GW from binaries
the inspiral and
the final stage

Theocharis Apostolatos

Summer school in Thessaloniki

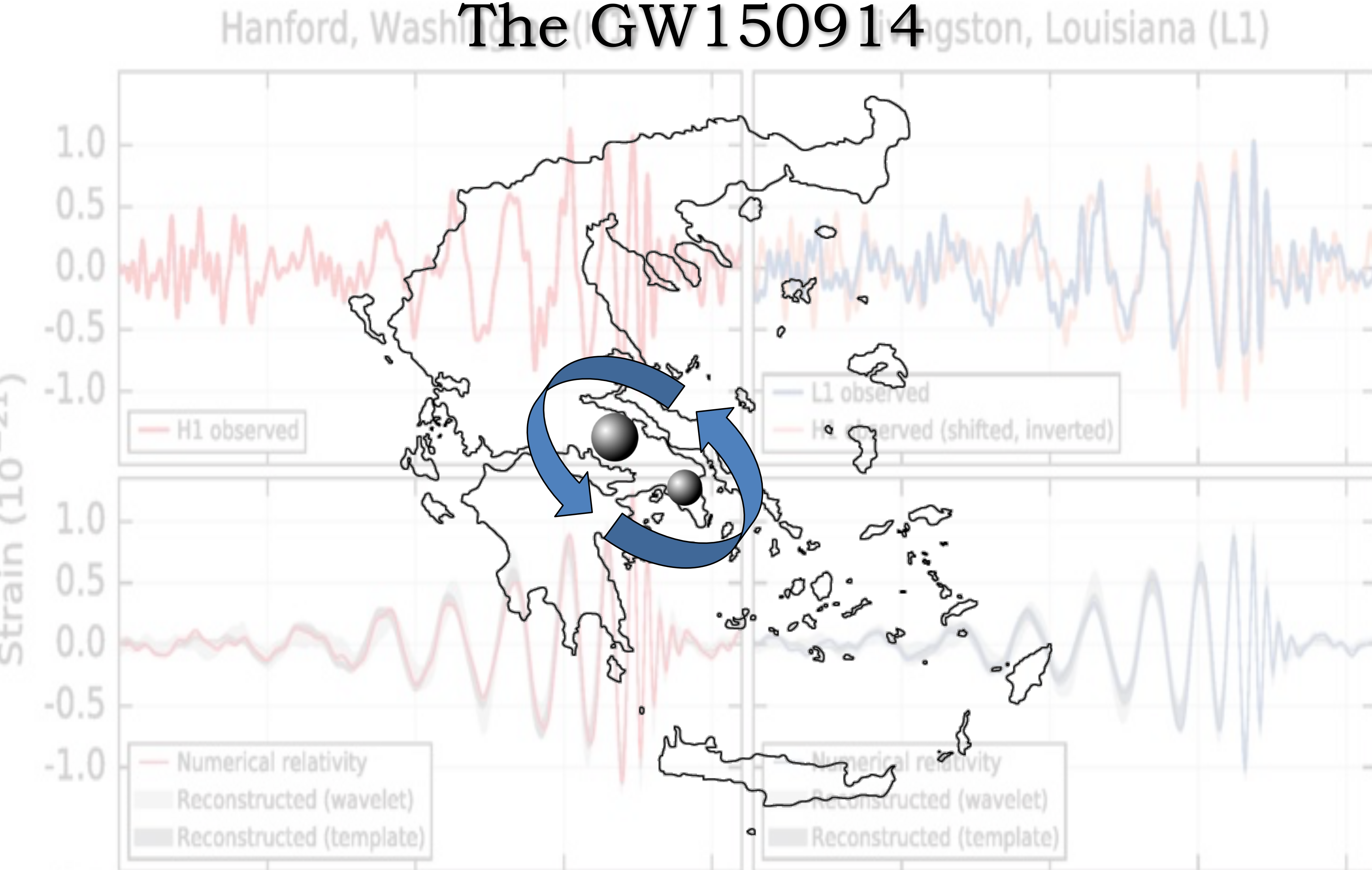
AUTH, 10/2018

Geometrized units

- One could use *G*, and *c* to transform units of mass to length and to time.
- Thus $G/c^2 = 7.4 \times 10^{-28}$ m/kg.
 and $G/c^3 = 2.5 \times 10^{-36}$ s/kg.

- Q1: What is the mass of sun in time and in length?
- Q2: Think of the physical interpretation of these quantities. How large is a black hole of 62 solar masses (*the remainder of the GW150914*)?

The GW150914



The quadrupole formula

- The quadrupole moment components of a binary consisting of masses m_1, m_2 revolving around each other in circular orbits, at a distance R from each other are

$$I_{xx} = -I_{yy} = \frac{1}{2}\mu R^2 \cos(2\omega t)$$

$$I_{xy} = \frac{1}{2}\mu R^2 \sin(2\omega t)$$

- The rate of loss of energy through GW (luminocity) is

$$\frac{dE_{GW}}{dt} = \frac{32G}{5c^5}\mu^2 R^4 \omega^6 = \frac{32G^4}{5c^5}\frac{M^3\mu^2}{R^5}$$

The time evolution of the orbit

- The distance between the 2 objects evolves according to

$$R^3 \frac{dR}{dt} = -\frac{64\, G^3}{5\, c^5} M^2 \mu$$

- Therefore it decreases at a rate

$$R = \left(\frac{256\, G^3}{5\, c^5} M^2 \mu (\mathrm{T} - t) \right)^{1/4}$$

while the rotation frequency evolves as

$$\omega = \sqrt{\frac{G\, M}{R^3}} = \cdots = \alpha (\mu^3 \mathrm{M}^2)^{-1/8} (\mathrm{T} - t)^{-3/8}$$

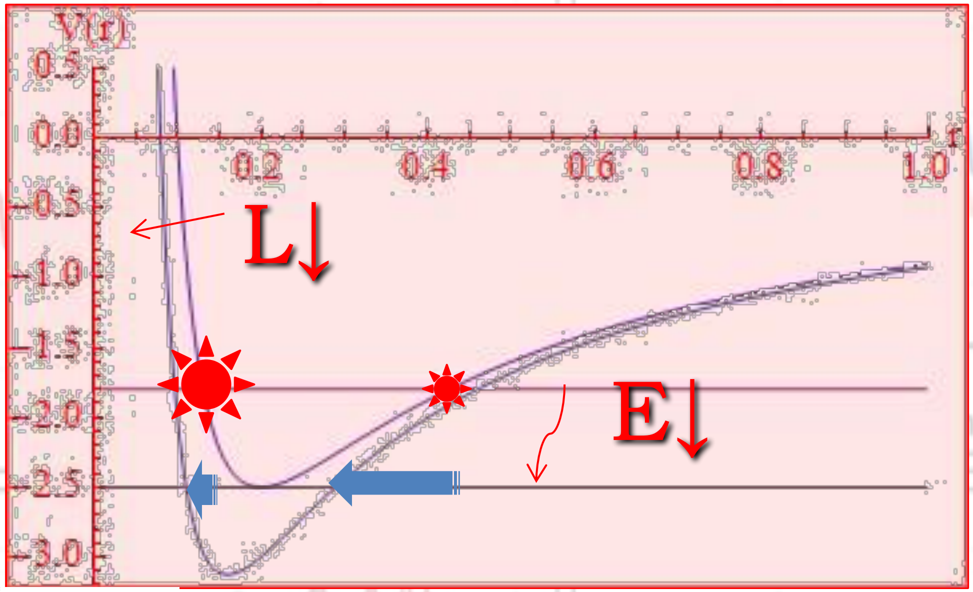
- This combination of mass $\mathcal{M} = (\mu^3 M^2)^{1/5} = (m_1^3 m_2^3 / M)^{1/5}$

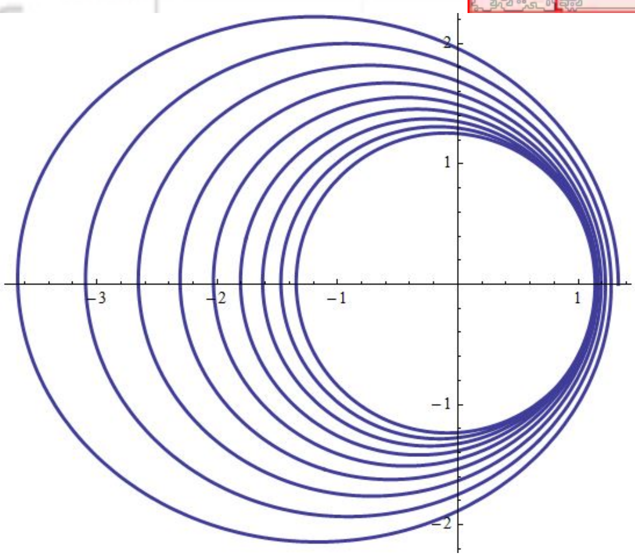
  is the so called chirp mass of the binary.

Mass of the source vs frequency band

- The masses of the source controls the whole frequency band which the signal will sweep in its final strongest stage.
- Thus if the masses go up by a factor of 10 the frequencies will go down by a factor of 10 (at the same stage of the signal, that is at a given number of cycles before the end).
- We need different detectors for different sources (sensitive at different frequency range).

The adiabatic inspiral in Newtonian terms.





- The binary loses energy and ang. momentum through gravitational radiation
- The orbit shrinks and become more circular
- The objects approach each other and revolve faster around each other

The waveform - amplitudes

- The GW strain from a binary will have the form

$$h(t) = \big(A_{+}(t) + A_{\times}(t)\big)e^{i2\varphi(t)}$$

where A_{+}, $A_{\times}$ are the amplitudes of the two wave polarizations, each one deforming spacetime in a different wave, while $\varphi(t)$ is the rotation angle of the binary itself:

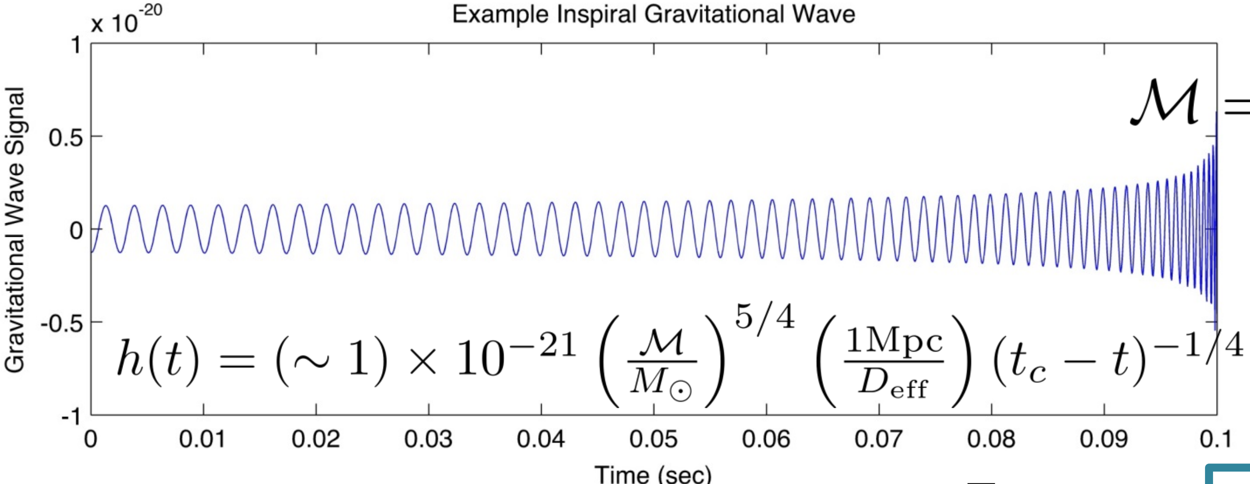
$$\phi(t) = \phi_0 + \int_0^t \omega(t)dt$$

The Amplitudes themselves vary with time since the emitter becomes stronger as time passes. They grow like

$$A(t) \sim \frac{\mathcal{M}^{5/4}(T-t)^{-1/4}}{D}$$

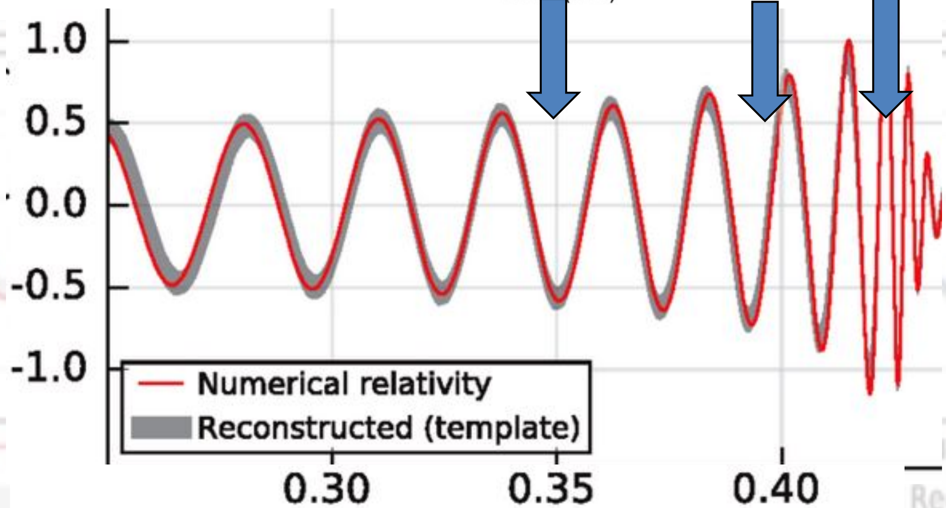
where D is the distance to the source. Again the chirp mass controls the time evolution.

The waveform - phase



$$\mathcal{M} = \left(\frac{M_1^3 M_2^3}{M_1 + M_2} \right)^{1/5}$$

$$h(t) = (\sim 1) \times 10^{-21} \left(\frac{\mathcal{M}}{M_\odot} \right)^{5/4} \left(\frac{1\text{Mpc}}{D_{\text{eff}}} \right) (t_c - t)^{-1/4} \cos(2\phi(t))$$



$t_1 = 0.35\text{s}$
$t_2 = 0.39\text{s}$
$t_c = 0.43\text{s}$

$$\mathcal{M}_{me} = 32 M_\odot$$
$$\mathcal{M}_{obs} = 28 M_\odot$$

$$\phi(t) = \phi_c - (\sim 760) \left(\frac{(t_c - t)[\text{s}]}{\mathcal{M}[\text{s}]} \right)^{5/8} [1 + \ldots (m_1, m_2, e, \mathbf{S}_{1,2})]$$

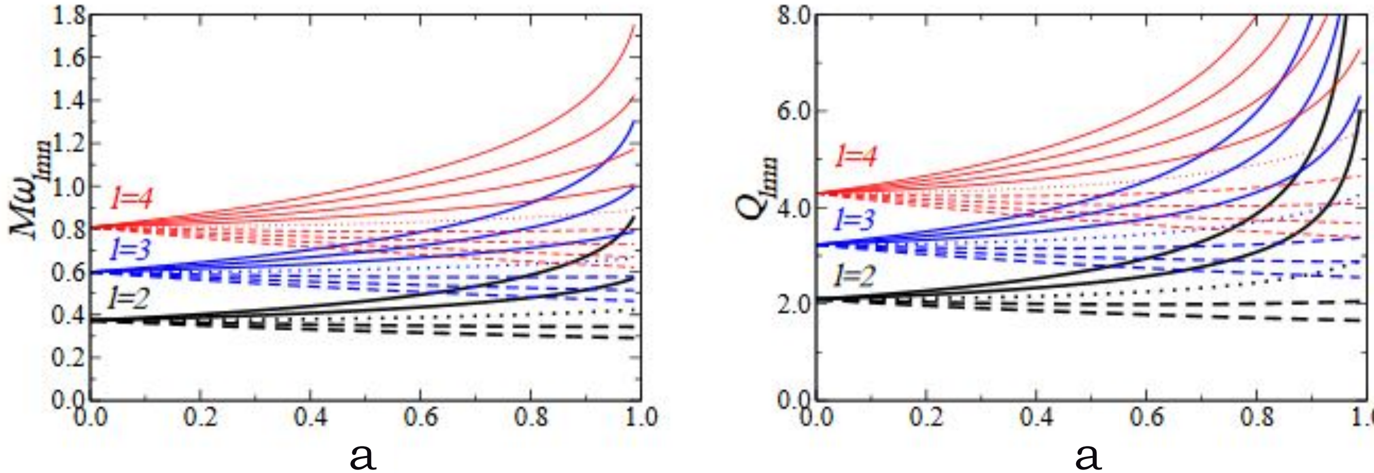
The final stage
a black hole rings down

- The final black hole has been created.
- Its horizon is still distorted. Not a Kerr black hole yet.
- The black hole (highly curved pure spacetime) oscillates.
- Part of GW propagate away, part of them fall inside the BH. Some GW interact with each other, and scatter.
- In the golden era of BHs (70's) perturbations of BHs were extensively studied. (Chandrashekhar, Price, Teukolsky, Zerilli, Hartle, Bekenstein).

The waveform of the final phase

- The gravitational signal has the form of exponentially decaying oscillations

$$h \propto \sum h_0 e^{-t/\tau_j} \sin(\Omega_j t)$$

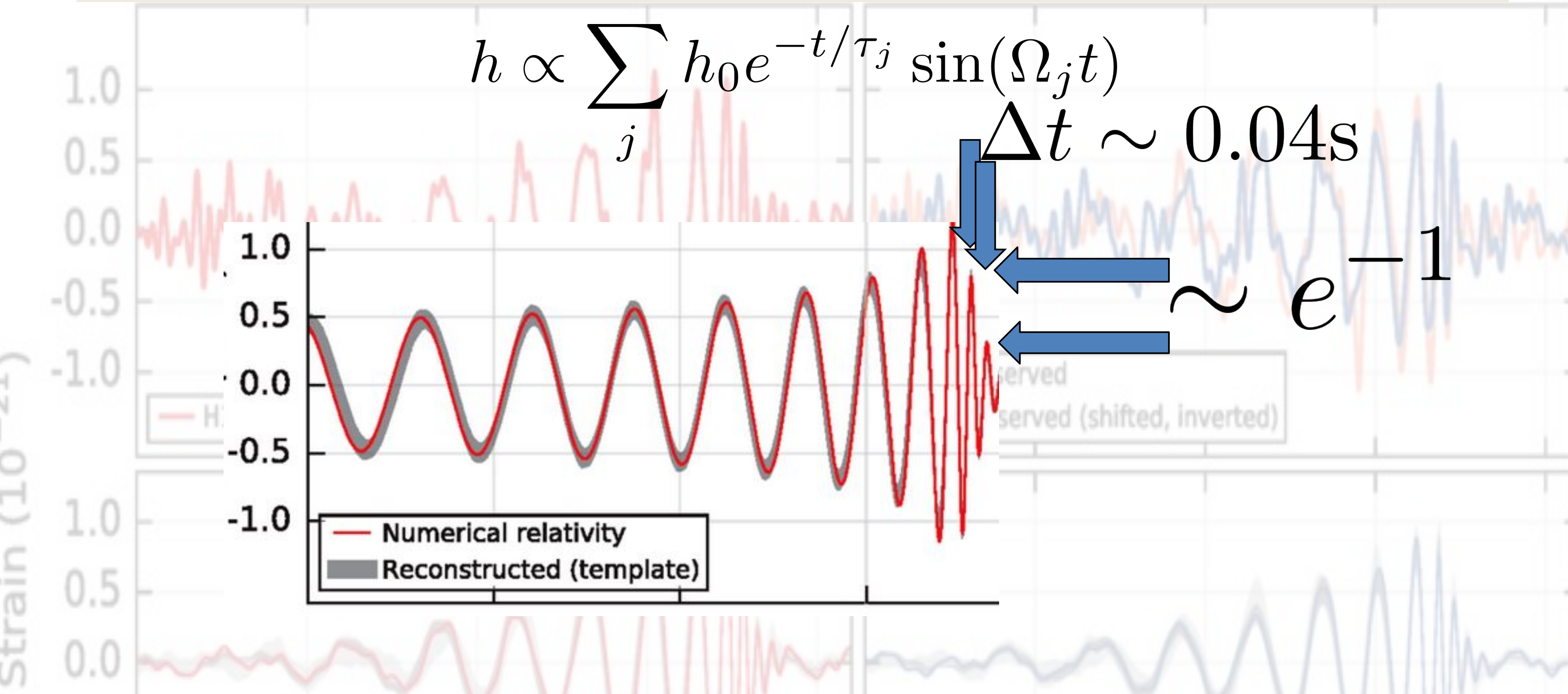


$$\omega_0 \simeq 0.02 \text{ MHz} \frac{M_\odot}{M} \left(1 - 0.63(1-a)^{0.3}\right)$$

$$\tau_0 \equiv \frac{2Q_0}{\omega_0} \simeq 200\ \mu\text{s} \frac{M}{M_\odot} \left(\frac{(1-a)^{-0.45}}{1-0.63(1-a)^{0.3}} \right)$$

The aveform of phase III (continued)

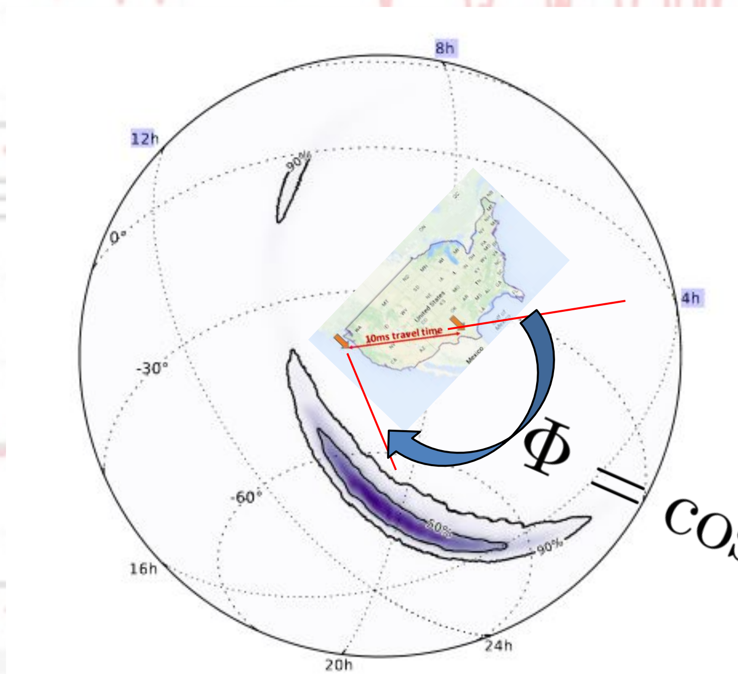
$$h \propto \sum_j h_0 e^{-t/\tau_j} \sin(\Omega_j t)$$



- A coarse estimation based on a single period gives T ≈1 τ_0: M=69 (M=62), a=0.6 (a=0.67)

Direction

- From the time delay between the 2 detectors 6.9 ms we could find the possible line of sight of the source.

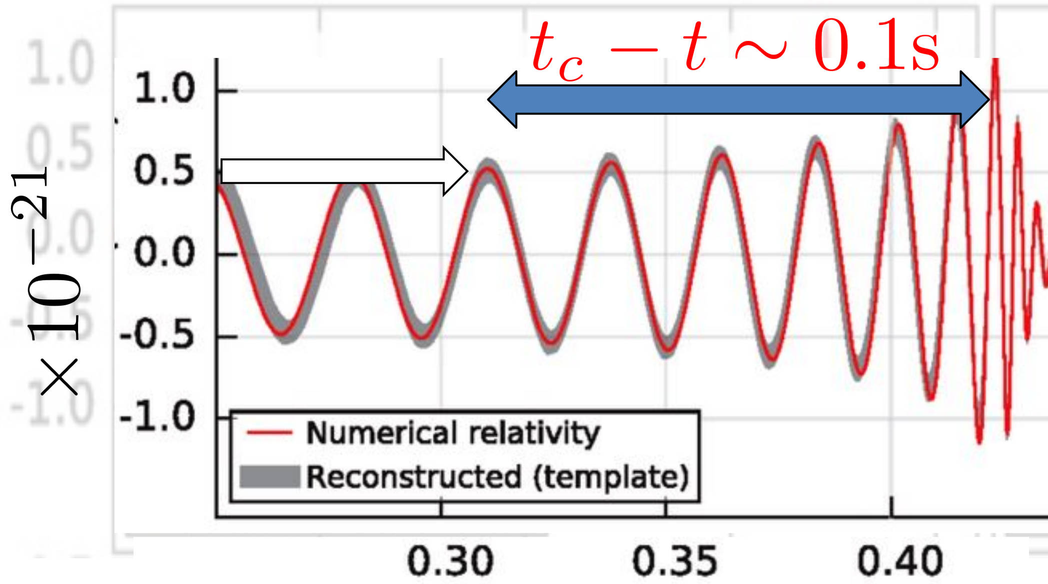


$$\Phi = \cos^{-1}\left(\frac{6.9\text{ms}}{10\text{ms}}\right)$$

- The fact that the probability curves have no annular shape is related to the non-isotropic distribution of the 2 polarizations and the corresponding direction of the arms of the 2 LIGOs.

Distance

- From the magnitude of the signal (h~10^{-21}) one can estimate the distance, which is comological (~Mpc)
- Knowing the masses and the time to coalesce we could measure the distance:



$$D = D_0(1+z(D_0))) \sim \frac{\mathcal{O}(1)\times 3\times 10^{-21}}{0.5\times 10^{-21}}\left(\frac{\mathcal{M}}{M_\odot}\right)^{5/4}\mathrm{Mpc}$$

$$\sim 390\mathrm{Mpc}$$

Compare to the measured 410±160 Mpc !

The Spin of the BHs

- To measure the spin of the BHs we need further accuracy of the phase of the waveform:

$$\phi(t) = \phi_c - (\sim 760)\left(\frac{(t_c - t)[\mathrm{s}]}{\mathcal{M}[\mathrm{s}]}\right)^{5/8}\left[1 + \ldots(m_1, m_2, e, \mathbf{S}_{1,2})\right]$$

- The time evolution of the waveform besides the dominant one $(\mathbf{t_c - t})^{\mathbf{5/8}}$ depends on the actual spin vectors.

- Besides the phase evolution, the LS and SS coupling leads to another signal modulation the one due to geometry modifications of the orbit (precession) and thus of the signal.

Bibliography I

1. (MTW) Gravitation (Misner, Thorne, Wheeler) *Freeman & Company, 1970,* [1280 pages, pedagogical, complete up to the 70s but too much time consuming].
2. The Classical Theory of Fields (Landau, Lifshitz) *Butterworth & Heinemann, 1951- reprint of the 4th edition 1998,* [deep physical approximation, the only book on GR without any mathematical introduction, insightful but extremely laconic and difficult.].
3. Gravitation & Cosmology (Weinberg) *John Wiley & Sons, 1972* [complete from a field-theoretical view of GR].
4. A First Course in General Relativity (Schutz) *Cambridge Univ. Press (2nd edition) 2009* [a very nice introductory book on GR; an abstract of MTW].
5. Spacetime and Geometry (Caroll) *Benjamin Cummings, 2003* [well written introductory book with physical approximation on the subject].

Bibliography II

7. Black Holes and Time Wraps: Einstein’s Outrageous Legacy (τoυ Thorne) *Norton & Company, 1995* [the prototype of a popular science book including history and physical interpretation of relativistic effects.].
8. Traveling at the speed of thought (D. Kennefick) *Princeton Univ. Press, 2007,* [excellent popular science book on the subject of GW with a fully explored history of the field].
9. The detection of gravitational waves (Blair) *Cambridge Univ. Press, 1991,* [a bunch of nice articles on GW, mostly on technical issues related to detection techniques].
10. Problem Book in Relativity and Gravitation (Lightman, Press, Price, Teukolsky) *Princeton Univ. Press, 1975,* [nice problems of relativity solved].
11. The article of R. C. Hilborna “*Gravitational waves from orbiting binaries without general relativity: a tutorial”* [public article were one could find all basic equations related to GW from binaries based on newtonian approximation].
12. The formal site of LIGO: https://www.ligo.caltech.edu/ [nice stories of people working in LIGO at the newsletter].

Problems on the theory of GW I

- (*Exerc. 8,9 of Schutz's chap. 9*) Does a free particle (i) feel an acceleration when hit by a GW? (ii) see any acceleration of nearby free particles (assume that light is used to detect motion and the distance is << the GW wavelength.
- Construct a circularly polarized GW.

Problems on compact object inspiral

- If an initially circular orbit evolves adiabatically (slowly) to smaller circular orbits find the rate of loss of angular momentum through GW from a binary, assuming the quadrupole formula (for the luminocity of GW) is known.
- Based on symmetry arguments, what kind of polarization do you expect from a binary seen edge on? [It is easier to think of an equal mass binary.] How are the amplitudes of the two polarizations compare for a binary seen from its axis of rotation?