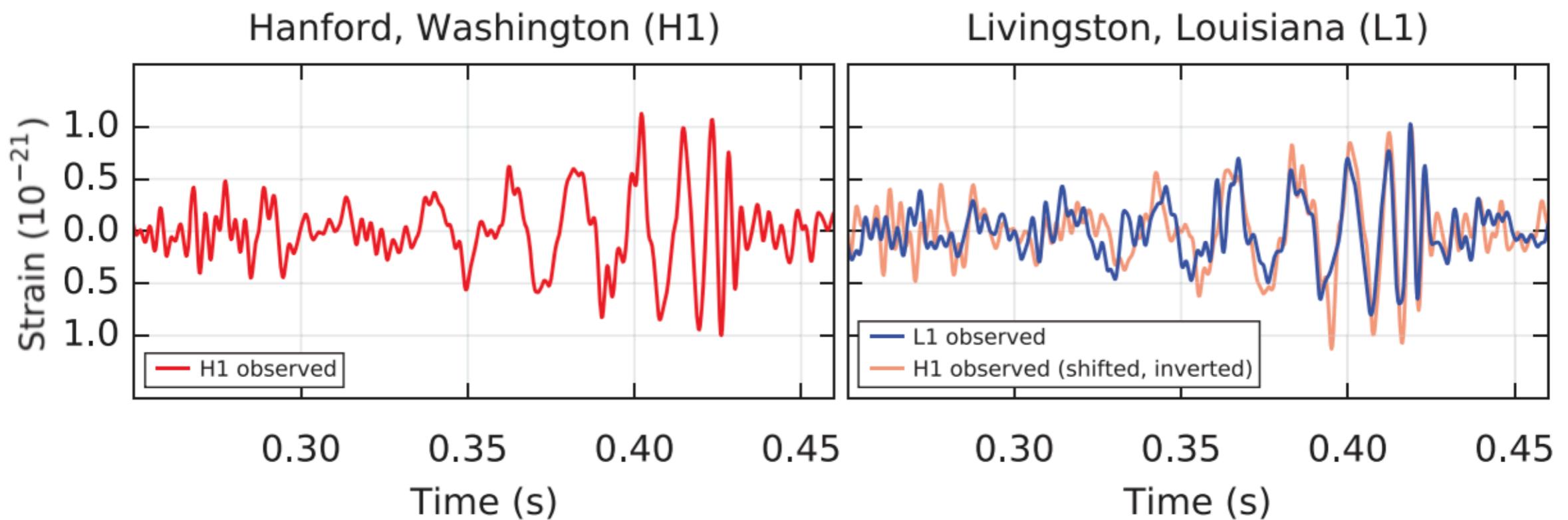


An introduction to CBC Parameter Estimation

Katerina Chatziliaoannou
Flatiron Institute

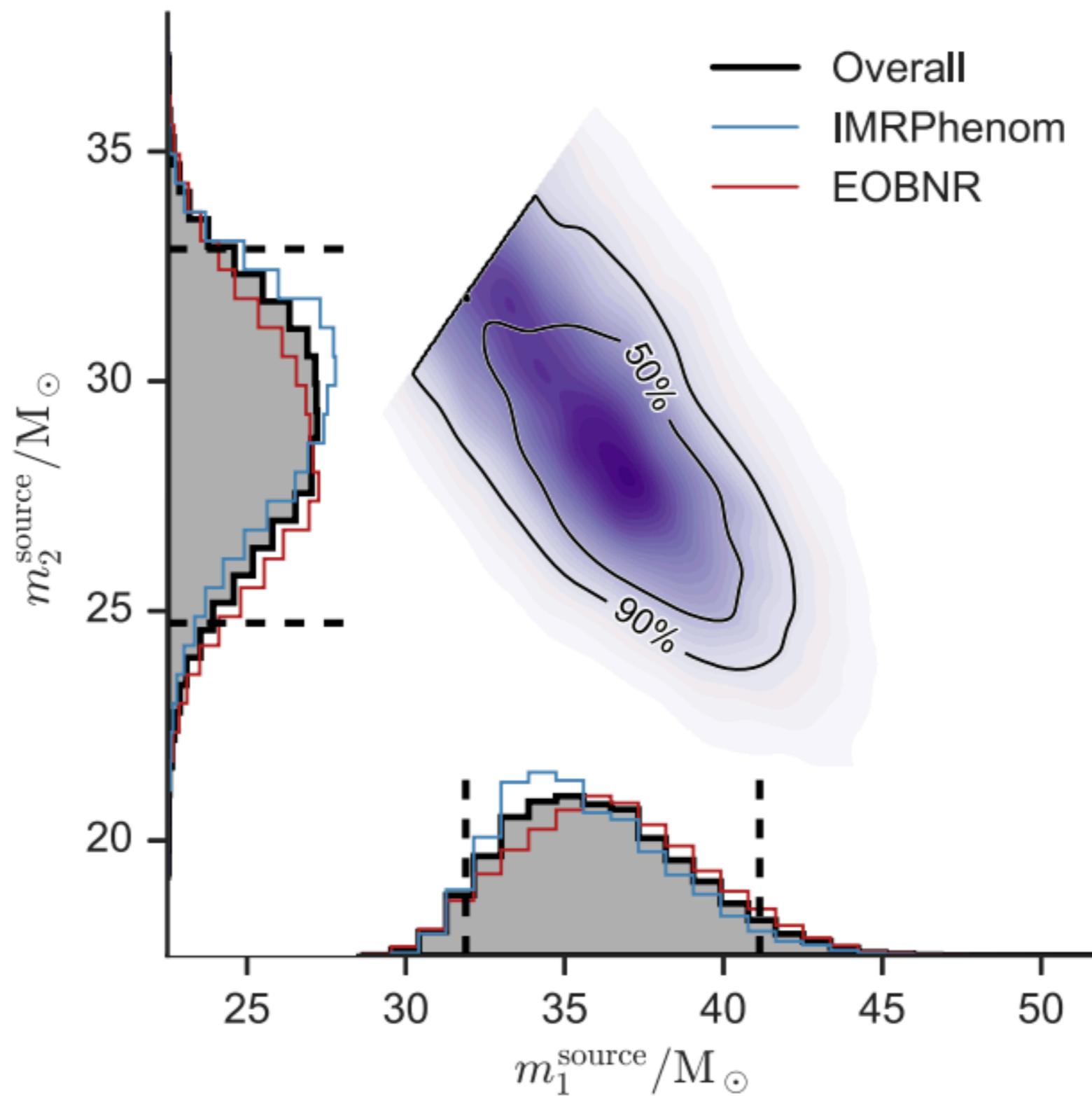
3rd HEL.A.S. and DAAD School, October 11th

From raw(ish) data



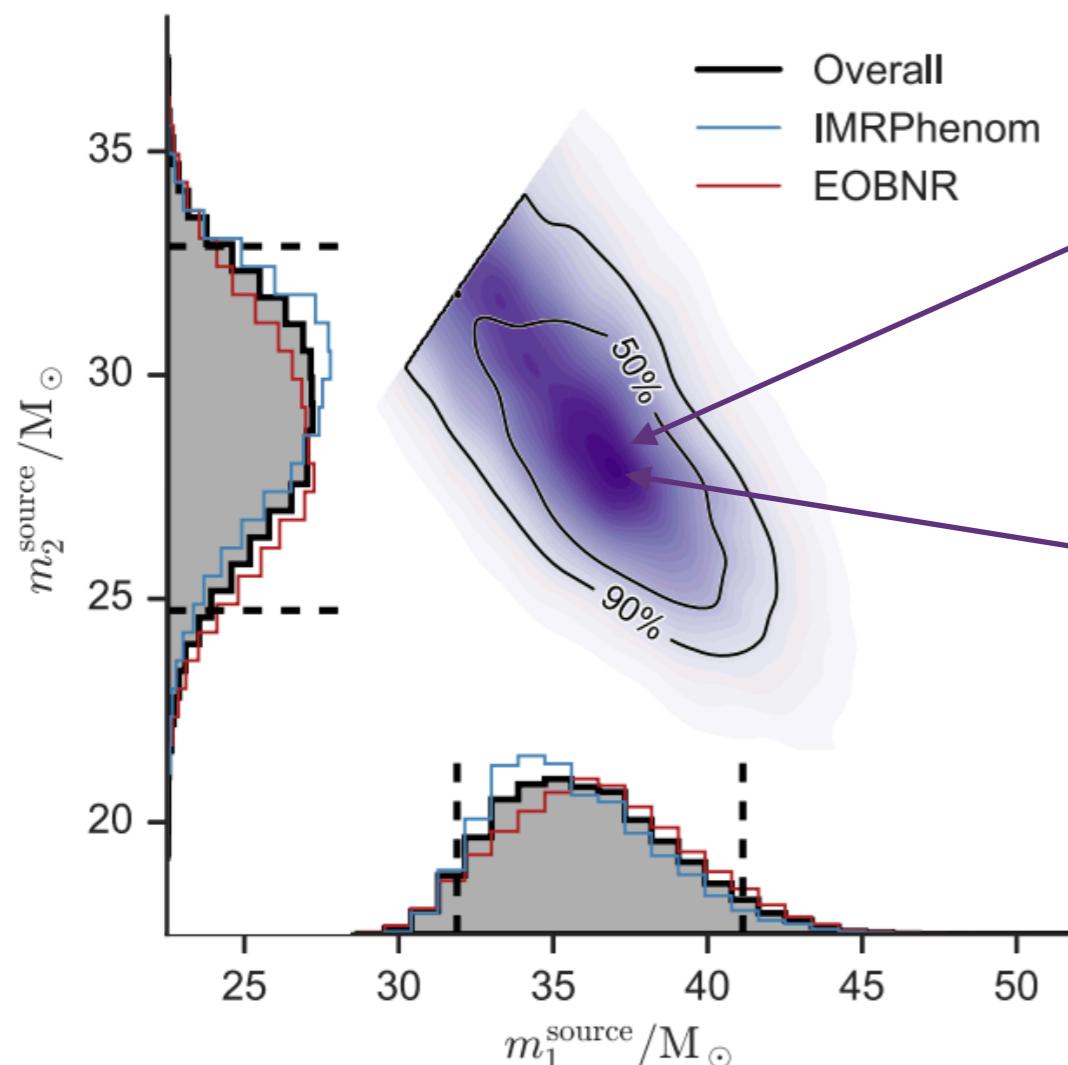
LVC (PRL:116, 061102)

to astrophysical parameters



Bayesian probability theory

Degree of belief interpretation of probability (vs rate of occurrence)



We are 50% sure that the true parameters are in there

If we got 100 identical binaries, 50 of them would have parameters in there

Mathematically rigorous and forces assumptions out

Bayes' theorem

Initial Understanding + New Observations = Updated Understanding

$$p(h'|d) = \frac{p(d|h')p(h')}{p(d)}$$

Likelihood **Prior**

Posterior **Evidence**

Likelihood

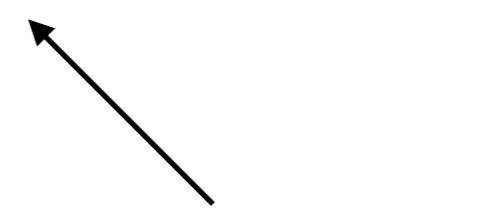
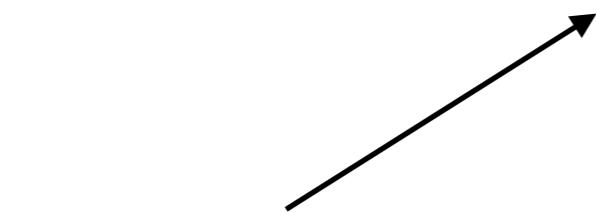
Likelihood

$$p(h'|d) = \frac{p(d|h')p(h')}{p(d)}$$

Data

A gravitational wave hits the detector, and the detector records

$$d = R[h] + n$$



available
data

detector
response
to GW

detector
noise

A single data point

$$d_1 = R[h_1] + n_1$$

Noise probability $p(n_1)$

Residual probability $p(d_1 - R[h'_1])$

Are they compatible? $p(d_1 - R[h'_1]) = p(n_1) = p(d_1 | h'_1)$

Probability of drawing $d_1 - R[h'_1]$ from the noise distribution under the null hypothesis

Many data points

discrete frequency bins

$$\begin{aligned} d &= \{d_1, d_2, \dots, d_{N_f}\} \\ &= \{R[h_1] + n_1, R[h_2] + n_2, \dots, R[h_{N_f}] + n_{N_f}\} \end{aligned}$$

Joint probability for the noise from all frequency bins

$$p(d|h') = p(d_1 - R[h'_1], d_2 - R[h'_2], \dots, d_{N_f} - R[h'_{N_f}]) = p(n_1, n_2, \dots, n_{N_f})$$

Two critical assumptions

Gaussian noise

Stationary noise

noise correlation
matrix

$$p(n_1, n_2, \dots, n_{N_f}) \sim e^{-\frac{1}{2} n_i C_{ij}^{-1} n_j}$$

$$C_{ij} = \frac{1}{2} S_n(f_i) \delta_{ij}$$

detector PSD
(no summation)

Likelihood

Probability of obtaining data d assuming signal h and that the noise is **stationary and gaussian**

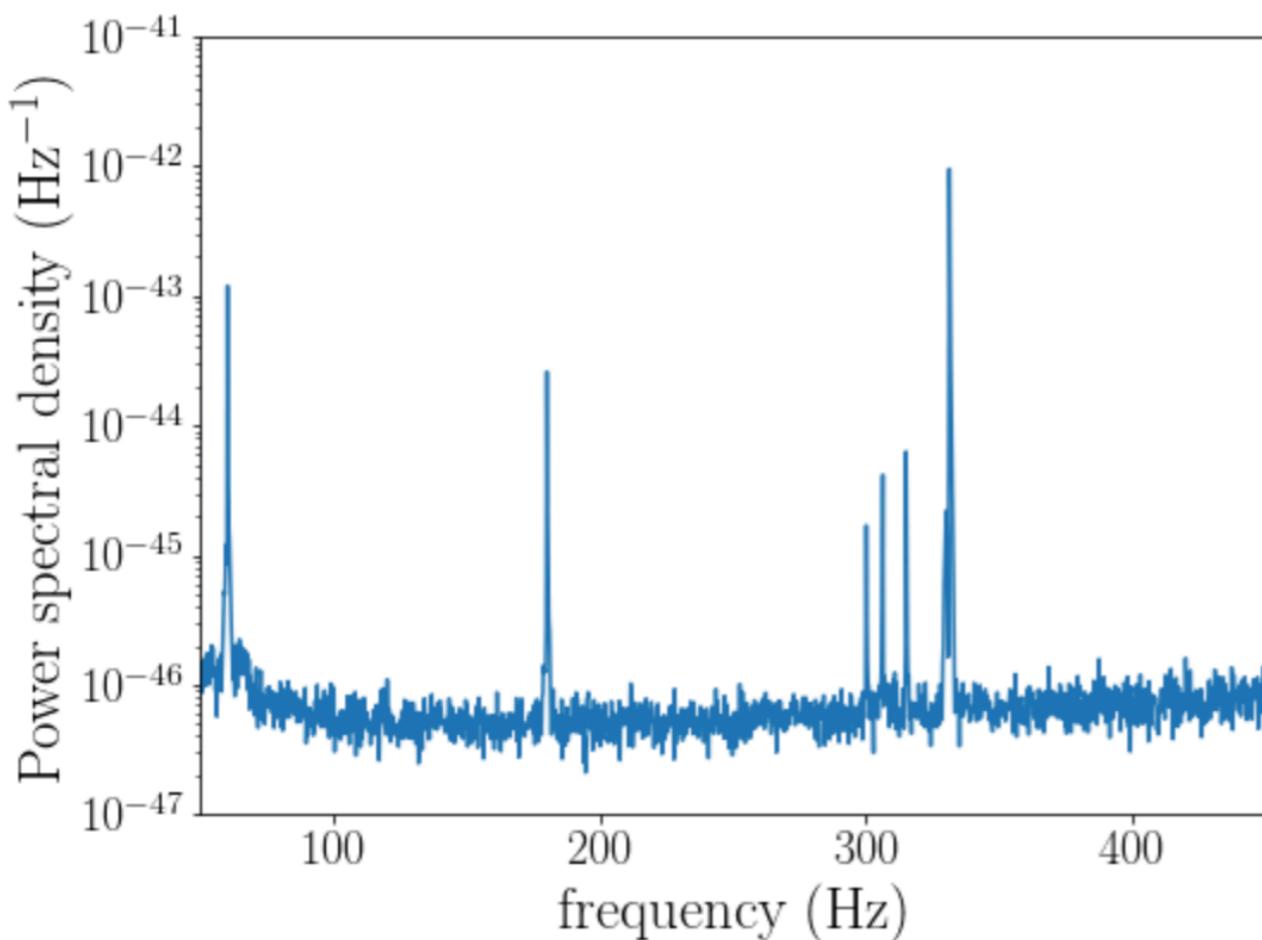
$$p(d|h) \sim e^{-\frac{1}{2}(d - R[h])^T(d - R[h])}$$

where the discrete frequency bins are now continuous

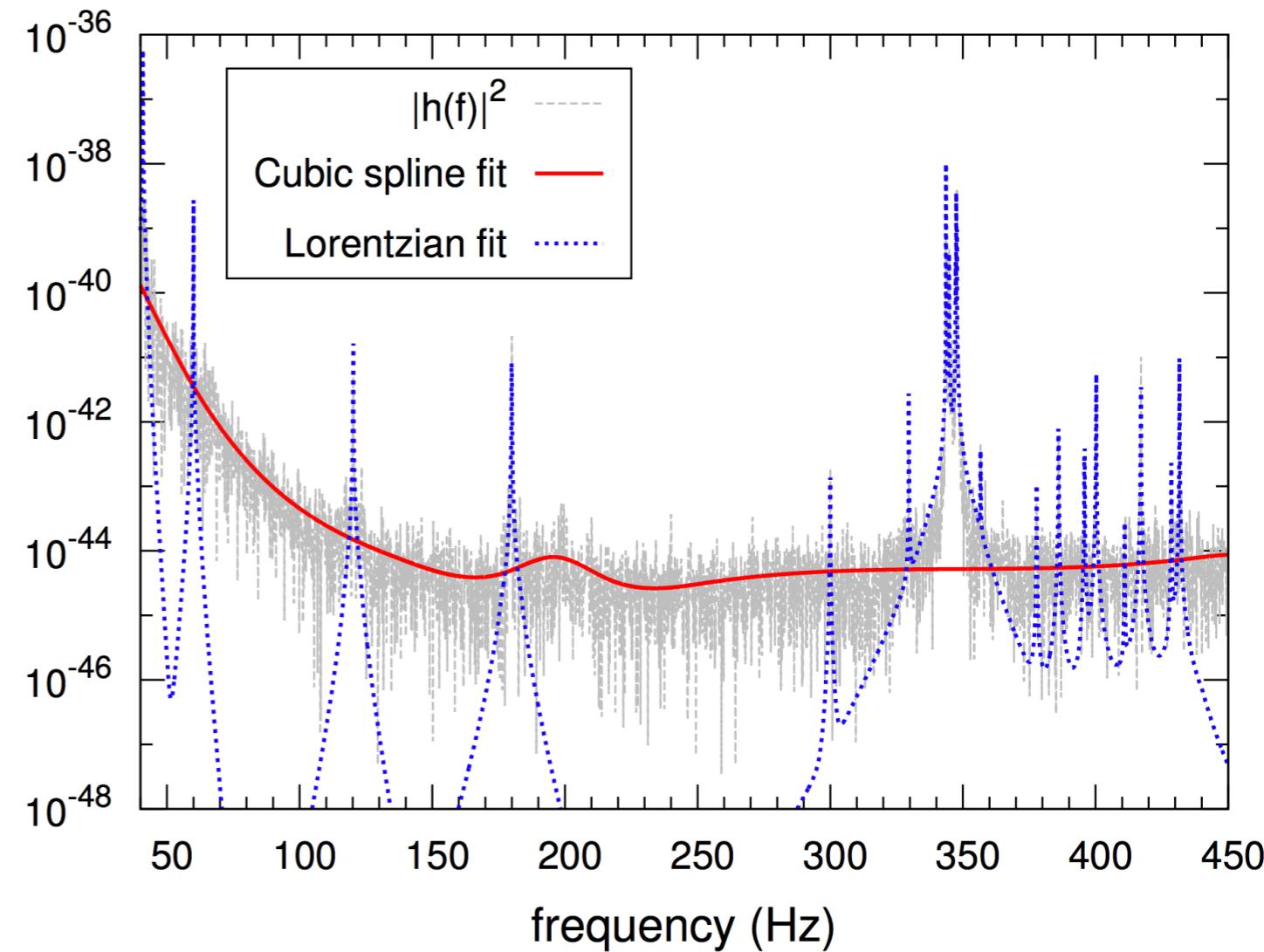
$$(a|b) = 4 \int \frac{a(f)b^*(f)}{S_n(f)} df$$

PSD Calculation

Off-source (average)



On-source



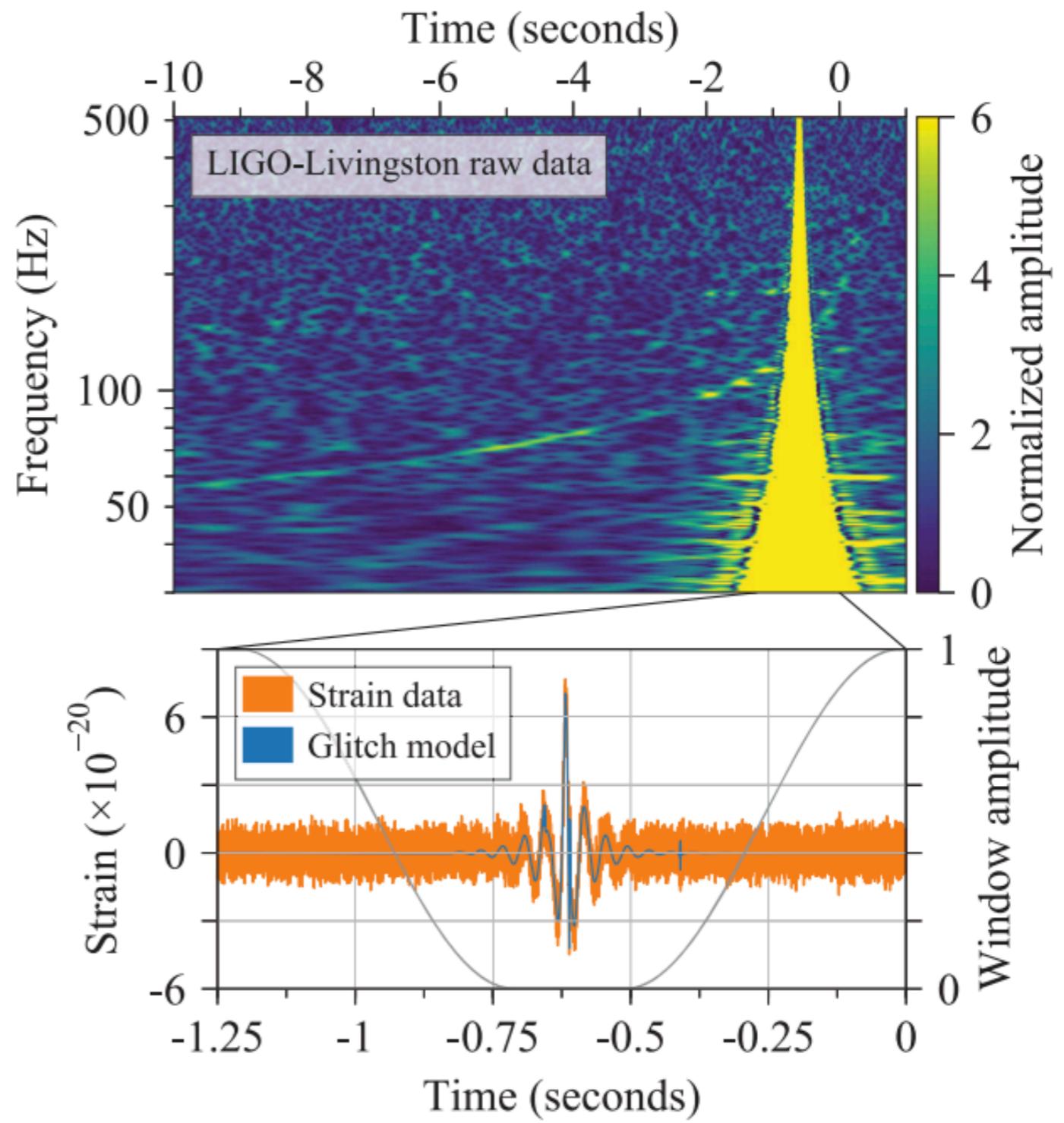
For more details see

Littenberg and Cornish (arxiv:1410.3852)

Gaussianity

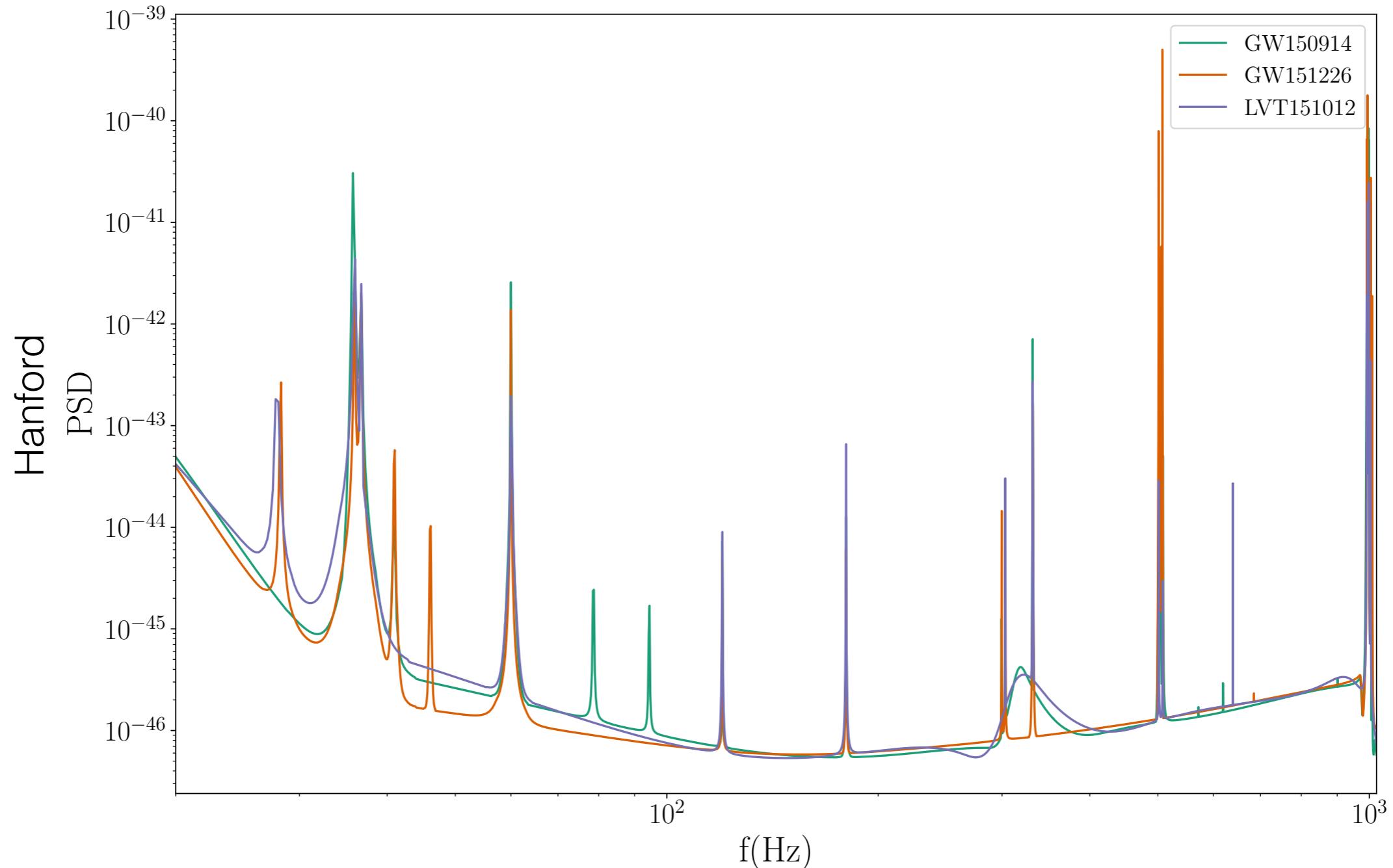
$$d = R[h] + n + g$$

Include glitch
in the likelihood or
remove it from the data



Stationarity

Recompute the PSD for every detector and for every event



For more details see
Littenberg and Cornish (arxiv:1410.3852)

Prior

Prior

$$p(h'|d) = \frac{p(d|h')p(h')}{p(d)}$$

Weak prior

Decompose the signal as a sum of bases functions

$$h' \rightarrow \sum^N w(\vec{y})$$

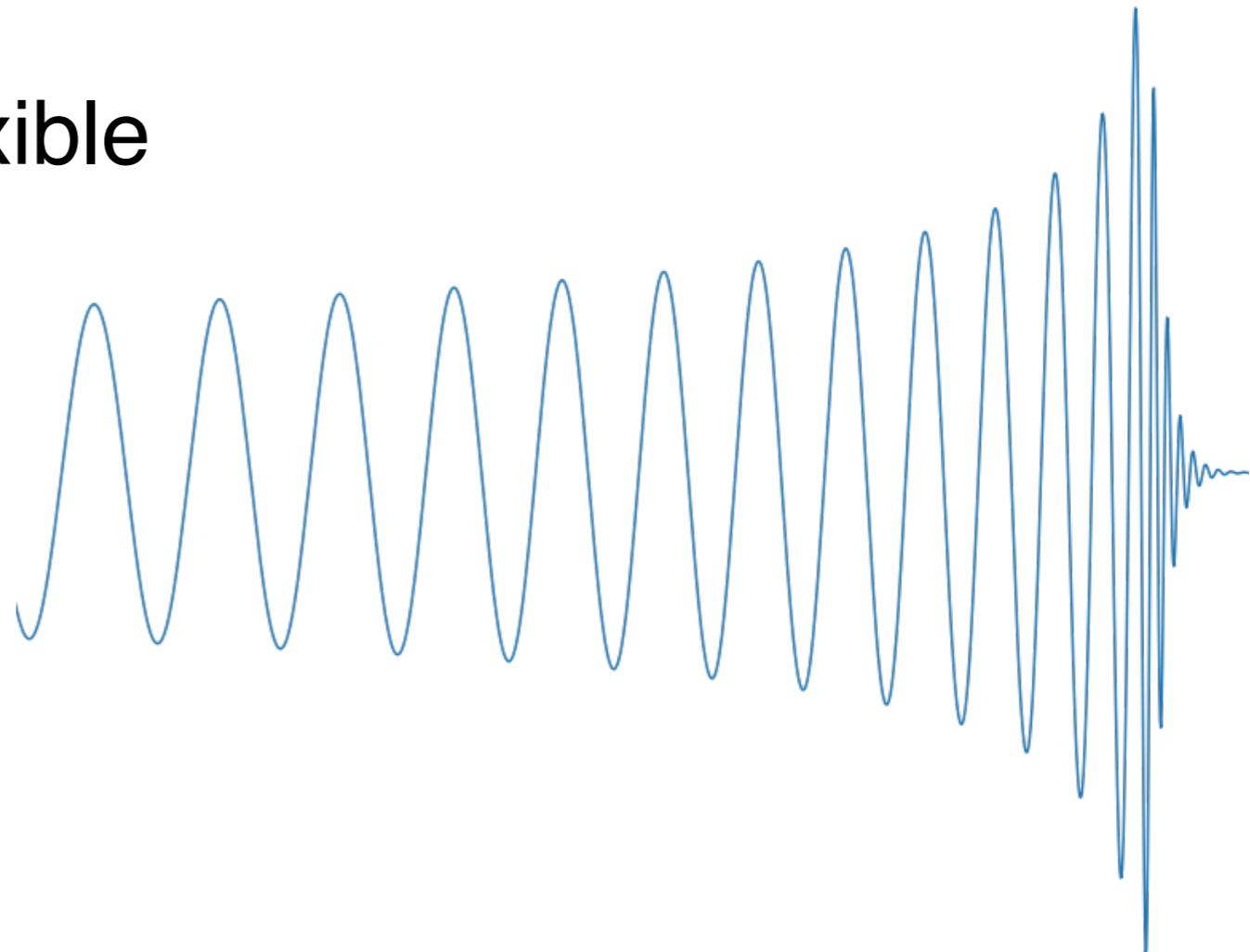
Less sensitive, more flexible

$$p(h') = \delta \left[h' - \sum^N w(\vec{y}) \right] p(\vec{y}, N)$$

Strong prior

More sensitive, less flexible

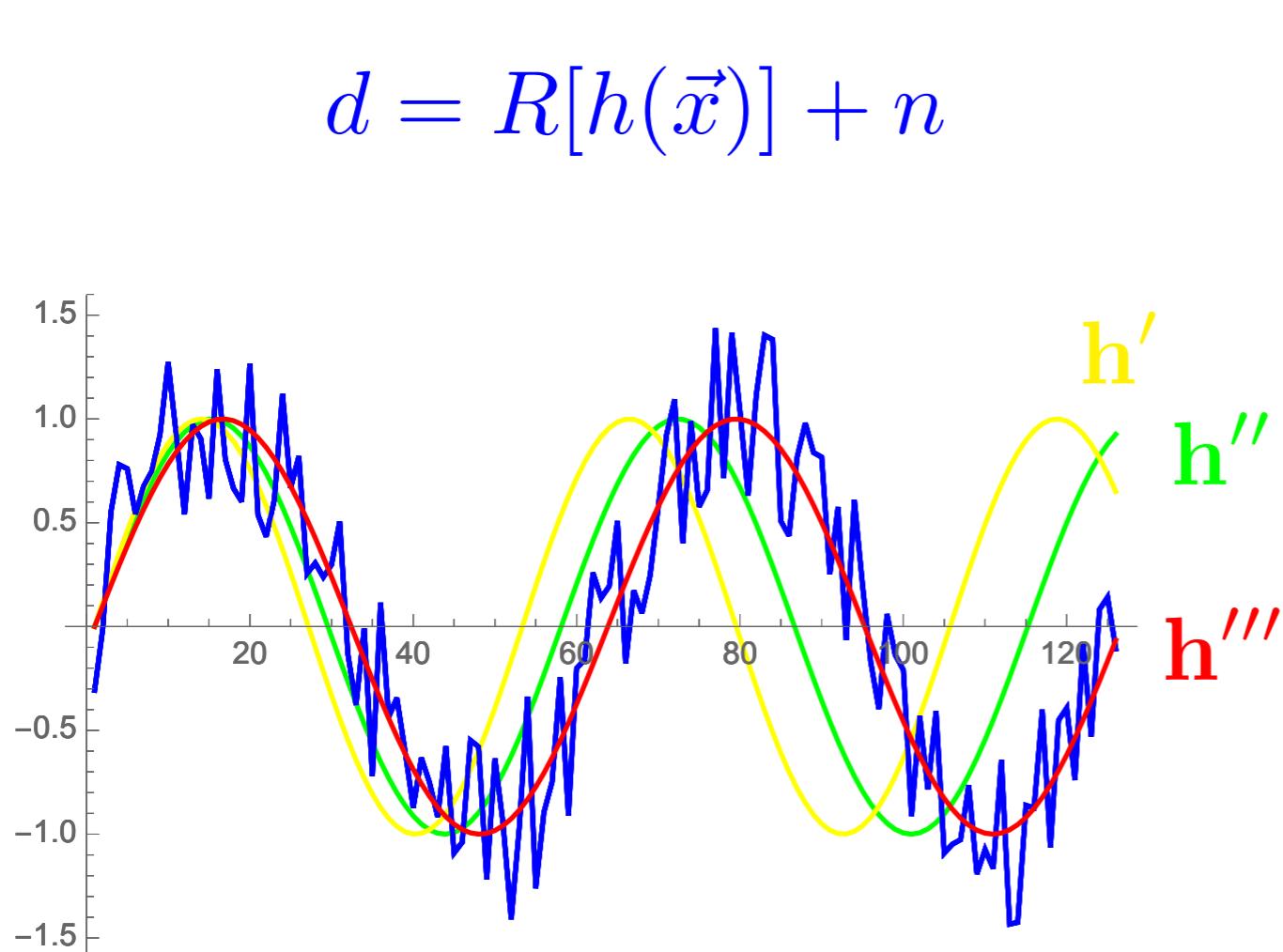
$$h' \rightarrow$$



Waveform models for the waves emitted from a compact binary coalescence: [IMRPhenomPv2](#), [IMRPhenomD](#), [SEOBNRv3](#), [SEOBNRv4](#)...

The various models differ both in the **physical effects they assume and the **methods** they use to describe the waveform**

Template-based analysis



Prior

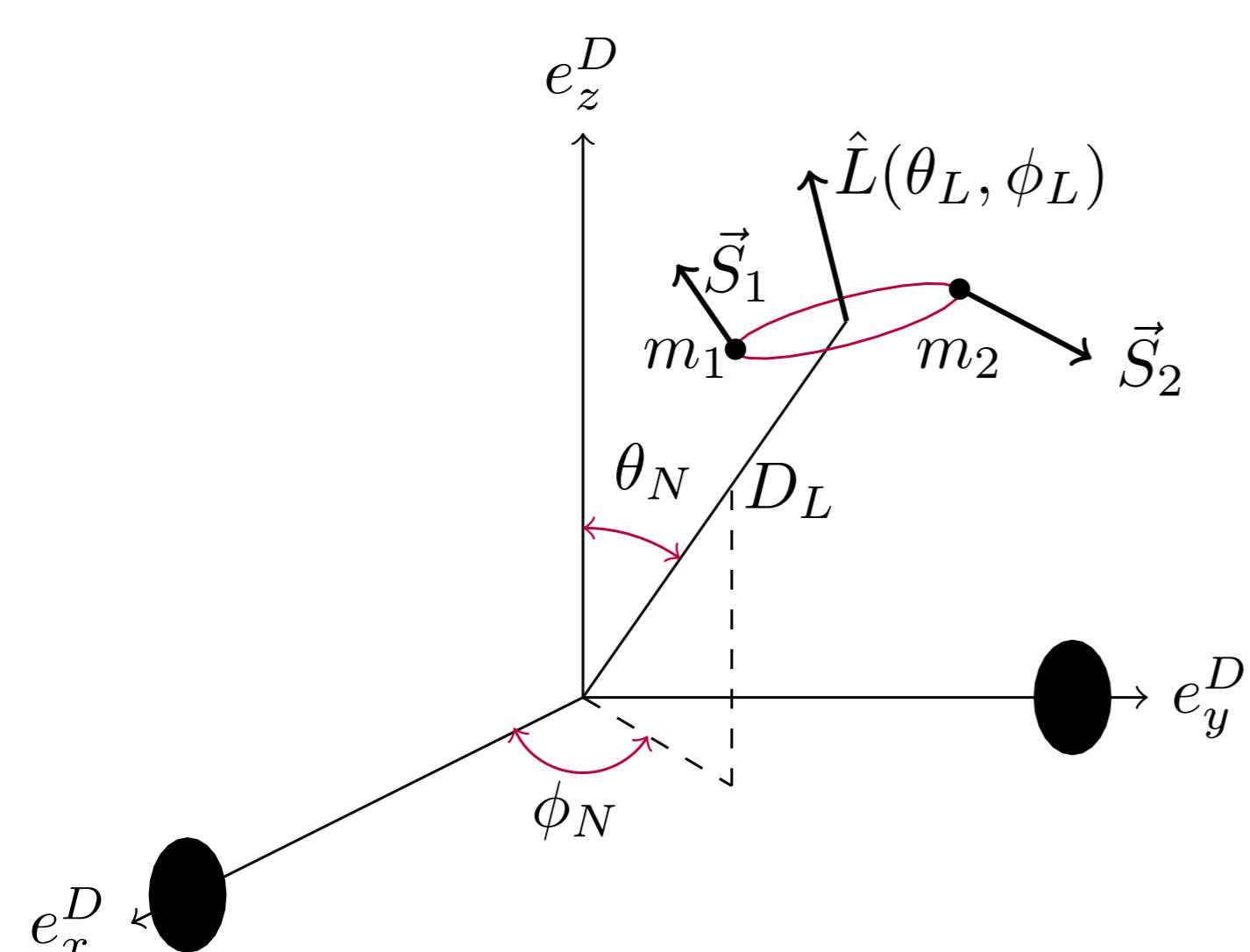
\downarrow

$$\left. \begin{aligned} p(d - R[h'(\vec{x})]) \\ p(d - R[h''(\vec{x})]) \\ p(d - R[h'''(\vec{x})]) \end{aligned} \right\} \sim p(n)$$

Likelihood

$$p(d|\vec{x})$$

Compact binaries

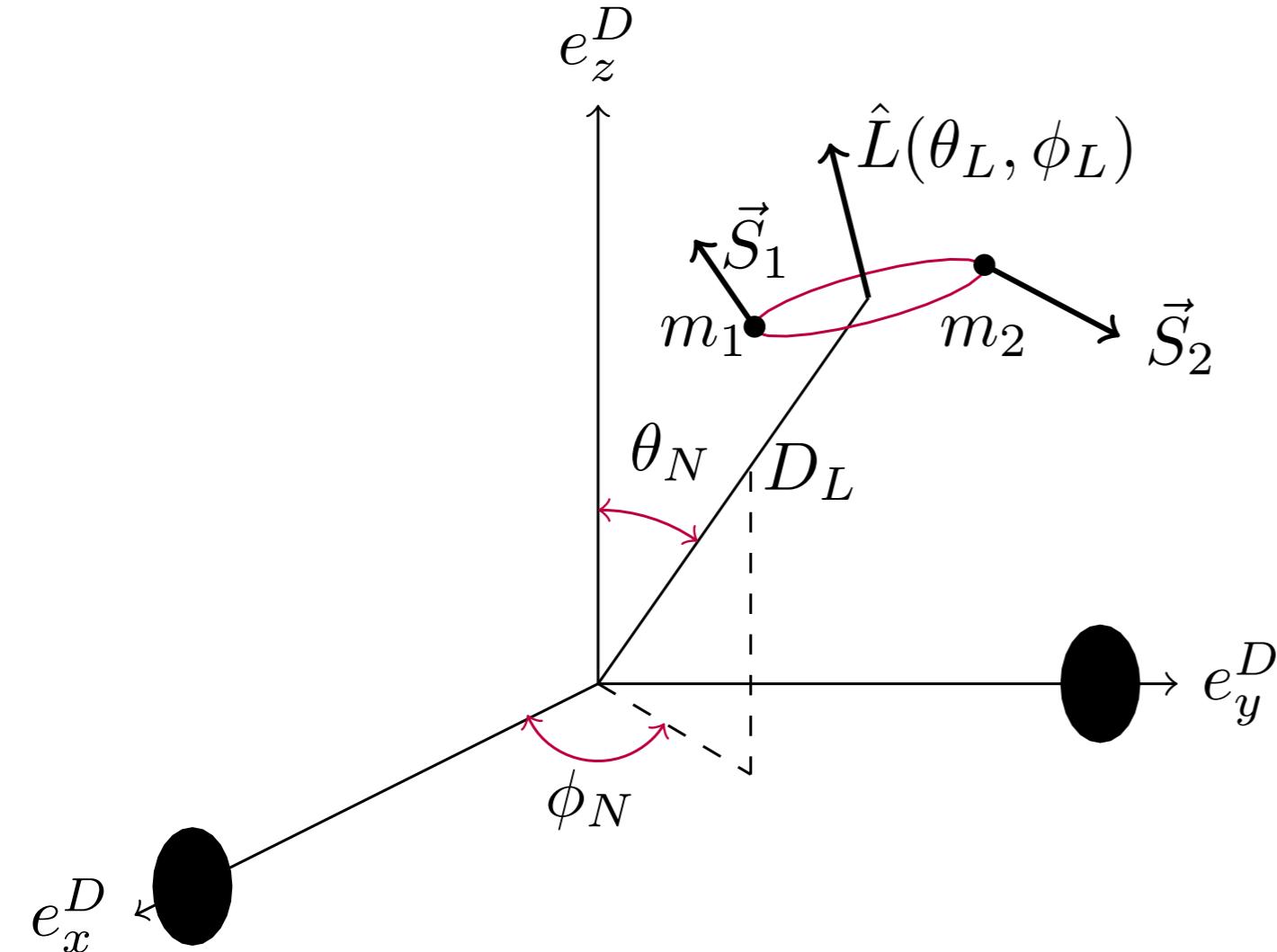


$$p(h') \sim \delta(h' - h'(\vec{\theta}))p(\vec{\theta})$$

priors on the system parameters

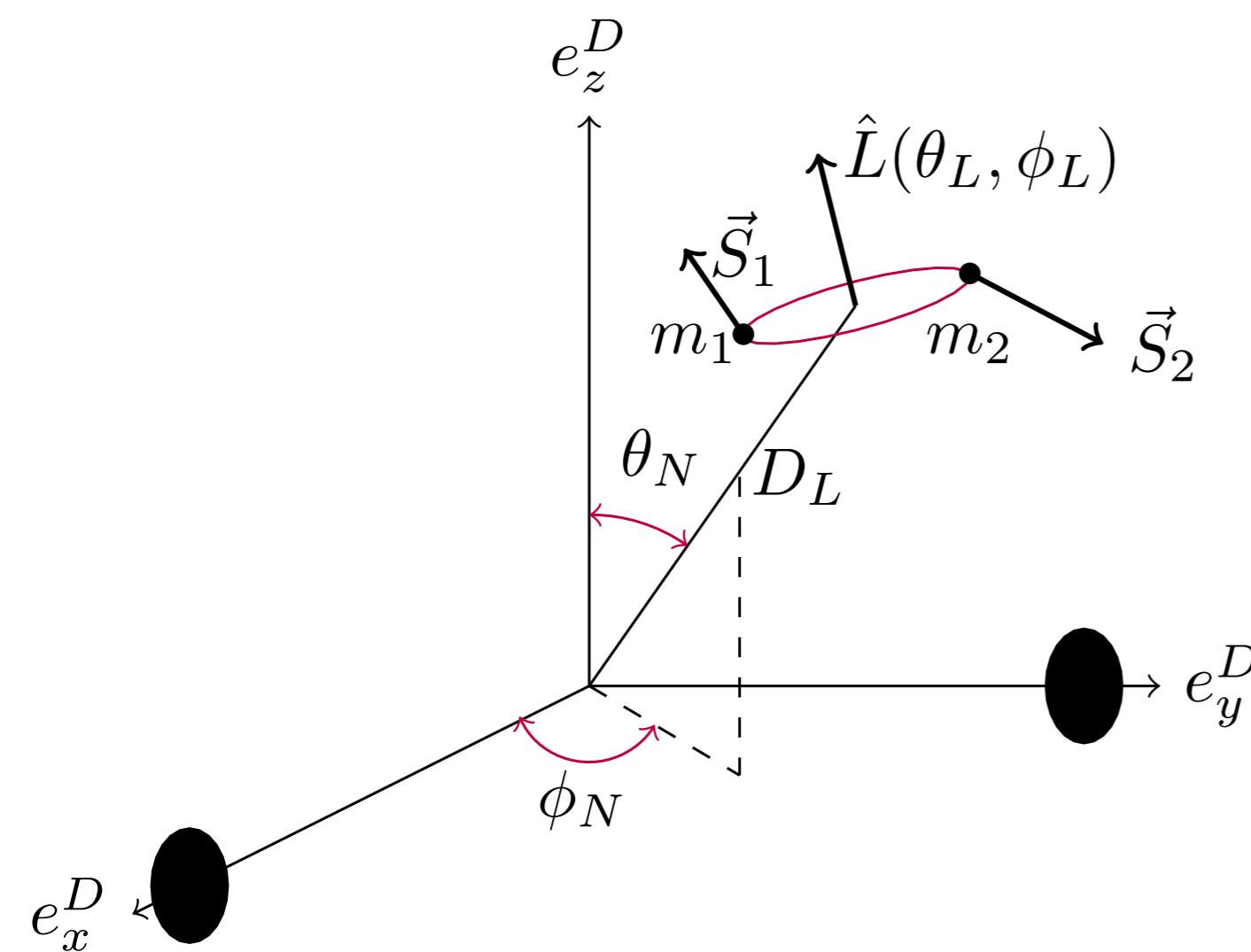
What we know about the parameters of the system without using the data

Extrinsic parameters



D_L	Uniform in volume
θ_N ϕ_N	Uniform in the sky
θ_L ϕ_L	Uniform in direction

Intrinsic parameters



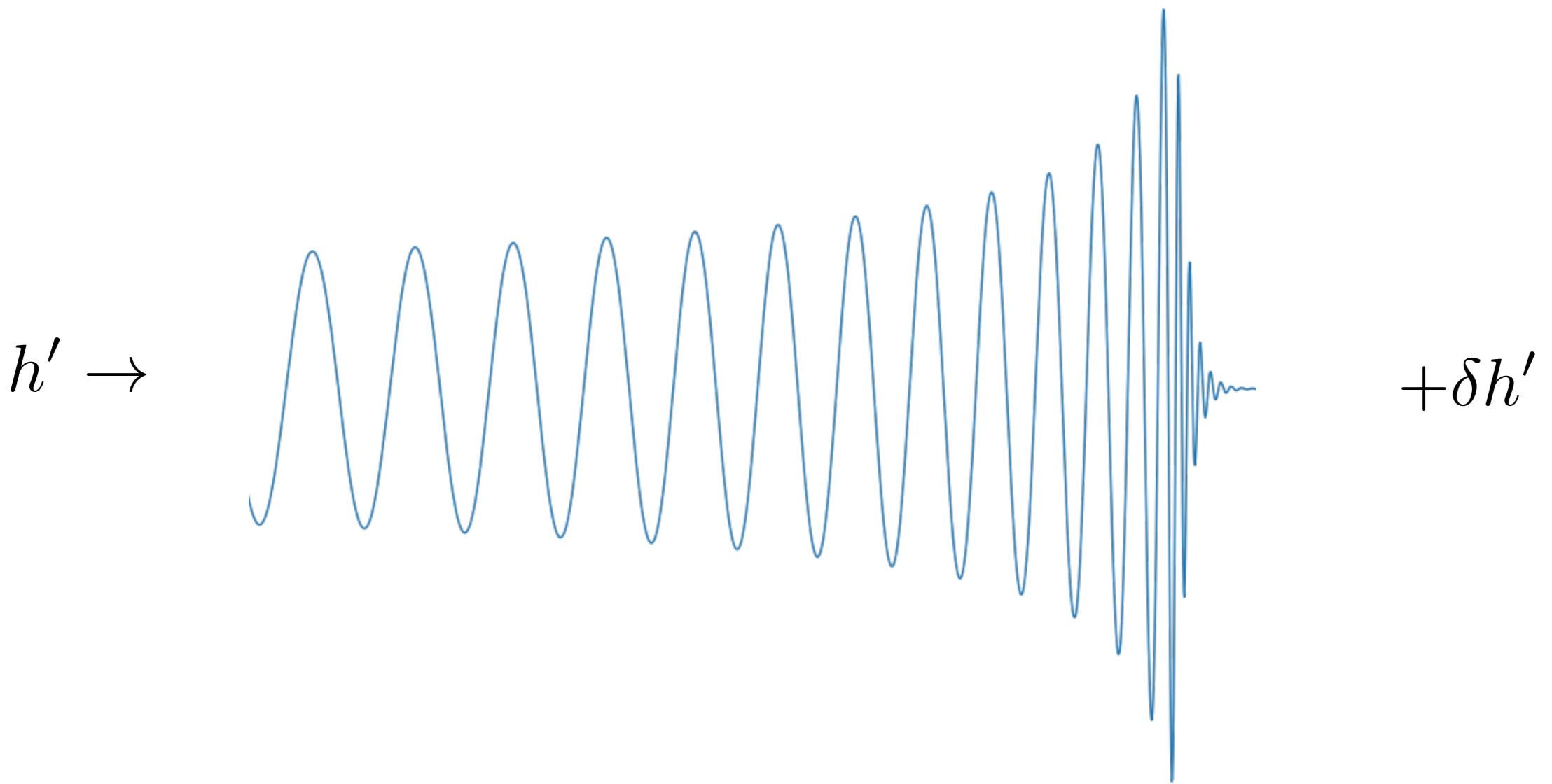
m_1 Uniform in some range
 m_2

\vec{S}_2 Uniform in direction and magnitude in $[0, m_i^2]$
 \vec{S}_1

λ_1 Uniform in $(0, 5000)$
 λ_2

Highly non unique and potentially **influential** choices

Calibration

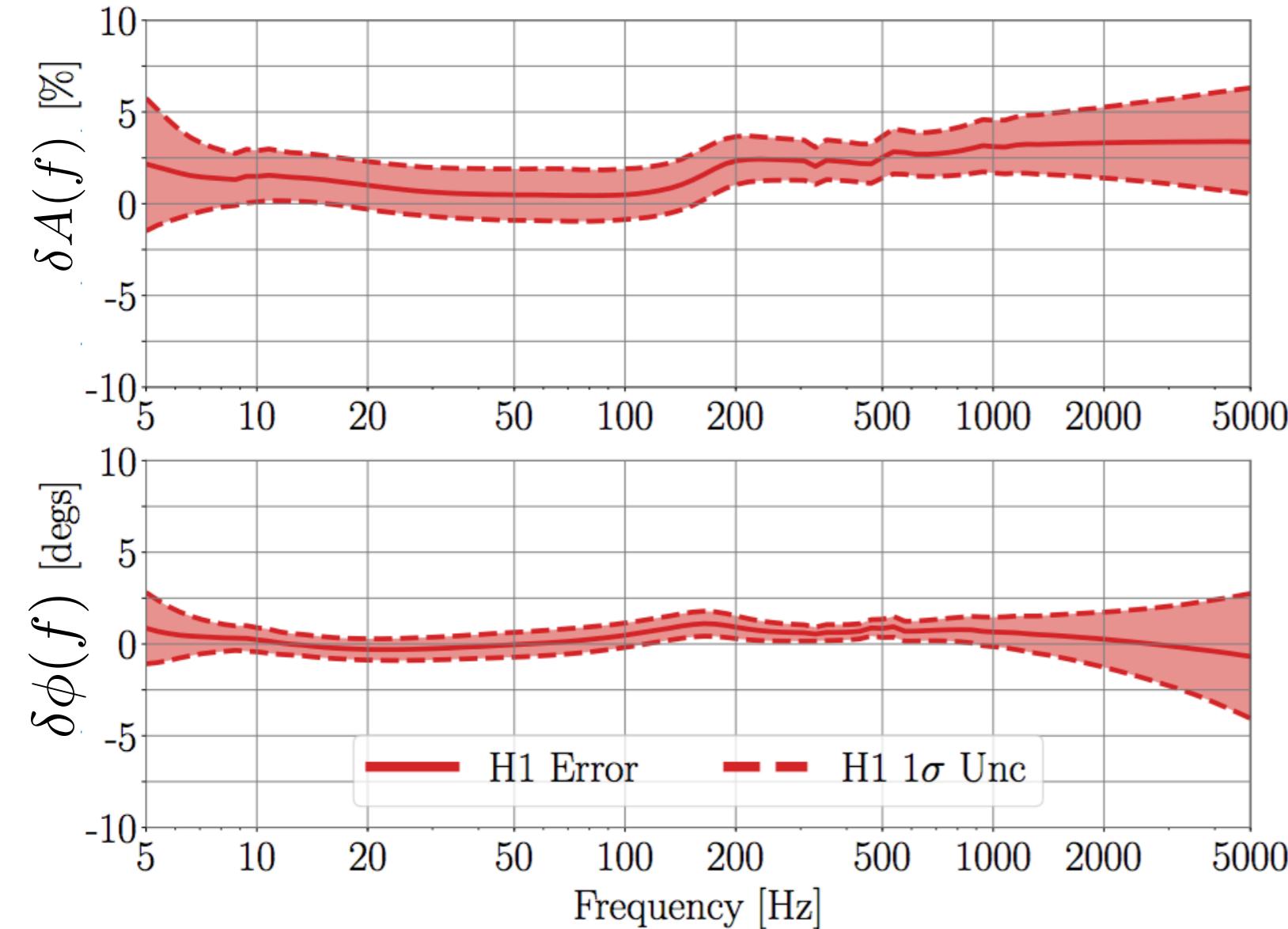


$$h' \rightarrow h'(1 + \delta A)e^{i\delta\phi}$$

Marginalize over
an amplitude and
a phase uncertainty

Calibration

$$h' \rightarrow h'(1 + \delta A)e^{i\delta\phi}$$



$$\delta A(f) = p_s(f; \{f_i, \delta A_i\})$$
$$\delta\phi(f) = p_s(f; \{f_i, \delta\phi_i\})$$

Interpolate with
cubic splines and
marginalize over
the calibration error

Farr+ LIGO Document T1400682-v1

Cahillane+ (PRD 96, 102001)

Evidence

$$p(h'|d) = \frac{p(d|h')p(h')}{\textcircled{p(d)}}$$

Evidence

Normalization factor for parameter estimation

Important for model selection

$$p(h'|d, M) = \frac{p(d|h', M)p(h'|M)}{p(d|M)}$$

M: any overall assumption or model (e.g. the signal is a GW, the BBH is spin-precessing, the binary components are NSs)

Odds ratio

Compare competing models, for example
‘GW170817 was a BNS’ vs ‘GW170817 was a BBH’

$$O_{ij} = \frac{p(M_i|d)}{p(M_j|d)}$$

$$= \frac{p(M_i)p(d|M_i)}{p(M_j)p(d|M_j)}$$

Bayes Factor
Likelihood of the models
Ratio of the evidences

(Really) hard to compute

Nested Sampling

Skilling (Bayesian Anal. 1, 833)

Thermodynamic Integration

Lartillot, Philippe (Systematic Biology 55, 195)

Reversible Jump Markov Chain Monte Carlo

Green (Biometrika 82, 711)

A number of approximations are commonly used
(maxL, Laplace approximation, AIC, BIC, DIC, FIC, HQC...)

Posterior

Posterior

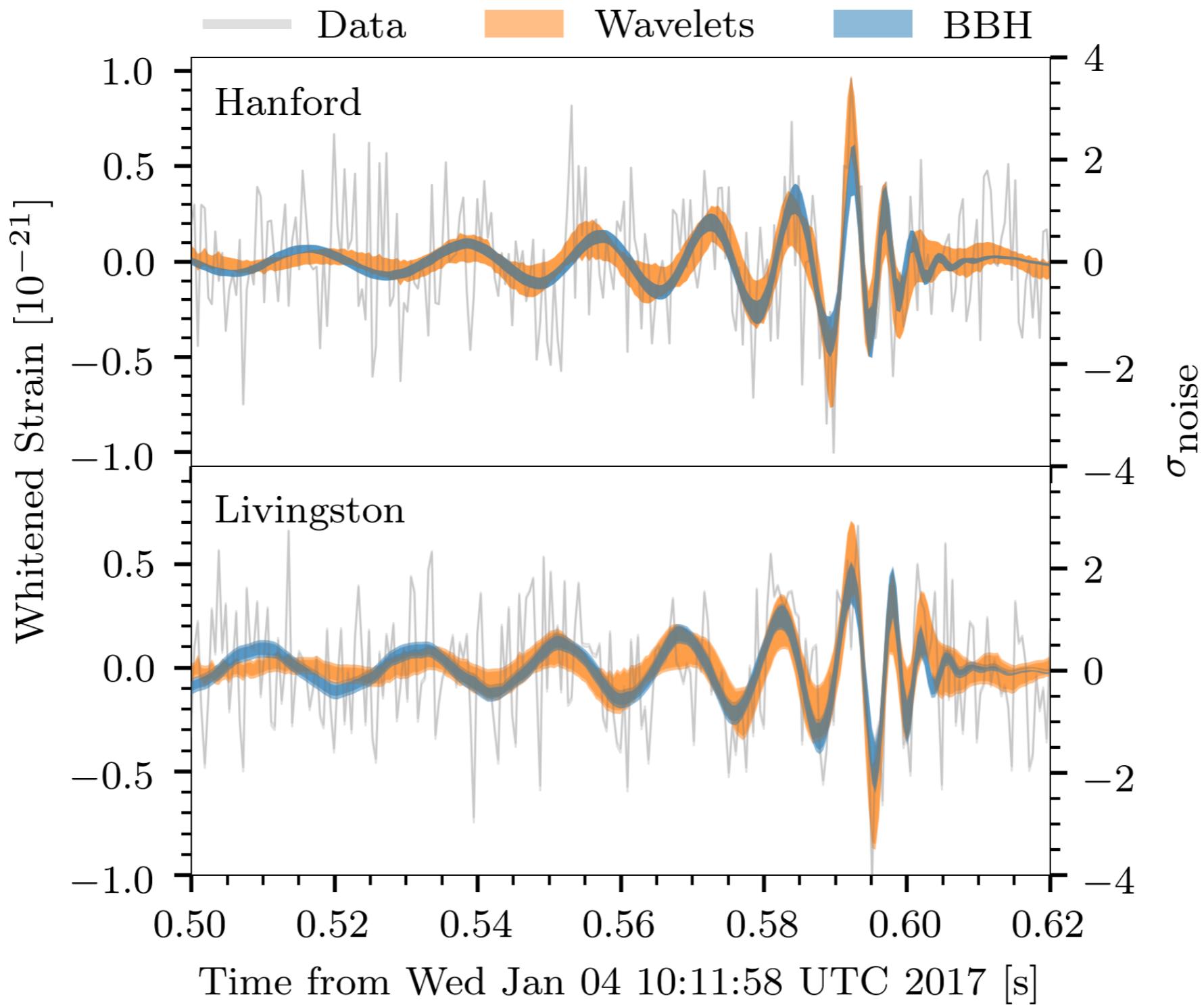
$$p(h' | d) = \frac{p(d|h')p(h')}{p(d)}$$

Bayesian Inference

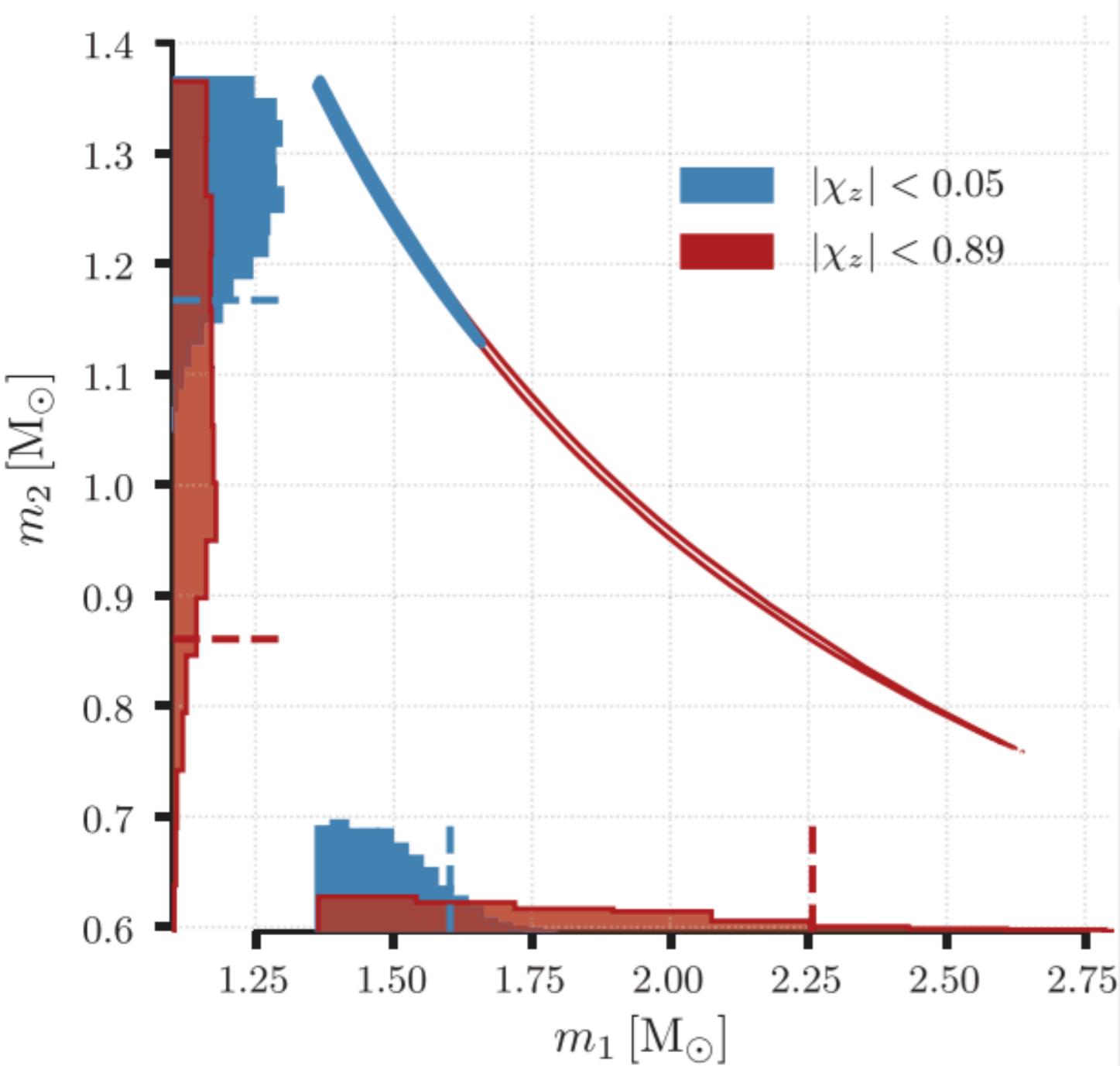
The posterior $p(h'(\vec{\theta})|d)$ gives the probability density that a model $h'(\vec{\theta})$ describes the data.

It is calculated with a likelihood and a prior and it is valid under the assumptions that were used when computing these two

Waveform posterior



Marginalized posterior



Integrate over all
'other' parameters

$$p(m_1, m_2 | d) = \int p(\vec{\theta}, m_1, m_2 | d) d\vec{\theta}$$

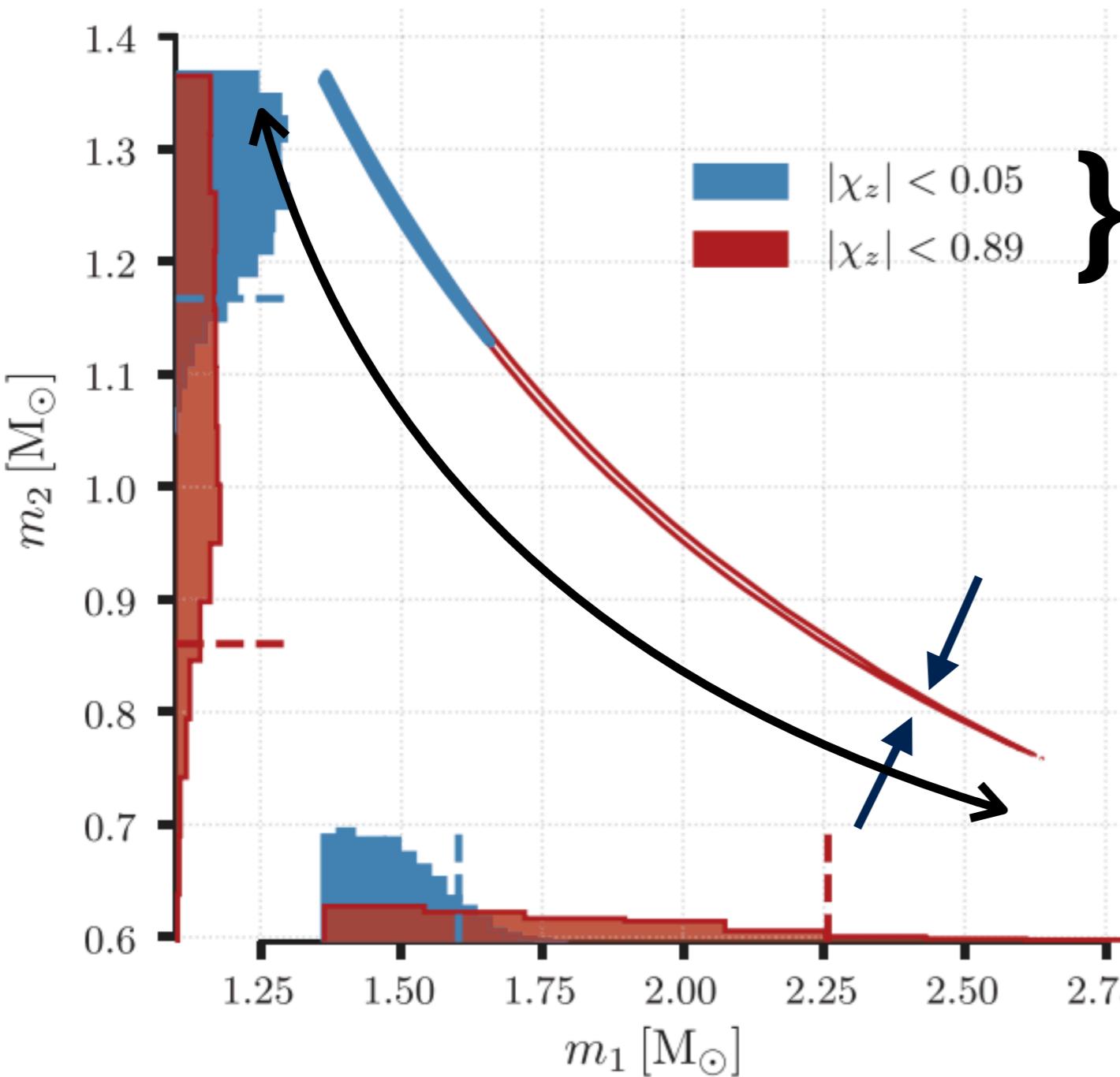
Compute expectation
values, quantiles etc.

$$E[m_1] = \int m_1 p(m_1, m_2 | d) dm_1 dm_2$$

Information gain
(Kullback–Leibler divergence)

$$KL = \int p(\vec{\theta} | d) \log_2 \left(\frac{p(\vec{\theta} | d)}{p(\vec{\theta})} \right) d\vec{\theta}$$

Inference from the posterior



The chirp mass is measured very well

The mass ratio is measured less well

The mass ratio is correlated with the spin

Hard(ish) to compute

Markov Chain Monte Carlo, Hamiltonian Monte Carlo,
Nested Sampling, Ensemble sampling, Genetic algorithms

LALsuite: <https://git.ligo.org/lscsoft/lalsuite>

Veitch+ (arxiv:1409.7215)

LVC (arxiv:1602.03840)

PyMC: <https://pymc-devs.github.io/pymc/>

emcee: <http://dfm.io/emcee/current/>

Kombine: <https://pages.uoregon.edu/bfarr/kombine/>

**The likelihood
is the noise model**

**Prior choices can
influence results**

Posterior

$$p(h' | d) = \frac{p(d | h') p(h')}{p(d)}$$

**The evidence is unimportant
for parameter estimation
(but not model selection!)**

Bibliography

1. D. S. Sivia (with J. Skilling) ***Data Analysis: A Bayesian Tutorial***

An introduction to Bayesian theory and data analysis

2. E. T. Jaynes ***Probability Theory: The Logic of Science***

More advanced theory of data analysis and probabilities

3. Michele Maggiore ***Gravitational Waves: Volume 1: Theory and Experiments***

The second half discusses gravitational wave detectors and data analysis

1. Veitch+ ([arxiv:1409.7215](https://arxiv.org/abs/1409.7215)): In-depth description of LALInference
2. LVC ([arxiv:1602.03840](https://arxiv.org/abs/1602.03840)): First parameter estimation on real data, includes a comprehensive discussion of methodology and assumptions
3. Thrane and Talbot ([arxiv:1809.02293](https://arxiv.org/abs/1809.02293)): Applications of Bayesian theory in gravitational wave inference
4. Hogg and Foreman-Mackey ([arxiv:1710.06068](https://arxiv.org/abs/1710.06068)): Practical guide to MCMC