The gravitational-wave damping timescale of f-modes in neutron stars

Universal and approximate relations

Georgios Lioutas

GSI Helmholtzzentrum für Schwerionenforschung

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Motivation

- Perturbed neutron stars have modes which can be damped through gravitational waves (GWs).
- Main damping mechanism should be GWs.
- The f-mode (fundamental) should be the most efficient emitter.
- Relevant/promising system: Binary neutron star mergers ($t_{\text{lifetime}} > \tau_{GW}$).
- For rapidly/differentially rotating stars, only rough estimates exist.
- Based on Gen. Rel. Gravit. (2018) 50:12 (GL & Stergioulas)

General picture

- Background:
  - Metric: $ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$.
  - Perfect fluid: $T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu + pg_{\mu\nu}$
  - TOV system to determine mass $m(r)$, pressure $p(r)$, relativistic energy density $\epsilon(r)$, etc.

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  - TOV system to determine mass $m(r)$, pressure $p(r)$, relativistic energy density $\epsilon(r)$, etc.
- Perturbations:
  - Expand everything in spherical harmonics e.g.

$$\delta\epsilon = r^l \delta\epsilon^{lm} Y_{lm} e^{i\omega t}.$$

  - For $l > 1$ non radial oscillations emit gravitational waves.
  - The modes are quasi-normal $\omega = \sigma + \frac{i}{\tau_{\rm GW}}$, where $\sigma = 2\pi f$.
  - Note:

$$e^{i\omega t} = e^{i\sigma t} e^{-t/\tau_{\rm GW}}.$$

Perturbed Quantities - Fluid

The perturbed pressure, relativistic energy density and $4-$velocity read:

$$\delta p(t,r,\theta,\phi) = -r^l \left(e^{-\nu}\, X^{lm} + \frac{e^{-\lambda}}{r}\, W^{lm}\, \frac{\partial p}{\partial r} \right) Y_{lm} e^{i\omega t},$$

$$\delta \epsilon(t,r,\theta,\phi) = \frac{\delta p(t,r,\theta,\phi)}{v_s^2},$$

$$\delta u^\mu = i\omega e^{-\nu} (i\delta\nu/\omega, \xi^r, \xi^\theta, \xi^\phi),$$

where

$$\xi_r = e^{\lambda} r^{l-1}\, W^{lm}\, Y_{lm} e^{i\omega t},$$

$$\xi_\theta = -r^l\, V^{lm}\, \partial_\theta Y_{lm} e^{i\omega t},$$

$$\xi_\phi = -r^l\, V^{lm}\, \partial_\phi Y_{lm} e^{i\omega t},$$

the covariant component of the Lagrangian displacement vector.

Perturbed Quantities - Spacetime

At the same time we have the metric perturbation

$$g_{\mu\nu} = g_{\mu\nu}^{BG} + h_{\mu\nu}$$

where

$$h_{\mu\nu} = -\begin{pmatrix} e^{2\nu} r^l H_0^{lm} & i\omega r^{l+1} H_1^{lm} & 0 & 0 \\ i\omega r^{l+1} H_1^{lm} & e^{2\lambda} r^l H_0^{lm} & 0 & 0 \\ 0 & 0 & r^2 r^l K^{lm} & 0 \\ 0 & 0 & 0 & r^2 r^l\, K^{lm} \sin^2\theta \end{pmatrix} Y_{lm} e^{i\omega t}.$$

Perturbations - Interior

The 4th-order system was first presented by Lindblom & Detweiler(1983,1985)

$$H_1^{lm\prime} = -\frac{1}{r}\left[l+1+\frac{2me^{2\lambda}}{r}+4\pi r^2 e^{2\lambda}(p-\epsilon)\right]H_1^{lm} + \frac{e^{2\lambda}}{r}\left[H_0^{lm}+K^{lm}-16\pi(\epsilon+p)V^{lm}\right],$$

$$K^{lm\prime} = \frac{1}{r}H_0^{lm} + \frac{l(l+1)}{2r}H_1^{lm} - \left[\frac{l+1}{r}-\nu'\right]K^{lm} - 8\pi(\epsilon+p)\frac{e^{\lambda}}{r}W^{lm},$$

$$W^{lm\prime} = -\frac{l+1}{r}W^{lm} + re^{\lambda}\left[\frac{e^{-\nu}}{v_s^2(\epsilon+p)}X^{lm} - \frac{l(l+1)}{r^2}V^{lm} + \frac{1}{2}H_0^{lm} + K^{lm}\right],$$

$$X^{lm\prime} = -\frac{l}{r}X^{lm} + \frac{(\varepsilon+p)e^{\nu}}{2}\left\{\left(\frac{1}{r}-\nu'\right)H_0^{lm} + \left(r\,\omega^2\,e^{-2\nu}+\frac{l(l+1)}{2r}\right)H_1^{lm} + \left(3\nu'-\frac{1}{r}\right)K^{lm} - \frac{2l(l+1)}{r^2}\nu' V^{lm} - \frac{2}{r}\left[4\pi(\epsilon+p)e^{\lambda}+\omega^2 e^{\lambda-2\nu} - \frac{r^2}{2}\left(\frac{2e^{-\lambda}}{r^2}\nu'\right)'\right]W^{lm}\right\},$$

where we also defined

$$X^{lm} = \omega^2(\epsilon+p)e^{-\nu}V^{lm} - \frac{1}{r}\frac{dp}{dr}e^{\nu-\lambda}W^{lm} + \frac{e^{\nu}}{2}(\epsilon+p)H_0^{lm}.$$

Perturbations - Exterior

The exterior problem is simpler and described by the Zerilli equation (1970),

$$\frac{d^2 Z^{lm}}{dr_*^2} + [\omega^2 - V_Z(r)]Z^{lm} = 0,$$

where

$$Z^{lm} = \frac{r^{l+2}}{nr + 3M}(K^{lm} - e^{2\nu} H_1^{lm}),$$

$$V_Z = e^{-2\lambda} \frac{2n^2(n+1)r^3 + 6n^2Mr^2 + 18nM^2r + 18M^3}{r^3(nr+3M)^2},$$

$$r_* = r + 2Mln(r/2M - 1),$$
$$n = (l-1)(l+2)/2.$$

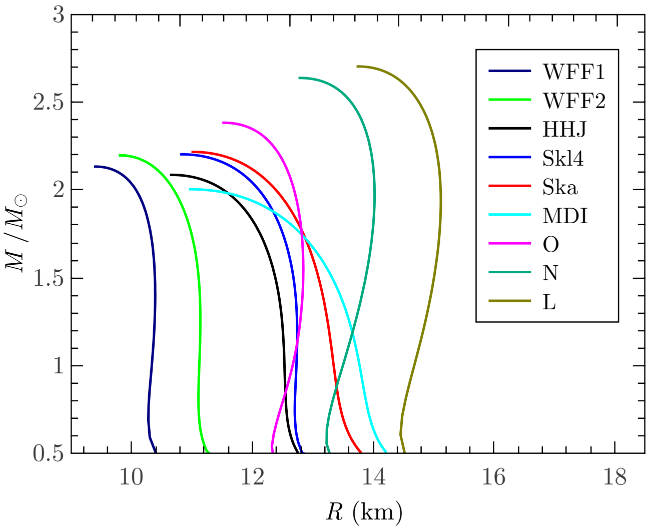
We adopted the approach by Andersson et al (1995) to solve this. They introduce a new variable

$$\Psi^{lm} = \left(1 - \frac{2M}{r}\right) Z^{lm},$$

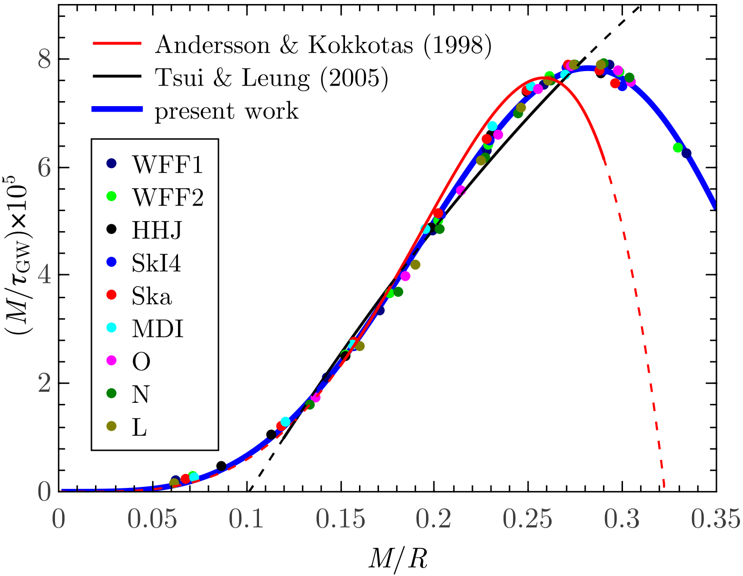
and reformulate the problem.

<u>Outcome</u>: Purely outgoing gravitational waves result in a discrete set of ω's.

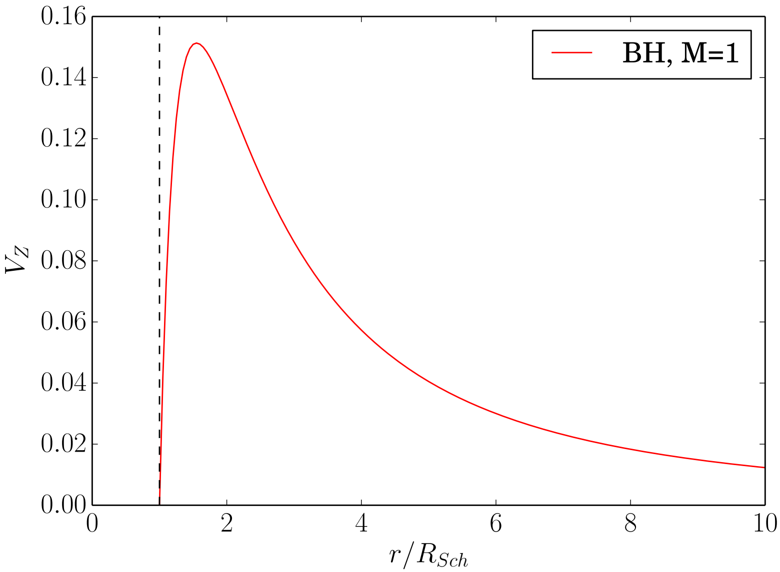
We took 9 different EoS into account and investigate the $(l, m) = (2, 0)$ case.



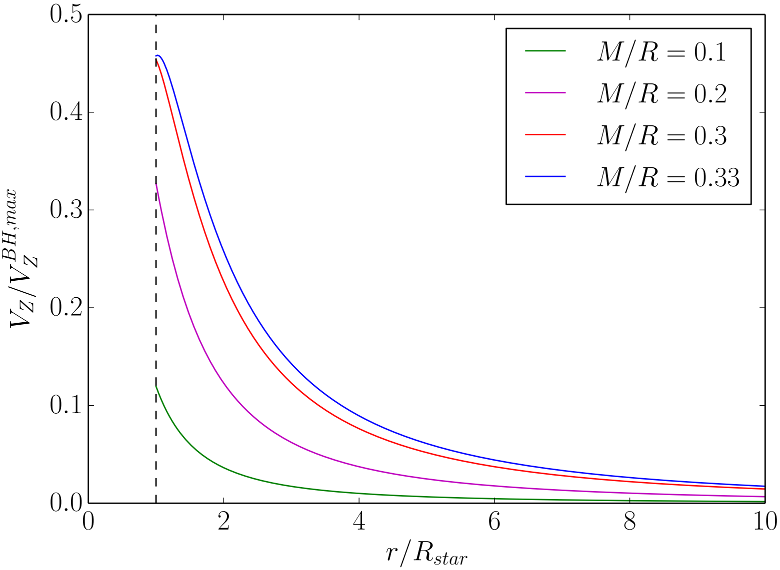
τ_{GW} of the $f-$mode vs compactness M/R



Zerilli potential for BH



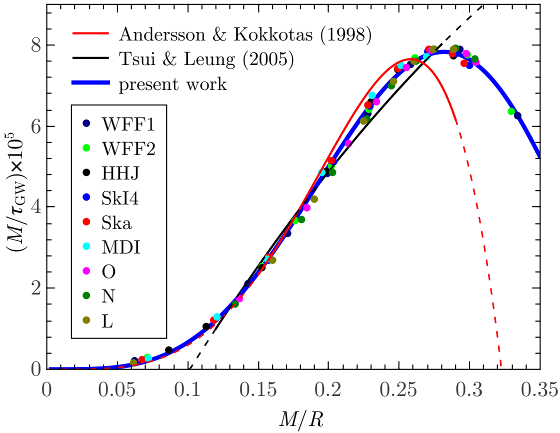
Zerilli potential for NS (WFF1 EoS)



The gravitational damping timescale satisfies the empirical relation

$$M/\tau_{\mathrm{GW}} = 0.112(M/R)^4 - 0.53(M/R)^5 + 0.628(M/R)^6,$$

which is accurate in the whole range $0 < M/R < 0.33$.



Approximate relations

The harmonic time dependence $e^{i\omega t}$ leads to the an $e^{2i\omega t}$ time dependence for the energy of the mode. Thus

$$E = E(0)e^{-2t/\tau_{GW}} \Rightarrow \frac{1}{\tau_{GW}} = -\frac{dE/dt}{2E}.$$

We will employ the quadrupole formula for gravitational wave luminosity

$$\left\langle \frac{dE}{dt} \right\rangle_{\mathrm{GW}} = -\frac{4\pi}{75}\sigma^6 \left(\int_0^R r^4 \delta\rho(r) dr \right)^2 .$$

Here R is the radius of the star and $\delta\rho$ the Eulerian perturbation of the rest-mass density.

Different choices can be made for $E, dE/dt$.

Choices for E:

- Relativistic: $(E^0_{\text{mode}})_R = \int_V \frac{1}{2}(\epsilon + p)\delta v^i \delta v_i^* (u^t)^{-1}\sqrt{^3g}d^3x$
- Newtonian:$(E^0_{\text{mode}})_N = \frac{1}{2}\int_V \rho\delta v^i \delta^* v_i dV$

Modifications of the standard quadrupole formula dE/dt through effective ρ:

	ρ_{eff}	$\delta\rho_{\text{eff}}$
SQF	ρ	$\delta\rho$
SQF1	$a^2\sqrt{\gamma}T^{tt}$	$2a\delta a\sqrt{\gamma}T^{tt} + a^2\frac{\delta\gamma}{2\sqrt{\gamma}}T^{tt} + a^2\sqrt{\gamma}\delta T^{tt}$
SQF2	$\sqrt{\gamma}W\rho$	$\frac{\delta\gamma}{2\sqrt{\gamma}}W\rho + \sqrt{\gamma}\delta W\rho + \sqrt{\gamma}W\delta\rho$
SQF3	$u^t\rho$	$u^t\delta\rho + \rho\delta u^t$
RQF	ϵ	$\delta\epsilon$

Approximate relations under investigation

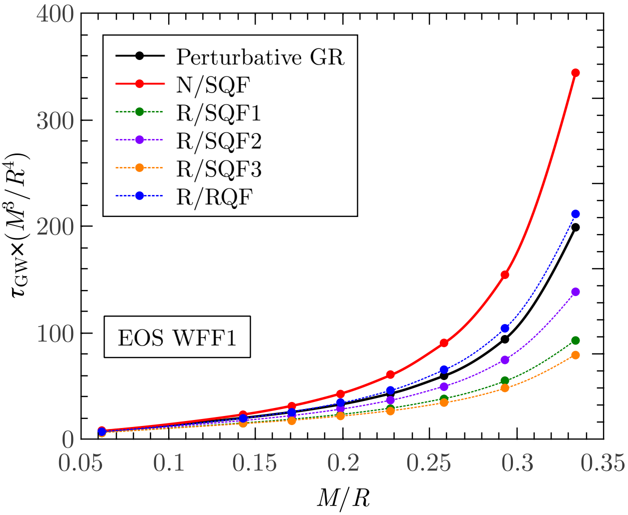
We pick the following combinations to investigate.

E^0_{mode}	$< dE/dt >_{\text{GW}}$	τ_{GW}
N	SQF	N/SQF
R	SQF1	R/SQF1
R	SQF2	R/SQF2
R	SQF3	R/SQF3
R	RQF	R/RQF

Here N →Newtonian, R →relativistic and the SQF's are as before.

Damping timescales results

Damping time computed through perturbative GR and the approximate relations.



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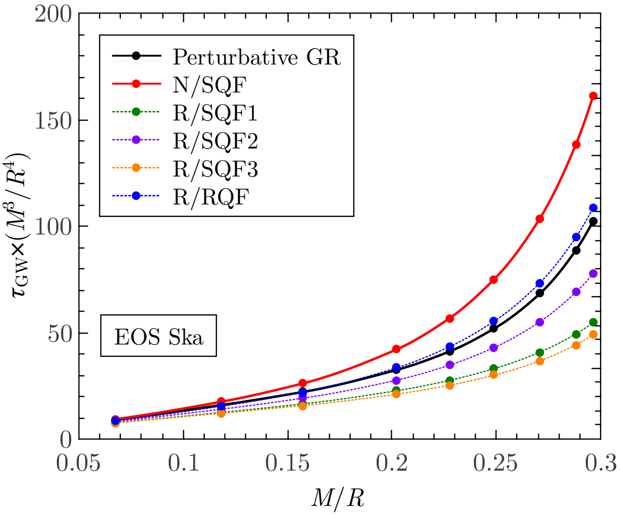

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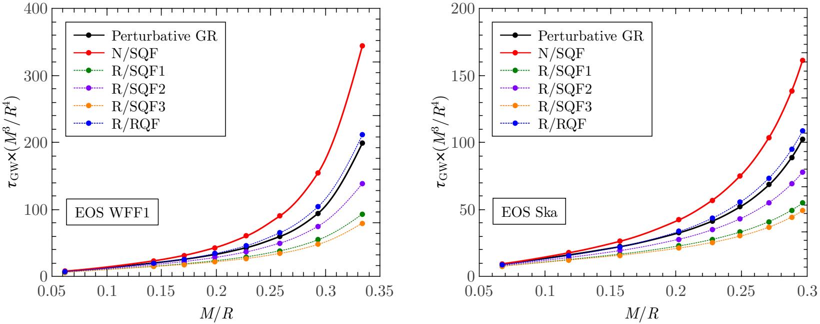
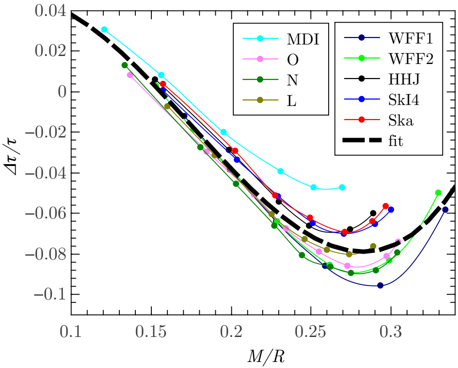


Define the relative difference as:

$$\frac{\Delta\tau}{\tau} = \left(\frac{\tau_{GW}^{\text{pert}} - \tau_{GW}^{\text{approx}}}{\tau_{GW}^{\text{approx}}}\right).$$

The approximate formula R/RQF yields gravitational-wave damping times that are within 10% of their exact value.

Relative difference vs Compactness



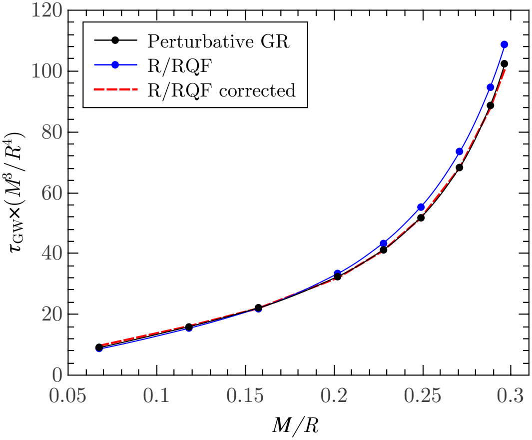
The relative difference can be described by an empirical relation in the case of R/RQF

$$\Delta\tau/\tau \simeq 1.47\left(\frac{M}{R}\right) - 13.3\left(\frac{M}{R}\right)^2 + 25.3\left(\frac{M}{R}\right)^3 .$$

Correction of R/RQF

Through the relation for $\Delta\tau/\tau$, we can "correct" the R/RQF result to be closer ($\lesssim 3\%$) to the perturbative one as

$$\tau_{\rm GW}^{\rm pert} \simeq \tau_{\rm GW}^{\rm R/RQF}\left(1 + \Delta\tau/\tau\right)$$



Conclusions

- We have constructed a universal empirical relation relating τ_{GW} scaled by M^3/R^4 to the compactness M/R, which is accurate in the range $0 < M/R < 0.33$.
- We constructed and investigated a number of choices for approximately computing the damping time τ_{GW} and identified the best candidate at the non-rotating regime.
- This candidate has been further improved to produce even more accurate results by the use of compactness M/R alone.

Future plans:

- Extend/verify the results in the rotating case as well.

The End

Thank you for your attention!