Probing the Nature of Compact Objects with QNMs

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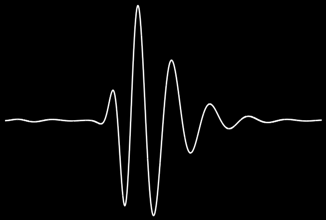

PART I

What are Compact Objects?

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- Are there **exotic compact objects**?[1]
  - gravastars, boson stars, wormholes, alternative black holes, ...
  - objects which could have $C \approx 0.3 - 0.5$

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- In contrast, **gravitational wave (GW) emission**
  - Our **new window** to the universe (**Nobel Prize in Physics 2017**)
  - **Testing compact objects** and **general relativity**

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  - **Numerical relativity** (solving the full Einstein equations numerically)

Perturbation Theory

- **Small perturbations** $h_{\mu\nu}$ to a known **background solution** $g^0_{\mu\nu}$

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu} \tag{1}$$

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  - $r^*(r) \equiv \int^r \sqrt{\frac{g_{11}(r)}{g_{00}(r)}}\mathrm{d}r$ **tortoise coordinate**

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PART II

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- **Numerical methods** (integrating wave equation, continued fraction, ...)
- **Approximate methods** (here WKB)
- Result of **WKB theory** are **Bohr-Sommerfeld (BS) rules**
- **Approximate** but useful for **semi-analytic studies**

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- **Important** for **quantum mechanics**, but also for **gravitational perturbations**
- **Asymptotic series**, does not necessarily converge!

The WKB Method II

- Time independent **Schrödinger equation** [3]

$$\varepsilon^2 y''(x) = Q(x)y(x), \qquad Q(x) \equiv V(x) - E_n \neq 0 \tag{3}$$

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- **Ansatz** of the form

$$y(x) \sim \exp\left(\frac{1}{\delta}\sum_{n=0}^{\infty} \delta^n S_n(x)\right), \qquad \delta \to 0 \tag{4}$$

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The WKB Method III

- **Inserting** and **ordering** in powers of ε and δ yields

$$S_0(x) = \pm \int^x \sqrt{Q(t)}\,\mathrm{d}t \tag{5}$$

$$S_1(x) = -\frac{1}{4}\ln\left(Q(x)\right) \tag{6}$$

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- Leading order WKB approximation for $y(x)$

$$y(x) \sim c_1 Q^{-1/4} \exp\left(\frac{1}{\varepsilon}\int_a^x \sqrt{Q(t)}\,\mathrm{d}t\right) \tag{7}$$

$$+ c_2 Q^{-1/4} \exp\left(-\frac{1}{\varepsilon}\int_a^x \sqrt{Q(t)}\,\mathrm{d}t\right), \qquad \varepsilon \to 0 \tag{8}$$

The WKB Method IV

- Valid for **slowly changing potentials** and **not too small** $Q(x)$

$$\left| \frac{\mathrm{d}}{\mathrm{d}x} \frac{1}{Q(x)} \right| = \left| \frac{Q'(x)}{Q(x)^2} \right| \ll 1 \tag{9}$$

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- Use **expanded potential around turning points** (Airy functions for linear expansion, "Kramer's relations")

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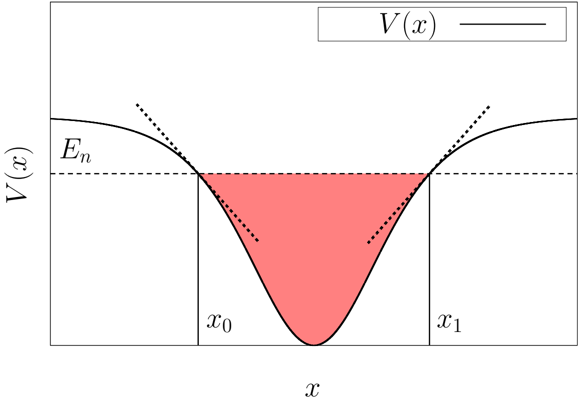
- Example: **potential well** $V(x)$ with **two turnings points** (x_0, x_1)



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- **Matching** solutions yields the classical **Bohr-Sommerfeld rule**

$$\int_{x_0}^{x_1} \sqrt{E_n - V(x)}\,\mathrm{d}x = \pi\left(n + \frac{1}{2}\right), \qquad n = 0,1,2,\ldots \tag{10}$$

Generalized Bohr-Sommerfeld Rules: I

- Generalization to **quasi-stationary states**[4]

$$\int_{x_0}^{x_1} \sqrt{E_n - V(x)}\mathrm{d}x = \pi\left(n + \frac{1}{2}\right) - \frac{i}{4}\exp\left(2i\int_{x_1}^{x_2} \sqrt{E_n - V(x)}\mathrm{d}x\right) \quad (11)$$

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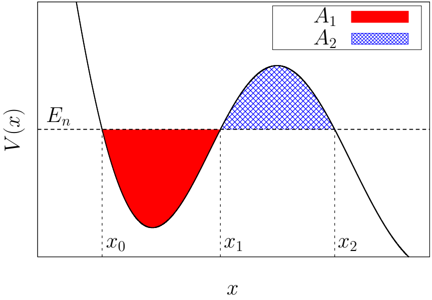

Figure 1: Völkel and Kokkotas (2017,1).

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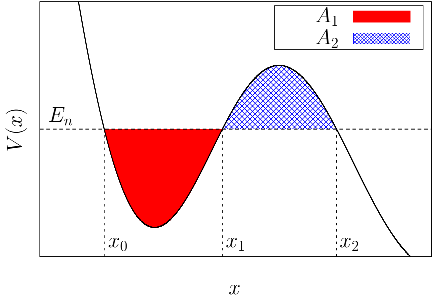

Figure 1: Völkel and Kokkotas (2017,1).

- 3 classical turning points
- **Complex** spectrum E_n
- Two widths $\mathcal{L}_1, \mathcal{L}_2$
- Application to ultra compact stars[5]

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Generalized Bohr-Sommerfeld Rules: II

- Estimate for repulsive potentials with a jump and QNM condition[6]

$$\int_{x_0}^{R} \sqrt{E_n - V(x)}\mathrm{d}x = \left(n + \frac{3}{4}\right) + i\,\mathrm{arctanh}\left(\frac{\sqrt{E_n - V(R_-)}}{\sqrt{E_n - V(R_+)}}\right) \tag{12}$$

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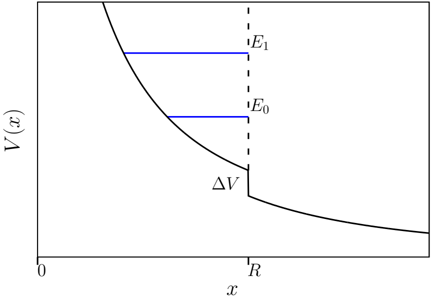

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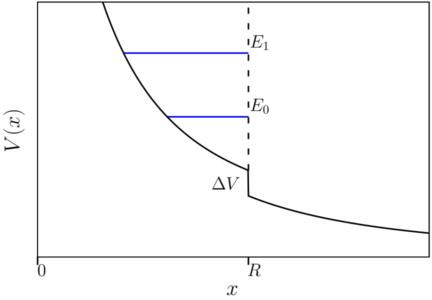

- 1 classical turning point
- **Complex** spectrum
- Asymptotic $\Delta\omega_{\mathbf{r},n} = \pi / R^*$[7]
- Applies to **neutron stars**

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PART III

Can one hear the shape of a drum?[8]

[8] Kac, M. (1966); Gordon et al. (1992)

Can on hear the shape of a drum?

Invert BS Rules

- Known for **single** wells and **single** barriers (**classical BS rule)**[9]

[9] Wheeler (1976); Chadan and Sabatier (1989); Lazenby and Griffiths (1980); Gandhi and Efthimiou (2006)
[10] Völkel and Kokkotas (2017,2); Völkel (2018,1); Völkel and Kokkotas (2018,2)
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$$\mathcal{L}_1(E) = x_1 - x_0 = 2\frac{\partial}{\partial E}\int_{E_{\text{min}}}^{E} \frac{n(E') + 1/2}{\sqrt{E - E'}} \mathbf{d}E' \tag{13}$$

$$\mathcal{L}_2(E) = x_2 - x_1 = -\frac{1}{\pi}\int_{E}^{E_{\text{max}}} \frac{(\mathbf{d}T(E')/\mathbf{d}E')}{T(E')\sqrt{E' - E}} \mathbf{d}E' \tag{14}$$

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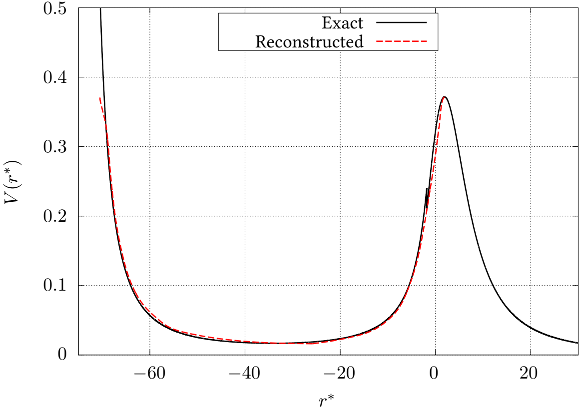
PART IV

Results

Ultra Compact Stars

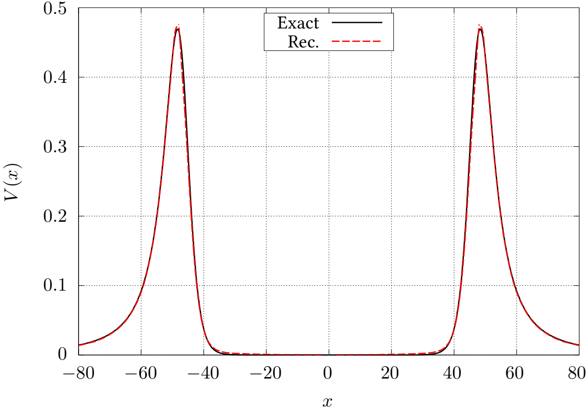
Example for ultra compact constant density star $C \approx 0.44$



Reconstructed axial perturbation potential, constant density star, $l = 3$, taken from Völkel and Kokkotas (2017,2).

Damour-Solodukhin Wormhole

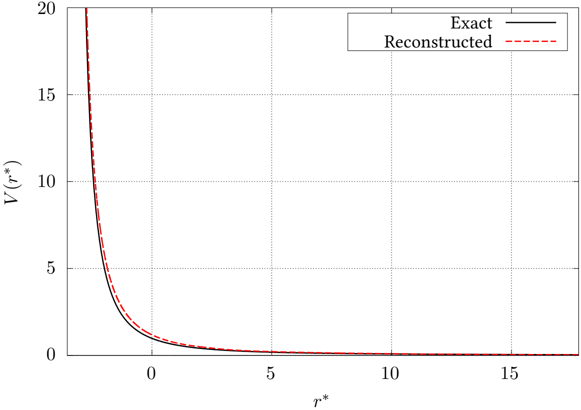
Example for Damour-Solodukhin wormhole $C \approx 0.5$



Reconstructed axial perturbation potential, Damour-Solodukhin wormhole, $l = 3$, taken from Völkel and Kokkotas (2018,2).

Neutron Stars

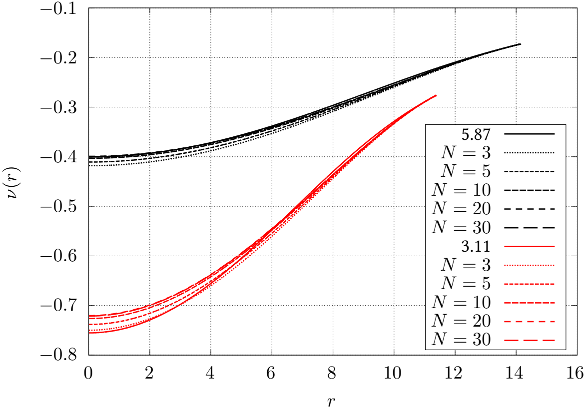
Example for neutron star polytrope $C \approx 0.15$



Reconstructed axial perturbation potential, neutron star polytrope, $l = 3$, Völkel and Kokkotas (2018 TBS).

Neutron Stars

Fit spectrum to analytic model potential for neutron star
Bohr-Sommerfeld rule to reconstruct internal space-time



Reconstructed metric function $\nu(r)$, for two neutron star polytropes, $l = 3$, Völkel and Kokkotas (2018 TBS).

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- **More information** is required for unique reconstruction, e.g.:
  - Birkhoff's theorem (ultra compact stars)
  - Symmetric potential (some wormholes)
  - Radius of the star (normal neutron stars)

Uniqueness?

- Inverted BS rules give **approximate results**
- Reconstruction of the **potential widths**
- Usually **no unique potential** possible (expected)
- **More information** is required for unique reconstruction, e.g.:
  - Birkhoff's theorem (ultra compact stars)
  - Symmetric potential (some wormholes)
  - Radius of the star (normal neutron stars)
- **One has to assume the underlying type of potential!**

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- **WKB/Bohr-Sommerfeld** rules are useful for semi-analytic work
- Can be inverted for the **inverse spectrum problem**
- **Potential** can be approximately **reconstructed**, depends on the type of object

More Details

Details for the presented work can be found in:

- Völkel and Kokkotas, *Class. Quantum Grav.* 34, 125006 (2017)
- Völkel and Kokkotas, *Class. Quantum Grav.* 34, 175015 (2017)
- Maselli, Völkel, and Kokkotas, Phys. Rev. D 96, 064045 (2017)
- Völkel, J. Phys. Commun. 2, 025029 (2018)
- Völkel and Kokkotas, *Class. Quantum Grav.* 35, 105018 (2018)

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