# Gravitational Waves (A General Overview)

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# Outline

# Electromagnetism

- The basic equation of E/M field (connecting the field with its sources)

$$F^{\mu\nu}_{\ ,\nu} = 4\pi J^{\mu}$$

where the whole information about the field is included in the components of $F^{\mu\nu}$ while $J^{\mu}$ describes the sources of the field.

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- Given the distibution of sources (currents, non-moving charges) the field $F^{\mu\nu}$ (or $A^{\mu}$) can be found.

# Electromagnetic waves

- Away from the sources the potential field $A^\mu$ satisfies the following equation

$$A^{\nu,\mu}_{\ \ ,\nu} - A^{\mu,\nu}_{\ \ ,\nu} = 0$$

which is not exactly a wave equation, except if the 1st term is 0. But $A^\mu$ is not unambiguously related with a real E/M field.

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offers an alternative description of the same E/M field.

- There is some freedom in defining $A^\mu$ (gauge). In the Lorentz gauge

$$A^\nu_{\ ,\nu} = 0$$

the propagation equation is a pure wave equation:

$$\Box A^\mu = 0$$

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  4. ... An isolated source is expected to produce a spherical E/M wave far from the source, and the field will scale like $A \approx 1/r$.
- Only accelerating charges generate waves. Linearly moving charges generate static fields (E and M).

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$$G^{\mu\nu} = \frac{4\pi G}{c^2} T^{\mu\nu}.$$
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$G^{\mu\nu}$, the Einstein tensor, comes from the Riemann tensor (describing all the details of spacetime curvature), while $T^{\mu\nu}$ includes all info about the distribution and motion of masses/energy. Spacetime is NOT fixed. Its own disturbances is the field itself.
- This is again a 2nd order p.d.e. with respect to the fundamental field, the metric $g_{\mu\nu}$:
$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R^{\kappa}_{\kappa}$$
and
$$R^{\mu\nu} = R^{\mu\alpha\nu\beta} g_{\alpha\beta}.$$

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- If we want to look for waves we should somehow "discriminate" them from the background. (An ocean wave with the dimensions of the Earth (tidal) is truely a wave?
- Waves in a flat background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

with $h_{\mu\nu} << 1$. $h$ could be "seen" as a field living on a flat spacetime as long as all equations are kept linearized with respect to $h$. [Not only $h$ but $h_{,a}$ should be small as well.]

# Wave equations for a suitable $h$

- Einstein equations for $g = \eta + h$:

$$
\begin{aligned}
R_{\alpha\beta\mu\nu} &= \frac{1}{2}(h_{\alpha\nu,\beta\mu} + h_{\beta\mu,\alpha\nu} - h_{\alpha\mu,\beta\nu} - h_{\beta\nu,\alpha\mu}) \Rightarrow \\
R_{\mu\nu} &= R^{\alpha}_{\mu\alpha\nu} = \frac{1}{2}(h_{\mu}^{\alpha}{}_{,\nu\alpha} + h_{\nu}^{\alpha}{}_{,\mu\alpha} - h_{\mu\nu,\alpha}{}^{\alpha} - h_{,\mu\nu}) \Rightarrow \\
G_{\mu\nu} &= \frac{1}{2}(h_{\mu\alpha,\nu}{}^{\alpha} + h_{\nu\alpha,\mu}{}^{\alpha} - h_{\mu\nu,\alpha}{}^{\alpha} - h_{,\mu\nu} \\
&- \eta_{\mu\nu}(h_{\alpha\beta}{}^{,\alpha\beta} - h_{,\beta}{}^{\beta}))
\end{aligned}
$$

But by using a new field alternative, $h$ with inverted trace

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h,$$

$$G_{\mu\nu} = \frac{1}{2}(-\bar{h}_{\mu\nu,\alpha}{}^{\alpha} - \eta_{\mu\nu}\bar{h}_{\alpha\beta,}{}^{\alpha\beta} + \bar{h}_{\mu\alpha,\ \nu}{}^{\alpha} + \bar{h}_{\nu\alpha,\ \mu}{}^{\alpha})$$

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$$\bar{h}_{\alpha\beta,}{}^{\alpha} = 0 \ldots$$

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- The new requirement reminds the Lorentz gauge ($A_{\mu,}{}^{\mu} = 0$) but with one more free index. Could this be somehow satisfied?
- YES. Just shift the coordinates a bit to redefine the almost flat metric.

$$\begin{aligned} x^{\mu\,(NEW)} &= x^{\mu\,(OLD)} + \xi^{\mu} \quad \text{with} \quad |\xi^{\mu}| << 1 \\ h^{(NEW)}_{\mu\nu} &= h^{(OLD)}_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu} \end{aligned}$$

# The Lorentz gauge

- Adjusting $\xi_\mu$ to nullify $\bar{h}_{\mu\nu,}{}^{\nu}$: One has to solve the system

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- In this type of coords the new metric field $\bar{h}$ satisfies the wave equation

  $$\Box \bar{h}_{\mu\nu} = 0$$

  far from any matter.

# The wavy solution

- The wave should have the following form

$$\bar{h}_{\mu\nu} = A_{\mu\nu} e^{ik_\alpha \boxed{x^\alpha}} \quad (1)$$

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On the other hand $\bar{h}_{\mu\nu}$ should be consistent with Lorentz gauge. Thus

$$A_{\mu\nu} k^\nu = 0.$$

$A_{\mu\nu}$ should be a symmetric 2nd order tensor orthogonal to a null vector.

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  Therefore only 2 components of $A_{\mu\nu}$ remain to be fixed. These are related to the polarizations of GW.

# A wise choice of freedom

- Choose the $\Xi$ solutions so that

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- In O's frame ($u'^{\nu} = (1,0,0,0)$)

$$A_{\mu 0} = 0$$

$$A_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{11} & A_{12} & A_{13} \\ 0 & A_{12} & A_{22} & A_{23} \\ 0 & A_{13} & A_{23} & A_{33} \end{pmatrix}$$

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- Require $A^{\mu}_{\mu} = 0$ (the 4th coordinate adjustment).

$$A_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{11} & A_{12} & 0 \\ 0 & A_{12} & -A_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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It is the metric in the TT (transverse-traceless) gauge; that is, in a frame of reference where no deviations from flatness arise in the direction of wave propagation, no deviations in time-components of the metric (nothing new about clocks besides SR) arise, and no area distortion show up in the transverse plane (traceless).

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- As the wave propagates, nearby points in the plane vertical to propagation approach or deviate from each other in an oscillatory manner. Actually they do NOT move. The spacetime background simply inflates and deflates, so the oscillating distance corresponds to proper distance $\int ds = \int \sqrt{g_{\mu\nu} dx^\mu dx^\nu}$.

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$$s = \int_0^{\epsilon} \sqrt{|g_{11}|}\, dx = \int_0^{\epsilon} \sqrt{1 + A_{11}\, e^{ik(z-ct)}}\, dx$$
for particles A, B at $(t, 0, 0, z)$ and at $(t, \epsilon, 0, z)$. Therefore this distance will oscillate like
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Therefore this distance will oscillate like
$$s \simeq \left[1 + \frac{A_{11}}{2}\right] e^{ik(z-ct)} \epsilon$$
- Similar for the cases:
  - B: $(t, 0, \epsilon, z)$ ($s$ as above with $A_{11} \to -A_{11}$),
  - B: $(t, \epsilon/\sqrt{2}, \epsilon/\sqrt{2}, z)$ ($s$ as above with $A_{11} \to A_{12}$)
  - B: $(t, \epsilon, \delta, z)$ ($s \simeq \sqrt{\epsilon^2 + \delta^2} \left[1 + \frac{A_{11}(\epsilon^2 - \delta^2) + 2A_{12}\epsilon\delta}{2(\epsilon^2 + \delta^2)}\right] e^{i\cdots}$)

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- A wave could have both polarizations (linear combination of them). Both carry information about the geometry of the source.

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- Is there a tool to measure proper distance? YES, light. The proper distance for a photon is 0 (null). Thus it could reveal metric disturbances by measuring travel time of light back and forth between two free particles.

$$0 = -c^2 dt^2 + g_{11} dx^2 \Rightarrow \epsilon = \int_0^{T_\rightarrow} \frac{c dt}{1 + \frac{A_{11}}{2} \cos[k(z - ct)]}$$

Note: If the distance is small compared to $\lambda_{GW}$ ($<< 10^6$m) $ct$ is almost constant along the flight.

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- We could measure $T_{\rightleftarrows}$ and monitor that time.

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- When the two beams interfere, if initially (before the wave) they were adjusted to interfere destructively, they will have partial positive interference when the 2 paths change (when GW present). The light output will be proportional to the small shift, thus to the amplitude of the wave and the length of the beams.

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The interferometers are broad-band detectors.

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  $$\bar{h}_{\mu\nu} = \frac{B_{\mu\nu}}{r} e^{i\Omega(t-r/c)}$$
  outside the source.
- A bit complicated calculation (even for low density-slowly moving objects) relates the amplitude $B_{\mu\nu}$ with the 2nd time derivative of the quadrupole moment of the source:
  $$B_{jk} e^{i\Omega t} = 2\frac{d^2}{dt^2} I_{jk}(t)$$
  where
  $$I_{jk} = \int T^{00} x^j x^k d^3 x$$
  is the quadrupole moment of the source.

## Energy radiated

- The amplitude of strong sources

# Estimation of strong energy emitters

- 

# Estimation of amplitude at Earth

-