau_i \left\langle \overline{\Psi}_i \gamma^0 \Psi_i \right\rangle = g_\rho \rho_I \ .$$

$$T^{\mu\nu} = \sum_{i=p,n} \frac{\kappa}{(2\pi)^3} \int_{|\vec{p}\,| \leq p_{F_i}} d^3p \, \frac{\Pi_i^\mu p^\nu}{\Pi_i^0} - g^{\mu\nu} \langle \mathcal{L} \rangle$$

$$\Pi_i^\mu = p_i^{*\mu} + m_i^* \left(\partial_p^\mu \Sigma_{si}\right) - \left(\partial_p^\mu \Sigma_{vi}^\beta\right) p_{i\beta}^*$$

$$\Sigma_{vi}^\mu = g_\omega \omega^\mu \quad + g_\rho \tau_i \rho^\mu \quad , \ \Sigma_{si} = g_\sigma \sigma$$

J.D. Walecka, Ann. Phys. (N.Y.) 83 (1974) 497

Relativistic Hadrodynamics:

Dirac equation

$$\boxed{\gamma_\mu \left(i\partial^\mu - g_\omega \omega^\mu\right) \Psi - \left(m - g_\sigma \sigma\right)\Psi = 0}$$

meson-field equations

$$m_\sigma^2 \sigma + \frac{\partial U}{\partial \sigma} = g_\sigma \sum_{i=p,n} \left\langle \overline{\Psi}_i \Psi_i \right\rangle = g_\sigma \rho_s$$

$$m_\omega^2 \omega = g_\omega \sum_{i=p,n} \left\langle \overline{\Psi}_i \gamma^0 \Psi_i \right\rangle = g_\omega \rho_0$$

$$m_\rho^2 \rho = g_\rho \sum_{i=p,n} \tau_i \left\langle \overline{\Psi}_i \gamma^0 \Psi_i \right\rangle = g_\rho \rho_I \; .$$

Equation of State (EoS)

$$\varepsilon = \sum_{i=p,n} \frac{\kappa}{(2\pi)^3} \int_{|\vec{p}\,| \le p_{F_i}} d^3p \, E(\vec{p}\,) - \langle \mathcal{L} \rangle$$

$$P = \frac{1}{3} \sum_{i=p,n} \frac{\kappa}{(2\pi)^3} \int_{|\vec{p}\,| \le p_{F_i}} d^3p \, \frac{\vec{\Pi}_i \cdot \vec{p}}{\Pi_i^0} + \langle \mathcal{L} \rangle$$

J.D. Walecka, Ann. Phys. (N.Y.) 83 (1974) 497

Relativistic Hadrodynamics: results for saturation

Parameters

	$\vec{\mathcal{D}}$	cut-off	Λ_s [GeV]	Λ_v [GeV]	g_σ	g_ω	g_ρ	b [fm^{-1}]	c	m_σ [GeV]	m_ω [GeV]	m_ρ [GeV]
NLD	$\dfrac{1}{1+\sum_{j=1}^{4}\left(\zeta_j^{\alpha}\, i\vec{\partial}_{\alpha}\right)^2}$	$\dfrac{\Lambda^2}{\Lambda^2+\vec{p}^{\,2}}$	0.95	1.125	10.08	10.13	3.50	15.341	−14.735	0.592	0.782	0.763

Some relativistic nuclear models:

Model	ρ_{sat} [fm^{-3}]	E_b [MeV/A]	K [MeV]	a_{sym} [MeV]	L [MeV]	K_{sym} [MeV]	K_{asy} [MeV]
NLD	0.156	−15.30	251	30	81	−28	−514
NL3*	0.150	−16.31	258	38.68	125.7	104.08	−650.12
DD	0.149	−16.02	240	31.60	56	−95.30	−431.30
D^{3}C	0.151	−15.98	232.5	31.90	59.30	−74.7	−430.50
DBHF	0.185	−15.60	290	33.35	71.10	−27.1	−453.70
	0.181	−16.15	230	34.20	71	87.36	−340
					88 ± 25	–	−550 ± 100

NS!

→ Gaitanos, Kaskulov

→ Lalazissis

→ Typel

→ Li, Machleidt, Brockmann

→ Fuchs

J.D. Walecka, Ann. Phys. (N.Y.) 83 (1974) 497

Relativistic Hadrodynamics: results for saturation

Parameters

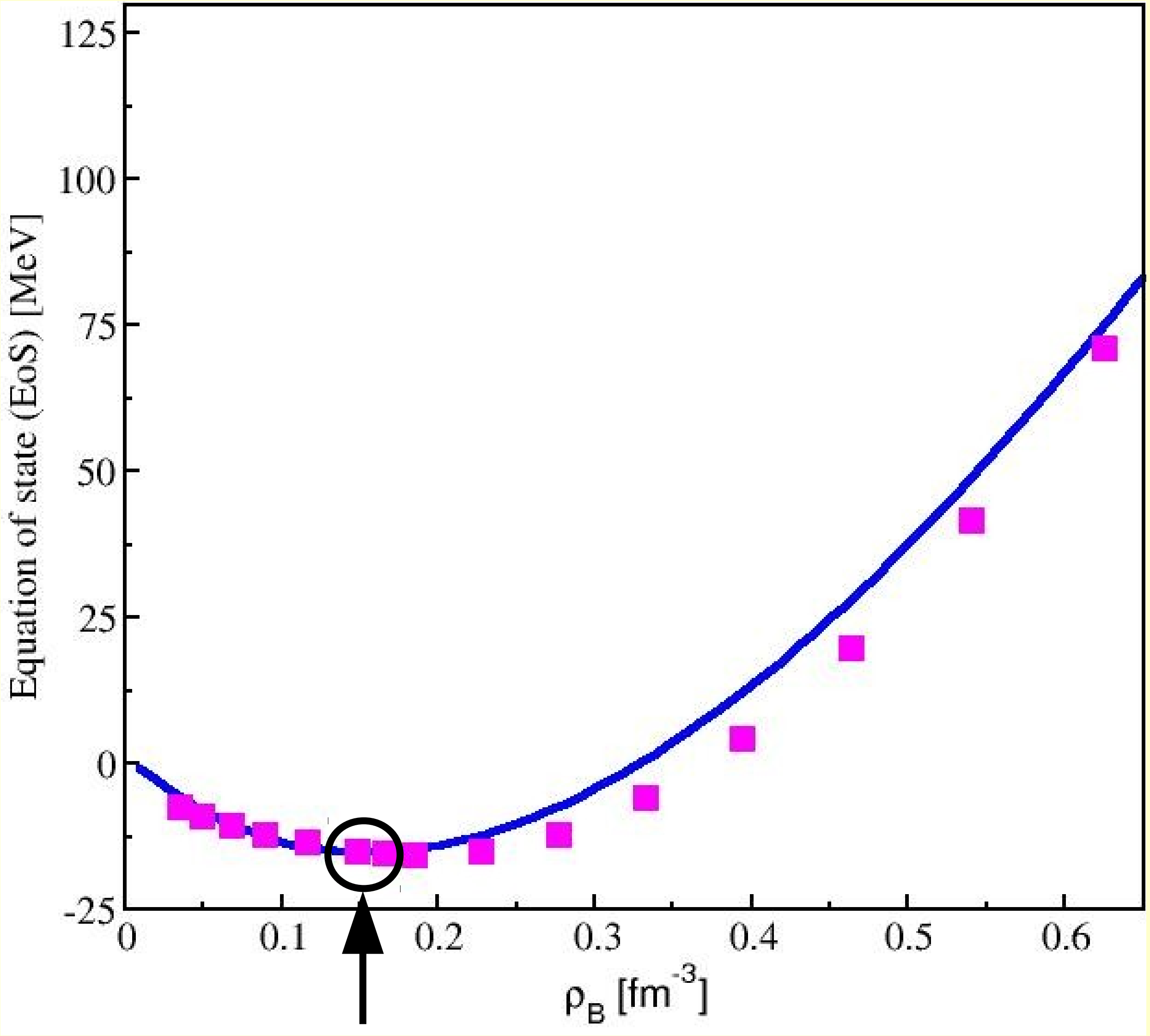
	$\vec{\mathcal{D}}$	cut-off	Λ_s [GeV]	Λ_v [GeV]	g_σ	g_ω	g_ρ	b [fm^{-1}]	c	m_σ [GeV]	m_ω [GeV]	m_ρ [GeV]
NLD	$\frac{1}{1+\sum_{j=1}^{4}\left(\zeta_j^\alpha i\vec{\partial}_\alpha\right)^2}$	$\frac{\Lambda^2}{\Lambda^2+\vec{p}^2}$	0.95	1.125	10.08	10.13	3.50	15.341	−14.735	0.592	0.782	0.763

Comparison with other models

soft EoS at ρ_{sat},
but stiff at high ρ relevant for NS!

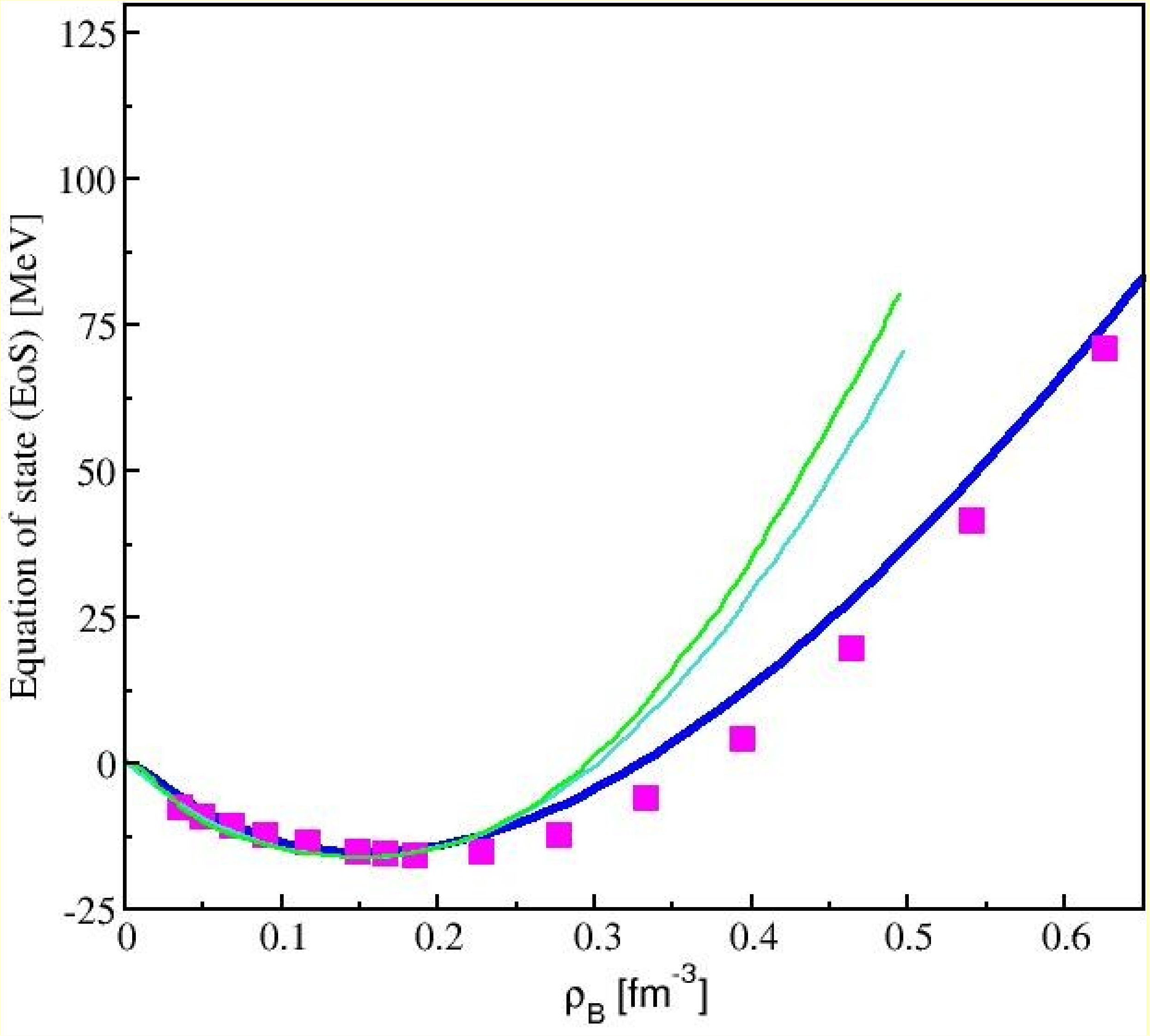
Model	ρ_{sat} [fm^{-3}]	E_b [MeV/A]	K [MeV]					
NLD	0.156	−15.30	251	30	81	−28	−514	→ Gaitanos, Kaskulov
NL3*	0.150	−16.31	258	38.68	125.7	104.08	−650.12	→ Lalazissis
DD	0.149	−16.02	240	31.60	56	−95.30	−431.30	→ Typel
D^{3}C	0.151	−15.98	232.5	31.90	59.30	−74.7	−430.50	
DBHF	0.185	−15.60	290	33.35	71.10	−27.1	−453.70	→ Li, Machleidt, Brockmann
	0.181	−16.15	230	34.20	71	87.36	−340	→ Fuchs
empirical	0.167 ± 0.019	-16 ± 1	230 ± 10	31.1 ± 1.9	88 ± 25	–	-550 ± 100	

Nuclear EoS = binding energy/nucleon

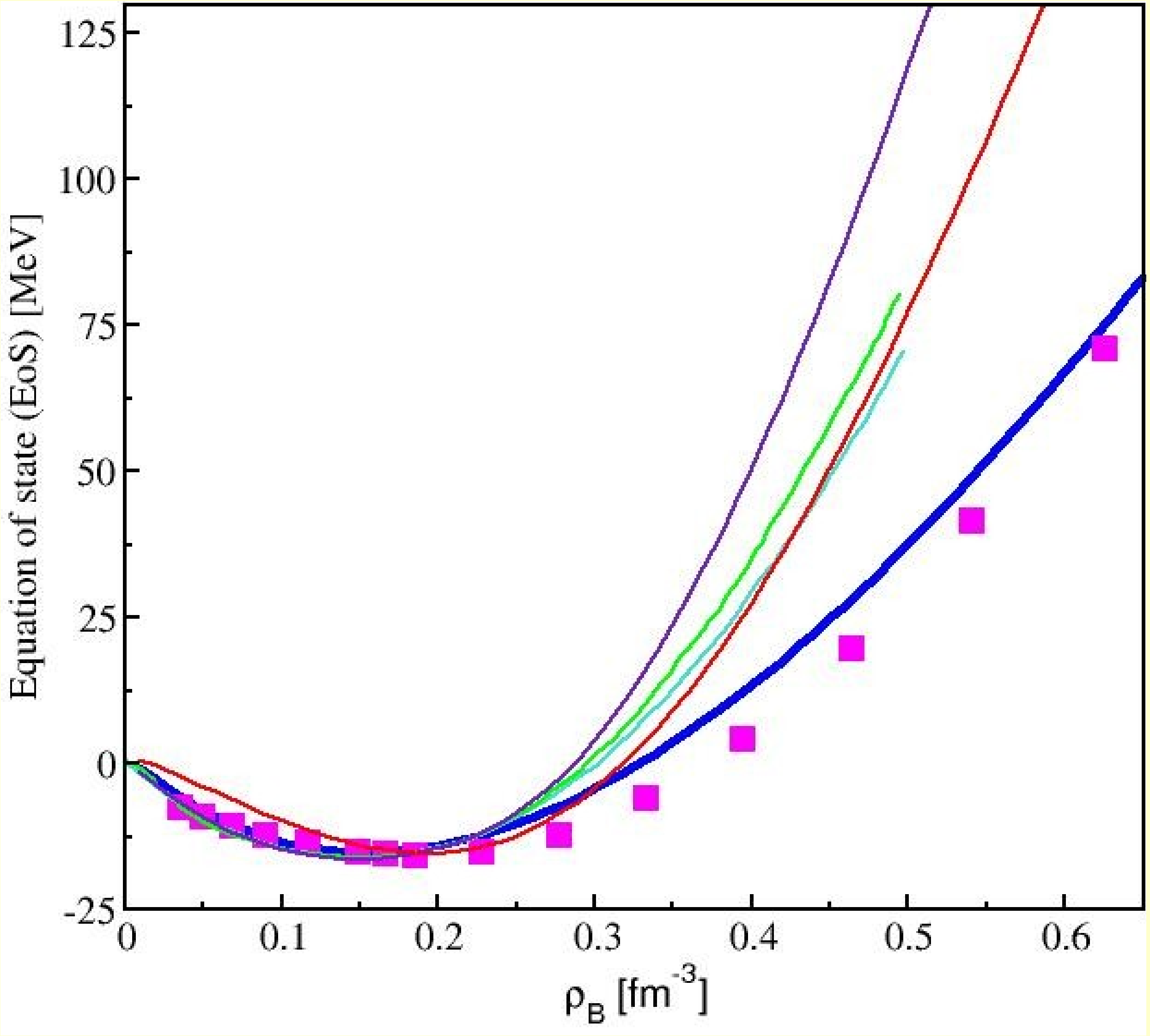


Minimum = stable nuclear matter (finite nuclei)

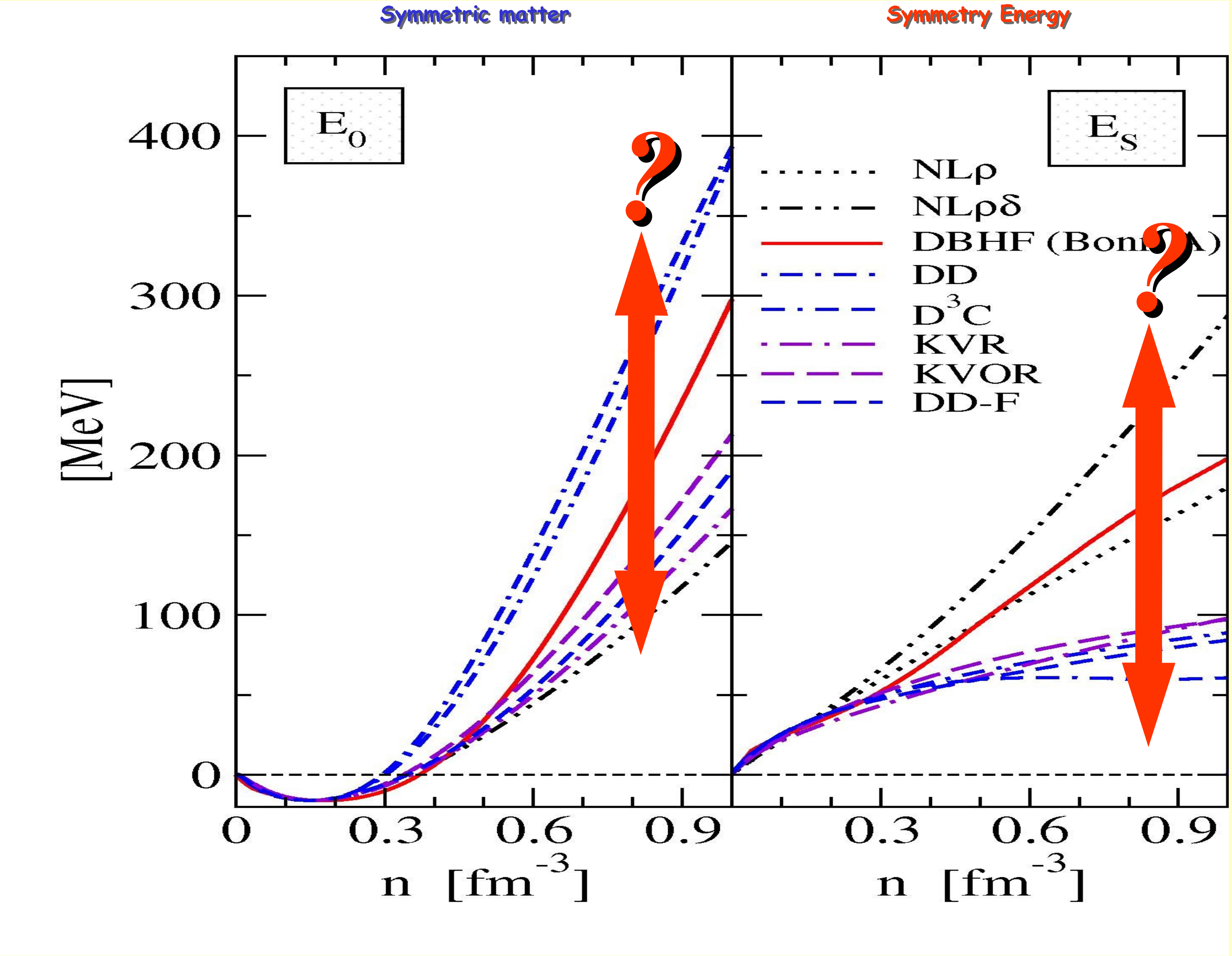
Nuclear EoS = binding energy/nucleon



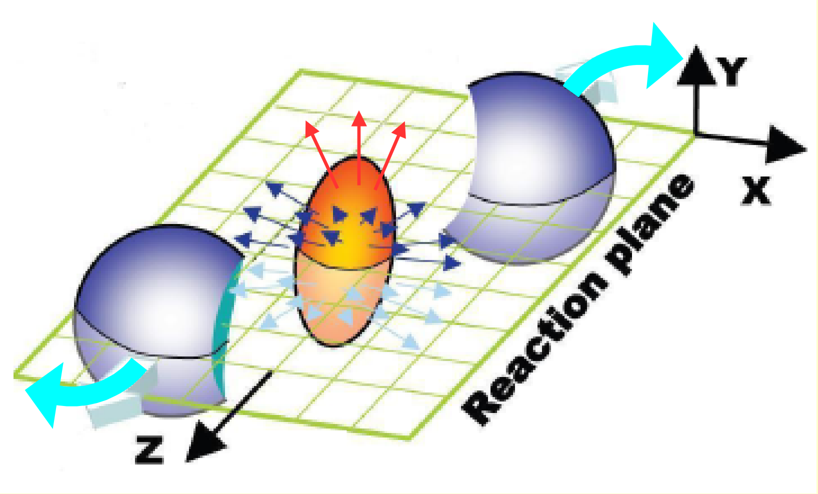
Nuclear EoS = binding energy/nucleon



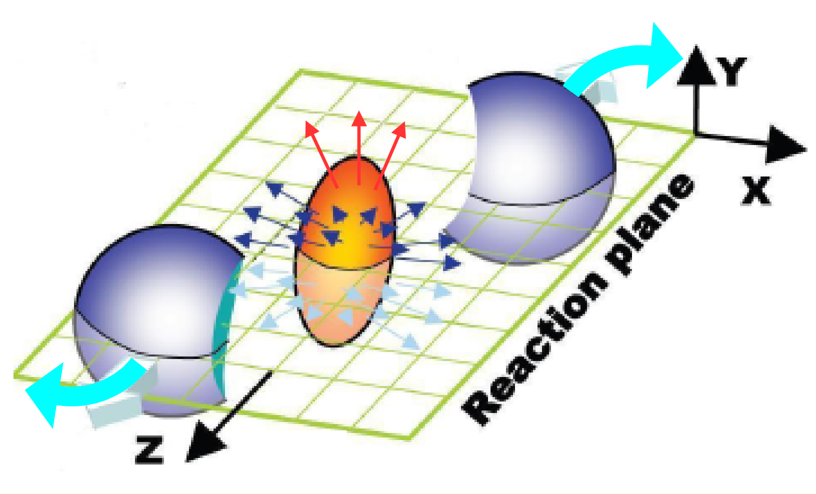
Nuclear EoS: more models...

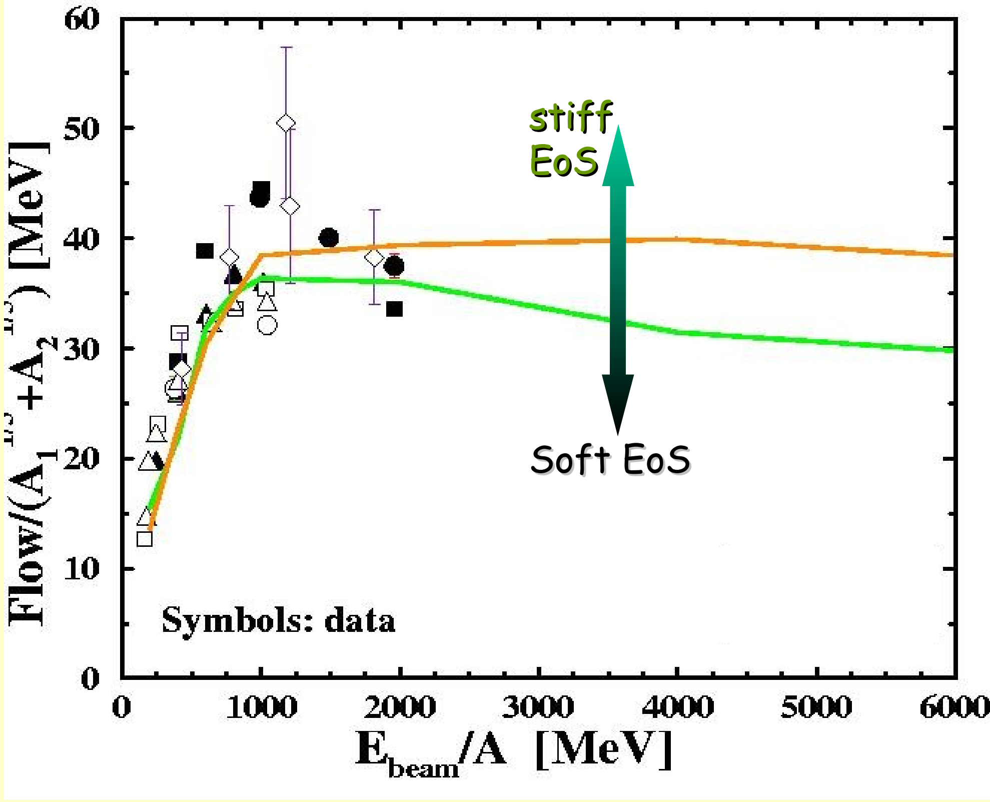


Nuclear EoS from HIC: Collective Flows

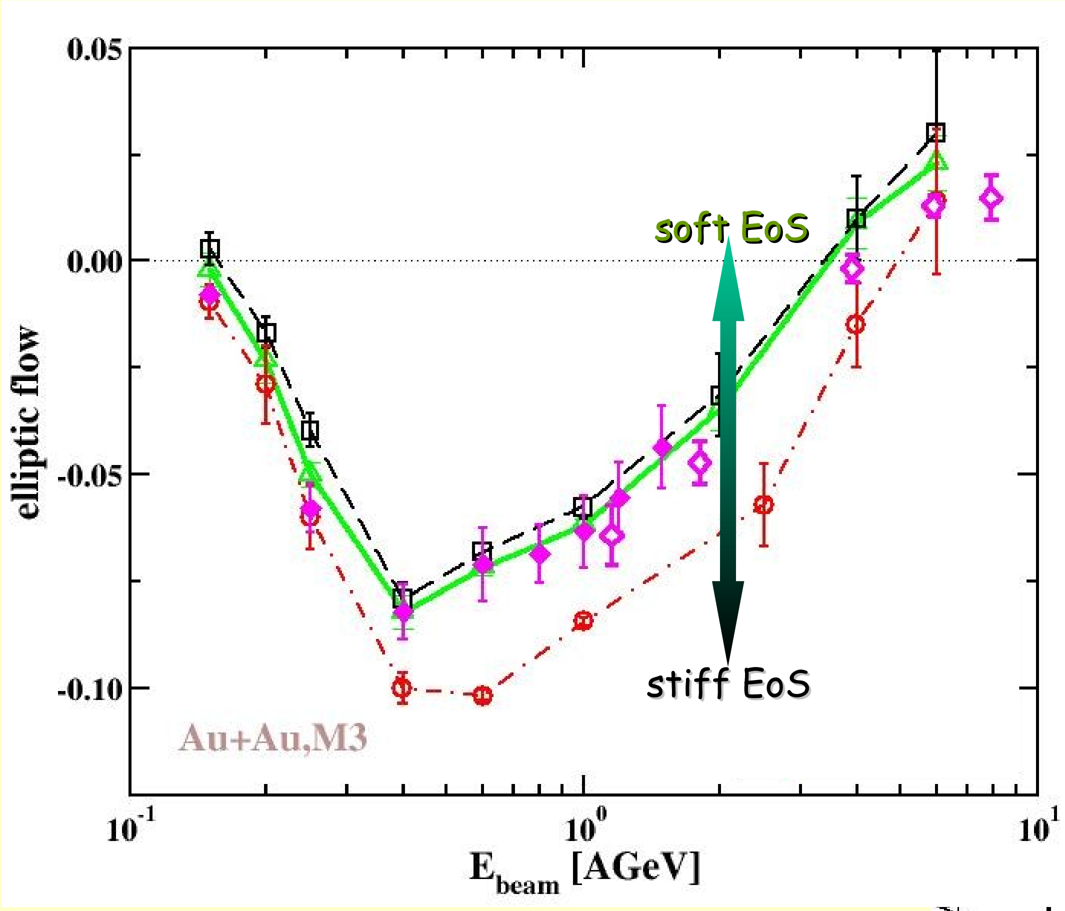


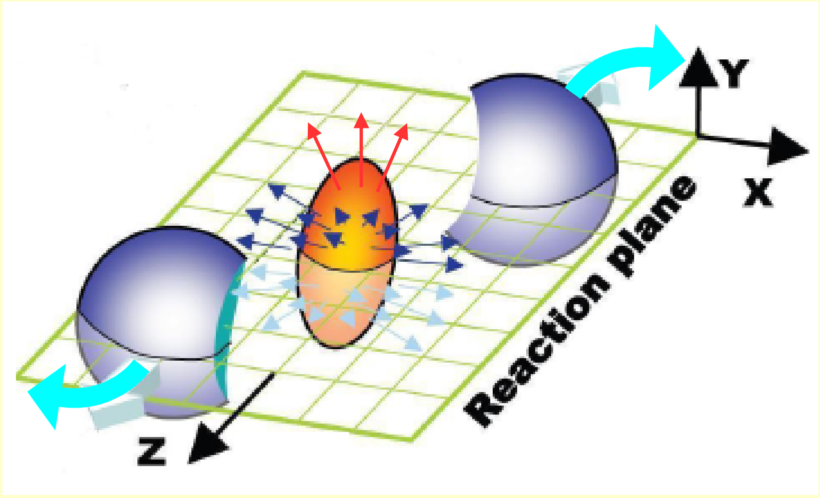
Nuclear EoS from HIC: Collective Flows

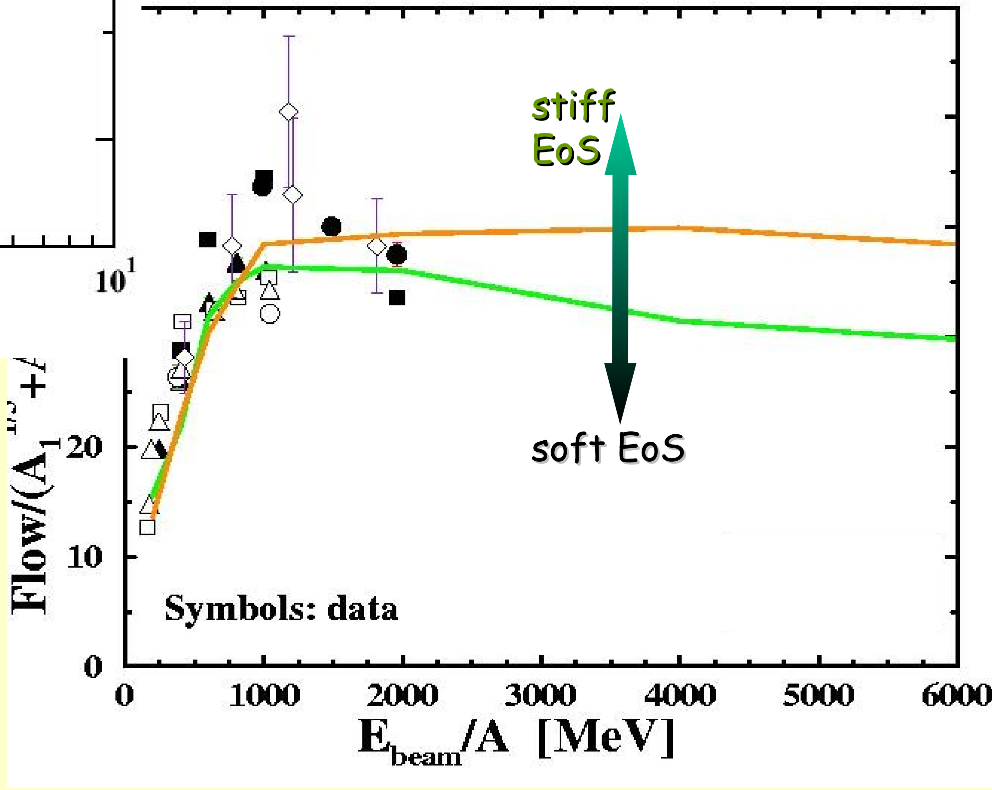




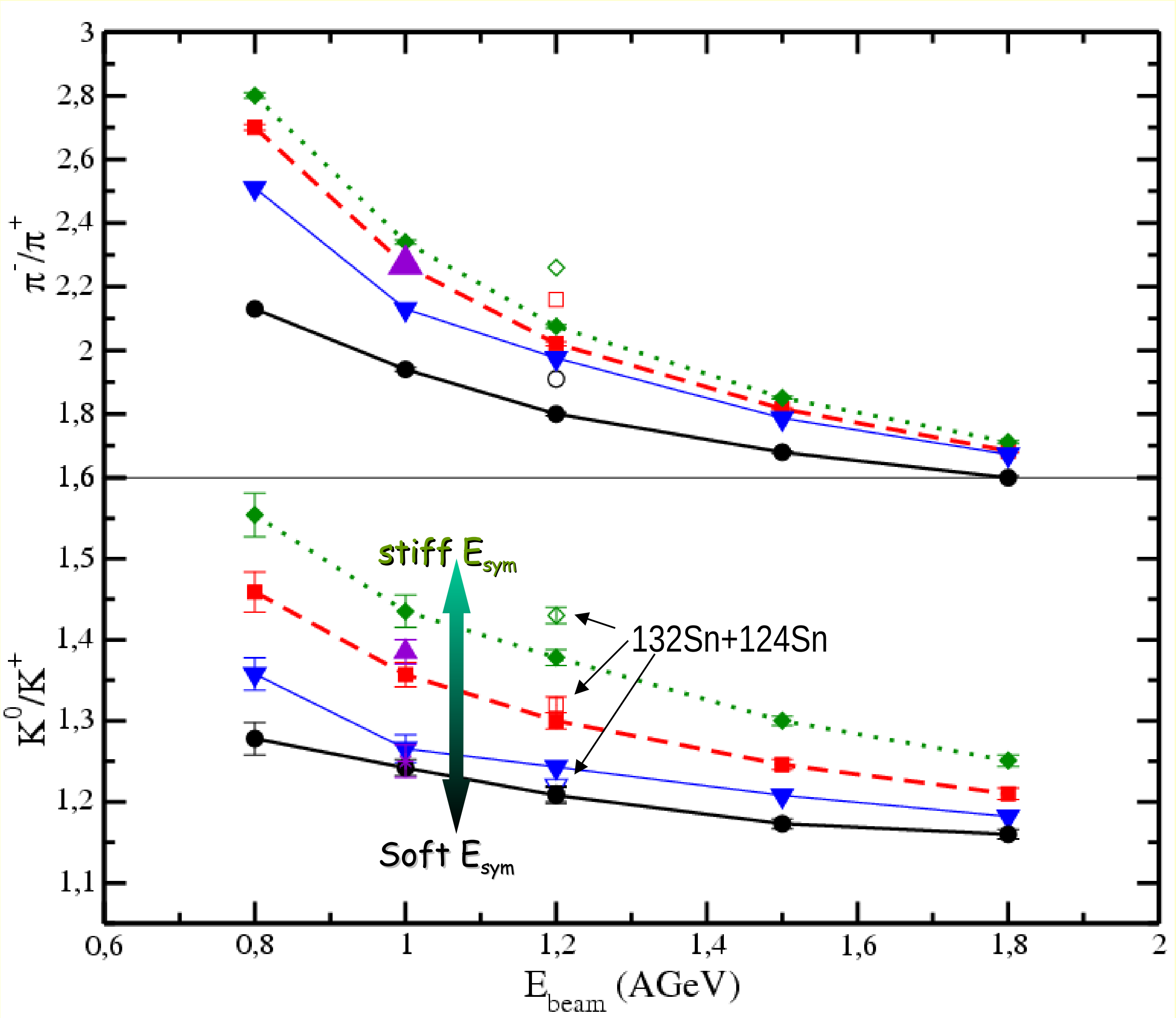
Nuclear EoS from HIC: Collective Flows







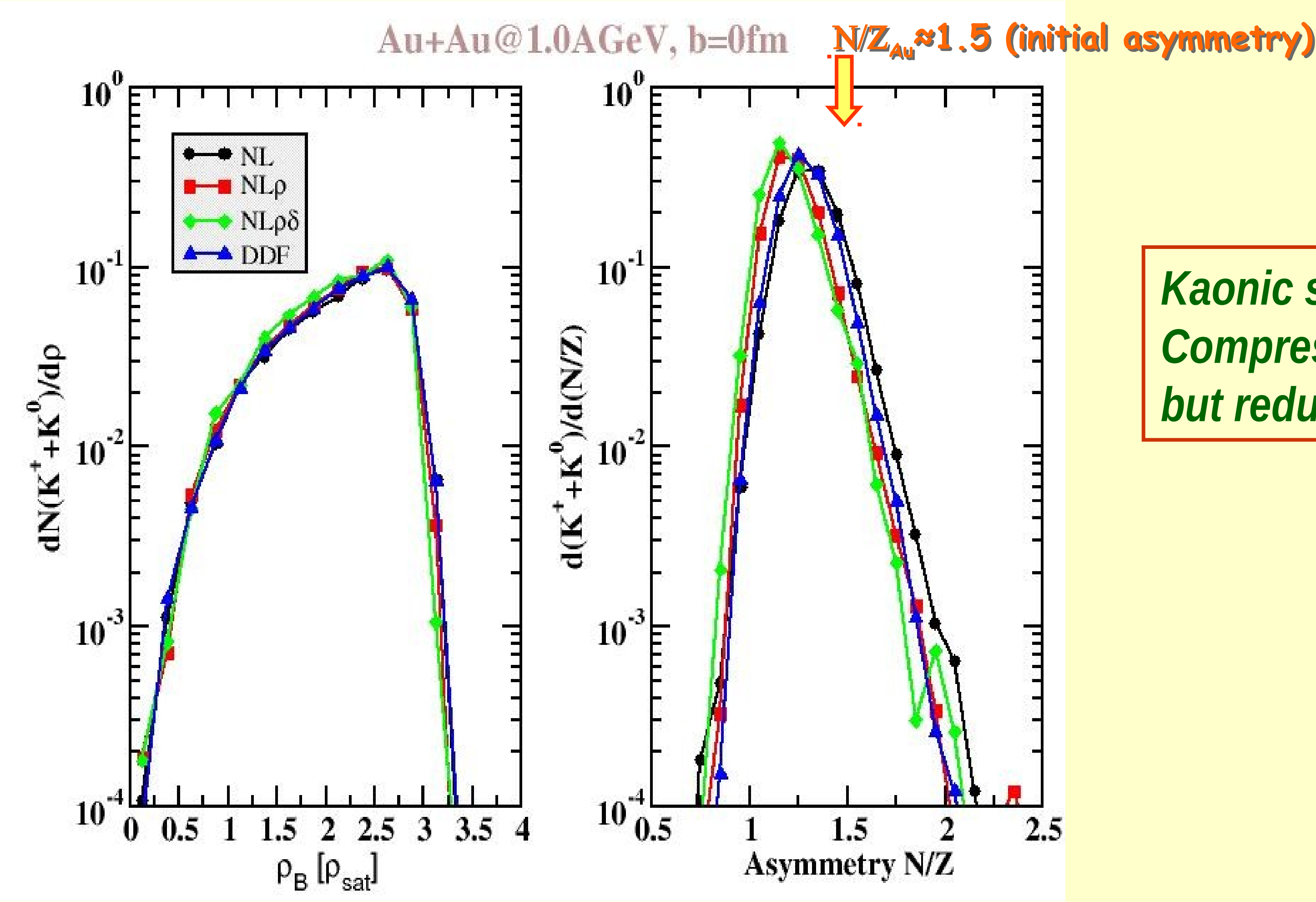
Nuclear E_{sym} from HIC: Particle Production



Kaons:
~15% difference between various E_{sym}'s
Data not yet conclusive

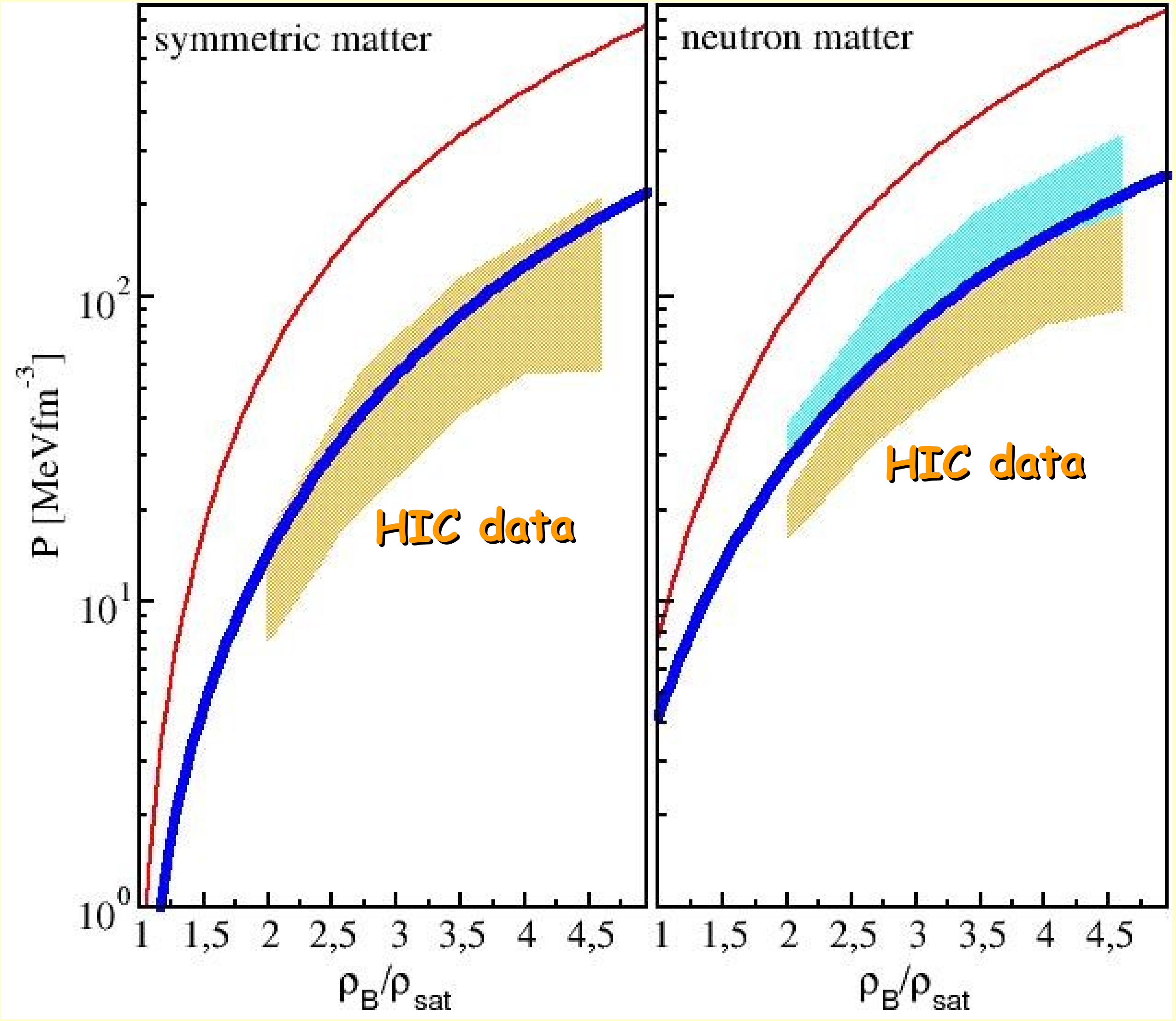
Nuclear EoS from HIC: Particle Production

Density & asymmetry of the K-source



Kaonic source:
Compressed fireball,
but reduced asymmetry

Nuclear EoS from HIC



Static Neutron Stars: theory

-Tolman-Oppenheimer-Volkov: attractive gravitation $\overset{\text{equilibrium}}{\leftrightarrow}$ repulsive interaction + relativity

$$\frac{dP}{dr} = -\frac{G\varepsilon(r)m(r)}{r^2}\left[1+\frac{P(r)}{\varepsilon(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{m(r)}\right]\left[1-\frac{2Gm(r)}{r}\right]^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon(r)$$

-Boundary conditions $P(r=0)=P_c, \quad P(r=R)=0$, R=radius (surface) of NS

-Total NS mass:

$$M(R) = \int_0^R dr 4\pi r^2 \varepsilon(r)$$

-Important input: Equation of State (EoS) E(ρ) of NS-matter at β-equilibrium (p,n,e,μ)

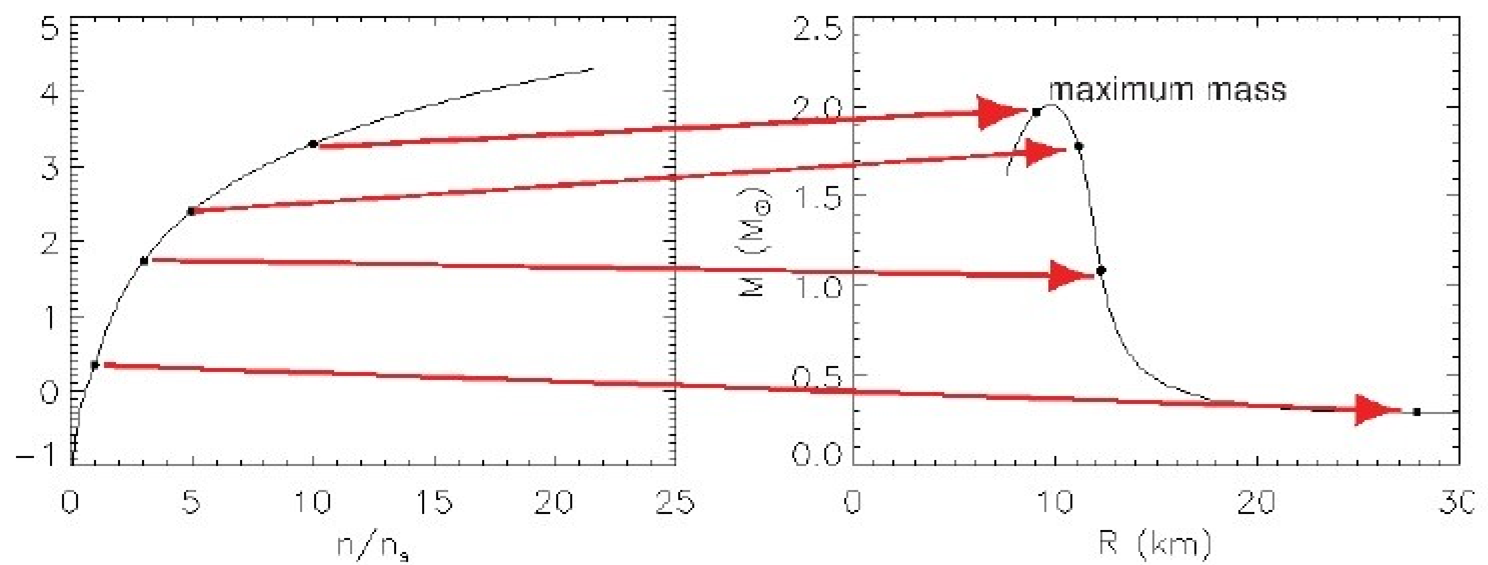
energy density $\varepsilon(\rho) = \rho E(\rho)$

pressure density $P(\rho) = \rho^2 \frac{\partial E}{\partial \rho}$

of nuclear matter at very high densities!

-Method: Solution of TOV for different central pressures P_c → Mass-radius relations

Static Neutron Stars: mass-radius relations

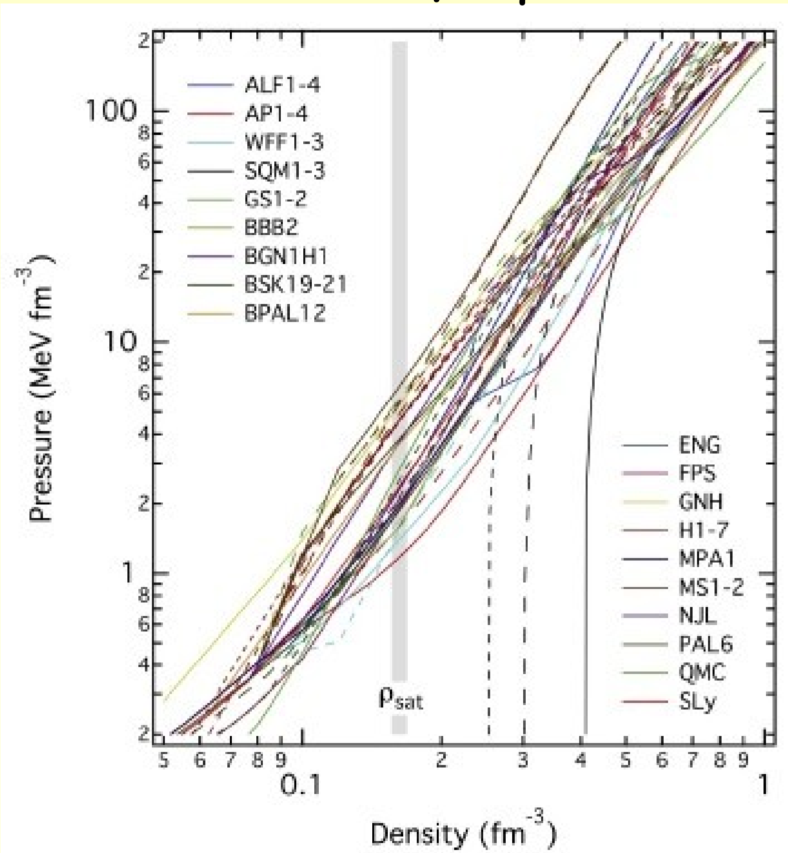


-At a given EoS: prediction of the maximum mass & radius of NS's

-Very high baryon densities! Needs information on baryonic EoS of compressed matter!

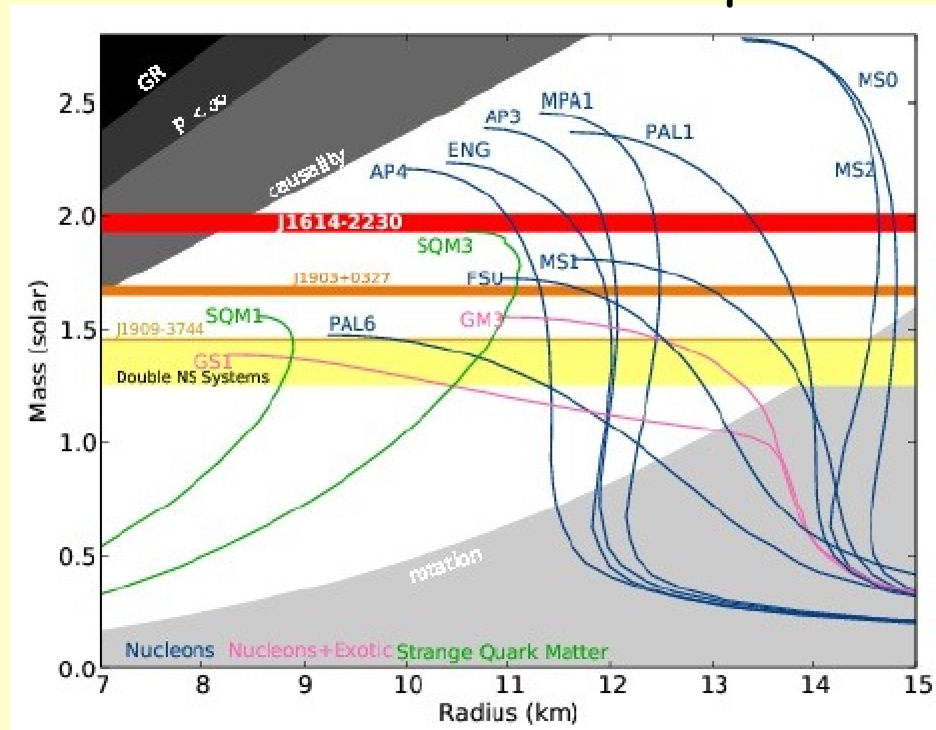
Static Neutron Stars: theory versus observations

Various EoS's at β-equilibrium



Özel, Freire, arXiv (2016)

Mass-Radius relations & exp. data



Demorest, *et al.*, Nature 467, 1081 (2010)

Maximum mass constrains model EoS's at high ρ, but still no exp. info on NS radius!

→ more constraints from Heavy-ion Collisions (HIC)

Relativistic EoS & Dynamics...

Literature on (relativistic) kinetic theory...

- 1. Primer L. Boltzmann, Wien. Ber. **66** (1872) 275
- 2. Primer...L. Nordheim, Proc. R. Soc. London **A119** (1928) 689

  E.A. Uehling, G.E. Uhlenbeck, Phys. Rev. **43** (1933) 552
- **Theoretical background**

  Non-Relativistic kinetic theory

  L.P. Kadanoff, G. Baym, „*Quantum Statistical Mechanics*" (Benjamin, N.Y. 1962)

  Relativistic kinetic theory

  S.R. de Groot, W.A. van Leeuwen, C.G. van Weert

  „*Relativistic kinetic theory*" (North Holland, Amsterdam, 1980)

  Modern Relativistic Quantum Transport Theory

  W. Botermans, R. Malfliet, Phys. Rep. **198** (1990) 115 (←difficult to understand...)
- **First applications to HIC...**

  P. Danielewicz, Ann. Phys. **152** (1984) 239 & 305

  G.F. Bertsch, S. Das Gupta, Phys. Rep. **160** (1988) 189

Relativistic applications to HIC...

B. Blättel, V. Koch, U. Mosel, Rep. Prog. Phys. **56** (1993) 1

Final remarks...

Neutron stars

→ most densest objects in universe consisting of baryonic matter!
→ precise astrophysical data for NS masses (not yet for NS-radii)
→ constraints on high-density baryonic EoS possible

Relativistic nuclear models

→ Relativity important, high densities/energies
→ Well fixed at saturation (nuclear structure) & at $\rho < (3\text{-}4)\rho_0$ (HIC)
→ Providing with EoS's for nuclear astrophysics (SN, neutron stars)

Heavy-Ion Collisions (fireballs)

→ violent dynamics (compression & expansion)
→ explore hot EoS of compressed symmetric matter
→ many data available (collective flows, particle production, meson-flows,...)
→ General conclusion: symmetric hot EoS at $\rho=(2\text{-}4)\rho_{sat}$ is soft
→ Asymmetric matter: data not conclusive yet,
still in progress & discussion