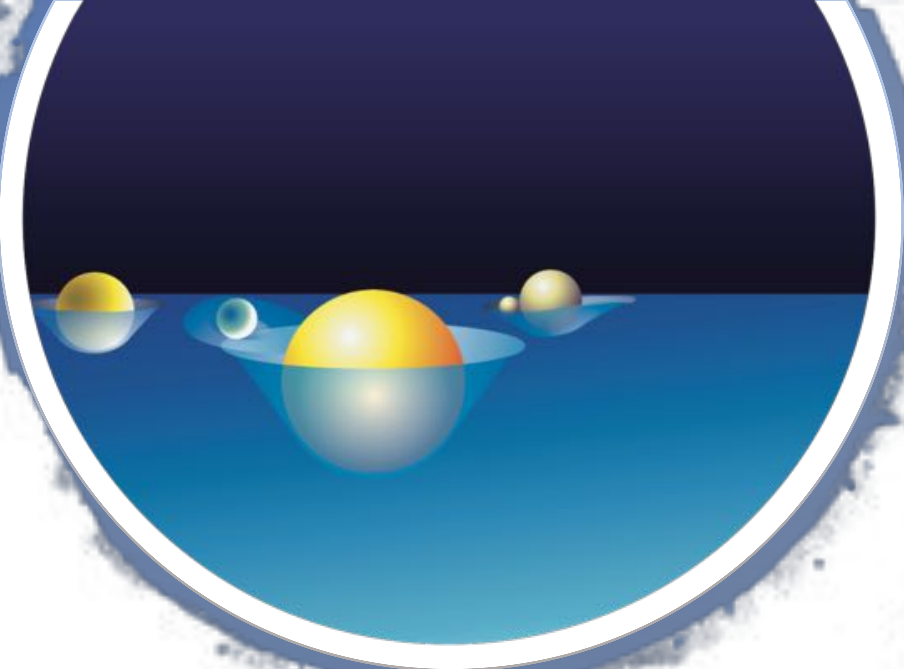
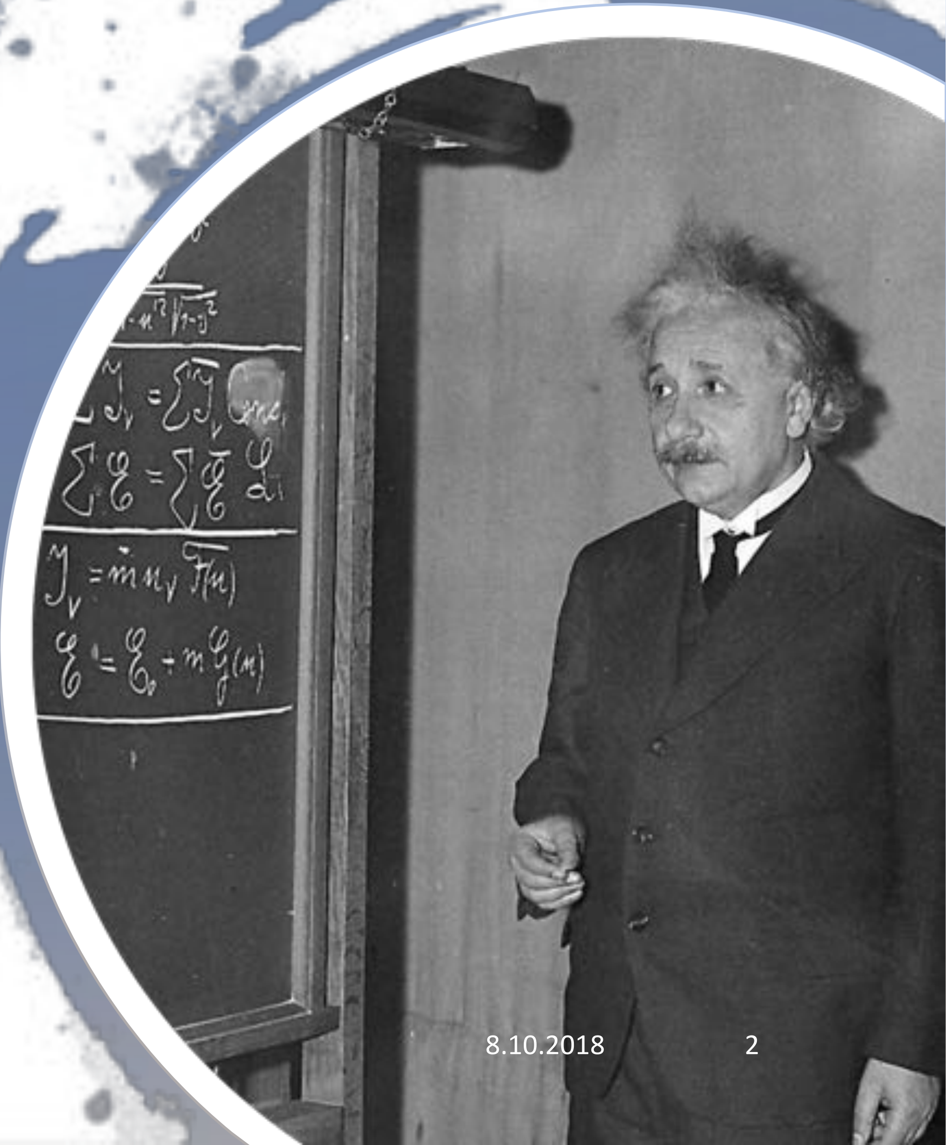
Neutron Stars & Gravitational Waves

Kostas Kokkotas

Theoretical Astrophysics

Eberhard Karls University of Tübingen





1915 -2015

General Theory of Relativity

matter tells spacetime how to curve

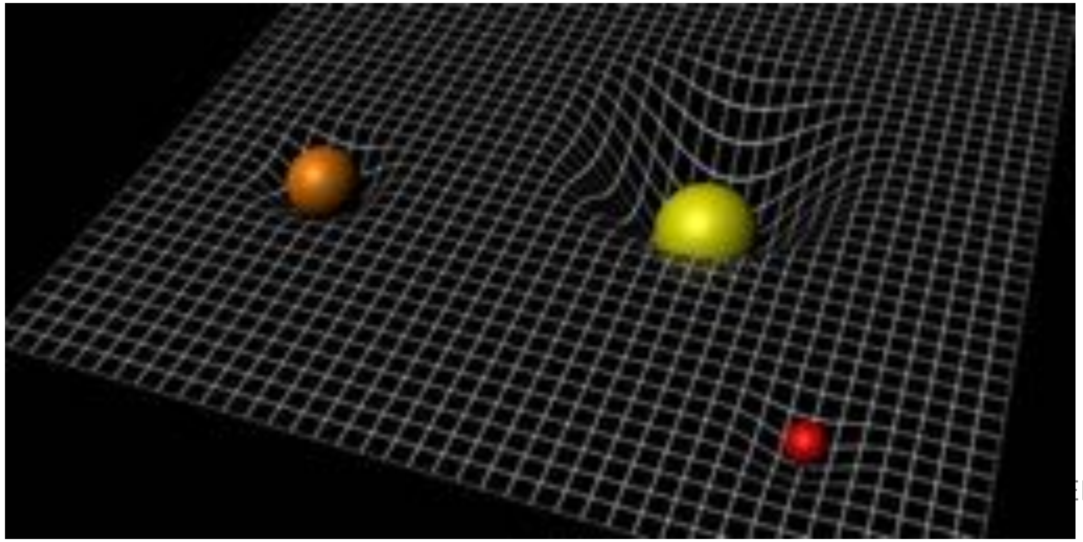
… and space tells matter how to move!

GEOMETRY

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

MATTER-ENERGY

Gravitational field is expressed via the spacetime curvature.



$$\frac{GM}{c^2 R} \sim 1$$

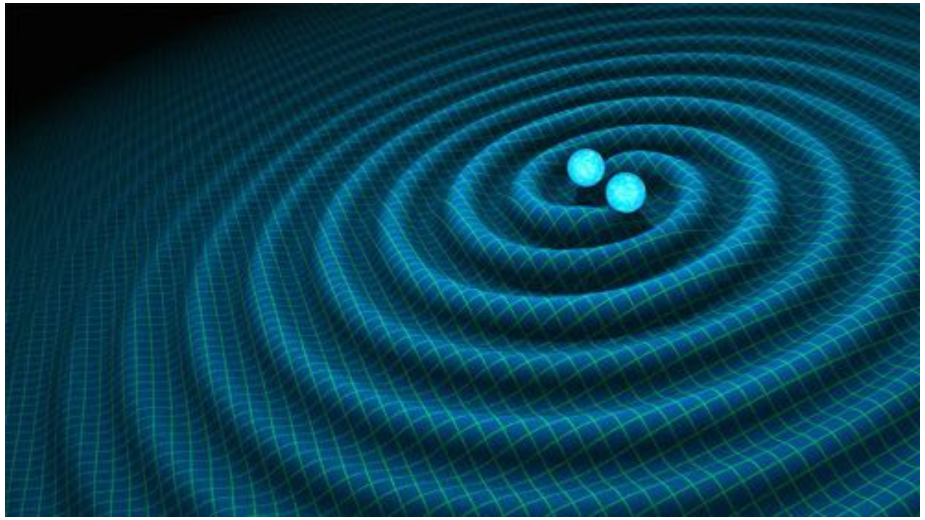
Sun~ 10^{-6}

Earth~10^{-9}

Gravitational Waves

- **RIPPLES** of the Spacetime Curvature
- Propagate with the **SPEED OF LIGHT**
- **EVERY (non-spherical) VARIATION** of the Gravitational Field Produces Gravitational Waves

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

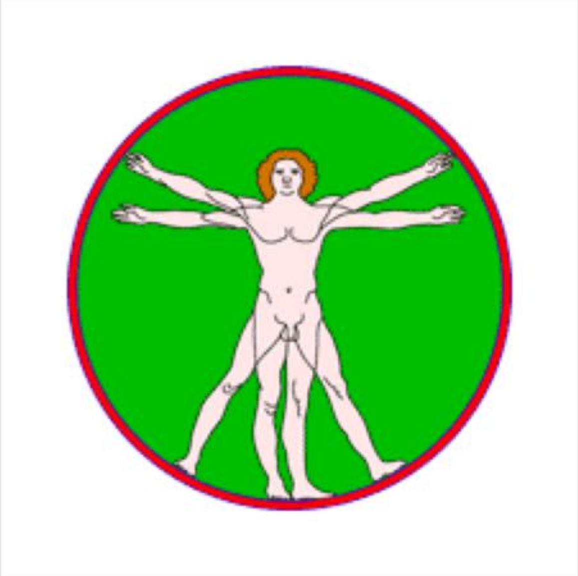


$$\boxed{\begin{aligned}&\left(\frac{1}{c^2}\frac{d^2}{dt^2} - \nabla^2\right) h^{\mu\nu} = \frac{4\pi G}{c^4} T^{\mu\nu} \\ &g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}\end{aligned}}$$

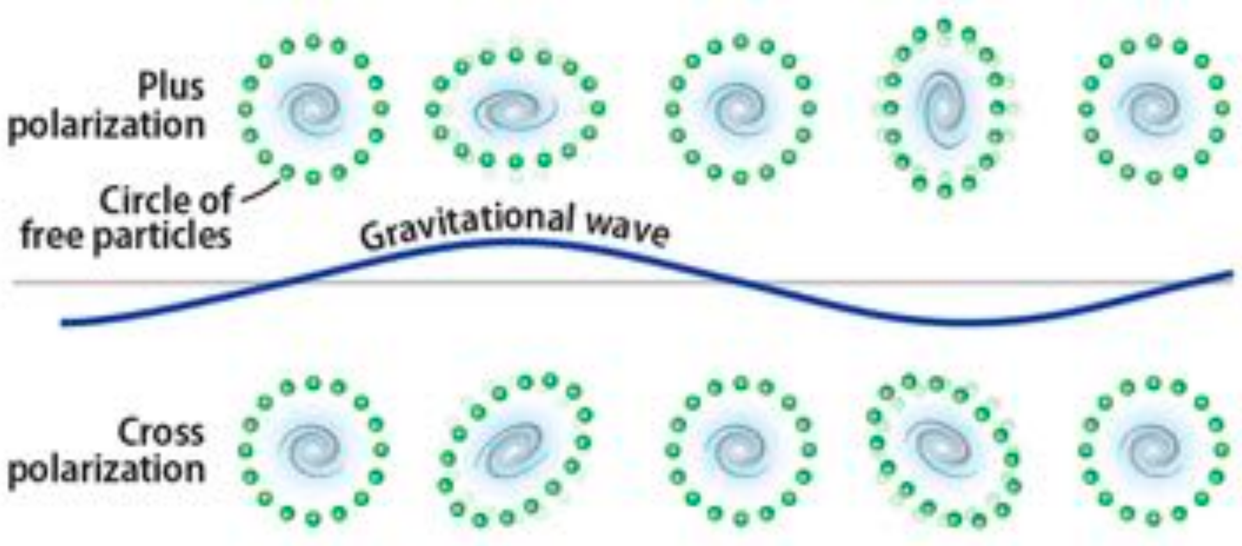
INTERACT WEAKLY WITH MATTER

What are Gravitational Waves

They produce **tidal deformations** on massive bodies.



… and deform periodically the spacetime geometry



$$\Delta L \sim h * L$$

Gravitational Waves: Primer

- Length Variation

$$\frac{\Delta\ell}{\ell} = h$$

G=c=1

- Amplitude

$$h^{jk} \approx \frac{2}{r}\ddot{Q}^{jk} \approx \varepsilon \cdot \left(\frac{M}{r}\right) \cdot \left(\frac{M}{R}\right)$$

- Luminosity

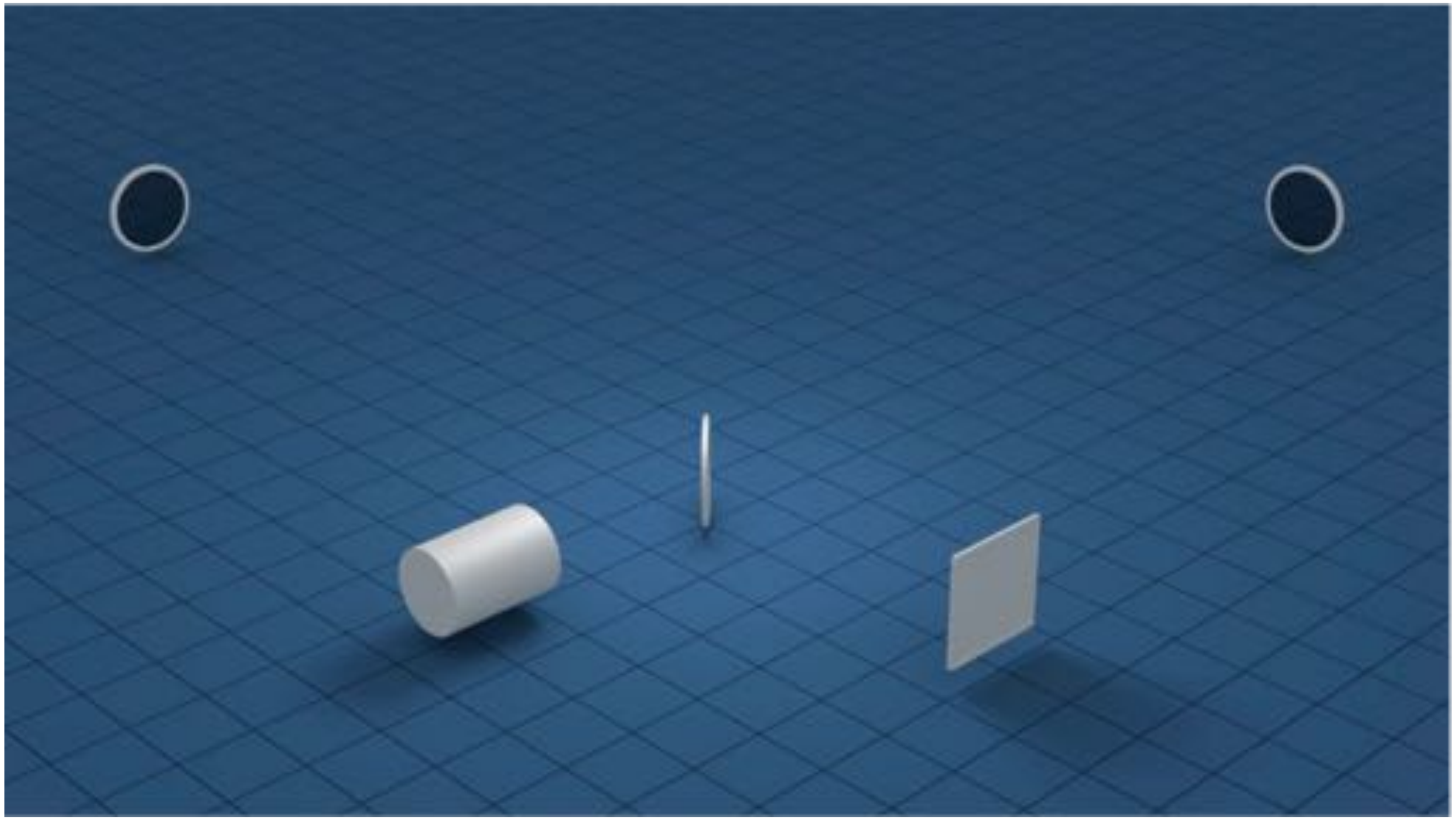
$$L_{GW} = -\frac{dE}{dt} = \frac{1}{5}\frac{G}{c^5}\sum_{ij}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle \approx \left(\frac{M}{R}\right)^5$$

BLACK HOLES:	M/R=0.5
NEUTRON STARS:	M/R~0.2
WHITE DWRFS:	M/R~10^{-4}

Gravitational Waves: Detectors

Rainer Weiss

Roland Drever



The “Flagship” Detectors

$$\frac{\Delta \ell}{\ell} = h$$

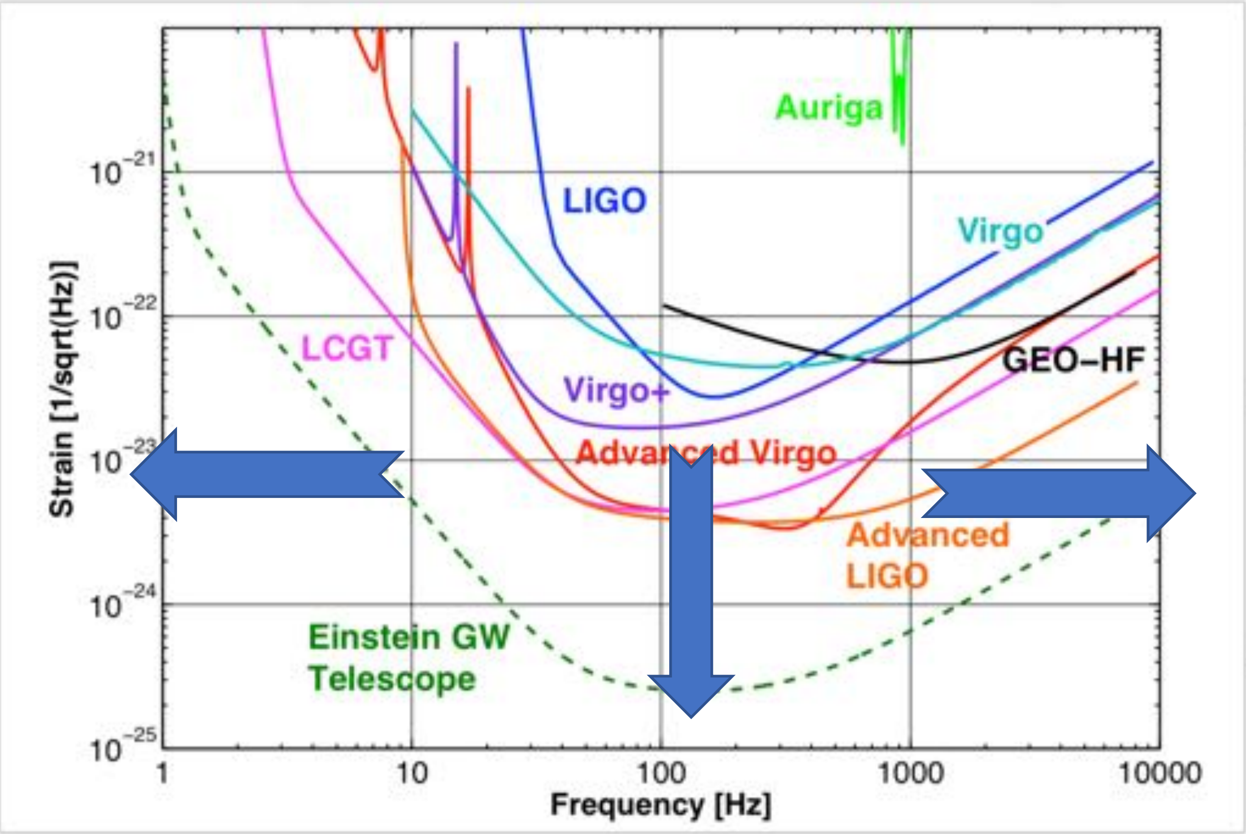
KAGRA:
Large-scale Cryogenic GW Telescope (Japan)

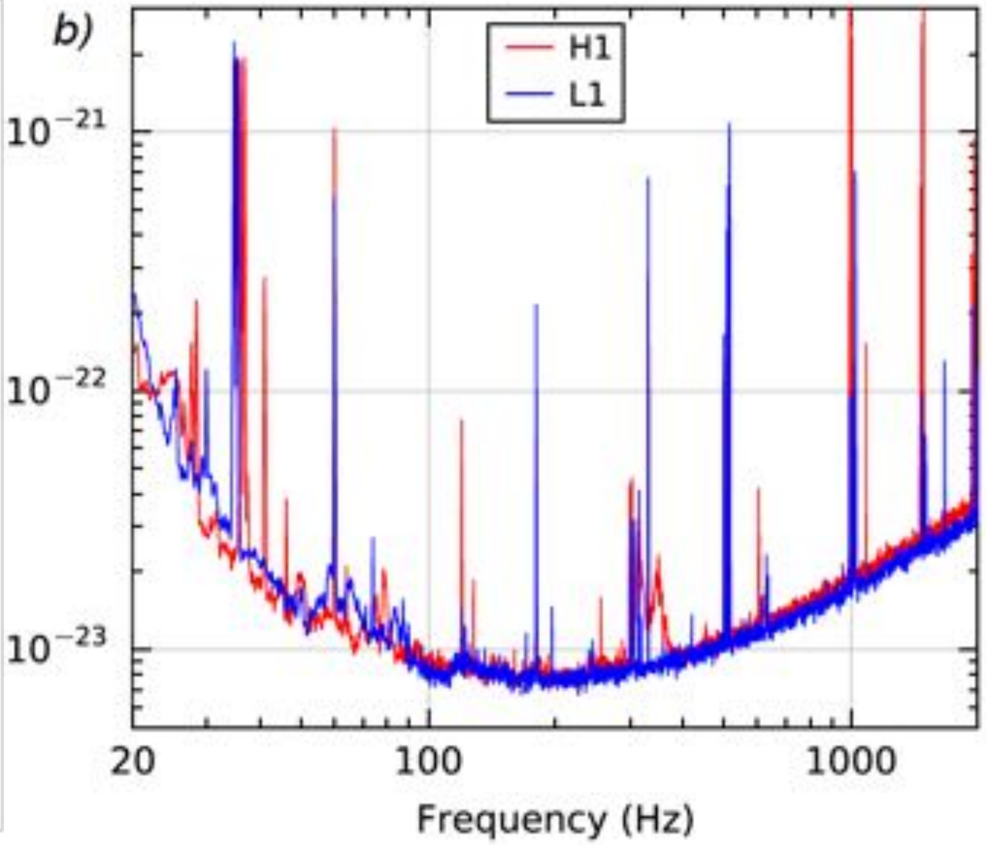
Maybe operational in 2019-20

- KAGRA consists of a modified Michelson interferometer with two 3-km long arms, located in the ground under Kamioka mine.
- The mirrors are cooled down to cryogenic temperature of -250 Celsius degree (20 Kelvin). Sapphire is chosen for the material of the mirror.

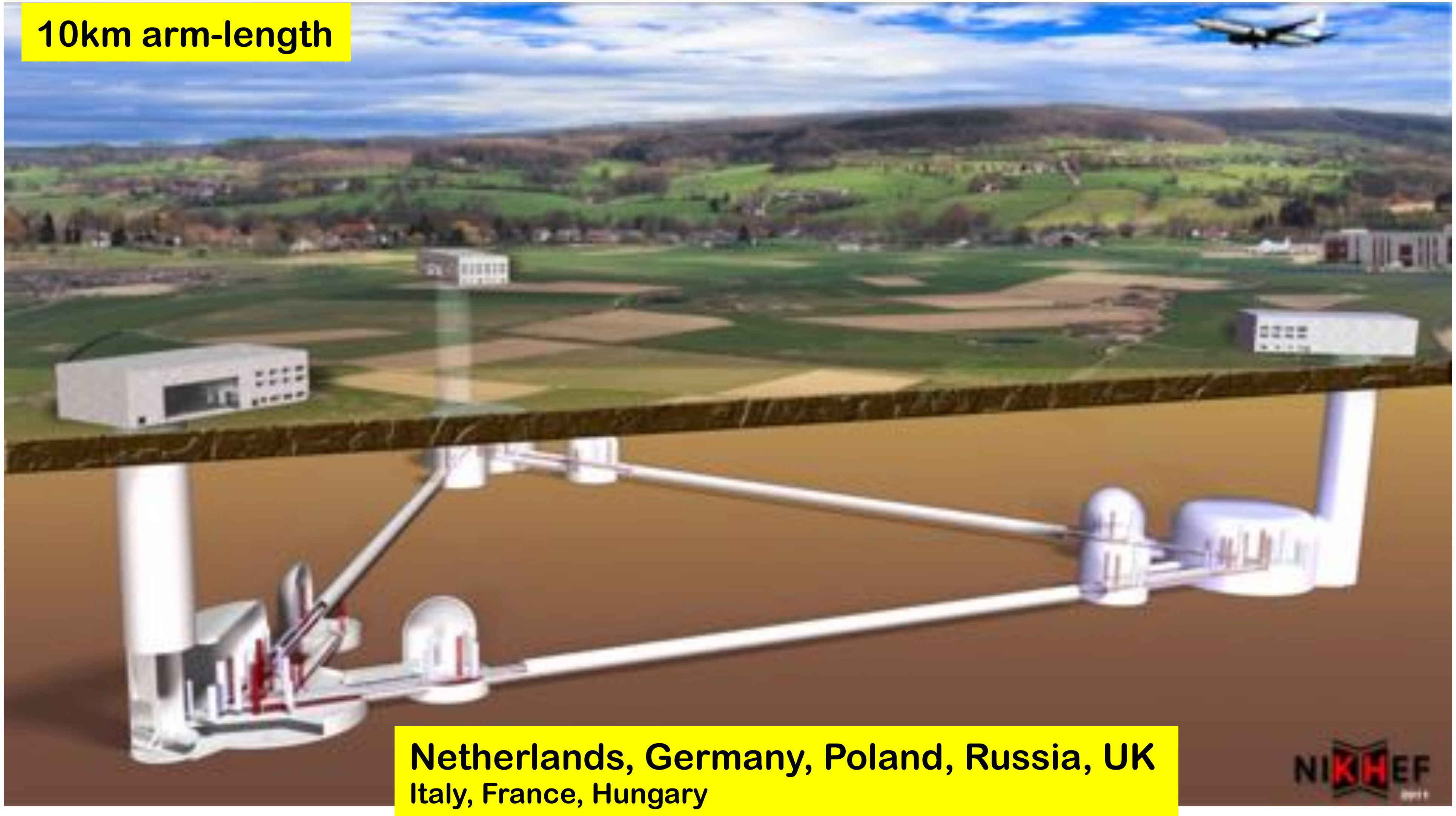
INDIGO (India-USA) : Similar to existing LIGOs

Detector Sensitivity Curves





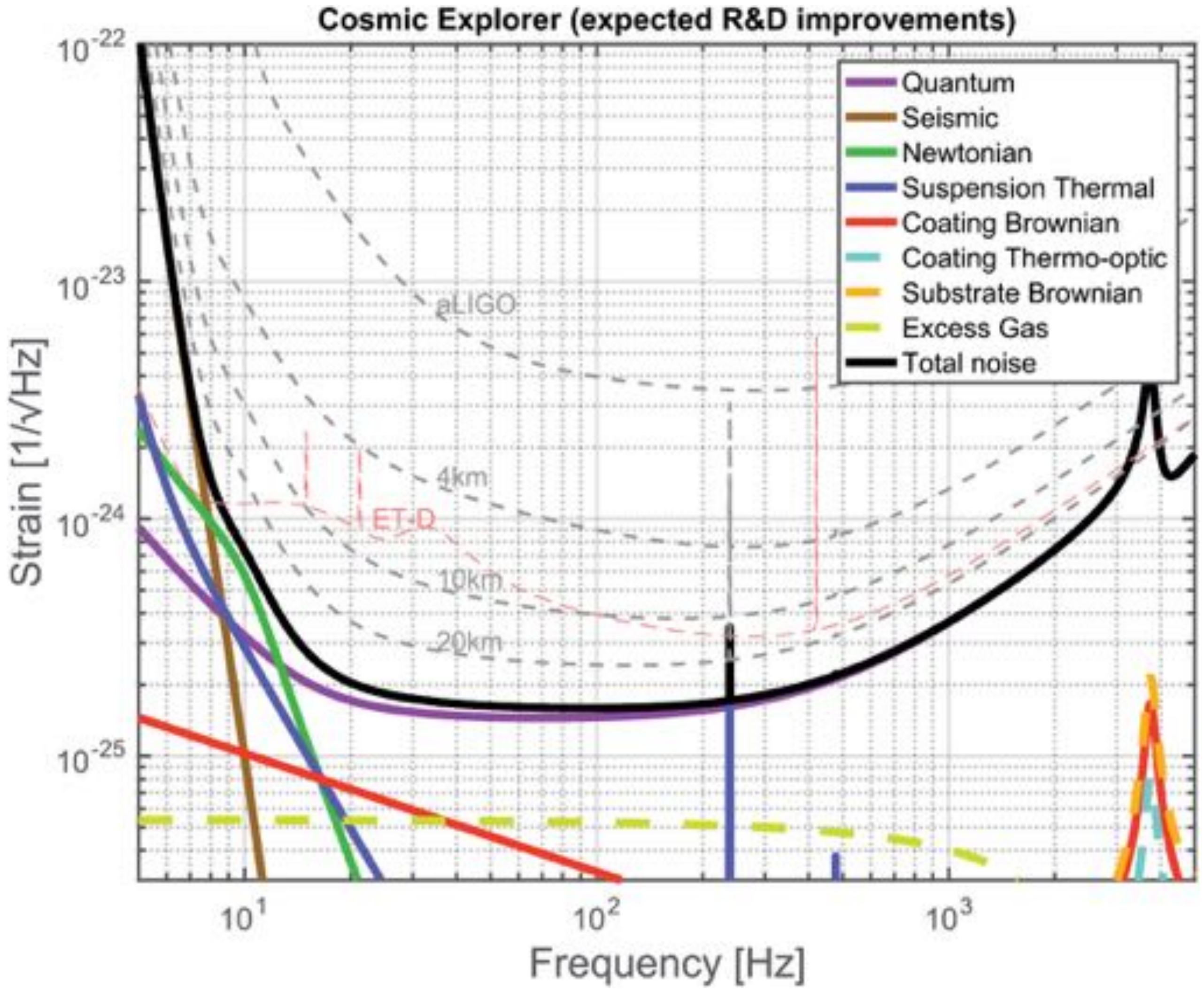
Einstein Telescope (2030+)



Cosmic Explorer (2030+)

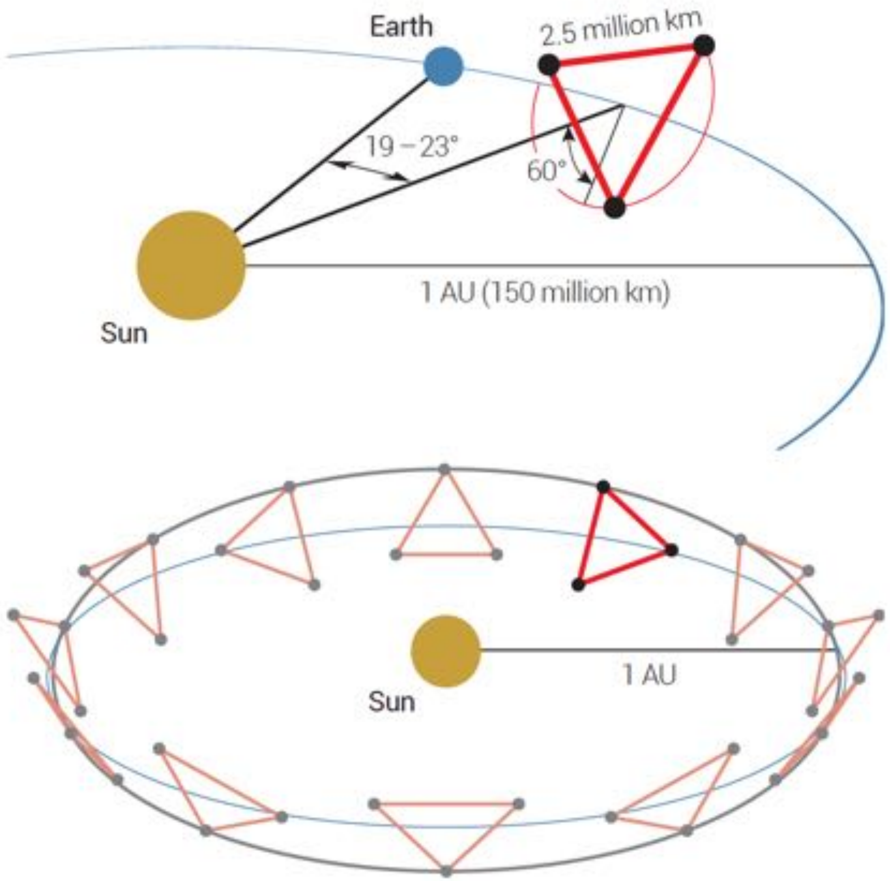
Exploring the sensitivity of next generation gravitational wave detectors

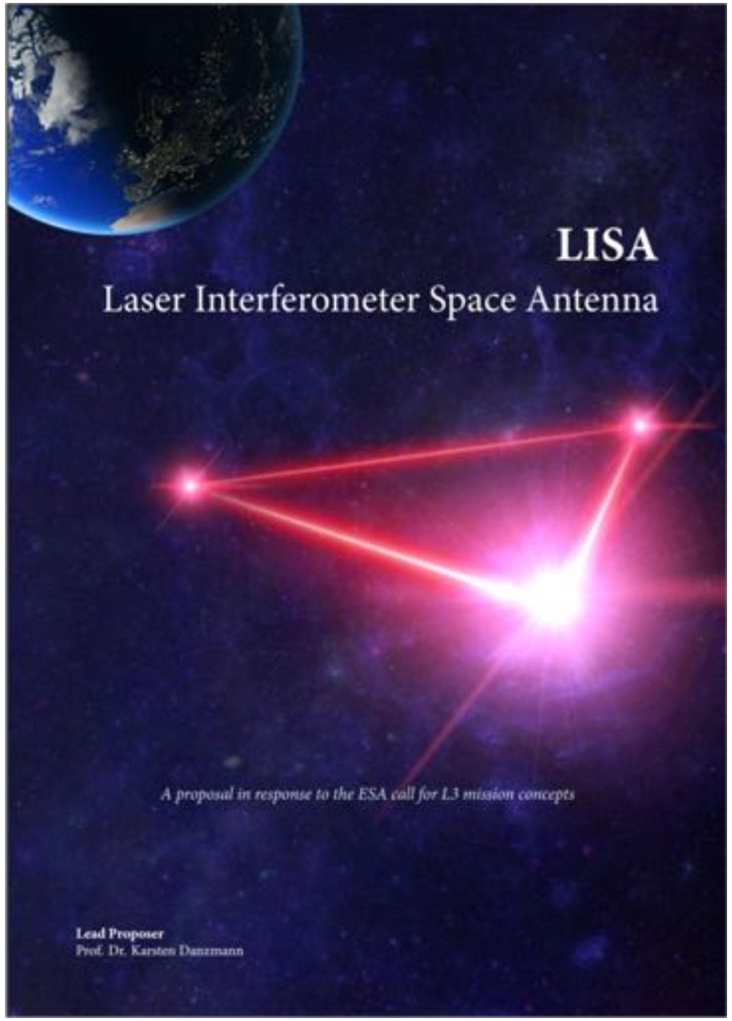
B P Abbott et al 2017 Class. Quantum Grav. 34 044001



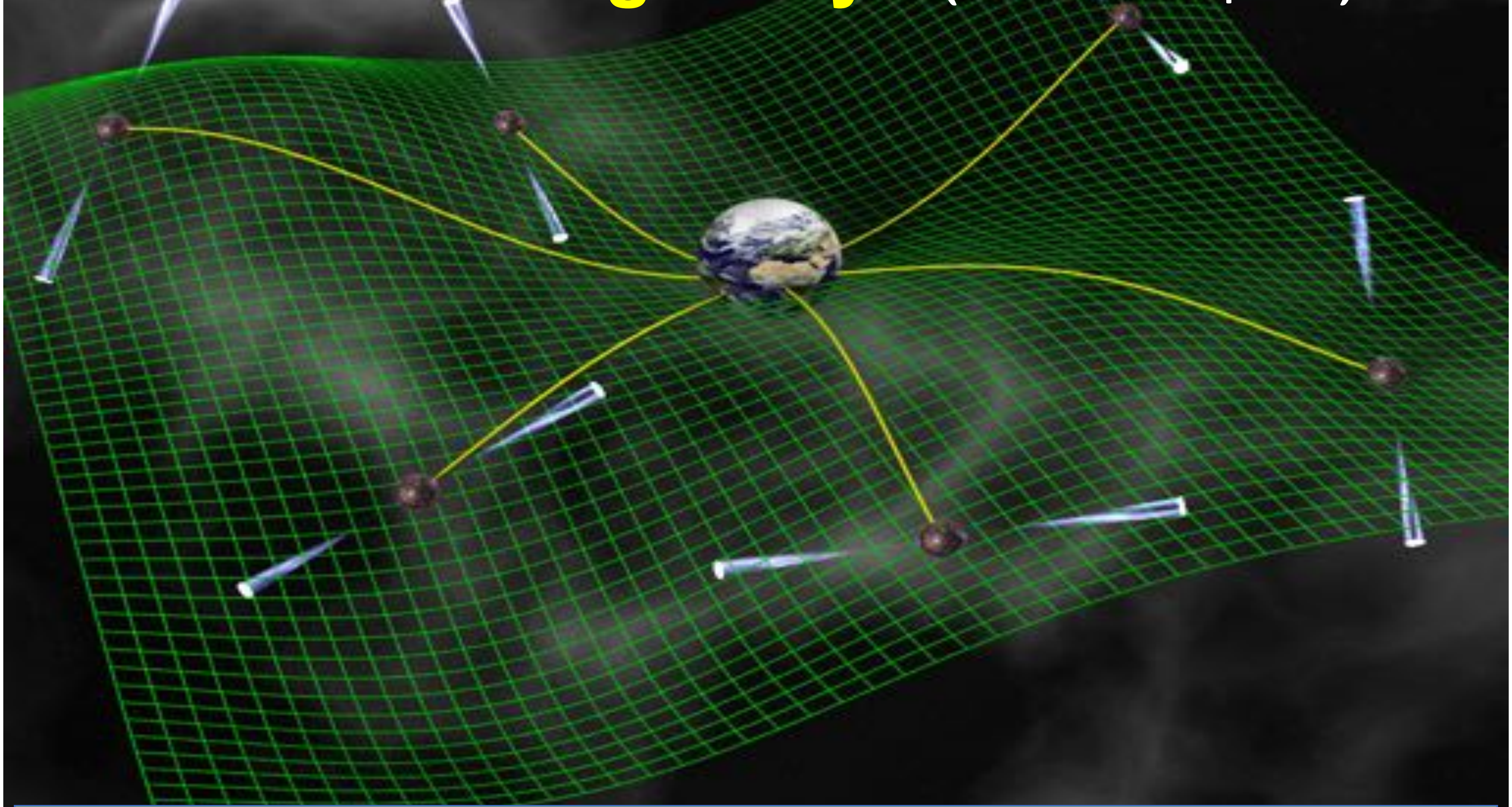
LISA: the Space Detector (2030+)

(January 2017)



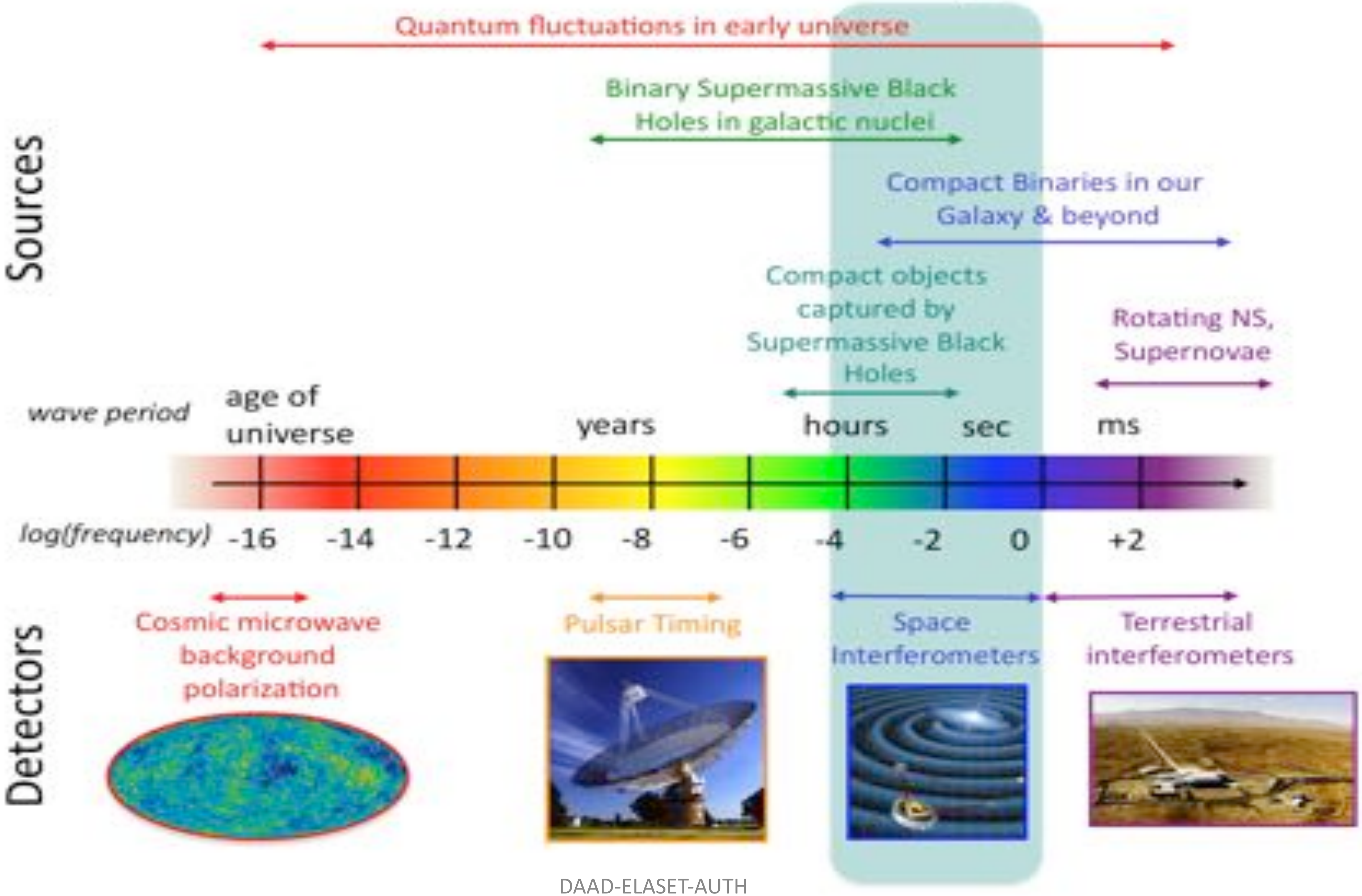


Pulsar Timing Arrays (10-1000 μHz)

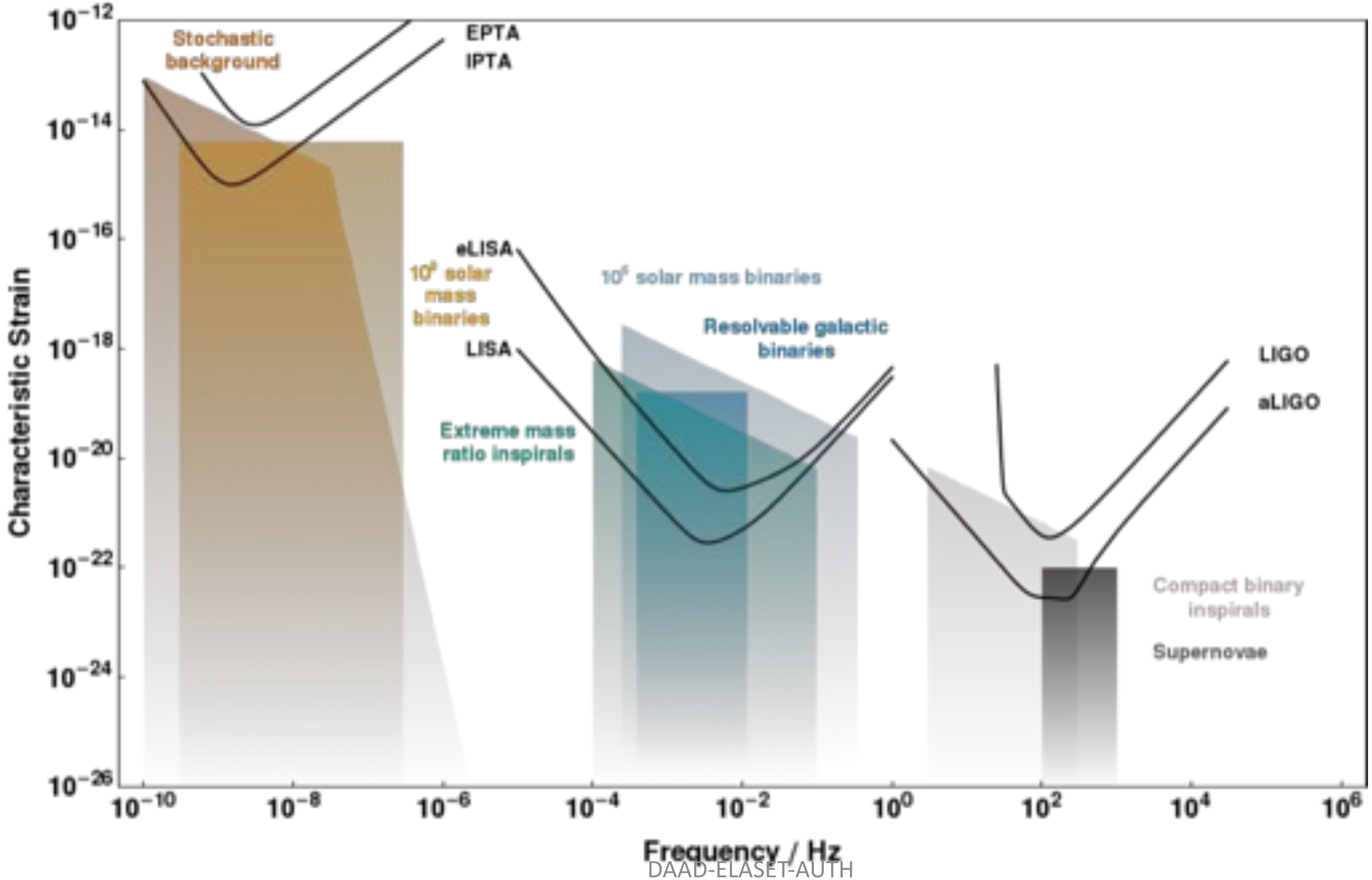


Available pulsar data have already placed limit of about **h<10^-13** on the amplitude of such low-frequency GWs

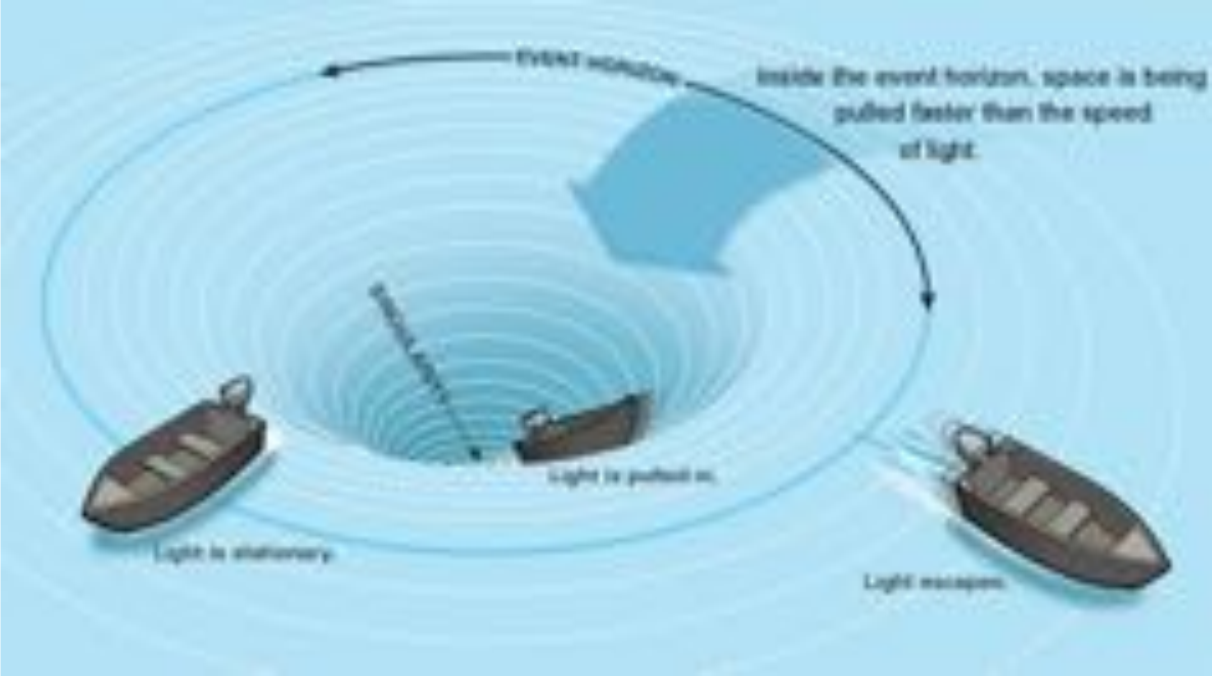
The Gravitational Wave Spectrum

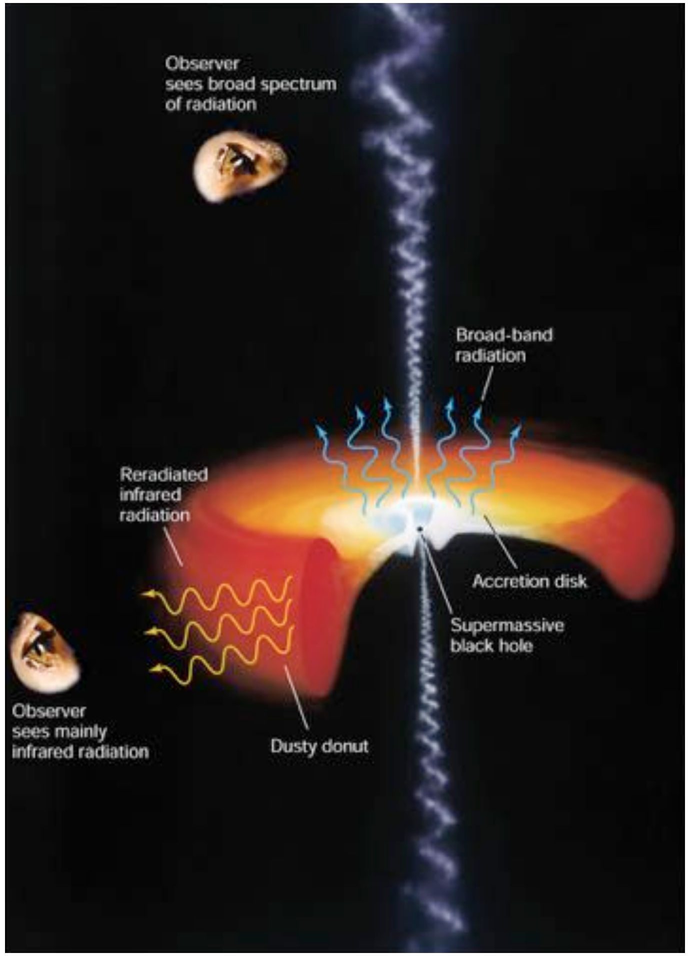


The Gravitational Wave Spectrum



Black Holes





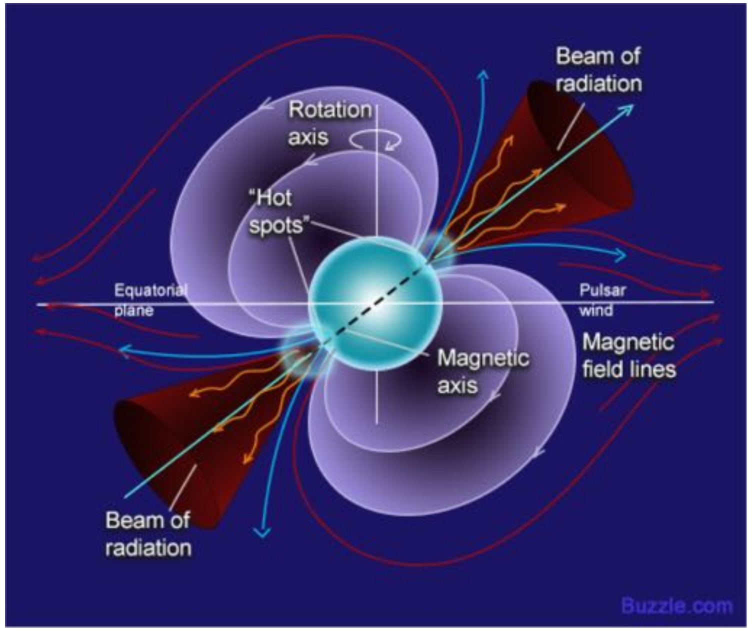
- **In the 60s : Suspicions**
- **In the 70s : Indications**
- **In the 80s : Proofs**

Most galaxies host a super-massive black hole in their center
Largest known black hole ~6.6×10^{10} solar masses

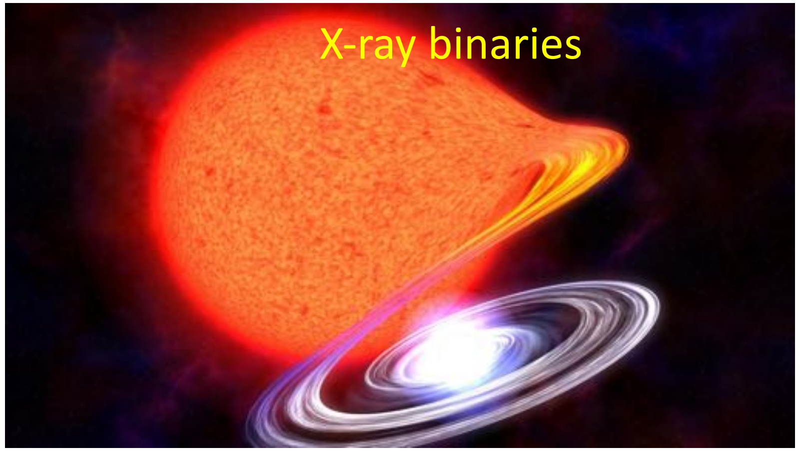
Neutron Stars

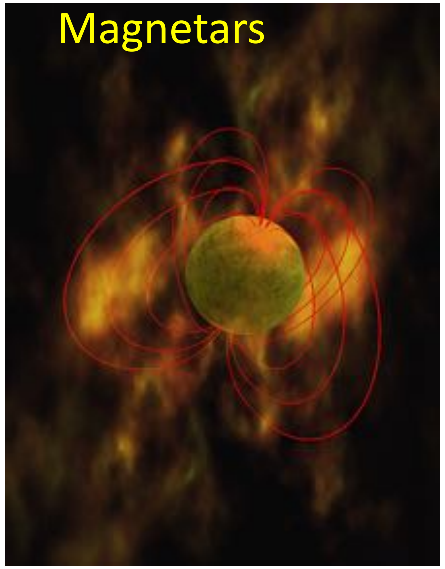
- They are the **most compact stars** known to exist in the universe.
- They have **densities equal to that of the early universe** and **gravity similar to that of a black hole.**
- Most extreme magnetic fields known in the universe up to **10^{16} G**.

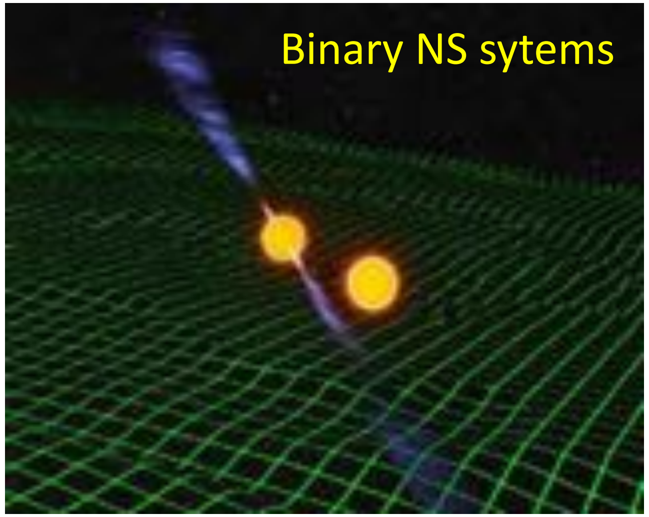
Conjectured	1931
Discovered	1967
Known	2500+
Mass	$1.2–2M_\odot$
Radius	8-14 km
Density	10^{15}g/cm^3
Spin	< 716 Hz
In our Galaxy	~10^8

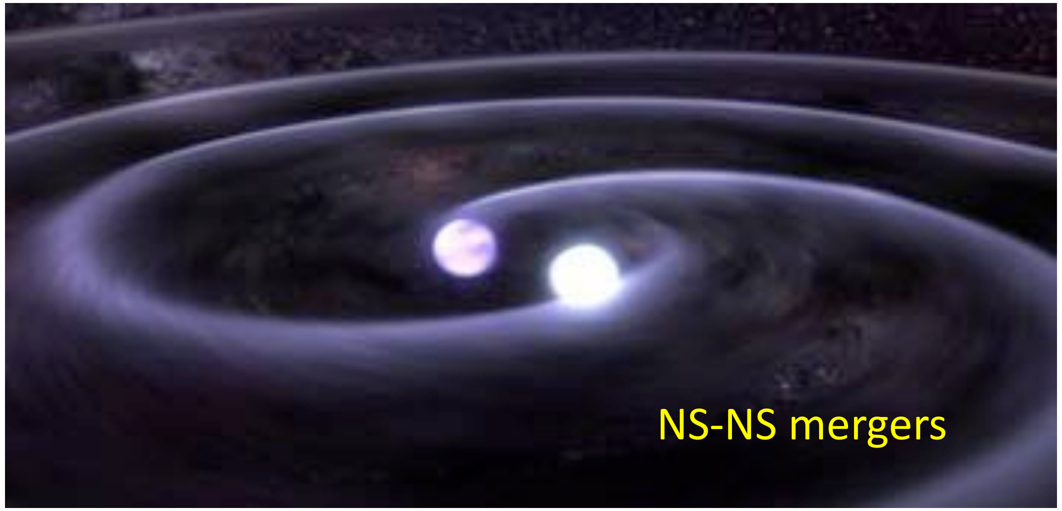


The Many Faces of Neutron Stars

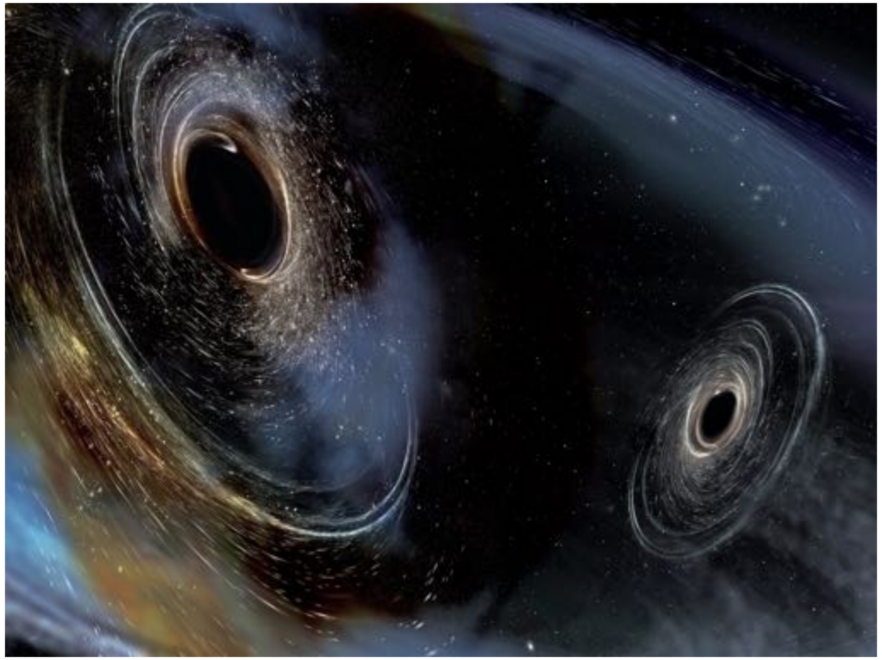


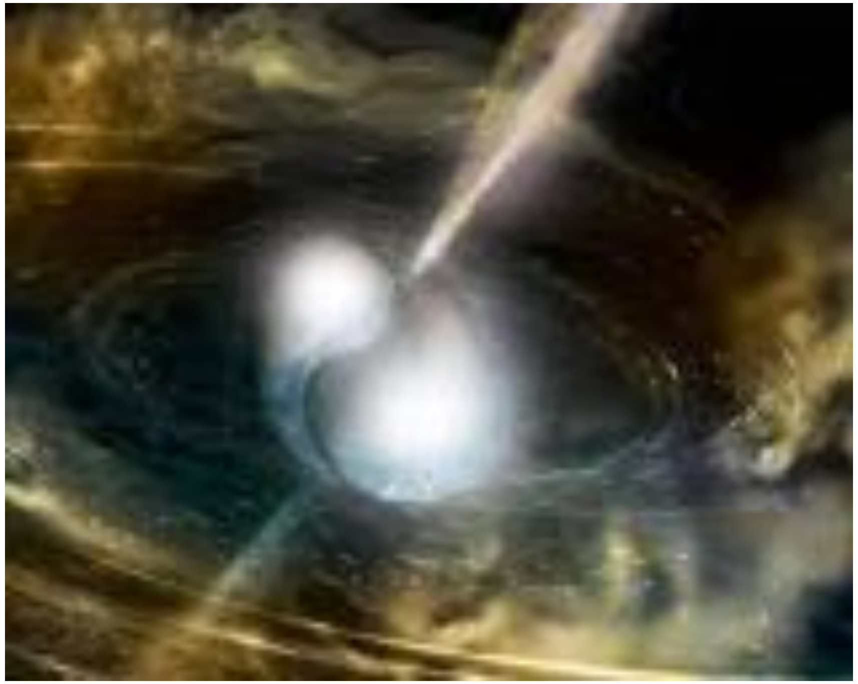






Typical masses ~1.2-2 $M_{\odot}$
Typical Radius ~9-14 km





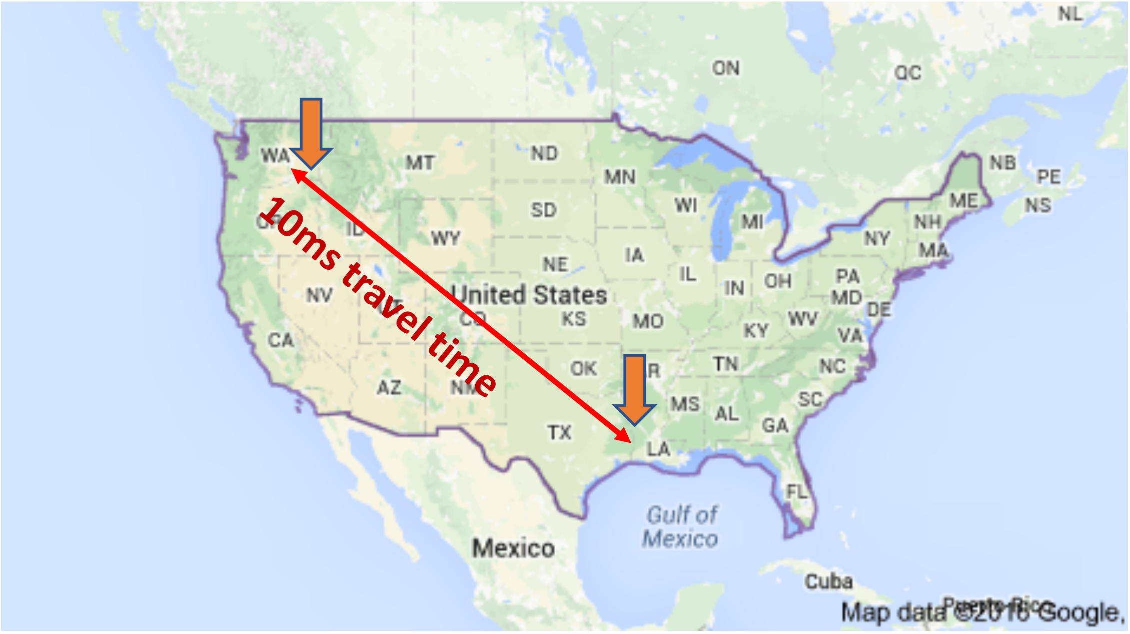
Recent Discoveries

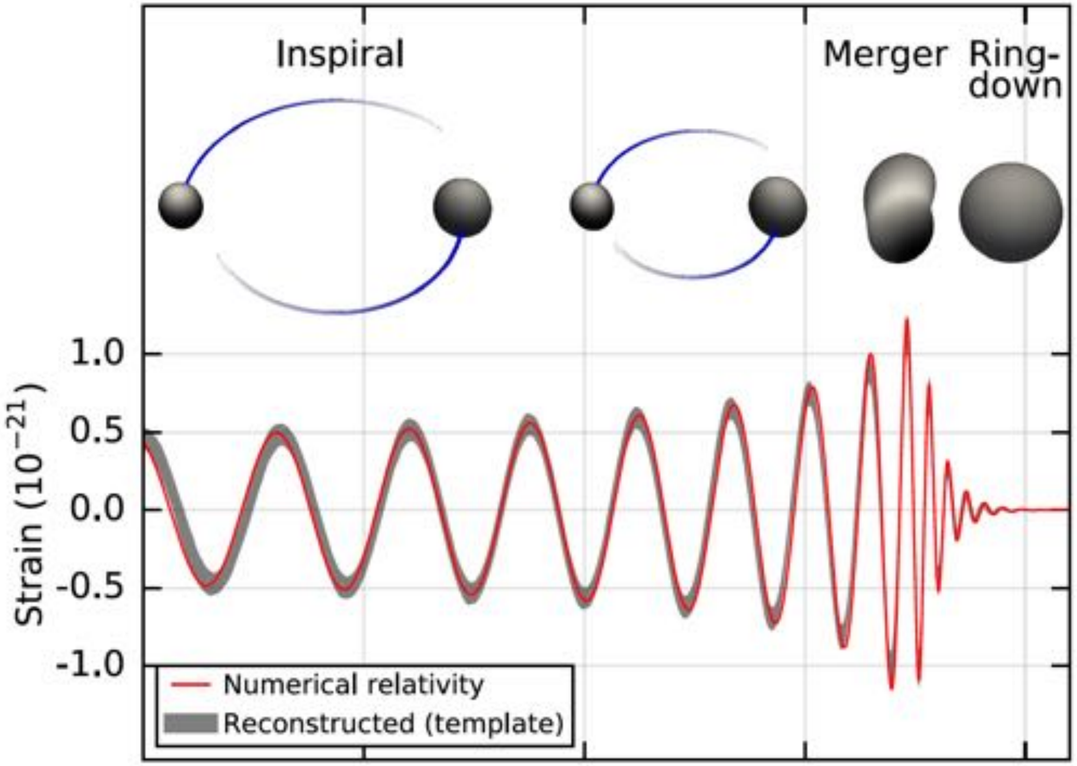
….our dream has come true

What Happened 1.3 Billion Years ago

Credits LIGO

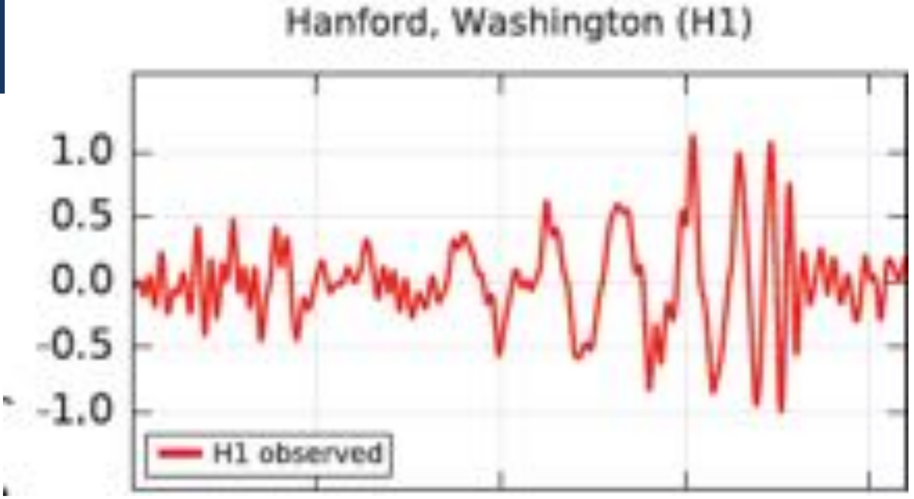
The Event: 14.9.2015 (09:50:45 UT)



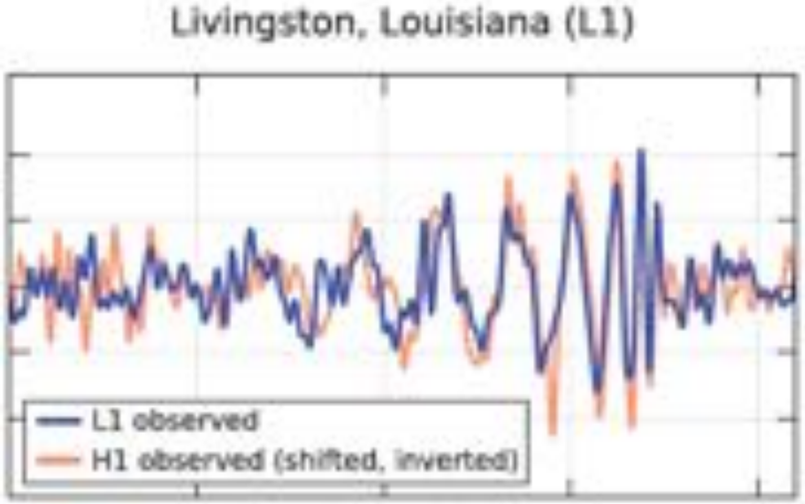


What did they Observe?

M_1	: $36^{+5}_{-4}\ M_{\odot}$
M_2	: $29^{+4}_{-4}\ M_{\odot}$
S/N	: **~24**
Spin	: 0.67
Final Mass	: $62^{+4}_{-4}\ M_{\odot}$
Distance	: $410^{+160}_{-180}\ Mpc$
	~1.3 x 10^9 light years
Redshift	: z~$0.09^{+0.03}_{-0.04}$



GW150914



SIGNAL from BINARIES: PRIMER

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$ Chirp mass

$$h \approx \frac{1}{D} M_c^2$$ AMPLITUDE

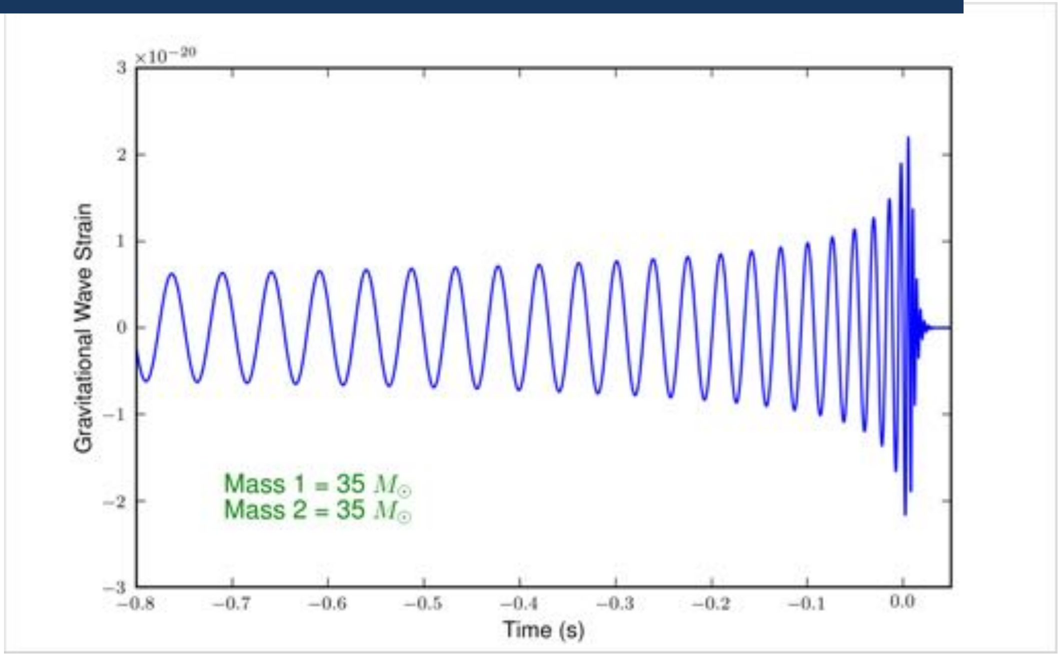
$$\dot{f} \approx f^{5/3} M_c^{-1/3}$$ Chirp

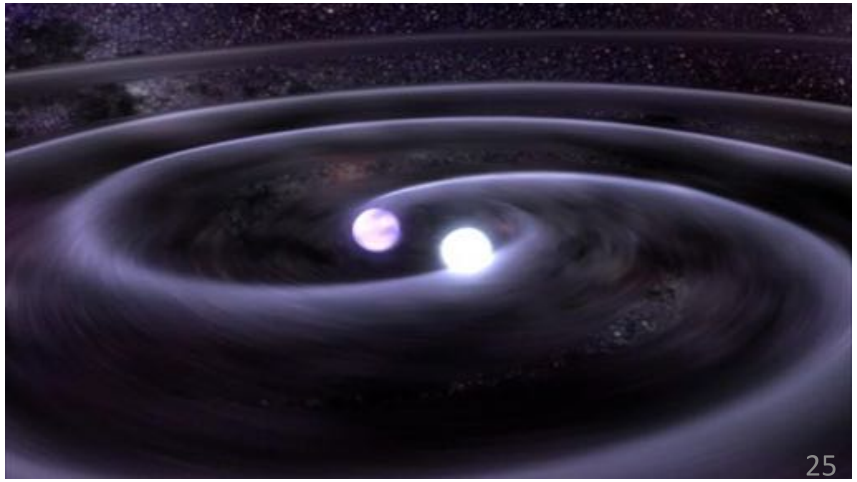
$$\boxed{M_c \approx \left(f^{-11/3}\dot{f}\right)^{3/5} = f^{-11/5}\dot{f}^{3/5}}$$ Chirp mass

$$\boxed{D \approx \frac{1}{h}\left(\frac{\dot{f}}{f^3}\right) = \frac{1}{h} M_c^{5/3} (f)^{2/3}}$$ DISTANCE

$$\mathrm{v} \approx f^{1/3} M^{1/3}$$ VELOCITY

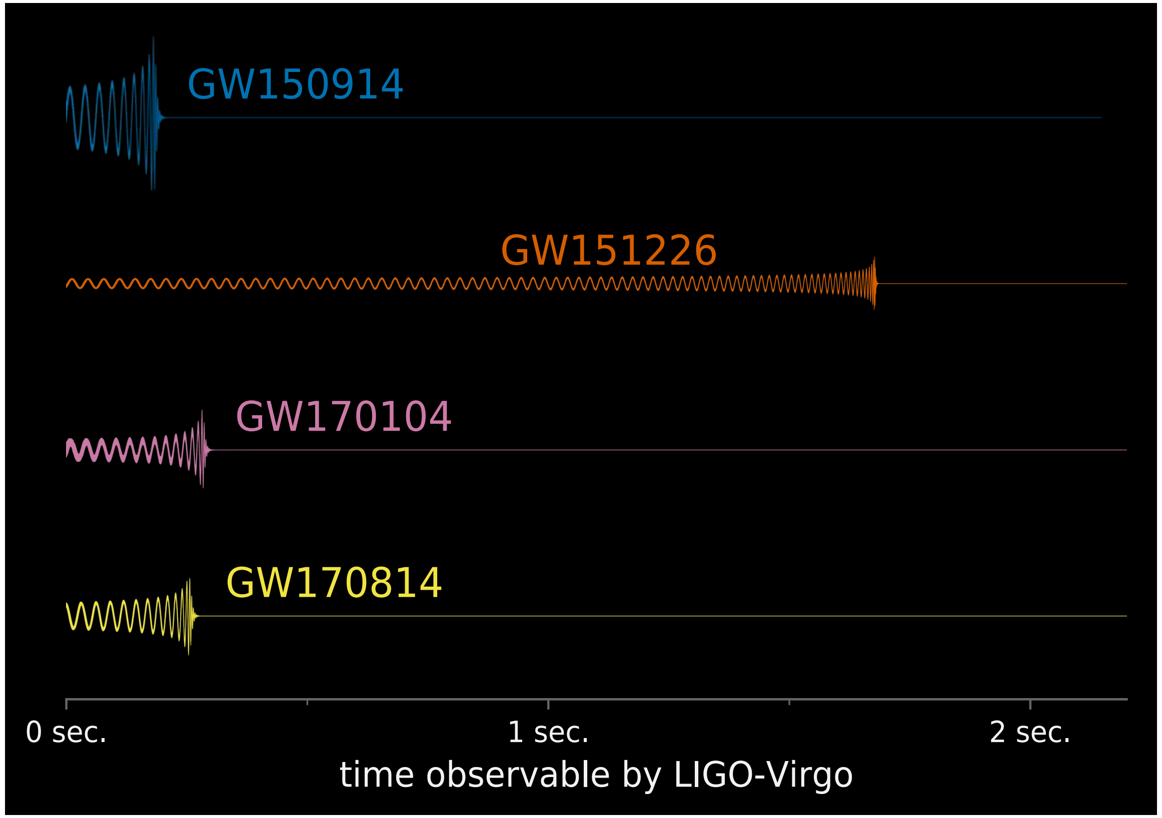
$$\eta = \frac{m_1 + m_2}{(m_1 \ + m_2\ )^2}$$





BORING: Binary BH mergers again and again!!

	SNR	M_1 ($M_\odot$)	M_2 ($M_\odot$)	D (Mpc)	Spin (final)
GW150914	23.7	36	29	440	68%
LVT151012	9.7	23	13	1000	66%
GW151226	13	14.2	7.5	440	74%
GW170114	10	31.2	19.4	880	64%
GW170608	13	12	7	340	69%
GW170814	(18)	30.5	25.3	540	70%



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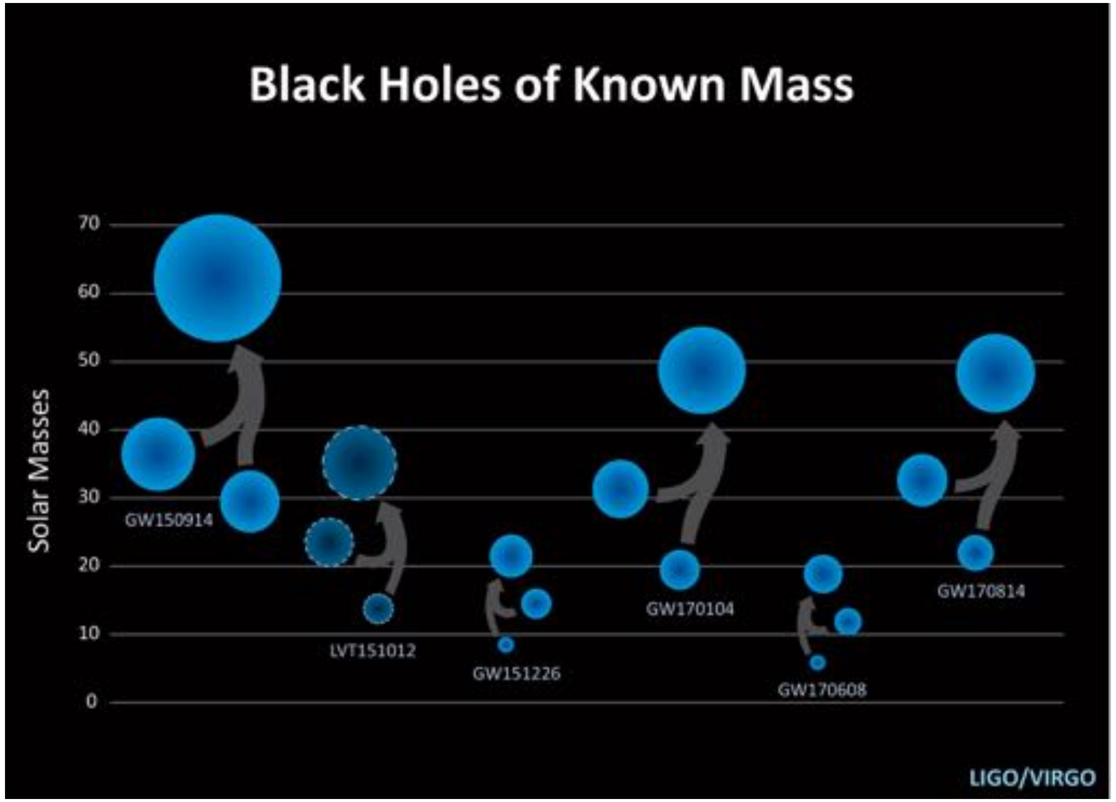
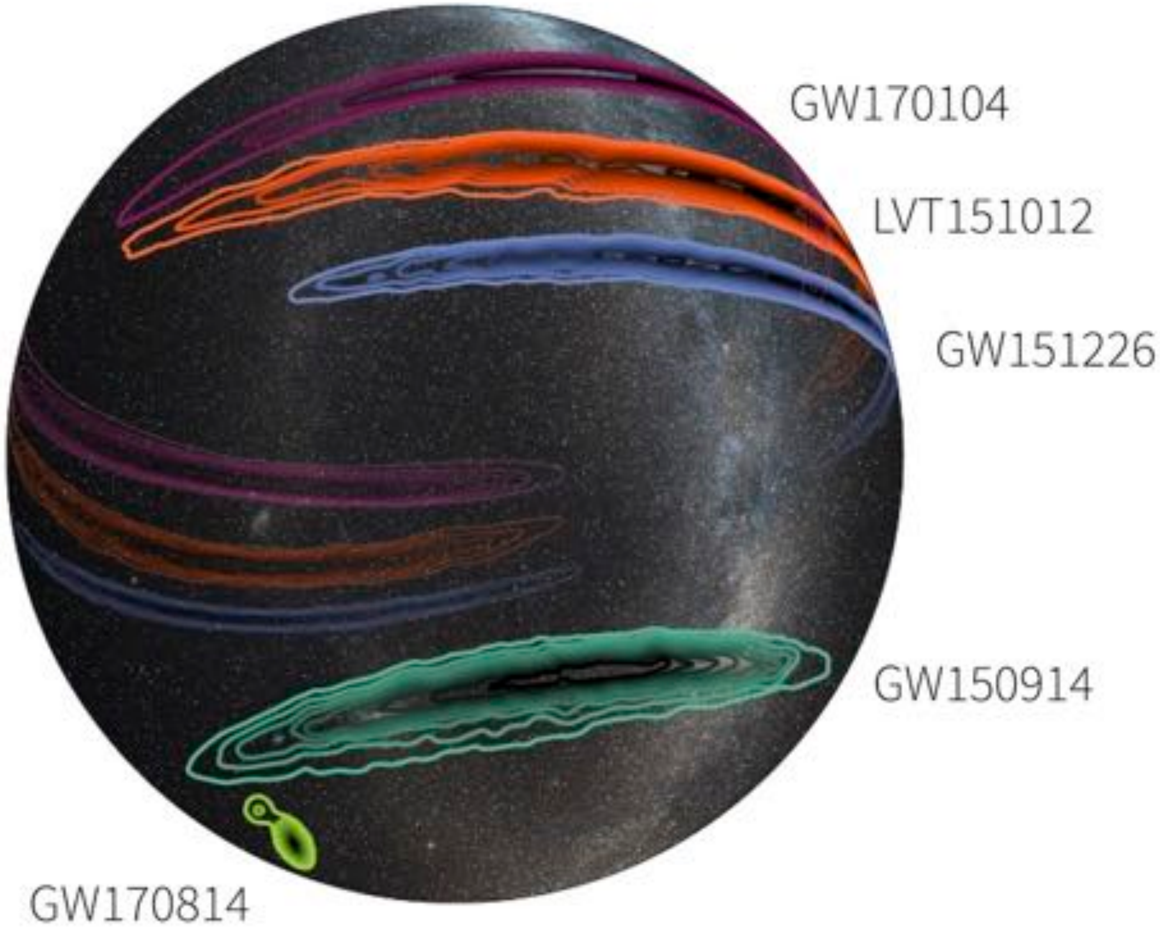


Positioning the Sources

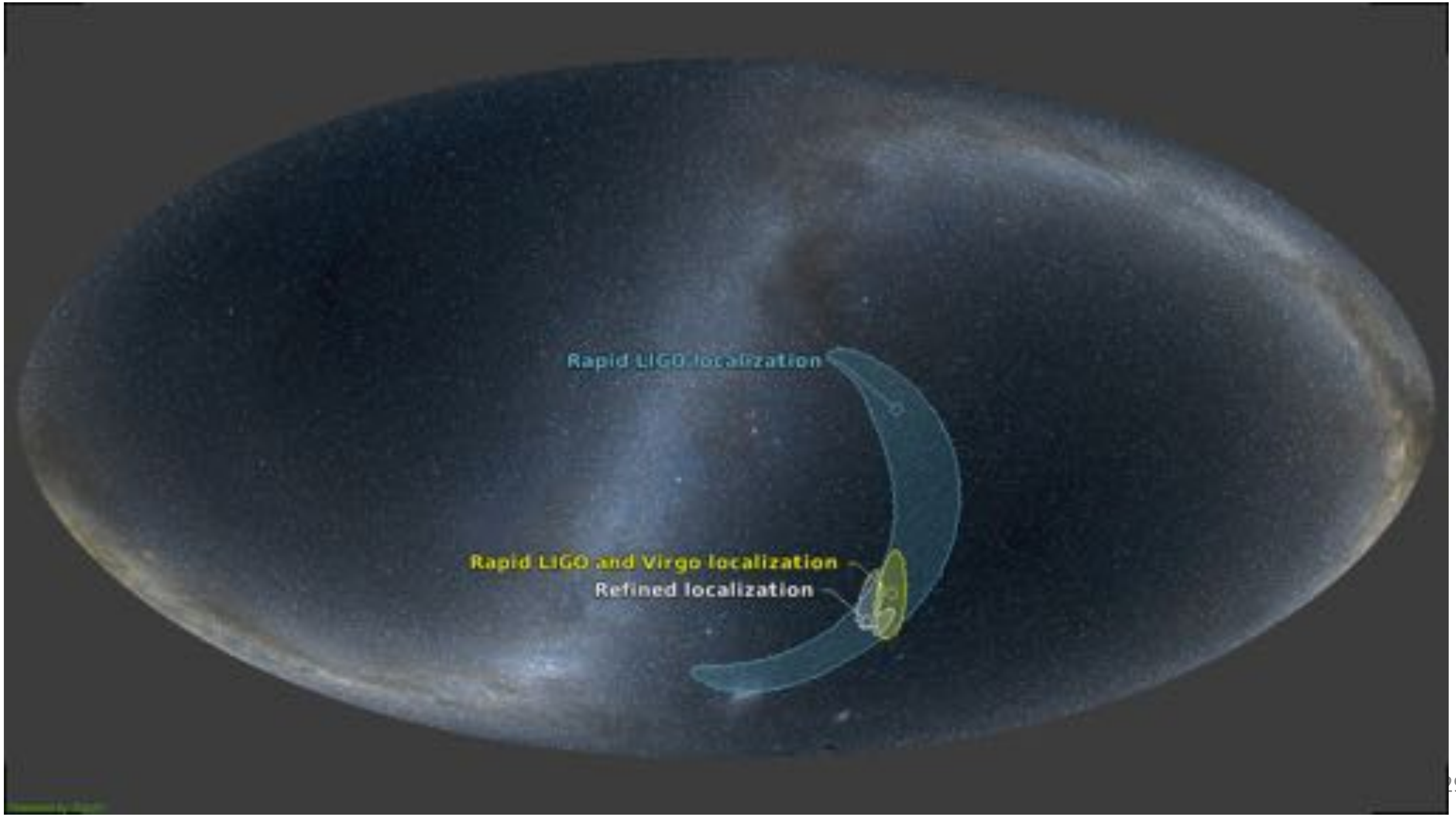


Positioning the Sources (GW170814)

1160 deg^2

60 deg^2

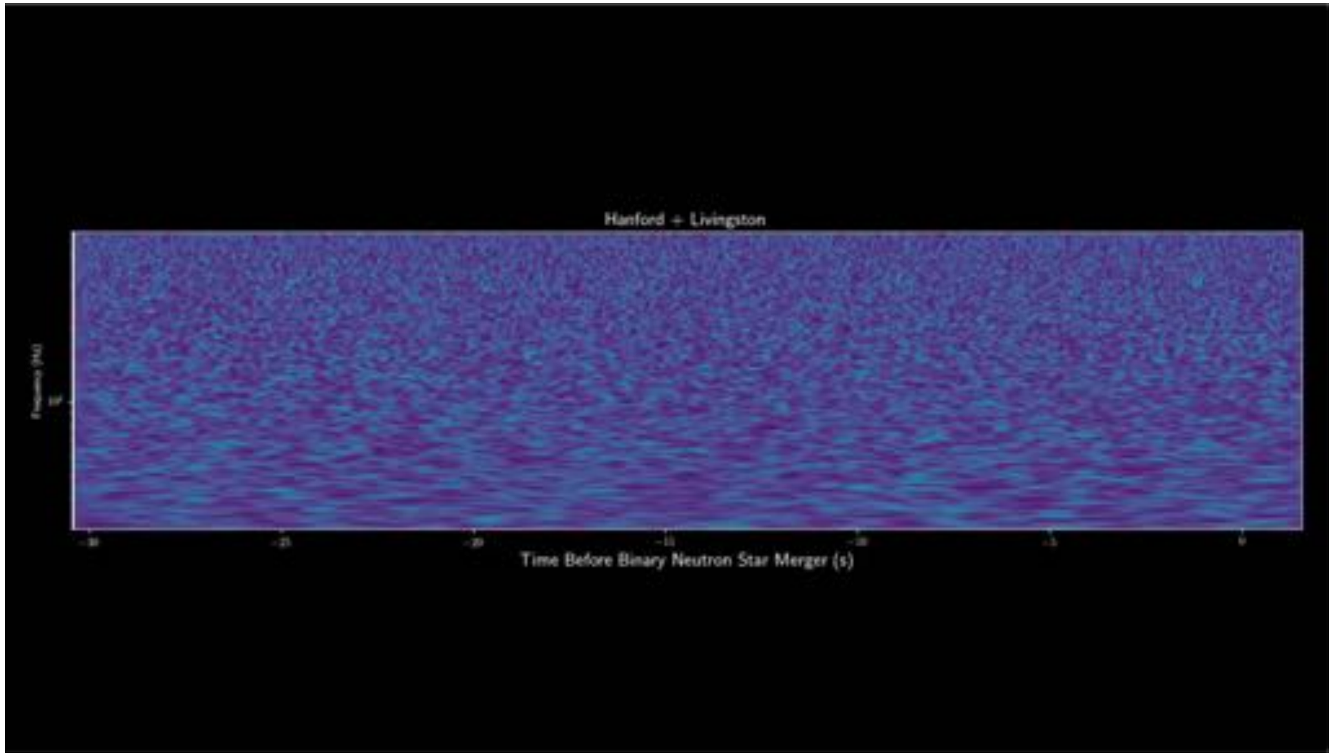
In the direction of Eridanus constellation

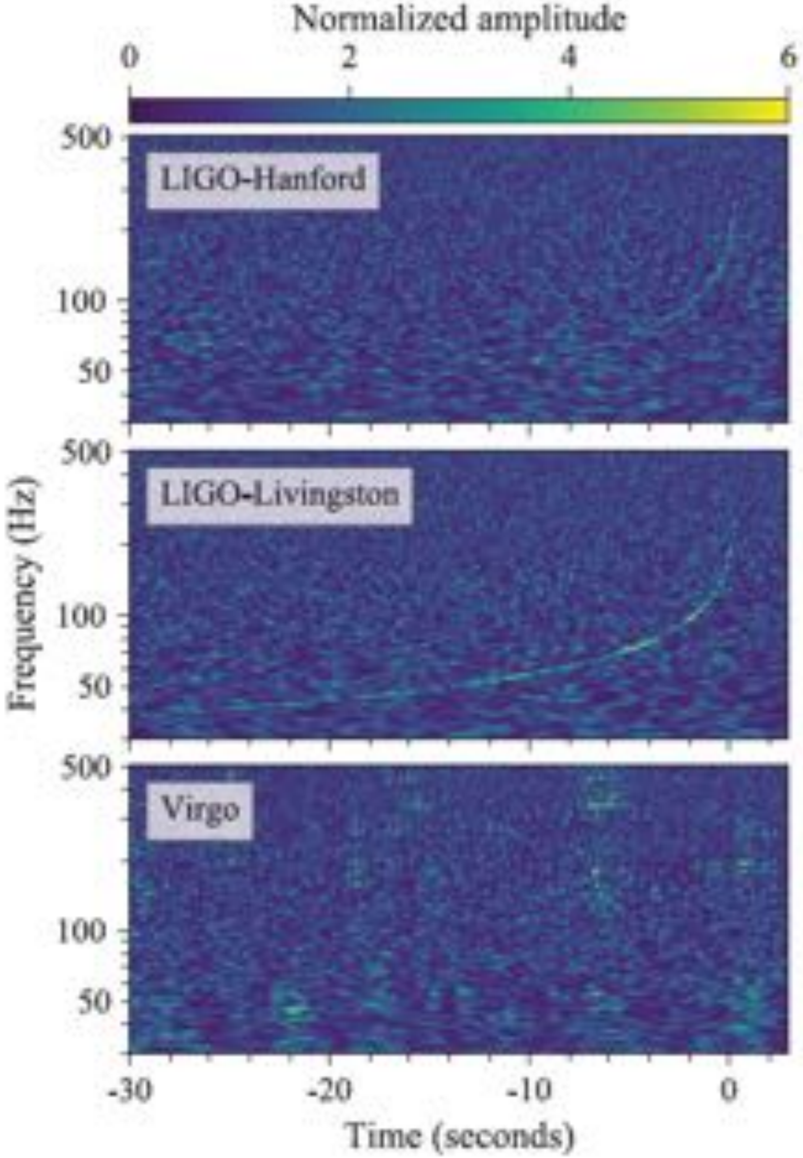


Prepare for the Unexpected

GW170817

Neutron Star Mergers
GW170817

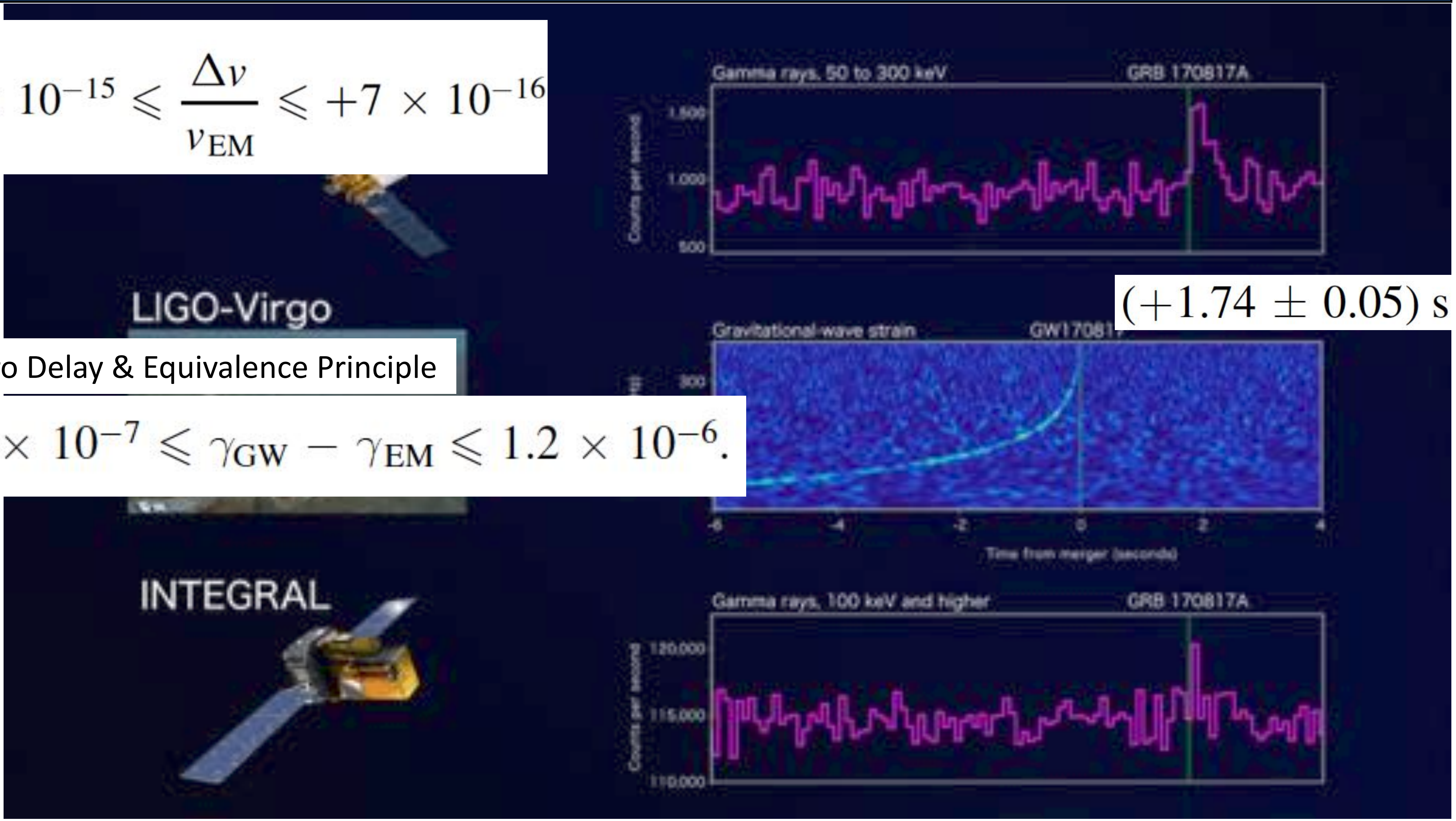




M_1~1.36-1.60 $M_{\odot}$
M_2~1.17-1.36 $M_{\odot}$
Radiated energy ~0.025 $M_{\odot}\ c^2$
Luminosity distance~ 40 Mpc

Neutron Star Mergers
constraining the speed of gravity

$$-3 \times 10^{-15} \leqslant \frac{\Delta v}{v_{\mathrm{EM}}} \leqslant +7 \times 10^{-16}$$

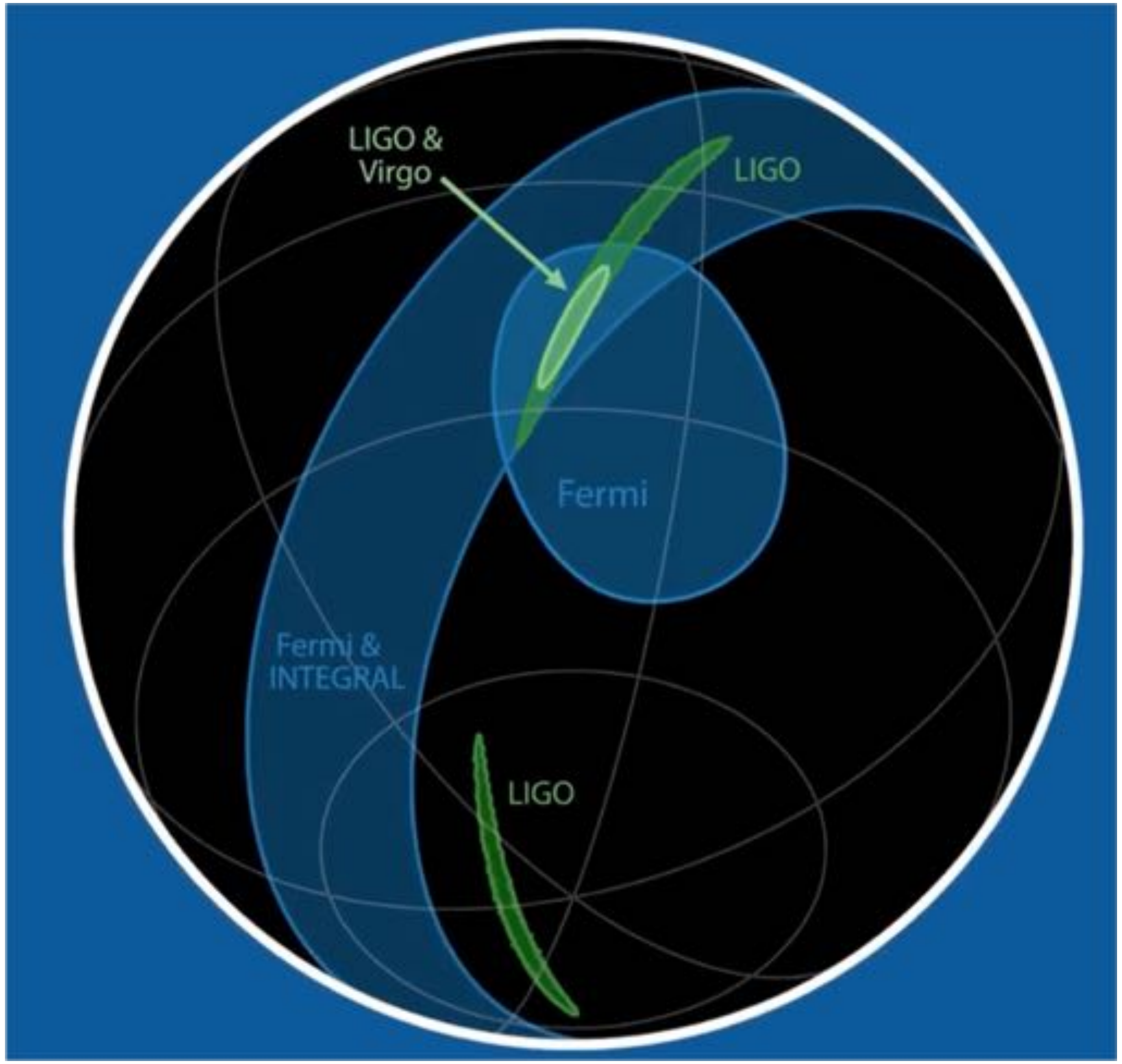


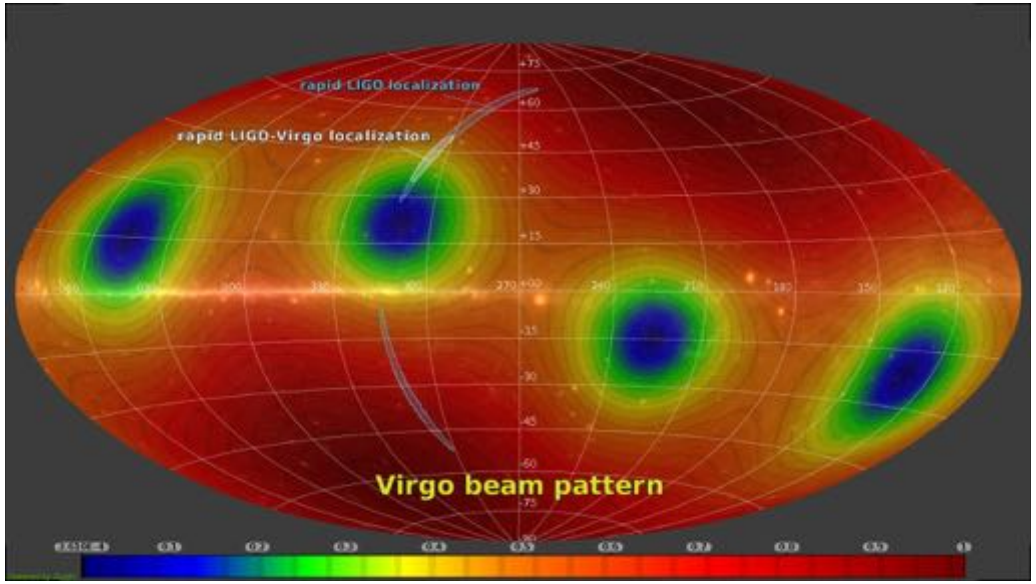
$(+1.74 \pm 0.05)$ s

Shapiro Delay & Equivalence Principle

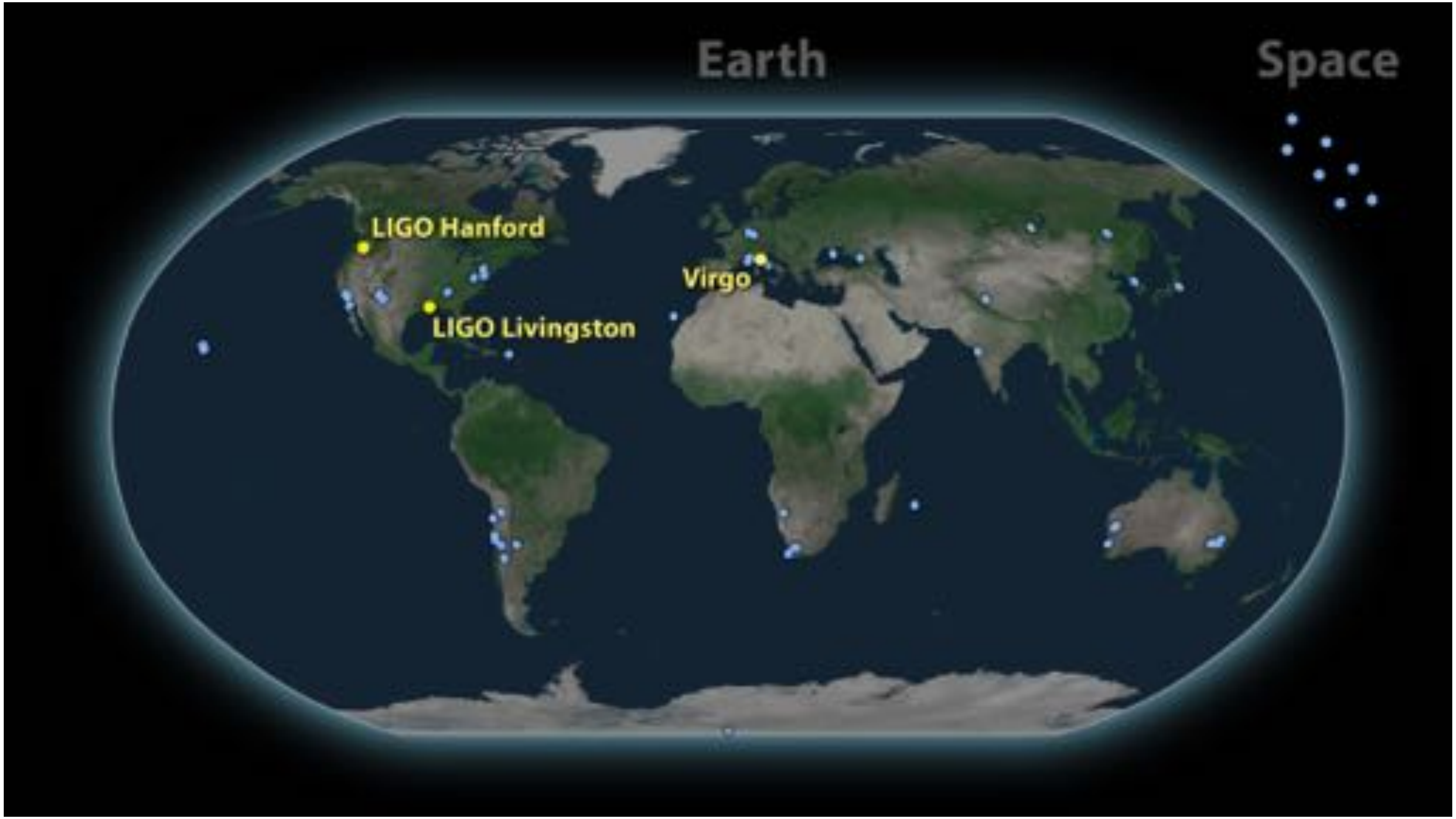
$$-2.6 \times 10^{-7} \leqslant \gamma_{\mathrm{GW}} - \gamma_{\mathrm{EM}} \leqslant 1.2 \times 10^{-6}.$$

Neutron Star Mergers
Positioning the source





Neutron Star Mergers
GW - EM Observatories

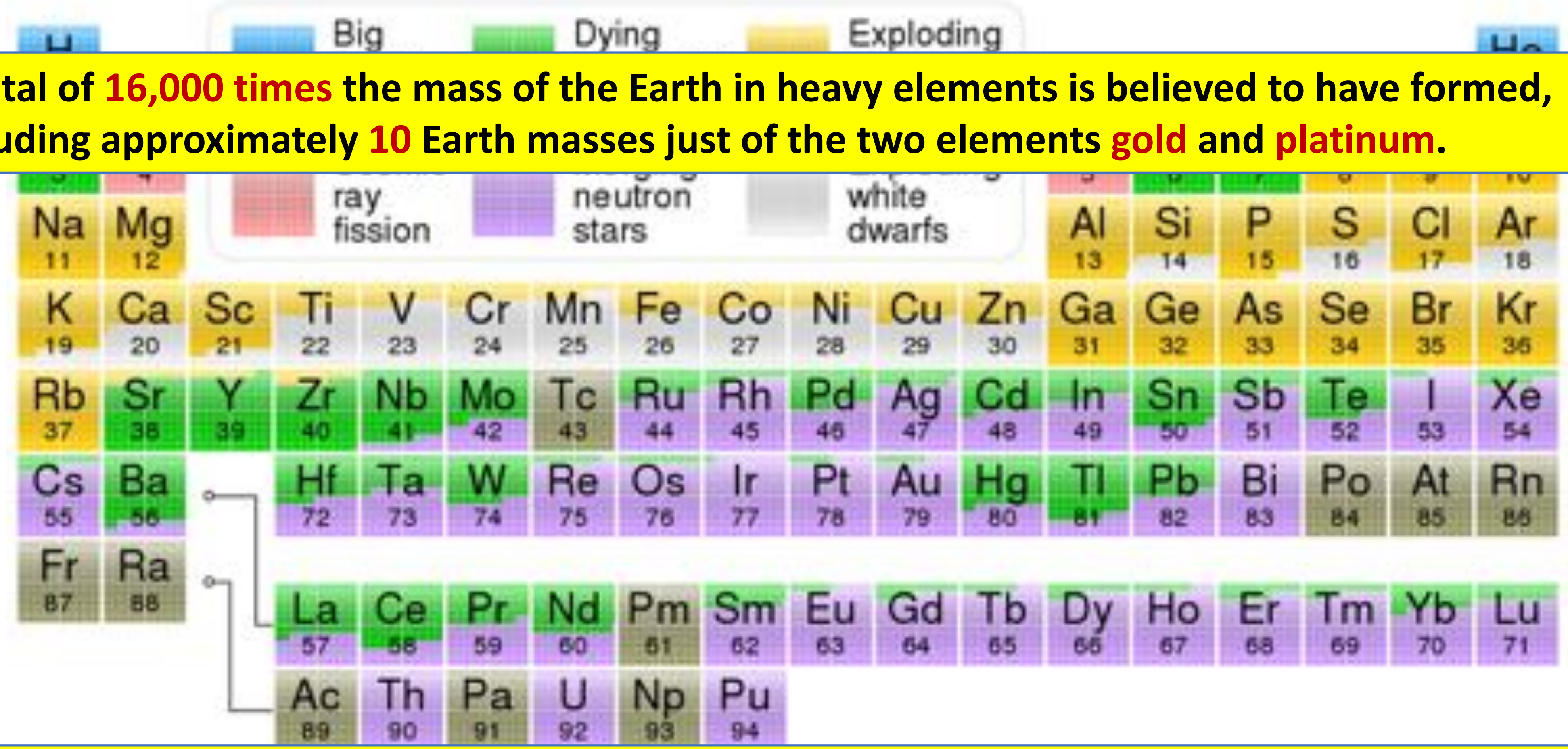


Neutron Star Mergers
Sequence of Events

12:41:04 UTC	A **Gravitational Wave** from binary NS merger is detected
+2 sec	A **Short Gamma ray** burst is detected
	GW + EM from the same source provide compelling evidence that **GWs travel with the speed of light**
	The two events allow for the **measure of the expansion rate of the Universe**
	Kilonova: neutron star mergers responsible for the production of heavy elements in the universe.
+10 h **52** min	A new bright source of **optical light** is detected in a galaxy called NGC 4993 (constellation of Hydra)
+11 h **36** min	**Infrared emission** observed
+15 h	**Bright ultraviolet** emission detected
+9 days	**X-ray** emission detected
+16 days	**Radio** emission detected

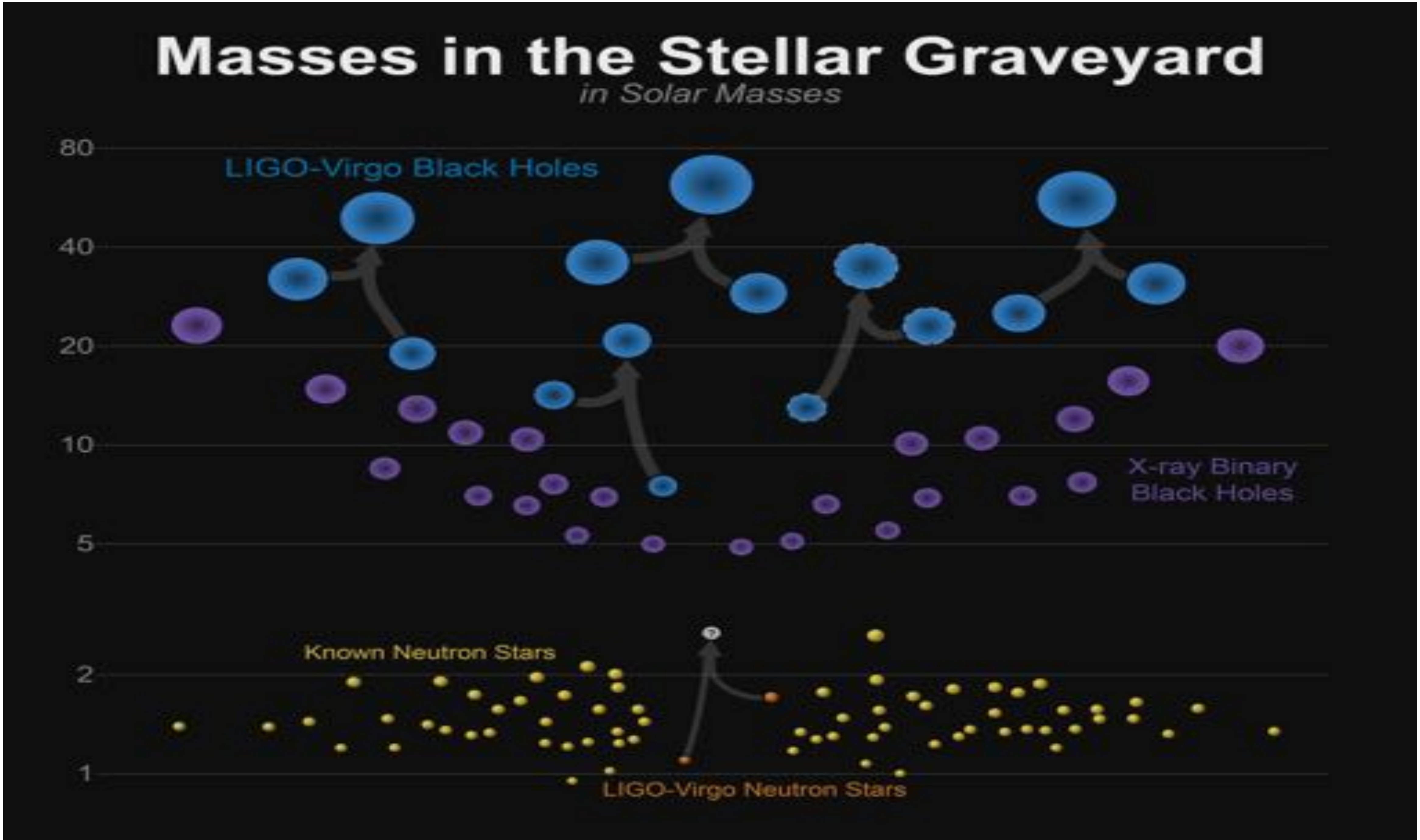
Neutron Star Mergers

Alchemy or Heavy Element Production



A total of 16,000 times the mass of the Earth in heavy elements is believed to have formed, including approximately 10 Earth masses just of the two elements gold and platinum.

Light emitted after a neutron star collision showed signs of heavy elements present in the aftermath, confirming that certain elements (purple) are produced in such mergers


Masses in the Stellar Graveyard
in Solar Masses
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
X-ray Binary Black Holes
Known Neutron Stars
LIGO-Virgo Neutron Stars
?

Science Objectives

Cosmology: Exploring black hole seeds

The origin and evolution of BHs that seem to populate galactic cores is ***one of the unsolved problems in modern cosmology****.*

Cosmography: Measuring the Universe with standard sirens

One of the most spectacular aspects of compact binary signals is that their amplitude is completely determined by GR, without the need for any complicated astrophysical modelling of their environments.

Fundamental physics: Testing gravity with black holes

Nearly a hundred years after its formulation, GR continues to be the preferred theory of gravity. ***However, the theory is yet to be tested in strong gravitational fields that occur in the vicinity of BH horizons****. Gravitational wave observations of compact binaries could facilitate many such tests*

Science Objectives

Astrophysics: Catching supernovae in their act

It is expected that the gravitational collapse and the ensuing explosion can be fully understood ***only by studying the deep interiors of the proto-neutron star that forms in the process****, which is inaccessible to electromagnetic observations. Modelling SNe involves inputs from almost all branches of physics and current simulations of the process are far from complete.*

Nuclear Physics: Probing neutron star cores

Neutron star cores are laboratories of extreme conditions of density, gravity and magnetic fields. The structure and composition of NS cores have largely remained unresolved even half-century after pulsars were first discovered. Their cores could be host to unknown physics and might be composed of quark-gluon plasma, hyperons or other exotica.

Confronting GR with Observations

The nature of gravity in the strong-field limit is so far largely unconstrained, leaving open several outstanding questions.

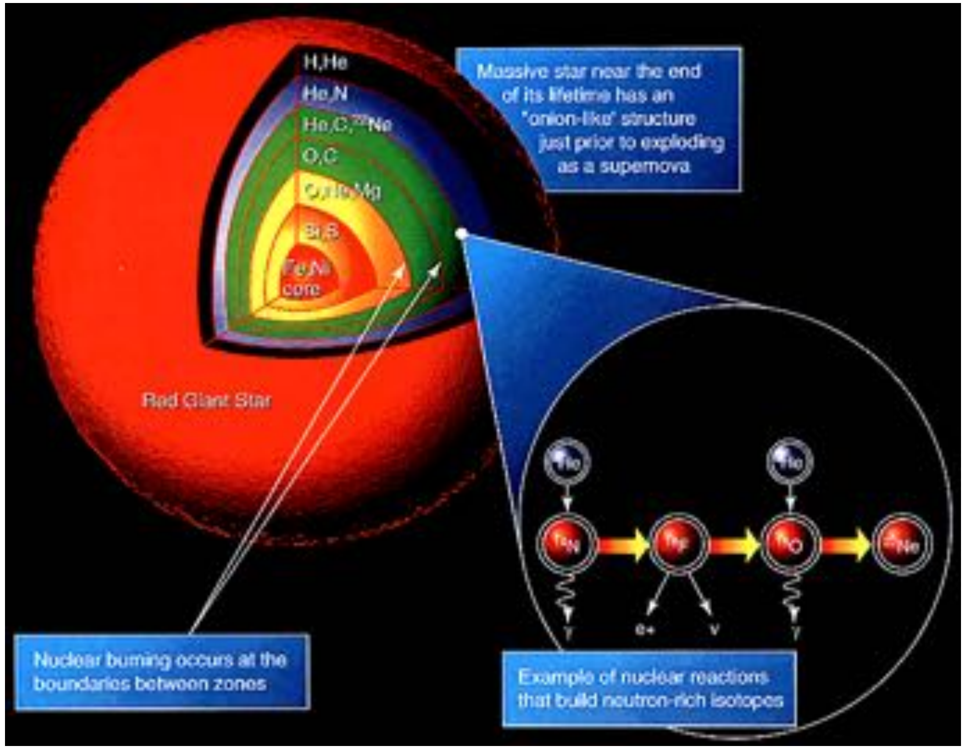
QUESTION	ANSWER
Does gravity travel at the speed of light? ➢Does the graviton have mass?	YES NO
How does gravitational information propagate: Are there more than two transverse modes of propagation (polarization)	Most probably NOT
Does gravity couple to other dynamical fields, e.g., massless or massive scalars?	?
What is the structure of spacetime just outside astrophysical black holes?	?
Do their spacetimes contain horizons?	Maybe
Are astrophysical black holes described by the Kerr metric, as predicted by GR?	Most probably YES … ??

Supernovae

Massive stars (10-20 $M_\odot$) fuse **H** into many heavier elements:

H → He → C → O → Si → Fe

- Fusion balances gravity
- But this can be done while the fuel supply lasts..
- Finally, you make more than a solar mass core made out of iron.
- Beyond iron fusion stops, and there is no balance for gravity



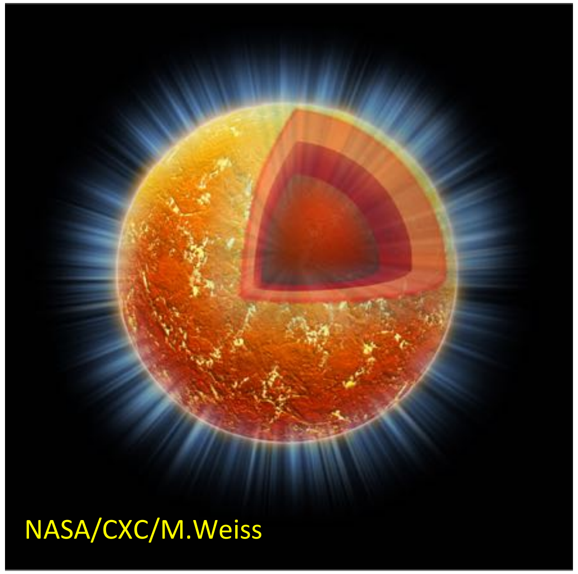
Dying Stars

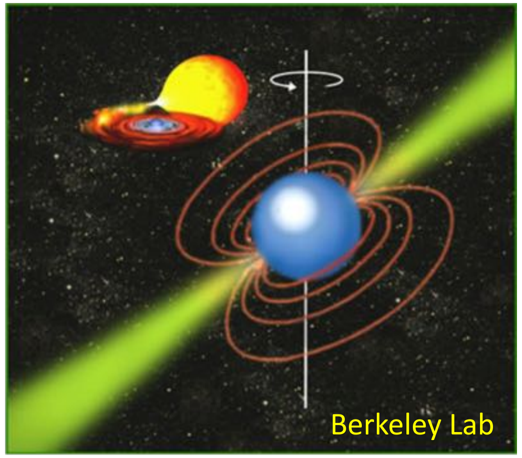
- Stars more massive than $\sim 8M_\odot$ end in core collapse (90% are stars with masses $\sim 8 - 20M_\odot$).
- Most of the material is ejected
- If $M > 20M_\odot$ more than 10 % falls back and pushes the PNS above the maximum NS mass leading to the formation of BHs (type II collapsars).
- If $M > 40M_\odot$ no supernova is launched and the star collapses to form a BH (type I collapsars)
- Formation rate: 1-2 per century / galaxy
- 5-40% of them produce BHs through the fall back material
- Limited knowledge of the rotation rate! Initial periods probably < 20ms.
  Maybe about 10% of pulsars are born spinning with millisecond periods.

Scientific Challenge

Neutron Stars - Physics in its extremes

- **Gravity** at its maximum (which gravity?)
- **Strong interactions** more important than in any other part of the present universe
- **Electroweak interactions** drive the astronomical emission



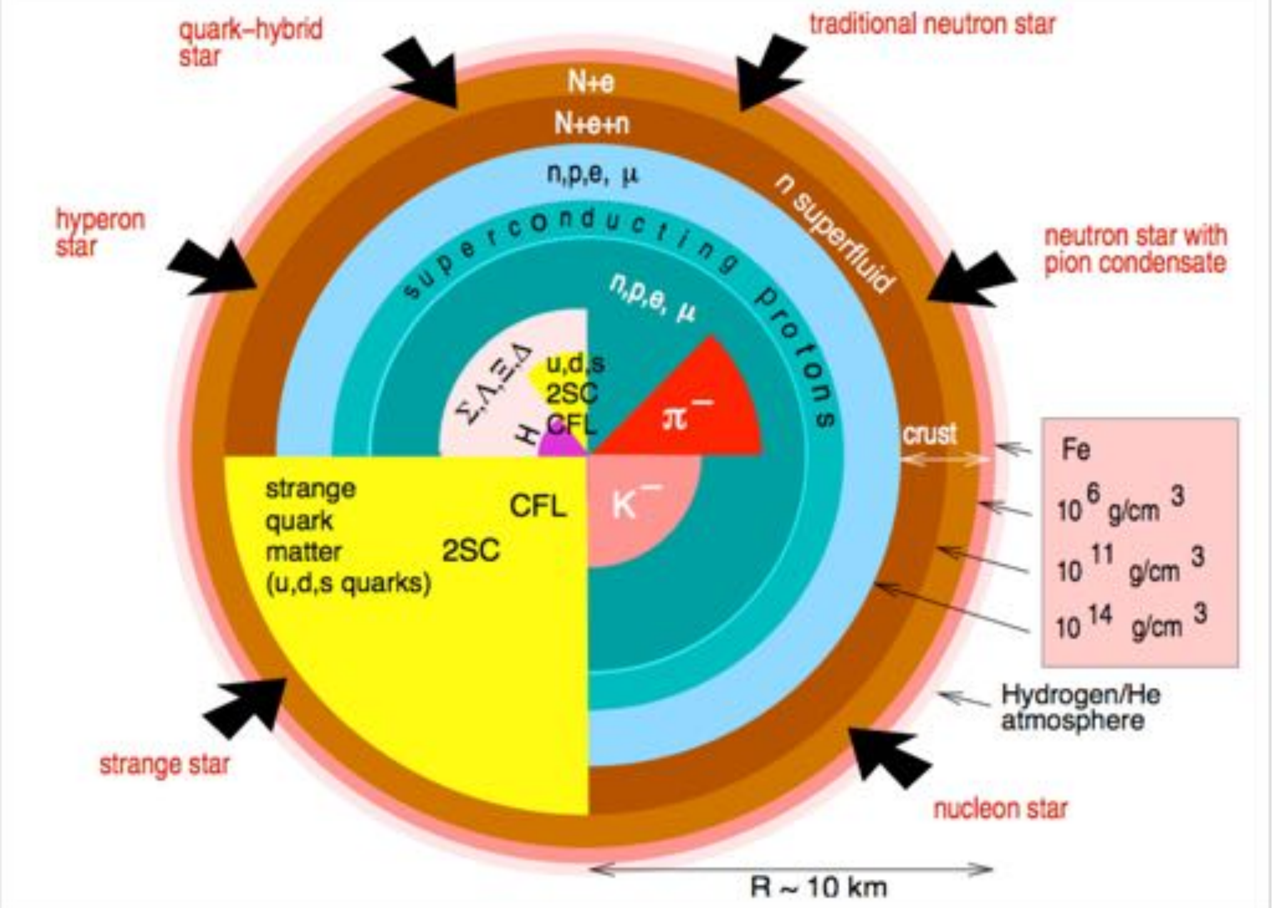


Neutron Stars - a unique interplay among

- **astrophysics**
- **gravitational physics**
- **nuclear physics**

After half a century since their discovery, we are still far from understanding the composition of matter in their cores!

Zooming into a Neutron Star



- **The holy grail of NS astrophysics…** is the **determination of the equation of state** (EOS) of matter at supra-nuclear densities.
- The most direct way of constraining the EOS is to **measure simultaneously** the neutron star **mass** and **radius**.

Neutron Stars -Observatories

Models of Neutron Stars

Choosing :

A) an equation of state

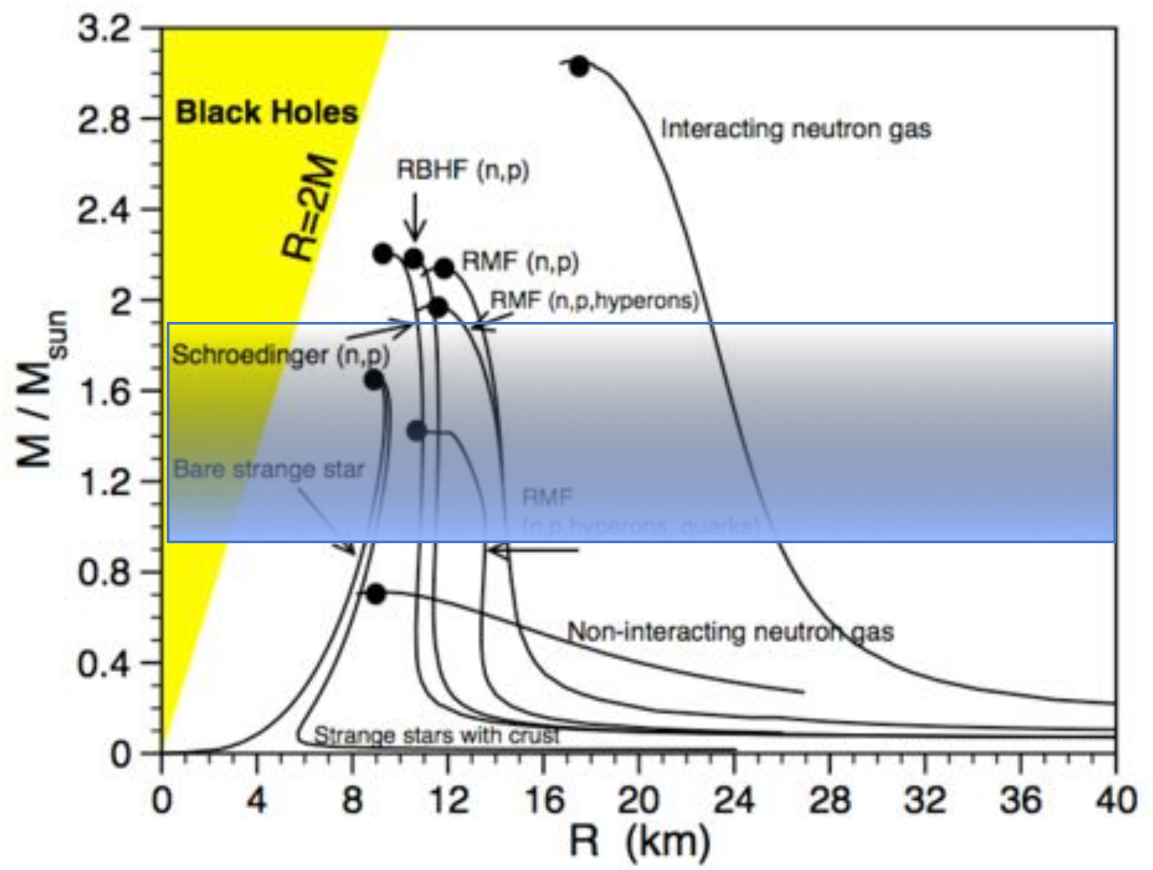
$$p = p(\rho, ..)$$

B) solving Einstein's equations

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

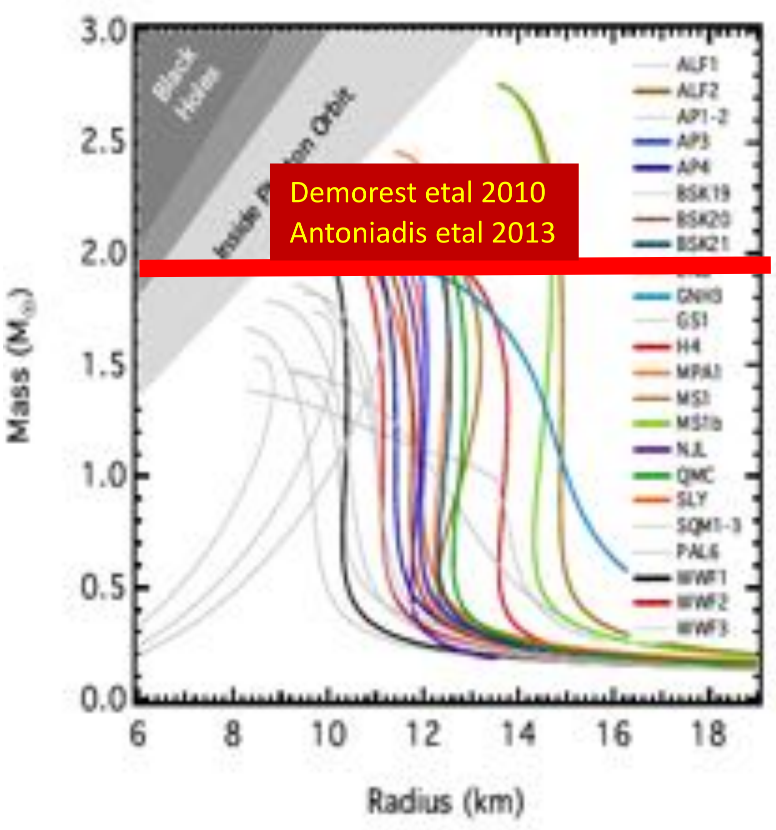
C) Solving the conservation of energy-momentum equations

$$\nabla^{\mu} T_{\mu\nu} = 0$$



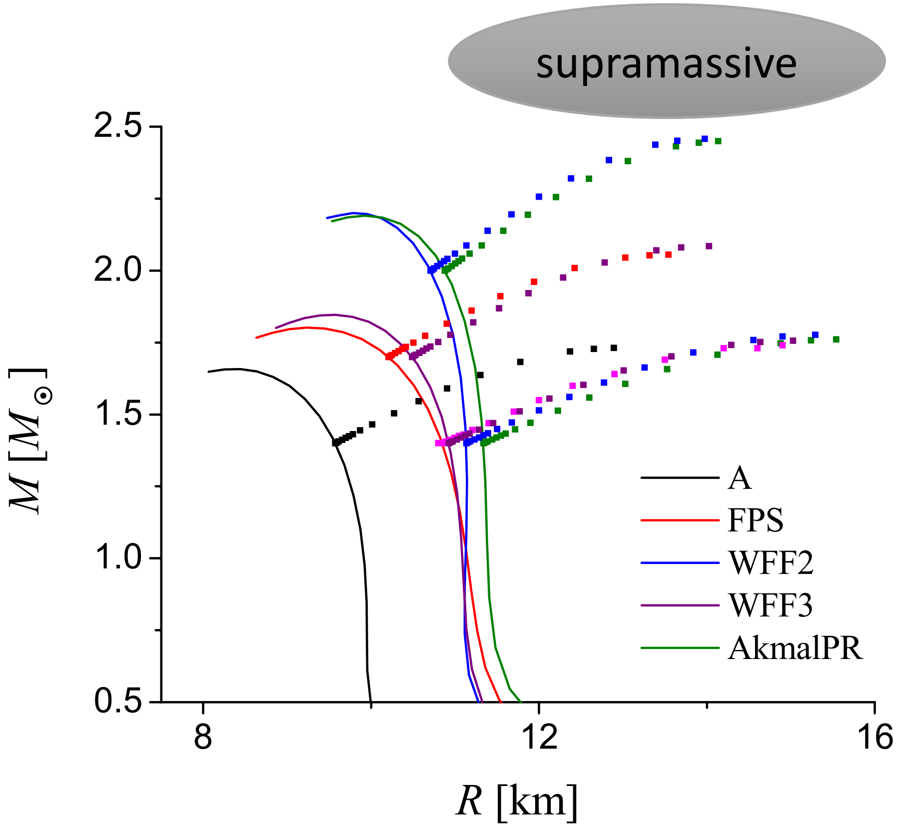
Neutron Stars: Mass vs Radius

Static Models



Özel & Freire (2016)

Rotating Models



$$M_{max} \simeq (1.1962 + 0.0108) M_{TOV}$$

Breu-Rezzolla 2015

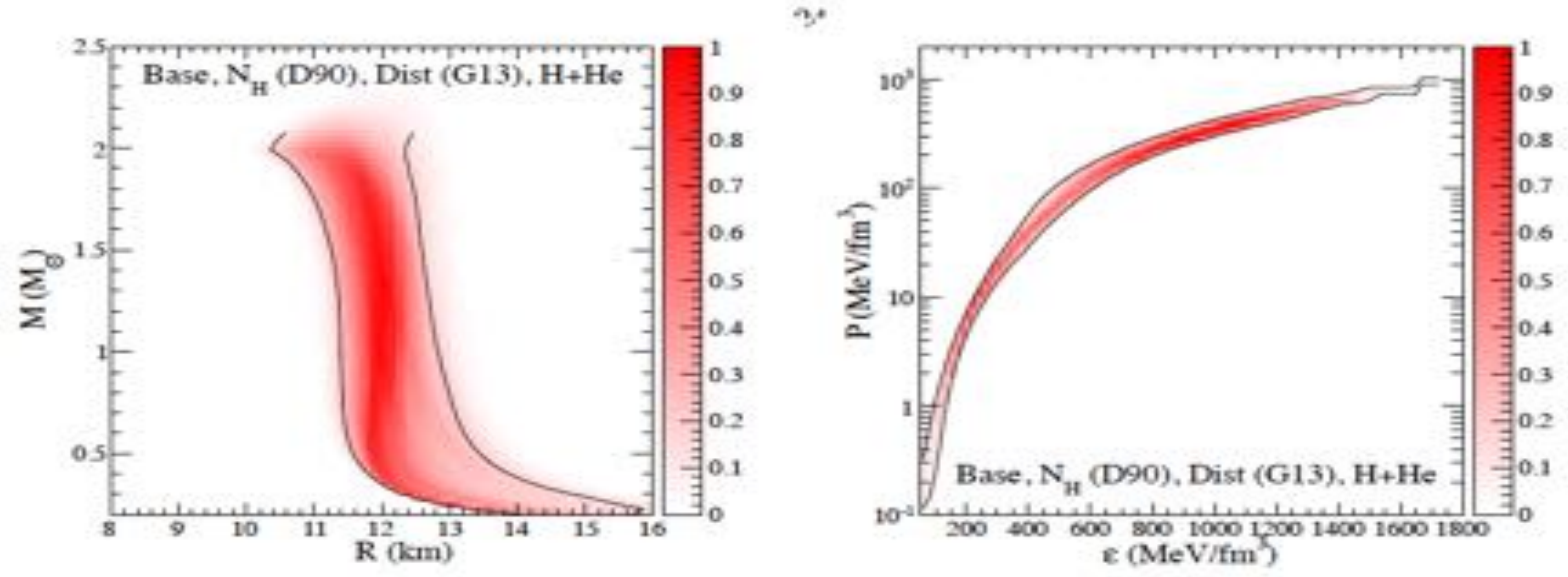
Constraints on Neutron Star Radius
EM observations

Main methods in EM spectrum:

- **Thermonuclear X-ray** bursts (photospheric radius expansion)
- **Burst oscillations** (rotationally modulated waveform)
- Fits of **thermal spectra** to cooling neutron stars
- **khZ QPOs** in accretion disks around neutron stars
- **Pericenter precession** in relativistic binaries (double pulsar J0737)

Observational Constraints on EOS

Attempts to constrain the EOS by combining observational data (LMXBs)
e.g. Lattimer-Steiner 2013



Constraints on Neutron Star Radius
GW observations

Main methods in GW spectrum:

- **Tidal effects on waveform during inspiral phase of NS-NS mergers**
- **Tidal disruption in BH-NS mergers**
- **Oscillations in post-merger phase of NS-NS mergers**
- **Oscillations after the gravitational collapse**

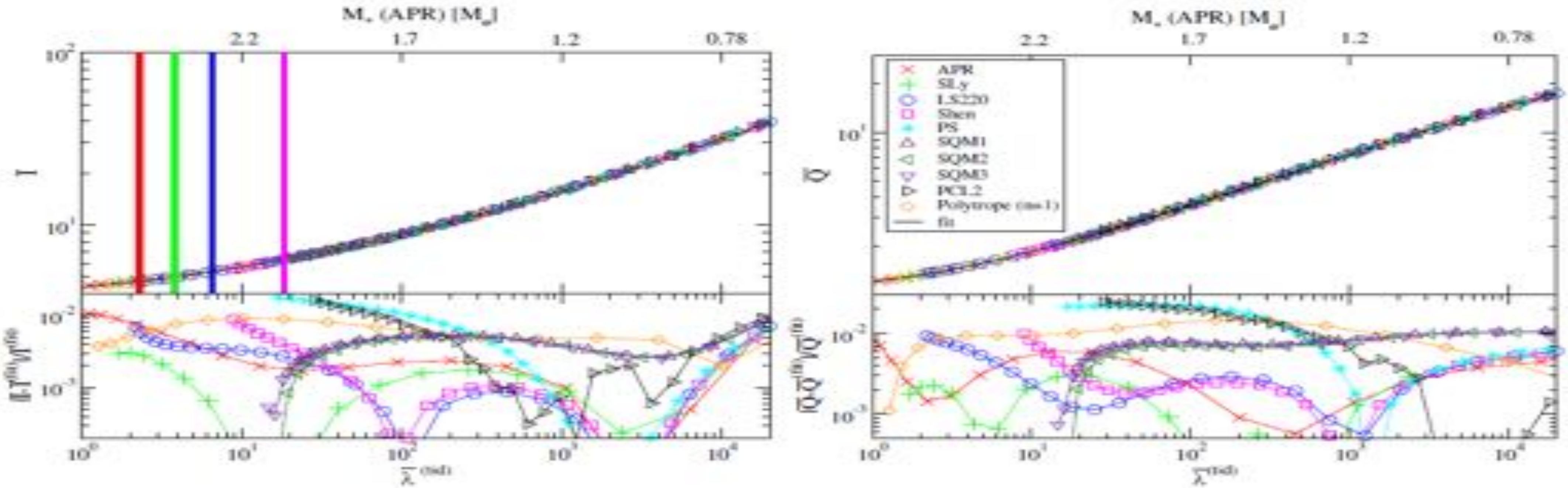
Neutron Stars & “universal relations”

Need for relations between the “observables” and the “fundamentals” of NS physics

Average Density	$\bar{\rho} \sim M / R^3$	
Compactness	$z \sim M/R$	$\eta = \sqrt{M^3 / I}$
Moment of Inertia	$I \sim MR^2$	$I \sim J / \Omega$
Quadrupole Moment	$Q \sim R^5\Omega^2$	$Q \sim MR^3$
Tidal Love Numbers	$\lambda \sim I^2 Q$	

I-Love-Q relations

EOS independent relations were derived by **Yagi & Yunes(2013)** for non-magnetized stars in the slow-rotation and small tidal deformation approximations.



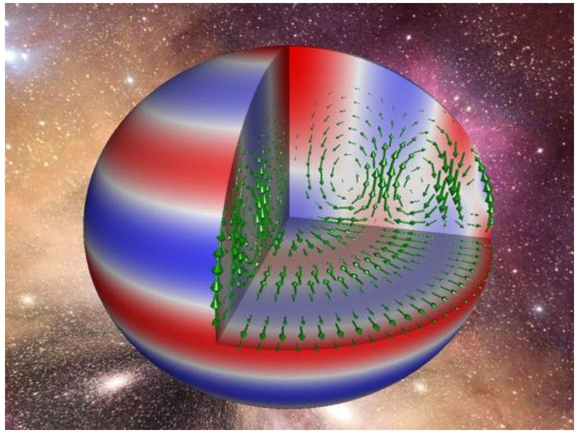
… the relations proved to be valid (*with appropriate normalizations*) even for *fast rotating* and *magnetized* stars

Latest developments: Yagi-Yunes arXiv:1601.02171 & arXiv:1608.06187

Oscillations & Instabilities

The most promising strategy for constraining the physics of neutron stars involves observing their "ringing" (oscillation modes)

- **f-mode :** scales with average density
- **p-modes:** probes the sound speed through out the star
- **g-modes :** sensitive to thermal/composition gradients
- **w-modes:** oscillations of spacetime itself.
- **s-modes:** Shear waves in the crust
- **Alfvèn modes:** due to magnetic field
- **i-modes:** inertial modes associated with rotation (r-mode)

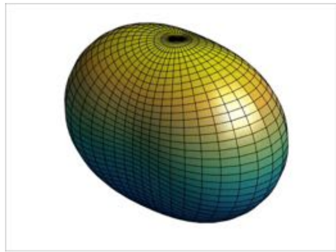


Typically **SMALL AMPLITUDE** oscillations –> weak emission of GWs

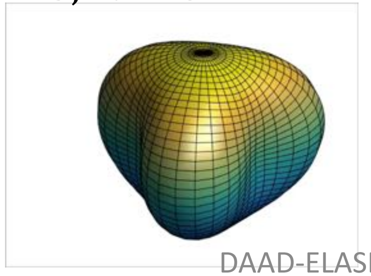
UNLESS

they become **unstable due to rotation** (r-modes & f-modes …)

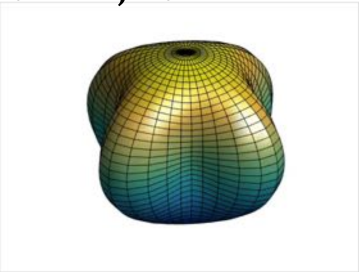
$l = 2, m = 2$



$l = 3, m = 3$

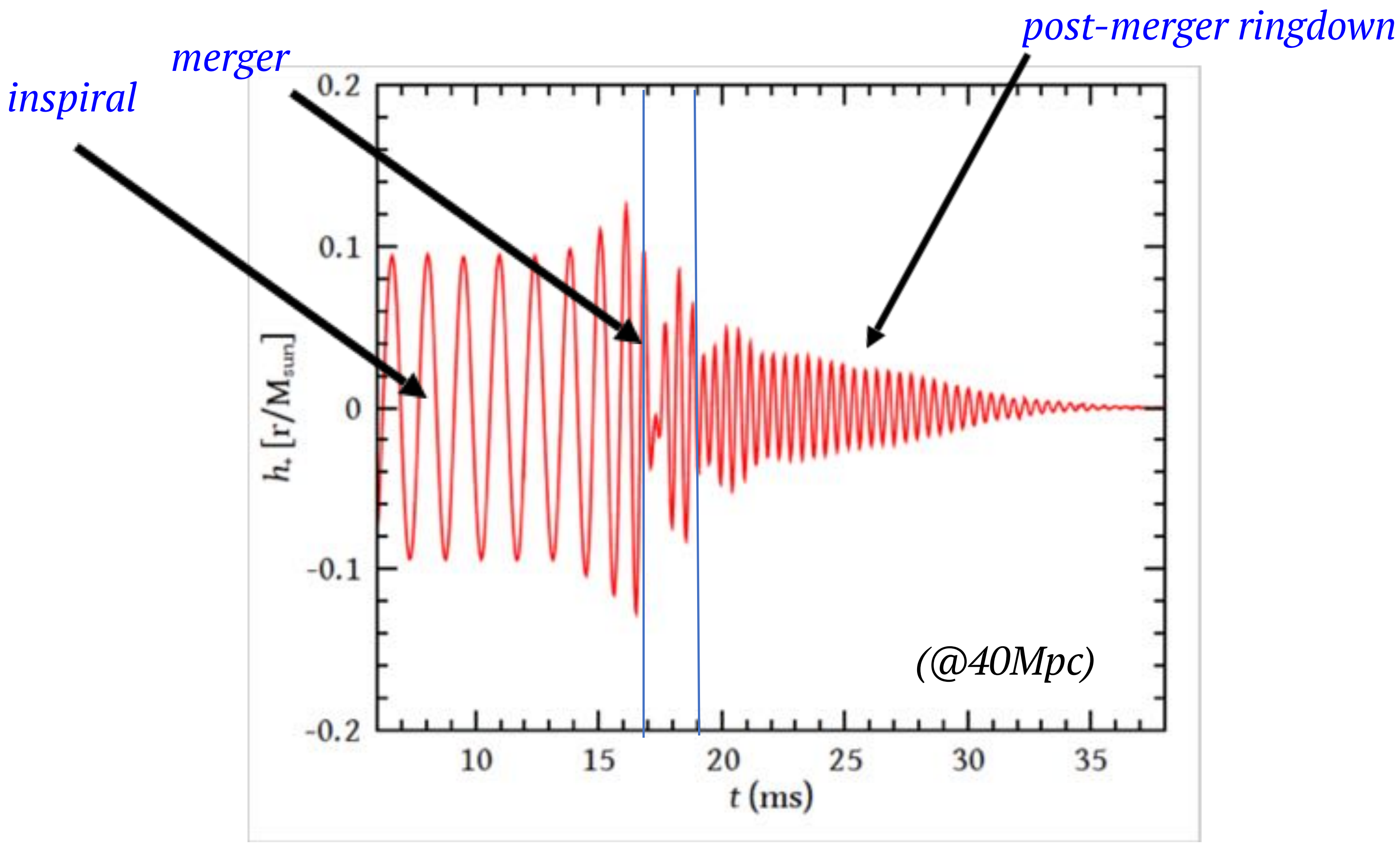


$l = 4, m = 4$



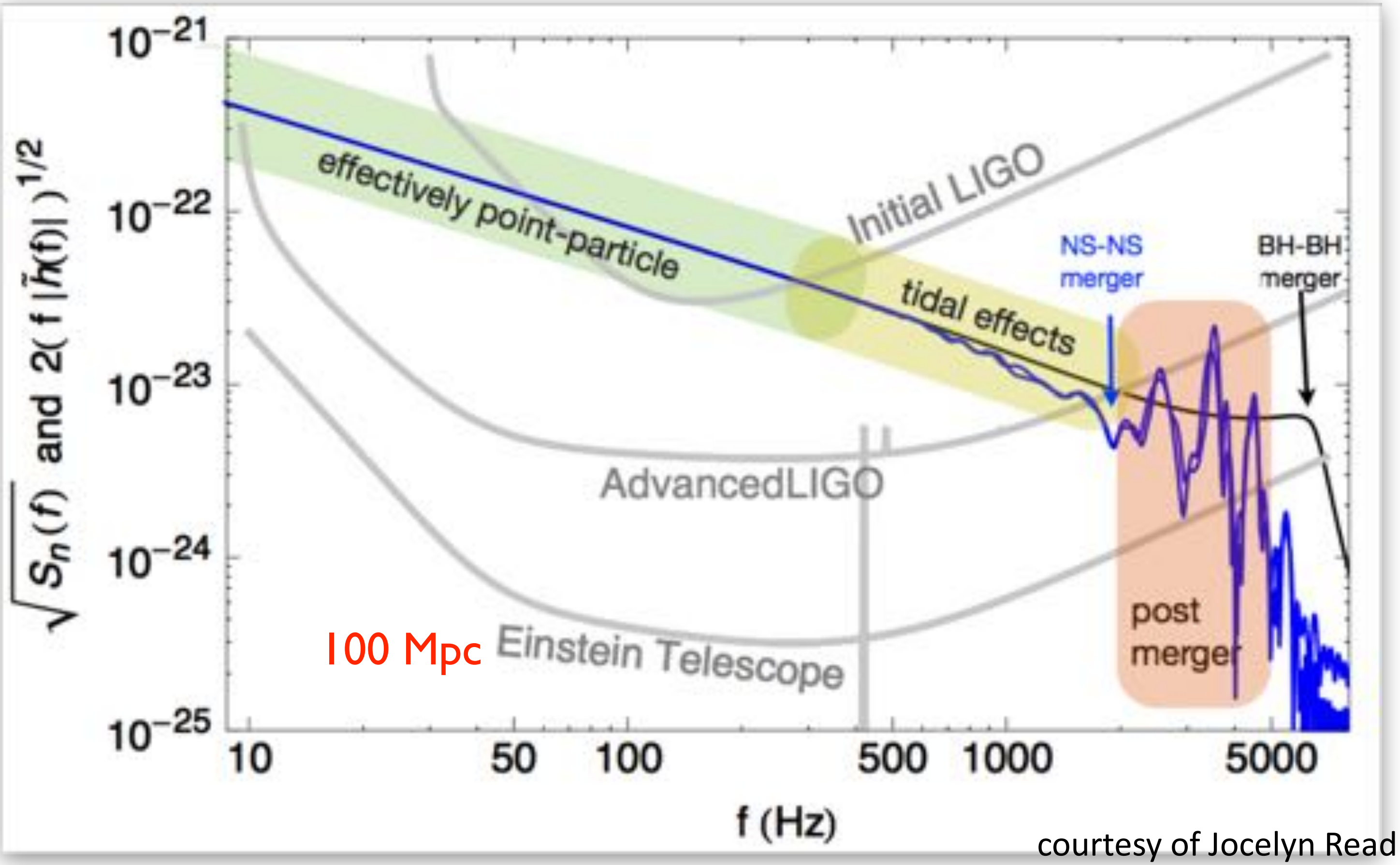
Binary Neutron Star Mergers
the standard scenario

The GW signal can be divided into **three distinct phases**



Binary Neutron Star Mergers
Tidal Interaction

Tidal interactions affect the last part of the inspiral, modifying the orbital motion and the GW emission.



courtesy of Jocelyn Read

NS as Continuous Sources
r-modes & “mountains”

“Mountains”: Deformations that are static (at dynamical timescales) typically in the crust. *The finite shear modulus of the crystalline crust offers the possibility of supporting a deformation* (Bildsten 1998)

- **Strong magnetic fields** can also confine material and lead to deformations that could be quite large (magnetars).
- In LMXBs **accretion process** can lead to material spreading equatorially and compressing the field making it locally strong enough to sustain sizable mountains.

- **Modes of oscillations** can grow to large amplitude and lead to GWs.
- The prime candidate in LMXBs is the **r-mode** (restoring force the Coriolis force).
- Primarily toroidal perturbations and the Eulerian velocity perturbation is:

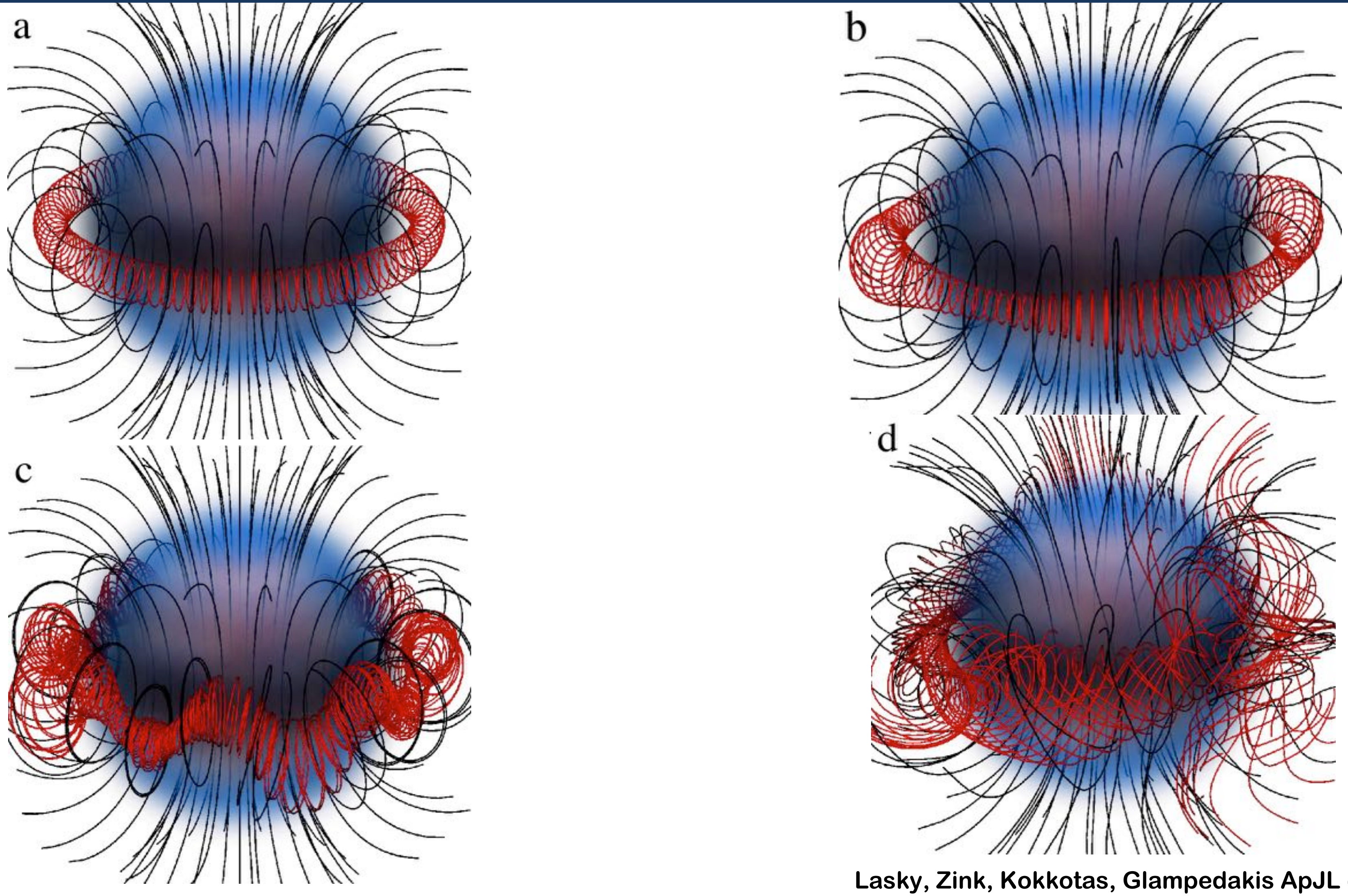
$$\delta\mathbf{v} = \alpha\left(\frac{r}{R}\right)^{l} R\Omega\mathbf{Y}^{B}_{lm}e^{i\omega t}$$

Observational constraints for ellipticity : **~8.5×10^{-6}**

Abbot at al arXiv:1607.02216

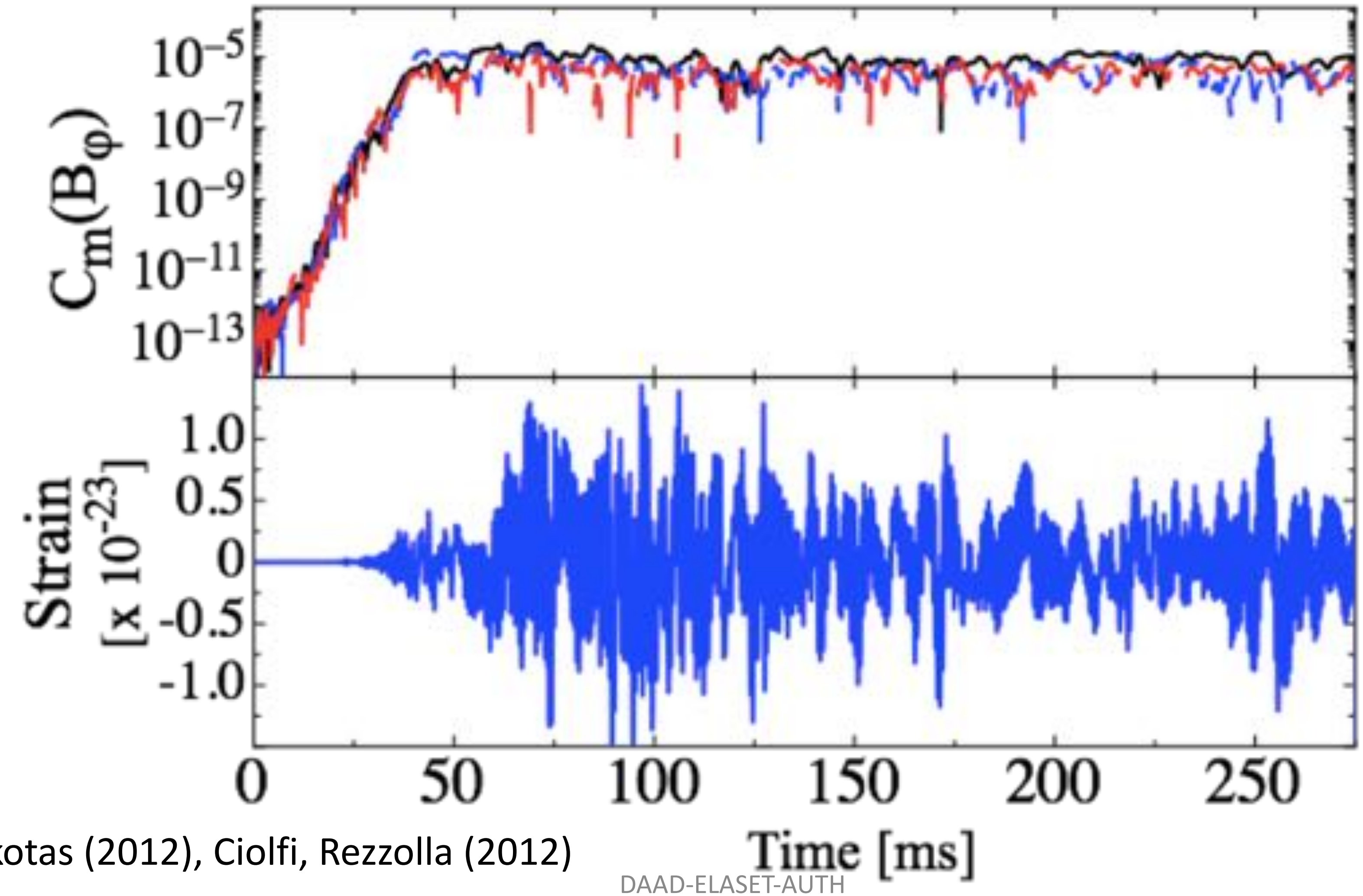
Magnetic Field Instability

Magnetars



Lasky, Zink, Kokkotas, Glampedakis ApJL (2011)

Gravitational Waves from Magnetars



Lasky, Zink, Kokkotas (2012), Ciolfi, Rezzolla (2012)

Conclusions

- ✓ **Binary Mergers (as of early 2017)**
  - ❖ **Tidal effects very promising, may be the first to be measured**
  - ❖ **Early post-merger phase**
    - Ideal field for asteroseismology and constraining the EOS
    - Towards the **end of next decade** will provide a wealth of information (earlier detection also possible).
  - ❖ **Late post-merger phase (afterglow)**
    - The GW emission depends strongly on the strength of the dipole component of the magnetic field ($\gtrsim 10^{14}$ Gauss)
    - All 3 cases (*boring* **B-field spindown**, **deformed magnetar** and **f-mode instability**) maybe observed towards the end of next decade.

- ✓ **Gravitational Collapse**
  - ❖ Weak local group of Galaxy sources for present generation detectors (ET, Cosmic Explorer)

- ✓ **Continuous Sources (mountains & R-modes)**
  - ❖ The details and nature of deformation (microphysics) need further study
  - ❖ Weak Galactic sources for present generation detectors