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Exercises:

1. Calculate the energy which is generated (per 1Kg) if Helium is burned to Carbon and Oxygen. Assume that Carbon and Oxygen are produced in equal amounts, i.e. they have the same final mass fraction.
2. How much energy is released by complete burning carbon and oxygen of a White Dwarf to nickel during a thermonuclear SN? Compare this to the binding energy of the White Dwarf. Assume that the White Dwarf has C and O mass fractions of 50% and the total mass is $M = 1M_\odot$.
3. During their lives stars lose material through stellar winds. Consider the evolution of a star with initial mass M_0. The star continuously burns lighter elements which release an amount of energy Q per gram of burnt material. The mass loss $\dot{M}$ due to the stellar wind is proportional to the luminosity L, which we assume to be constant: $\dot{M} = -\alpha L$ The burnt material is lost from the envelope and adds to the mass of the core.

   a) Find the mass of the core as a function of time, $M_C(t)$, assuming that $M_C(0) = 0$

b) Find the mass of the envelope as a function of time, $M_E(t)$,assuming that $M_E(0) = M_0$.
c) What is the final mass of the core, once the entire envelope has vanished?
d) Can you give an upper estimate of M_0, for which the star will become a White Dwarf with $Q = 5 \times 10^{14} \mathrm{J/kg}$ and $\alpha = 10^{-14}$ kg/J. Remember that the maximum mass of a stable White Dwarf is the Chandrasekhar mass of about $M = 1.4 M_\odot$

4. How much binding energy is in the core of a massive star with $M = 20 M_\odot$? Assume $M_{\rm core} = 1.6 M_\odot$ and $R_{\rm core} = 3000$ km . Can you estimate the binding energy the envelope of the same star has (i.e. outside of its core)?
   How much binding energy does a NS have? $M_{\rm NS} = 1.4 M_\odot$ and $R_{\rm NS} = 10$ km
5. How much mass loses the Sun every second if the density of the solar wind at Earth is 7 particles per cm^3 and the wind velocity is v=400 km / s. Assume spherically symmetric wind and wind containing only H.
6. Assume that the Sun mainly produces energy through the He fusion
   $4p \rightarrow ^4 He + 2e^+ + 2\nu + Q$
   where Q~25 MeV. Estimate the total number of neutrinos emitted by the Sun. Estimate the solar neutrino flux through a human being on Earth.
   The solar luminosity is: $L_\odot = 3.828 \times 10^{26}$W
   Assuming a mean free path of $\lambda \sim 10^{20}$ cm in water, estimate the amount of water necessary to achieve a reasonable reaction rate in the order of 1 reaction per day.
7. During the collapse of a massive star, the optical depth of neutrinos crossing the gain layer behind the shock is roughly described by the following formula. By assuming typical absorption rates of about $0.1 \ldots 0.5 M_\odot/\mathrm{s}$ and taking the reference values for T, R_S, R_g and M_{NS}, estimate the optical depth of the outstreaming neutrinos. How much of the total outstreaming neutrino luminosty can thus be absorbed in the gain layer?
   $$\tau \approx 0.026 \left(\frac{k_{\rm B} T_\nu}{4\,\mathrm{MeV}}\right)^2 \left(\frac{\dot{M}}{0.1\, M_\odot\,\mathrm{s}^{-1}}\right) \left(\frac{R_{\rm s}}{200\,\mathrm{km}}\right)^{3/2} \left(\frac{R_{\rm g}}{100\,\mathrm{km}}\right)^{-2} \left(\frac{M_{\rm NS}}{1.5\, M_\odot}\right)^{-1/2}.$$
8. Starting with the equation of motion $\ddot{r}(m,t) = -\dfrac{Gm}{r(m,t)^2}$ for freely falling material, and assuming that during collapse the density of the core will remain uniform (homologous contraction), show that the solution of the equation of motion leads to $|v(r)| \sim r$
9. Show that the free-fall collapse of a stellar core of initial uniform density is indeed homologous, i.e. the density structure will stay uniform. Hint: consider particular mass elements at given radii:
   $m = \dfrac{4\pi}{3} r_0^3 \rho_0$ and make use of the substitution $\cos x^2 = \dfrac{r}{r_0}$
10. Based on the quadrupole formula, the gravitational wave signal of a rotating ellipsoid can be estimated to $h_{\rm gw} = \dfrac{G\epsilon I}{c^4 D}\Omega^2$, where G is the gravitational constant, c the speed of light, D the distance to the source, ε the ellipticity, I the moment of inertia, Ω rotation frequency of the source. Can you estimate the gravitational wave signal amplitude of a rotating ellipsoid with the quadrupole formula with an ellipticity of $\epsilon = 0.001$? Which frequency would the gravitational wave have?