Open source codes: a glance at RNS

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Outline

- What is RNS?
- What does it do?
- How does it do it?
- Does it do it well?
- extensions and modifications

Where can i find it?

- early versions of the code publicly available
- `http://www.gravity.phys.uwm.edu/rns/`
- See Stergioulas & Friedman (1994) for details

Wait...1994?

Hold on to that thought...

Ok, ok...What is it for?

- RNS is a code written in C
- models rapidly rotating, relativistic, compact stars
- use either polytropic or tabulated EOS
- compute individual models as well as sequences of rest mass or angular momentum
- assume uniform rotation

Compiling the code

Make sure that you have a C compiler installed, such as `gcc`. Then you can use a suitable `Makefile` to compile and obtain the executable.

```
make rns
./rns + various options (execution)
```

Check the `Makefile` to see if it uses the desired compiler, the correct flags and possible optimizations. If not, the `Makefile` should be edited.

Test run

- Download RNS
- Compile it
- Run a simple model

How does it do it?

General idea

To construct a model, solve iteratively the field equations and the hydrostatic equilibrium equation, until the coordinate equatorial radius changes by less than a specified relative accuracy.

How does it do it?

essential references

The code follows the KEH method (Komatsu, Eriguchi & Hachisu, 1989) and implements modifications introduced by Cook, Shapiro & Teukolsky (1994).

No, i mean really...how does it do it?!

- specify EOS and rotation law
- matter is approximated by a perfect fluid
- a model is defined uniquely by specifying two parameters: the central energy density ϵ_c and the axis ratio r_p/r_e (polar coordinate radius to the equatorial coordinate radius)

No, i mean really...how does it do it?!

The coordinates of the stationary, axisymmetric spacetime used to model the compact star are defined through the metric:

$$ds^2 = -e^{\gamma+\rho}\, dt^2 + e^{2\alpha}\left(dr^2 + r^2 d\theta^2\right) + e^{\gamma-\rho} r^2 \sin^2\theta \left(d\phi - \omega dt\right)^2.$$

where the potentials γ, ρ, α and ω are functions of r and θ only ($\mu = \cos\theta$).

CST compactification

- The CST variable s is introduced as an improvement upon the original KEH method via the transformation

$$r = r_e \frac{s}{1-s}$$

in order to map radial infinity to $s = 1$.

- This choice improves accuracy in calculations of radial integrals and the boundary conditions are satisfied exactly.

Numerical grid

- RNS uses a uniform 2-D grid in the variables $s = r/(r + r_e)$ and $\mu = \cos\theta$
- The center is at $s = 0$, the equator at $s = 0.5$ and infinity at $s = 1$.
- The equatorial plane is at $\theta = \pi/2$ while the pole is at $\theta = 0$.
- The grid size must be specified in the `Makefile`.

Algorithm description

Step 1

Begin the iterative procedure with a guess for the four metric functions $\rho, \gamma, \omega, \mu$, the energy density ϵ and the angular velocity $\Omega(\varpi)$. This can be either a nonrotating model (TOV) or a previously computed model.

Step 2

Rescale the potentials ρ, γ and μ by r_e^2, while ω is rescaled by $1/r_e$.

Algorithm description

Step 3

Equate the 1st integral of the hydrostationary equilibrium at the location of the energy density maximum to its value at the pole, to obtain a new value of the coordinate radius r_e.

Step 4

Equate the 1st integral of the hydrostationary equilibrium at the equator to its value at the pole, to obtain a new value for Ω_e (angular velocity at the equator).

Algorithm description

Step 5

The new value for Ω_e is used to find a new value for Ω_c and thus a new value for $\Omega(\varpi)$ everywhere inside the star.

Step 6

Using the EOS and solving the first integral of the hydrostationary equilibrium everywhere inside the star (equating it to its value at the pole), we obtain new distributions of the enthalpy, energy density and pressure.

Algorithm description

Step 7

New distributions of the metric functions ρ, γ, ω and μ are obtained from their respective expressions.

Step 8

Repeat the iteration from step 2, until convergence is achieved.

Options

RNS has several options which allow you to specify model parameters.

- `-e` central energy density in $\mathrm{gr/cm^3}$
- `-r` axes ratio
- `-m` mass in $\mathrm{M_\odot}$
- `-z` rest mass in $\mathrm{M_\odot}$
- `-o` angular velocity in $10^4 \mathrm{s^{-1}}$
- `-j` angular momentum in $\mathrm{GM_\odot^2/c}$

Check the supplied documentation for more options and tasks.

Output

- central energy density ϵ_c
- maximum energy density ϵ_max
- gravitational mass M
- rest mass M_0
- angular momentum $cJ/GM_\odot^2$
- rotational/gravitational energy T/W
- angular velocity on symmetry axis Ω_c
- angular velocity at density maximum Ω_max
- angular velocity at equator Ω_e
- Keplerian angular velocity Ω_K
- circumferential radius at equator R_e
- coordinate radius at equator r_e
- axes ratio (polar to equatorial) r_p/r_e

Does it do it well?

- robust and well tested against similar codes
- RNS has been used extensively by the community in the last 20 years
- no extreme demands in computational power (constructs equilibrium models)
- easy to modify
- implemented in Einstein Toolkit as RNSID, for initial data construction

Don't reinvent the wheel!

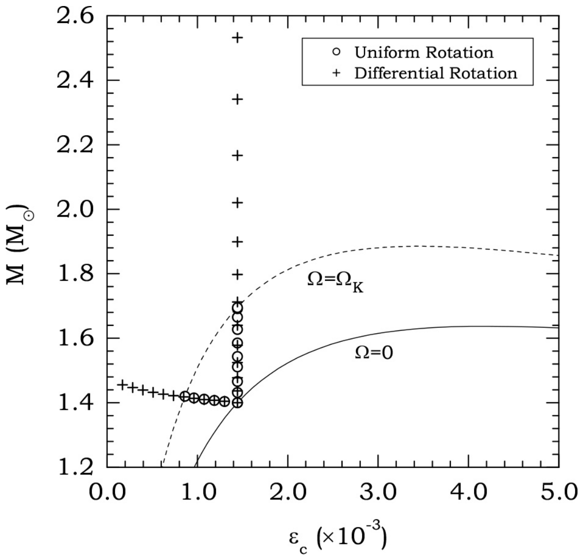
Perfect it: Extend and modify

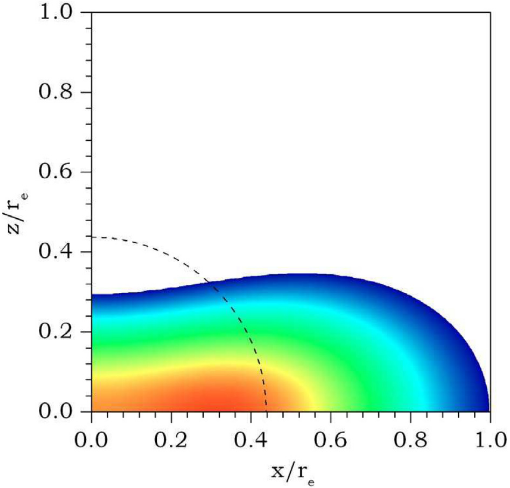
- differential rotation: Stergioulas, Apostolatos & Font (2004)
- conformal flatness approximation: Iosif & Stergioulas (2014)
- scalar–tensor theories of gravity: Doneva, Yazadjiev, Stergioulas & Kokkotas (2013)
- new differential rotation law: Bauswein and Stergioulas (2017)

RNS + differential rotation

- Stergioulas, Apostolatos & Font (2004)
- rotation law $F(\Omega) = A^2(\Omega_c - \Omega)$
- sequences A and AU: constant rest mass $M_0 = 1.506 M_\odot$
- sequences B and BU: constant central density $\rho_c = 1.28 \times 10^{-3}$

RNS + differential rotation

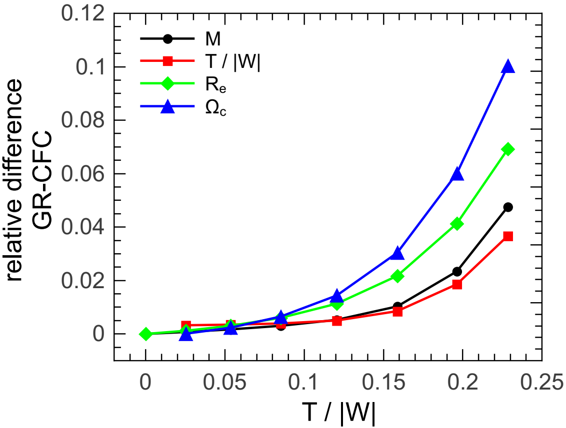




- Density stratification in the fastest differentially rotating model of sequence A.
- The maximum density appear off center.

RNS + CFC

- Iosif & Stergioulas (2014)
- determine the accuracy of the CFC approximation for differentially rotating relativistic stars.
- replace equation for metric function μ with $\mu = \dfrac{\gamma - \rho}{2}$
- for the fastest rotating and most relativistic polytropic models, the deviation from full GR is below 5% for integrated quantities and below 10% for local quantities, such as the angular velocity



- Differentially rotating polytropes with N = 1 and K = 100.
- The central energy density is fixed at $\epsilon_c = 3.3 \times 10^{-3}$.

RNS + scalar tensor theories

- Doneva, Yazadjiev, Stergioulas & Kokkotas (2013)
- first presentation of the field equations governing rapidly rotating neutron stars in scalar-tensor theories of gravity
- extension of RNS to solve these equations numerically
- numerical results showed that rapidly rotating neutron star models with a nontrivial scalar field can exist and that the effect of the scalar field is stronger for rapid rotation.

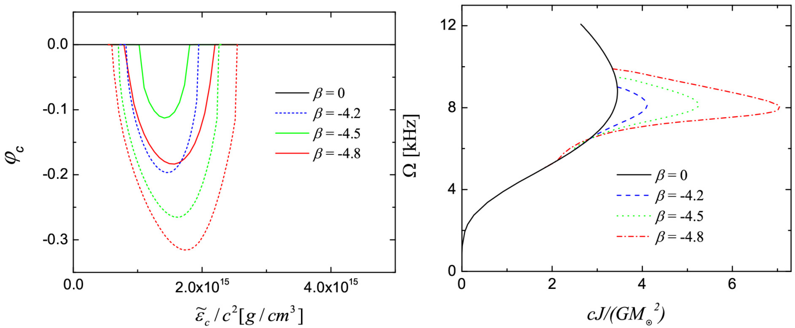


FIG. 3: *(left panel)* The central value of the scalar field as a function of the central energy density for static sequences of neutron stars (solid lines) and sequences of stars rotating at the mass-shedding limit (dotted lines). For $\beta = -4.2$ nontrivial solutions do not exist in the nonrotating case. *(right panel)* The angular velocity as a function of the angular momentum for sequences of stars rotating at the mass-shedding limit.

RNS + 3 parameter rotation law

- Bauswein and Stergioulas (2017)
- extension of RNS to a 3-parameter rotation law
- allows for a different rotational description of the envelope, compared to the core of the star
- Semi-analytic model reproduces collapse behavior

$$F(\Omega) = \begin{cases} {A_1}^2 \left(\Omega_c - \Omega\right) + \left({A_2}^2 - {A_1}^2\right)\left(1 - b\right)\Omega_c, & \Omega \leq b\,\Omega_c \\ {A_2}^2 \left(\Omega_c - \Omega\right), & b\,\Omega_c \leq \Omega \leq \Omega_c \end{cases}$$

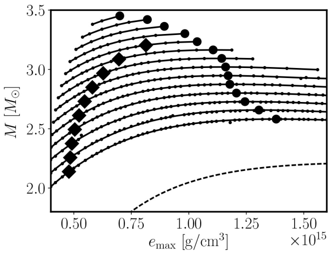


Figure 2. Gravitational mass M as function of the maximum energy density e_{max} of differentially rotating NSs with various fixed values of angular momentum J for the TM1 EoS. The solid lines show fifth-order polynomial least-square fits to the sequences with $J = 4, 4.5, 5, 5.5, ..., 10.5$ (in geometrical units). Up to $J = 8.5$, filled circles mark the turning point of each J-constant sequence. For the five highest values of J the numerical code produces equilibrium sequences that only come close to the turning point (see text). In these cases the filled circles still represent a good approximation of the maximum mass that can be reached for each value of J because the J-constant sequences have a small slope. The filled diamonds are a sequence of models that satisfy the empirical relation (4) for binary NS merger remnants. The dashed line corresponds to the non-rotating limit.

References

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Thank you!