Electronic Supplement to "On the Estimation of Ground-Motion Duration Models with an Application to the M9 Simulations"

This electronic supplement contains additional plots and information.

1 Graphical Posterior Predictive Checks

First, we show graphical posterior predictive checks, similar to Figure 8. Here, we compare the median, 5$ and 95% fractile of simulated data sets with the corresponding values of the M9 simulations for data in distance of 25km.

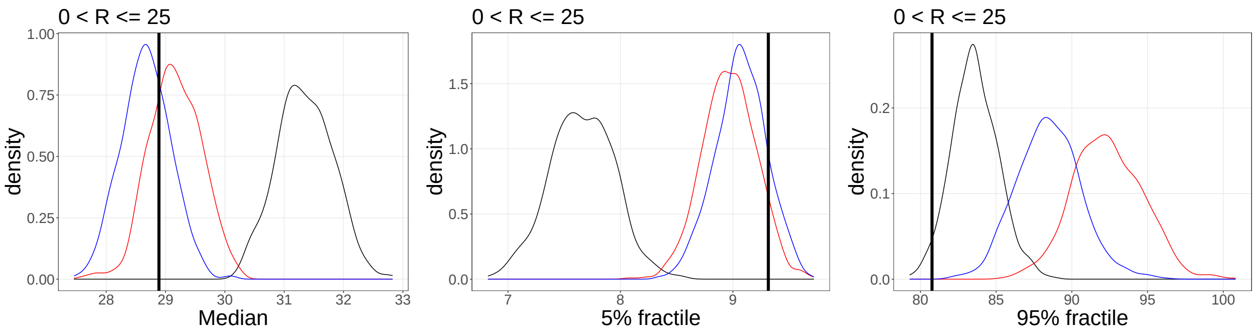

Figure S1: Posterior predictive checks for data in the range $0 < R \leq 25$km. Shown are densities of the median and 5%/95% fractiles of 800 simulated data sets. The black vertical line displays the median or fractile of the M9 data.

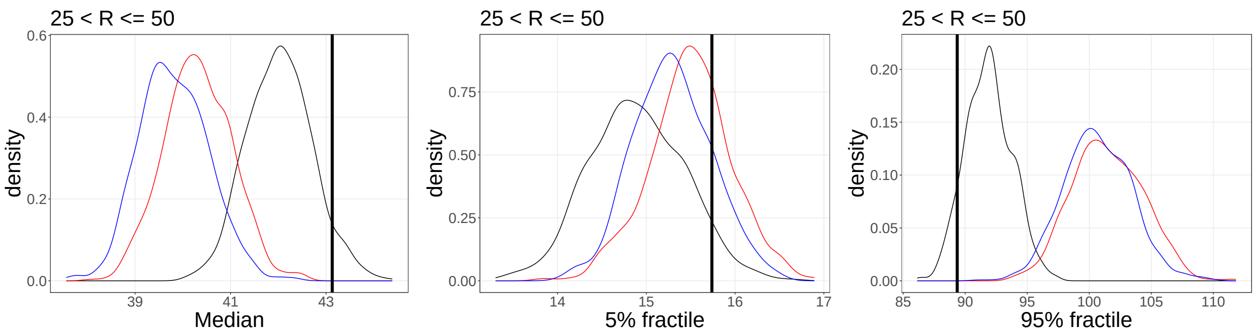

Figure S2: Posterior predictive checks for data in the range $25 < R \leq 50$km. Shown are densities of the median and 5%/95% fractiles of 800 simulated data sets. The black vertical line displays the median or fractile of the M9 data.

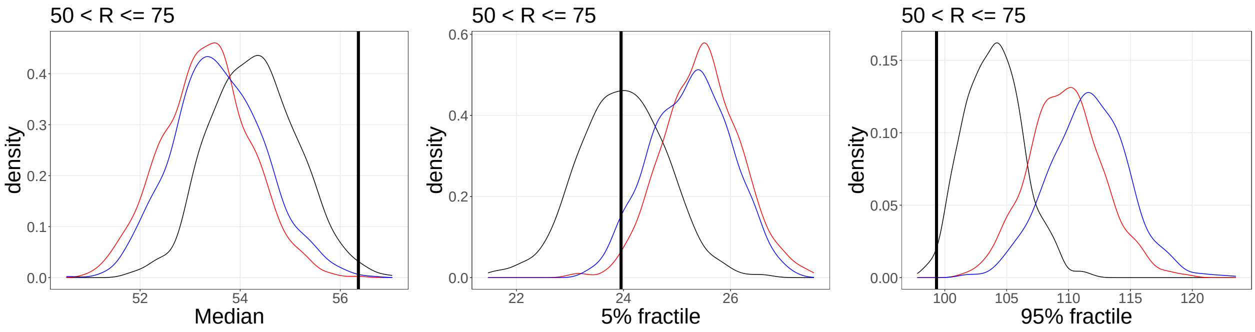

Figure S3: Posterior predictive checks for data in the range $50 < R \leq 75$km. Shown are densities of the median and 5%/95% fractiles of 800 simulated data sets. The black vertical line displays the median or fractile of the M9 data.

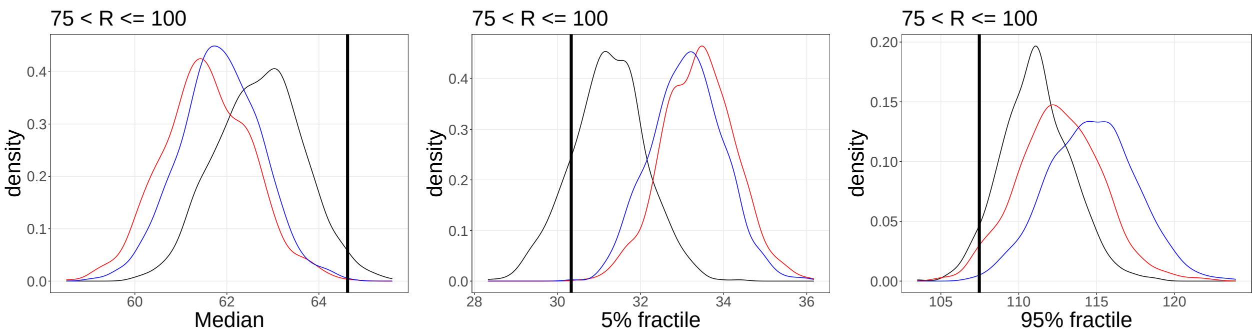

Figure S4: Posterior predictive checks for data in the range $75 < R \leq 100$km. Shown are densities of the median and 5%/95% fractiles of 800 simulated data sets. The black vertical line displays the median or fractile of the M9 data.

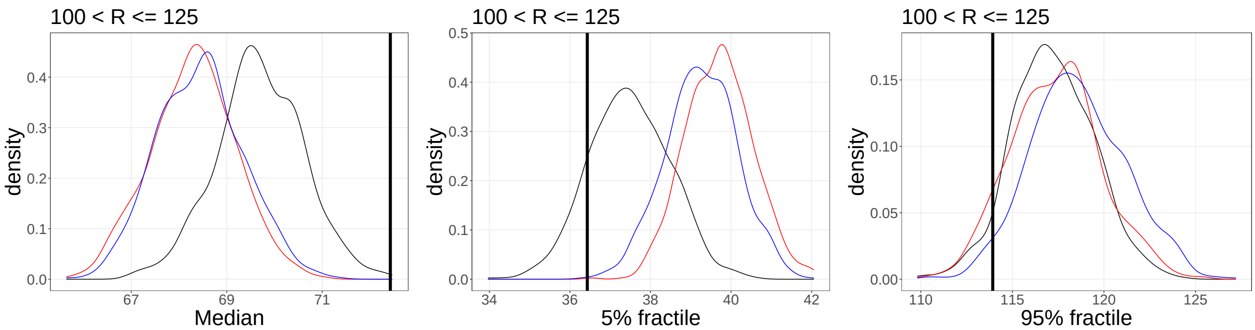

Figure S5: Posterior predictive checks for data in the range $100 < R \leq 125$km. Shown are densities of the median and 5%/95% fractiles of 800 simulated data sets. The black vertical line displays the median or fractile of the M9 data.

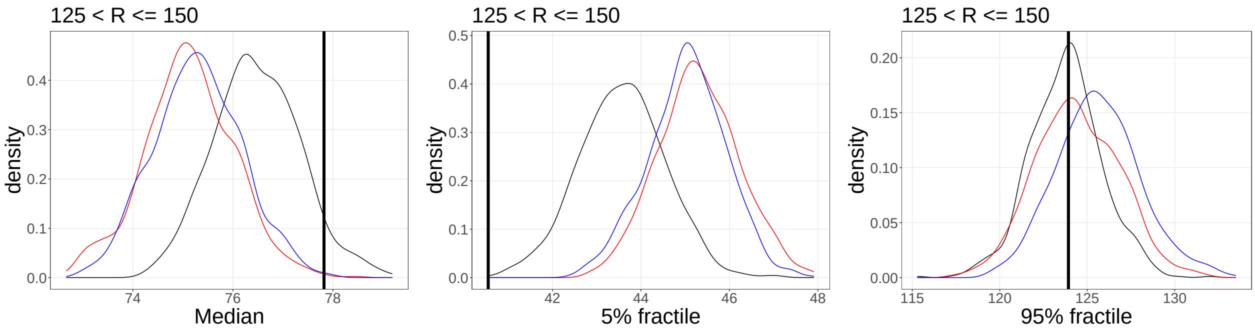

Figure S6: Posterior predictive checks for data in the range $125 < R \leq 150$km. Shown are densities of the median and 5%/95% fractiles of 800 simulated data sets. The black vertical line displays the median or fractile of the M9 data.

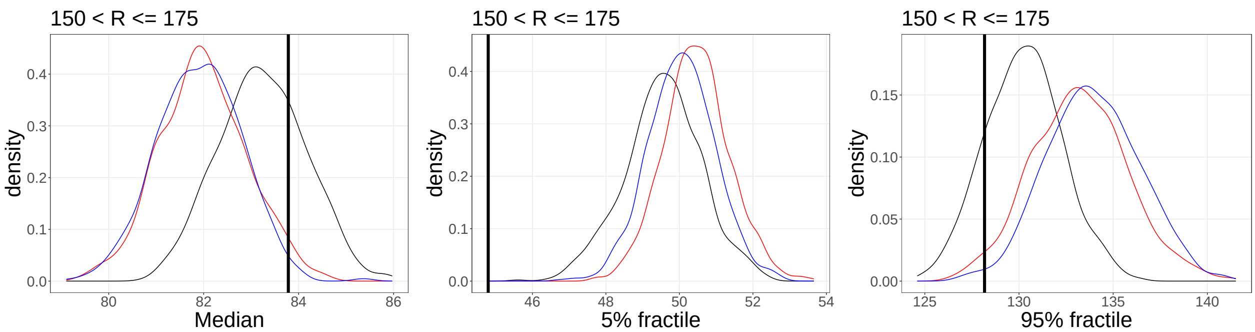

Figure S7: Posterior predictive checks for data in the range $150 < R \leq 175$km. Shown are densities of the median and 5%/95% fractiles of 800 simulated data sets. The black vertical line displays the median or fractile of the M9 data.

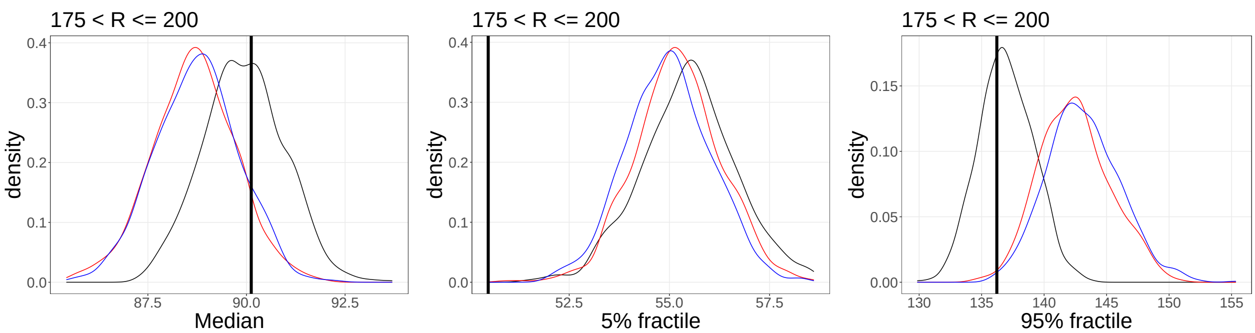

Figure S8: Posterior predictive checks for data in the range $175 < R \leq 200$km. Shown are densities of the median and 5%/95% fractiles of 800 simulated data sets. The black vertical line displays the median or fractile of the M9 data.

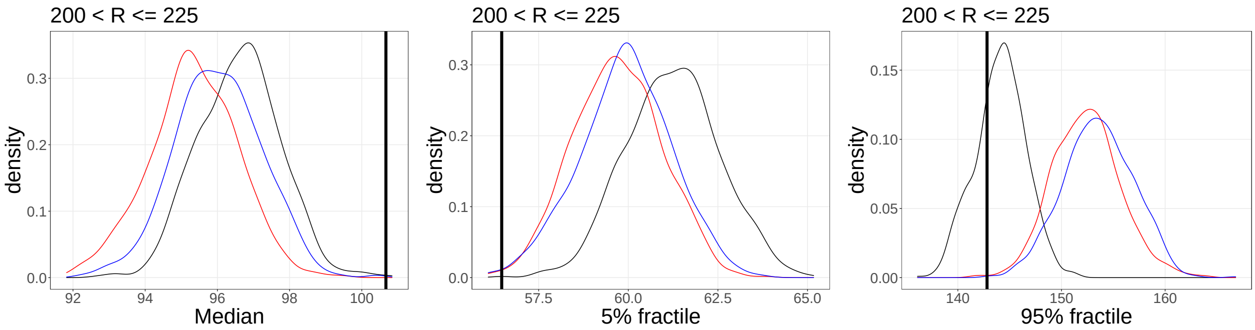

Figure S9: Posterior predictive checks for data in the range $200 < R \leq 225$km. Shown are densities of the median and 5%/95% fractiles of 800 simulated data sets. The black vertical line displays the median or fractile of the M9 data.

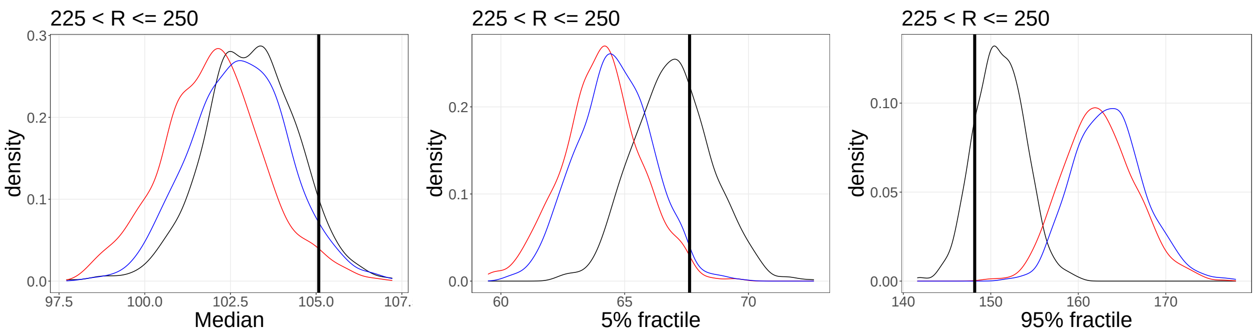

Figure S10: Posterior predictive checks for data in the range $225 < R \leq 250$km. Shown are densities of the median and 5%/95% fractiles of 800 simulated data sets. The black vertical line displays the median or fractile of the M9 data.

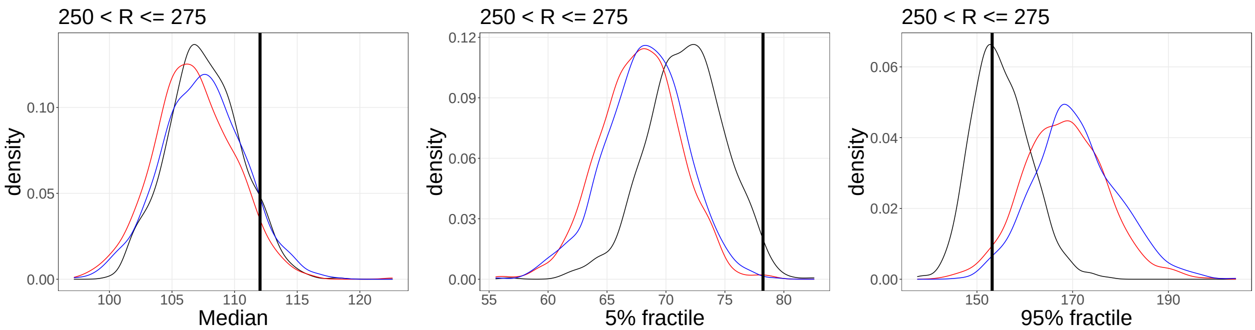

Figure S11: Posterior predictive checks for data in the range $250 < R \leq 275$km. Shown are densities of the median and 5%/95% fractiles of 800 simulated data sets. The black vertical line displays the median or fractile of the M9 data.

2 Calculation of Leave-One Event-Out Log-Likelihood

In order to compare the models in terms of their log-likelihood values on test data (where the test data is ll data from one event), the log-likelihood values need to be calculated. Since the event terms/source durations are unknown, this requires integration over the source duration uncertainty. The log=likelihood value for a single observation D_{obs} can be calculated as

$$\ln \mathcal{L} = \ln \int_0^\infty p(D_{obs}|D_S)p(D_S)dD_s \tag{1}$$

For the mixed model this can be calculated as

$$\ln \mathcal{L}(D_{Obs}) = p_{LN}(D_{obs}|\mu, \sigma_T) \tag{2}$$

where μ is the median prediction for the observation, and $\sigma_T^2 = \sigma_P^2 + \sigma_S^2$.

For the partially decoupled models, we use Monte Carlo Integration. We randomly sample a value of the source duration 1000 times, and calculate the likelihood of D_{obs} for each sample. We then average the likelihood values to get $\mathcal{L}(D_{Obs}) = \frac{\sum_{k=1}^{1} 000 \mathcal{L}_k(D_{Obs}|D_{S,k})}{1000}$, where $\mathcal{L}_k(D_{Obs}|D_{S,k})$ is the likelihood value for the kth sampled source duration $D_{S,k}$.

For the estimation, we run 3 chains with 300 warm-up and 200 sampling iterations, with the number of chains reduced compared to the paper for computational reasons. We calculate a likelihood value for each posterior sample, and average over samples to get a likelihood value for each observation (thus integrating over the source variability and the posterior uncertainty). We then calculate the sum of all log-likelihood values, which is reported in Table 1. The standard errors of the difference in log-likelihood values is calculated according to Equation (24) of Vehtari et al. (2017).

3 Epistemic Uncertainty

Figure S12 shows the epistemic uncertainty associated with the *mean* model predictions. For each of the posterior samples, we calculate a prediction for a given distance, which results in 800 model predictions. We then calculate mean, median, and 5%/95% fractiles of those 800 predictions and plot them against distance in Figure S12.

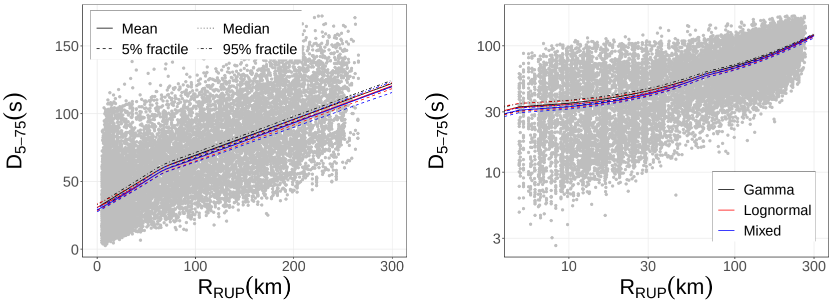

Figure S12: Epistemic uncertainty in mean model predictions based on posterior samples.

4 Residuals to NGA-Sub Data

This section shows residuals to the observed NGA-Sub data for the lognormal and mixed model, similar to Figure 9.

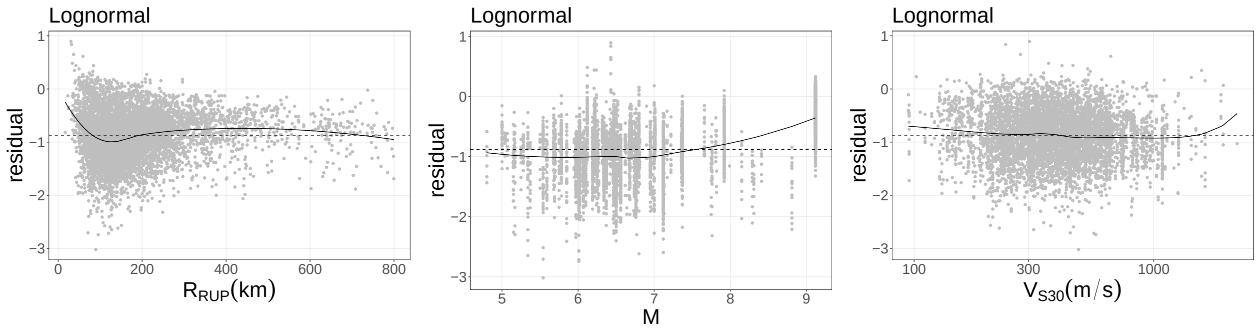

Figure S13: Residuals to Interface NGA-Sub data from KBCG with the lognormal model. The soild black lines show a loess fit to the residuals, the dashed black lines are the mean of the residuals.

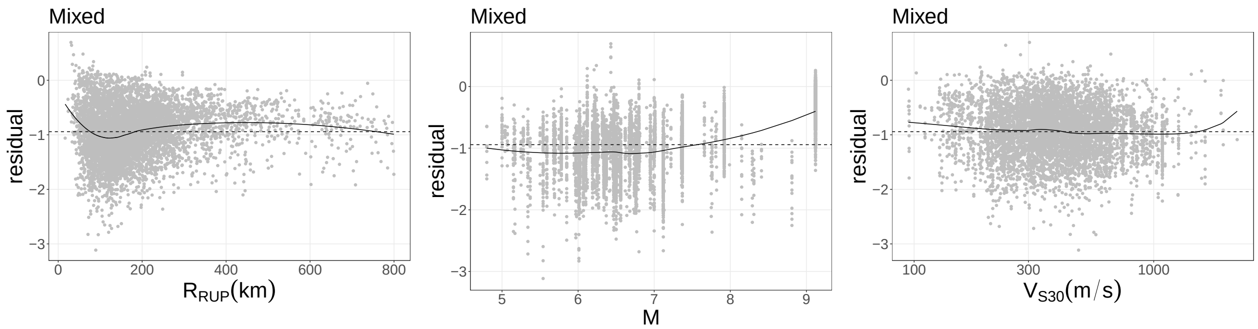

Figure S14: Residuals to Interface NGA-Sub data from KBCG with the mixed model. The soild black lines show a loess fit to the residuals, the dashed black lines are the mean of the residuals.

References

Vehtari, A., A. Gelman, and J. Gabry (2017, sep). Practical Bayesian model evaluation using leave-one-out cross-validation and WAIC. *Statistics and Computing 27*(5), 1413–1432.