Discrete-Time Design and Applications of Uncertainty and Disturbance Estimator

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Outline

Part 1: Introduction

Motivation

Digital Control:-

- Modern digital computers cannot process continuous-time data.
- However, physical systems are CT systems.
  - Must sample output/error signals to generate a digital control law.
  - Use a zero–order–hold (ZOH)/DAC to drive the original system.

Robust Control:-

- Uncertainties in mathematical models and unmodeled dynamics.
- External, unmeasurable disturbances.
- System faults, other unknown inputs and measurement noise.

Important to design robust, discrete-time systems.

- Design a DT control law robust to uncertainties and disturbances.
- Can easily control CT systems using sampling and ZOH.

Related Work (1)

- Many strategies for robust control exist:
  - Sliding Mode Control (SMC), Time–Delay Control (TDC), Disturbance Observer (DO), $\mathcal{H}_\infty$ control.
- Other than SMC, not many of these are standard in discrete-time.
- Advantage of DT-SMC: No chattering in control signal, compared to CT-SMC [1].
- Extension to discretized CT systems: assumption of small sampling time [2].
- Recent work in DT-SMC [3, 4]: Focus on modifying original laws in [1].
- Drawback of DT-SMC: Requires knowledge of bounds on uncertainties.
- Drawback of TDC: Requires all states *and* derivatives to be available for feedback.

Related Work (2)

- Focus of this work: Uncertainty and Disturbance Estimator (UDE).
- Originally proposed for CT systems in 2004 [5].
  - Estimate overall disturbance on system using LPF.
  - Compensate using this estimate in control law.
- Advantages:
  - Knowledge of bounds on uncertainties not required.
  - Only states required for feedback, and not even this if an observer is used.
- No complete, self-contained prior work on UDE for DT systems.
  - In [6], UDE used for a DT system, but final control law based on DT-SMC.
- DT control laws: easier to implement.
- Can control CT systems using sampling and a ZOH.

Contributions

- A new error-based control law for CT-UDE.
- Design of a novel digital filter for disturbance estimation in DT-UDE.
- Deriving control law for DT systems.
- Analyzing stability of entire system.
- Extending to control of CT systems.
- Extending to control of nonlinear systems.

Part 2: UDE for Continuous-time Systems

Problem Setup

- Main idea: Reference-model following = Stabilization of state tracking error: Feedback control.
- Plant:

$$\begin{aligned}\dot{x}(t) &= (A_n + \Delta A)\, x(t) + (b_n + \Delta b)\, u(t) + d(x,t)\\ &= A_n x(t) + b_n u(t) + D(x,t)\end{aligned} \tag{1}$$

- Objective: States of (1) must follow states of reference model:

$$\dot{x}_m(t) = A_m x_m(t) + b_m r(t) \tag{2}$$

- State tracking error $e(t) \triangleq x(t) - x_m(t)$; $\dot{e}(t) = \dot{x}(t) - \dot{x}_m(t)$:

$$\begin{aligned}\dot{e}(t) &= A_n x(t) + b_n u(t) + D(x,t) - A_m x_m(t) - b_m r(t)\\ &= A_n e(t) + b_n u(t) + L(t)\end{aligned} \tag{3}$$

Design of Control Law (1)

- If $L(t) = 0$, $u(t) = -Ke(t)$ stabilizes error.
- With non–zero $L(t)$, we assume disturbances are matched, i.e. $L(t) = b_n \psi(t)$.
  - Disturbances lie in range space of input matrix.
  - Easily satisfied for common canonical forms.
- Technique of UDE:
  - Estimate $L(t)$ using LPF.
  - Compensate $L(t)$ in control law.
- Estimation: Let $\hat{L}(t) = G_f(s)L(t)$, where $G_f(s) = \dfrac{1}{1+\tau s}$.
- Compensation: $u(t) = -Ke(t) + u_d(t)$ and $b_n u_d(t) = -\hat{L}(t)$; $u_d(t) = -b_n^{\dagger}\hat{L}(t)$
  - $u_d(t)$: The robust control, augmenting feedback control.

Design of Control Law (2)

Substitute $u(t) = -Ke(t) + u_d(t)$ in (3):

$$\dot{e}(t) = A_n e(t) - b_n K e(t) + b_n u_d(t) + L(t) \tag{4}$$

$$\implies L(t) = \dot{e}(t) - (A_n - b_n K)e(t) - b_n u_d(t)$$

Multiply throughout by $b_n^\dagger G_f(s)$:

$$b_n^\dagger \hat{L}(t) = -u_d(t) = G_f(s) b_n^\dagger \Big(\dot{e}(t) - (A_n - b_n K)e(t)\Big) - G_f(s) u_d(t) \tag{5}$$

Solve for $u_d(t)$:

$$\begin{aligned} u_d(t) &= \frac{G_f(s)}{G_f(s) - 1} b_n^\dagger \Big(\dot{e}(t) - (A_n - b_n K)e(t)\Big) \\ &= -\frac{1}{\tau} b_n^\dagger \left[e(t) - (A_n - b_n K) \int e(t)\mathrm{d}t\right] \end{aligned} \tag{6}$$

Design of Control Law (3)

Finally:

$$u(t)_{CT-UDE} = -Ke(t) - \frac{1}{\tau} b_n^{\dagger} \left[e(t) - (A_n - b_n K) \int e(t)\mathrm{d}t \right] \tag{7}$$

For MIMO systems:

$$u(t)_{CT-UDE} = -Ke(t) - \frac{1}{\tau} B_n^{\dagger} \left[e(t) - (A_n - B_n K) \int e(t)\mathrm{d}t \right]$$

- The above control law is:
  - Time-domain law, not frequency domain.
  - Based on error dynamics, not directly on states.
- However, condition for stability remains the same.

Stability Analysis (1)

From (4):

$$\dot{e}(t) = (A_n - b_n K)e(t) + \tilde{L}(t) \tag{8}$$

$\tilde{L}(t) = L(t) - \hat{L}(t)$: disturbance estimation error.

$\hat{L}(t) = G_f(s)L(t)$:

$$\hat{L}(t) + \tau\dot{\hat{L}}(t) = L(t)$$

or,

$$\dot{\hat{L}}(t) = \frac{1}{\tau}\tilde{L}(t)$$

$$\implies \dot{\tilde{L}}(t) = -\frac{1}{\tau}\tilde{L}(t) + \dot{L}(t) \tag{9}$$

Stability Analysis (2)

From (8) and (9):

$$\begin{bmatrix} \dot{e}(t) \\ \dot{\tilde{L}}(t) \end{bmatrix} = \begin{bmatrix} A_n - b_n K & I_n \\ \mathbf{0} & T \end{bmatrix} \begin{bmatrix} e(t) \\ \tilde{L}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ I_n \end{bmatrix} \dot{L}(t) \tag{10}$$

where $T = -\frac{1}{\tau} I_n$.

Stability: Roots of $\left| sI - (A_n - b_n K) \right| \left(s + \frac{1}{\tau} \right)^n = 0$ must be in LHP.

$$\implies \tau > 0 \tag{11}$$

(10) depends on $\dot{L}(t)$. If $L(t)$ is slowly time-varying, $\dot{L}(t) \approx 0$: asymptotic stability. Else, only BIBO stability.

Part 3: UDE for Discrete-time Systems

Problem Setup

- Plant:

$$
\begin{aligned}
x(k+1) &= (F_n + \Delta F)\,x(k) + (g_n + \Delta g)\,u(k) + d_d(x,k) \\
&= F_n x(k) + g_n u(k) + D_d(x,k) \qquad (12)
\end{aligned}
$$

- Objective: States of (12) must follow states of reference model:

$$
x_m(k+1) = F_m x_m(k) + g_m r(k) \qquad (13)
$$

- State tracking error $e(k) \triangleq x(k) - x_m(k)$; $e(k+1) = x(k+1) - x_m(k+1)$:

$$
\begin{aligned}
e(k+1) &= F_n x(k) + g_n u(k) + D_d(x,k) - F_m x_m(k) - g_m r(k) \\
&= F_n e(k) + g_n u(k) + L_d(k) \qquad (14)
\end{aligned}
$$

Design of Control Law (1)

- If $L_d(k) = 0$, $u(k) = -K_d e(k)$ stabilizes error.
- With non–zero $L_d(k)$, assume matched disturbances, i.e. $L_d(k) = g_n \psi_d(k)$.
  - Disturbances lie in range space of input matrix.
  - Easily satisfied for common canonical forms.
- Technique of UDE:
  - Estimate $L_d(k)$ by a novel digital filter.
  - Compensate $L_d(k)$ in control law.
- Estimation: Let $\hat{L}_d(k) = G_f(\gamma) L_d(k)$, where $G_f(\gamma) = \dfrac{1}{1+\tau\gamma}$; $\gamma = \dfrac{z-1}{T_s}$
  - γ is similar to analog frequency-domain operator s.
  - $\gamma\left[m(k)\right] = \frac{m(k+1)-m(k)}{T_s}$
- Compensation: $u(k) = -K_d e(k) + u_d(k)$ and $g_n u_d(k) = -\hat{L}_d(k)$; $u_d(k) = -g_n^{\dagger} \hat{L}_d(k)$
  - $u_d(k)$: The robust control, augmenting feedback control.

Design of Control Law (2)

Substitute $u(k) = -K_d e(k) + u_d(k)$ in (14):

$$e(k+1) = F_n e(k) - g_n K_d e(k) + g_n u_d(k) + L_d(k) \tag{15}$$

$$\implies L_d(k) = e(k+1) - (F_n - g_n K_d)e(k) - g_n u_d(k)$$

Multiplying throughout by $g_n^{\dagger} G_f(\gamma)$:

$$g_n^{\dagger} \hat{L}_d(k) = -u_d(k) = G_f(\gamma) g_n^{\dagger} \Big(e(k+1) - (F_n - g_n K_d)e(k) \Big) - G_f(\gamma) u_d(k) \tag{16}$$

Solve for $u_d(k)$:

$$\begin{aligned} u_d(k) &= \frac{G_f(\gamma)}{G_f(\gamma) - 1} g_n^{\dagger} \Big(e(k+1) - (F_n - g_n K_d)e(k) \Big) \\ &= -\frac{T_s}{\tau} \left(\frac{1}{1 - z^{-1}} \right) g_n^{\dagger} \Big(e(k) - (F_n - g_n K_d)e(k-1) \Big) \end{aligned} \tag{17}$$

Design of Control Law (3)

This results in:

$$u(k)_{DT-UDE} = \begin{cases} -\left(K_d + \dfrac{T_s}{\tau} g_n^{\dagger}\right) e(0) & \text{if } k = 0 \\ -K_d e(k) - \dfrac{T_s}{\tau}\left(\dfrac{1}{1 - z^{-1}}\right) g_n^{\dagger}\Big(e(k) - (F_n - g_n K_d) e(k-1)\Big) & \text{if } k > 0 \end{cases} \tag{18}$$

For MIMO systems:

$$u(k)_{DT-UDE} = \begin{cases} -\left(K_d + \dfrac{T_s}{\tau} G_n^{\dagger}\right) e(0) & \text{if } k = 0 \\ -K_d e(k) - \dfrac{T_s}{\tau}\left(\dfrac{1}{1 - z^{-1}}\right) G_n^{\dagger}\Big(e(k) - (F_n - G_n K_d) e(k-1)\Big) & \text{if } k > 0 \end{cases}$$

Stability Analysis (1)

From (15):

$$e(k+1) = (F_n - g_n K_d)e(k) + \tilde{L}_d(k) \tag{19}$$

$\tilde{L}_d(k) = L_d(k) - \hat{L}_d(k)$: disturbance estimation error.

Now, $\hat{L}_d(k) = G_f(\gamma)L_d(k)$:

$$\hat{L}_d(k) + \tau\gamma\hat{L}_d(k) = L_d(k)$$

After a few simple algebraic manipulations:

$$\tilde{L}_d(k+1) = \left(1 - \frac{T_s}{\tau}\right)\tilde{L}_d(k) + \Delta L_d(k) \tag{20}$$

Stability Analysis (2)

From (19) and (20):

$$\begin{bmatrix} e(k+1) \\ \tilde{L}_d(k+1) \end{bmatrix} = \begin{bmatrix} F_n - g_n K_d & I_n \\ \mathbf{0} & T_d \end{bmatrix} \begin{bmatrix} e(k) \\ \tilde{L}_d(k) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ I_n \end{bmatrix} \Delta L_d(k) \tag{21}$$

where $T_d = \left(1 - \frac{T_s}{\tau}\right) I_n$.

Stability: Roots of $\left|zI - (F_n - g_n K_d)\right| \left(z + \frac{T_s}{\tau} - 1\right)^n = 0$ must be within unit disk.

$$\implies \frac{T_s}{\tau} - 1 < 1, \text{ or, } \tau > \frac{T_s}{2} \tag{22}$$

(21) depends on $\Delta L_d(k)$. If slowly time-varying, $\Delta L_d(k) \approx 0$: asymptotic stability. Else, only BIBO stability.

Part 4: Discrete-time UDE for Continuous-time Systems

Introduction

Motivation:

- Physical systems: Continuous-time systems.
  - Cannot physically process CT/analog signals.
  - Requirement of very small sampling time.
- In contrast, digital control laws: easily implementable.

Two-step procedure:

- Discretize system states/outputs/error signals.
  - A microcontroller can use these to generate digital control law.
- Drive original CT system using the digital control law and a D/A converter (ZOH).

Procedure (1)

Consider the CT error system (3):

$$\dot{e}(t) = A_n e(t) + b_n u(t) + L(t)$$

Sample CT error signal with sampling time T_s:

$$e(k+1) = F_n e(k) + g_n u(k) + L_d(k) \tag{23}$$

where

$$F_n = \exp(A_n T_s); \qquad g_n = \int_{\theta=0}^{T_s} \exp(A_n \theta) b_n \mathrm{d}\theta;$$

$$L_d(k) = \int_{\theta=0}^{T_s} \exp(A_n \theta) L((k+1)T_s - \theta) \mathrm{d}\theta$$

Procedure (2)

Are matching conditions still satisfied?

Assuming $L(t) = b_n\psi(t)$, is $L_d(k) = g_n\psi_d(k)$?

YES.

- Proved by Tesfaye and Tomizuka [2].
- Assume that T_s is "small" such that powers beyond T_s^2 can be neglected.
- Can be shown that $L(t) = b_n\psi(t) \implies L_d(k) = g_n\psi_d(k)$.

Hence,

- Matching conditions hold after discretizing error.
- Design $u(k)$ to nullify effect of $L_d(k)$.
- $e(k)$ is asymptotically stable $\implies e(t)$ is asymptotically stable.

Block Diagram

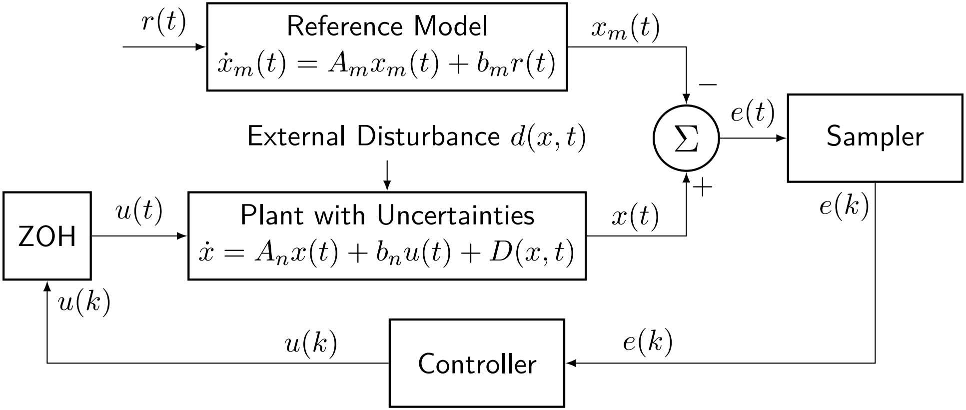


Figure 1: Block Diagram

Part 5: Controlling a Class of Nonlinear Systems

Introduction

Motivation:

- Physical systems are nonlinear in nature.
- Efficacy of proposed scheme can be evaluated in real-world.

Considered system: Phenomenon of wing-rock motion.

- Observed in high-performance combat aircraft, with slender delta wings.
- At high angles of attack: displays limit cycle behaviour: may result in instability.
- Controlling wing-rock motion is of enormous significance in high-performance aircraft, with manoeuvres requiring high angles of attack.
- Robustness: an important consideration, due to uncertainties in mathematical models and nonlinearities.

Related Work

Prior strategies using CT-UDE:

- Robustification of an input-output linearization (IOL) controller using UDE [7].
- A controller-observer structure for UDE-based IOL [8].
- Controlling wing-rock motion under time-varying angle of attack [9].

Prior discrete-time strategies:

- A DT SMC law using a first-order Taylor series expansion for discretization [10].
- Discretizing the original nonlinear wing-rock dynamics with varying angle of attack [11].

Problem Setup and Solution (1)

Consider dynamics from [12]:

$$\begin{aligned} \dot{\phi} &= p \\ \dot{p} &= c_1 + c_2\phi + c_3 p + c_4|\phi|p + c_5|p|p + c_6 u \\ y &= \phi \end{aligned} \tag{24}$$

ϕ	:	Roll angle
p	:	Roll rate
u	:	Aileron deflection angle (input)
c_1 to c_6	:	Aerodynamic coefficients depending on angle of attack

Technique: Combine all nonlinear terms together as disturbances.

Problem Setup and Solution (2)

Let $c_2 = c_{2n} + \Delta c_2$, $c_3 = c_{3n} + \Delta c_3$, $c_6 = c_{6n} + \Delta c_6$. (Nominal + Uncertainty). Then,

$$\begin{aligned} \dot{\phi} &= p \\ \dot{p} &= c_{2n}\phi + c_{3n}p + c_{6n}u + d(t) \\ y &= \phi \end{aligned} \tag{25}$$

with $d(t) = c_1 + \Delta c_2\phi + \Delta c_3 p + c_4|\phi|p + c_5|p|p + \Delta c_6 u$.

Can be expressed as $\dot{x}(t) = A_n x(t) + b_n u(t) + D(x,t)$:

$$x = \begin{bmatrix} \phi \\ p \end{bmatrix}, \quad A_n = \begin{bmatrix} 0 & 1 \\ c_{2n} & c_{3n} \end{bmatrix}, \quad b_n = \begin{bmatrix} 0 \\ c_{6n} \end{bmatrix}, \text{ and } D(x,t) = \begin{bmatrix} 0 \\ d(t) \end{bmatrix}$$

Problem Setup and Solution (3)

With a stable reference model:

$$
\begin{aligned}
\dot{\phi}_m &= p_m \\
\dot{p}_m &= c_{2m}\phi_m + c_{3m}p_m + c_{6m}r \\
y_m &= \phi_m
\end{aligned}
\tag{26}
$$

i.e. $\dot{x}_m(t) = A_m x_m(t) + b_m r(t)$:

$$
x_m = \begin{bmatrix} \phi_m \\ p_m \end{bmatrix}, \quad A_m = \begin{bmatrix} 0 & 1 \\ c_{2m} & c_{3m} \end{bmatrix}, \quad b_m = \begin{bmatrix} 0 \\ c_{6m} \end{bmatrix}
$$

- Construct CT error dynamics (3) with $e(t) = x(t) - x_m(t)$.
- Sample to obtain DT error dynamics (23).
- Design $u(k)$, and use a ZOH to control the original CT system.

Part 6: Simulation Examples

Example 1 (1)

Example 1:

- Controlling a CT system using DT-UDE.
- Comparing proposed DT-UDE with CT-UDE.

Plant: $\dot{x}(t) = (A_n + \Delta A)\, x(t) + (b_n + \Delta b)\, u(t) + d(x,t)$, where $x(0) = [3, -1]^T$;

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}; \quad A_n = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}; \quad \Delta A = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}; \quad b_n = \begin{bmatrix} 0 \\ 2 \end{bmatrix}; \quad \Delta b = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}$$

$$d(x,t) = 2\left(\sin(t)x_1^2(t) + \cos(t)x_2(t) + 1\right)$$

Reference Model:

$$\begin{bmatrix} \dot{x}_{1m}(t) \\ \dot{x}_{2m}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_{1m}(t) \\ x_{2m}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t); \quad r(t) = 5\sin(2\pi 0.3t); \quad x_m(0) = [-2, 2]^T$$

Example 1 (2)

Error: $\dot{e}(t) = A_n e(t) + b_n u(t) + L(t)$, where
$L(t) = \Delta A x(t) + \Delta b u(t) + d(x,t) + (A_n - A_m)x_m(t) - b_m r(t)$.

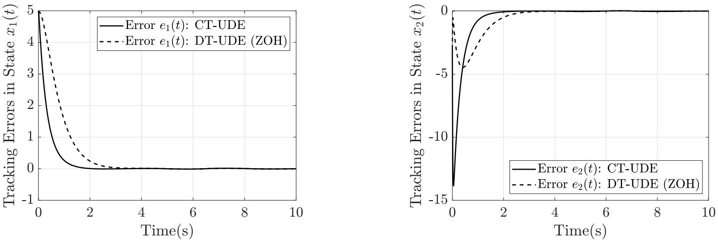
CT-UDE: Filter time-constant $\tau = 0.01 > 0$.
$u(t) = -Ke(t) + u_d(t)$. Choose K such that $\text{eig}\,(A_n - b_n K) = \text{eig}\,(A_m)$: $K = [2, 4]$.

DT-UDE: Discretize error: $e(k+1) = F_n e(k) + g_n u(k) + L_d(k)$, where

$$F_n = \begin{bmatrix} 1 & 0.01 \\ -0.02 & 1.03 \end{bmatrix}; \quad g_n = \begin{bmatrix} 0 \\ 0.02 \end{bmatrix}; \quad T_s = 0.01\text{s}$$

$u(k) = -K_d e(k) + u_d(k)$. Choose K_d such that $\text{eig}\,(F_n - g_n K_d) = \text{eig}\,(F_m)$ where $F_m = \begin{bmatrix} 1 & 0.01 \\ -0.06 & 0.95 \end{bmatrix} \implies K_d = [1.88, 3.91]$. $\tau = 0.01 > \frac{T_s}{2}$. Use ZOH to control system.

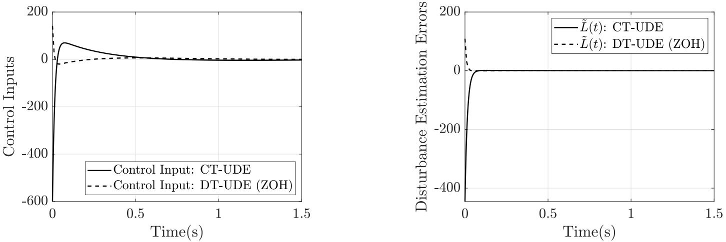
Results for Example 1 (1)



(a) Error $e_1(t)$ (b) Error $e_2(t)$

Figure 2: Comparing CT-UDE and DT-UDE (1)

Results for Example 1 (2)



(a) Control Inputs　　(b) Disturbance Estimation

Figure 3: Comparing CT-UDE and DT-UDE (2)

DT-UDE: Improved transient error performance and lesser control energy.

Example 2 (1)

Example 2: Phenomenon of wing-rock motion.

Plant: As in (25). For angle of attack $\alpha = 30^\circ$ (see [12]), $c_{1n} = 5, c_{2n} = -26.67$ s^{-1}, $c_{3n} = 0.765$ s^{-1}, $c_{4n} = -2.92$ rad s^{-1}, $c_{5n} = -2.5, c_{6n} = 0.75$.
$\phi(0) = \pi/9; \;\; p(0) = 0.$

Uncertainties: $c_2 = c_{2n} + \Delta c_2 = -20$, $c_3 = c_{3n} + \Delta c_3 = 1$, $c_6 = c_{6n} + \Delta c_6 = 0.5$.
Then:

$$\begin{bmatrix} \dot{\phi} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -20 & 1 \end{bmatrix} \begin{bmatrix} \phi \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} u + \begin{bmatrix} 0 \\ d(t) \end{bmatrix}$$

where $d(t) = \Delta c_2 \phi + \Delta c_3 p + \Delta c_6 u + c_1 + c_4 |\phi| p + c_5 |p| p$. Choose $c_1 = 5 \sin(15t), c_4 = 3 \cos(5t), c_5 = 10 \sin(10t)$ as in [7, 8].

Example 2 (2)

Reference Model:

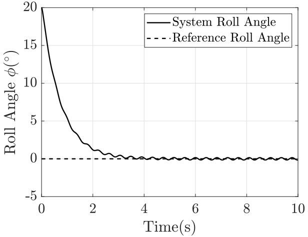
$$\begin{bmatrix} \dot{\phi}_m(t) \\ \dot{p}_m(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} \phi_m \\ p_m \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

$r(t)$ depends on whether stabilization or tracking is desired. $\phi_m(0) = p_m(0) = 0$.

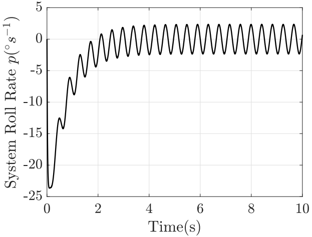
Obtain $e(t) = [\phi(t) - \phi_m(t), p(t) - p_m(t)]^T$, discretize with $T_s = 0.01\text{s}$:
$e(k+1) = F_n e(k) + g_n u(k) + L_d(k)$, where $F_n = \begin{bmatrix} 1 & 0.01 \\ -0.27 & 1.01 \end{bmatrix}$, $g_n = \begin{bmatrix} 0 \\ 0.43 \end{bmatrix}$.

$u(k) = -K_d e(k) + u_d(k)$. Choose K_d using DT-LQR: Minimize
$J = \sum_{k=0}^{\infty} \left(e(k)^T Q e(k) + u(k)^T R u(k) \right)$ with $Q = I_2$ and $R = 2$. More weight to the input term. Results in $K_d = [0.18, 0.63]$. $\tau = 0.01 > \frac{T_s}{2}$. Use ZOH to control system.

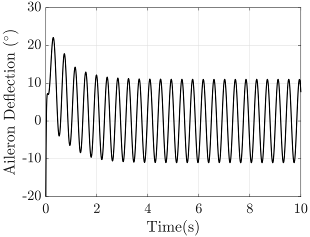
Results for Example 2 (1)



(a) Stabilization of ϕ

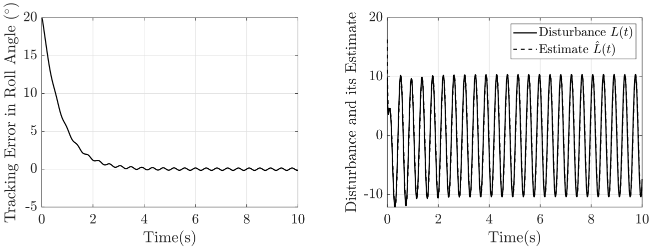


(b) Roll rate p



(c) Aileron Deflection (input)

Figure 4: Stabilization of roll angle (1) $(r(t) = 0)$

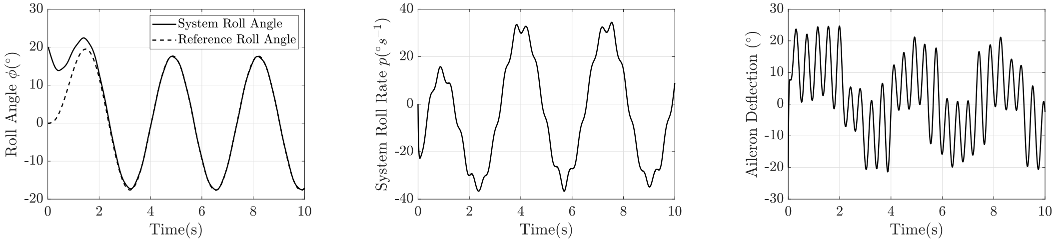


(a) Tracking error in ϕ (b) Disturbance Estimation

Figure 5: Stabilization of roll angle (2) ($r(t) = 0$)

Stabilization of roll angle and accurate disturbance estimation achieved with DT-UDE + ZOH.

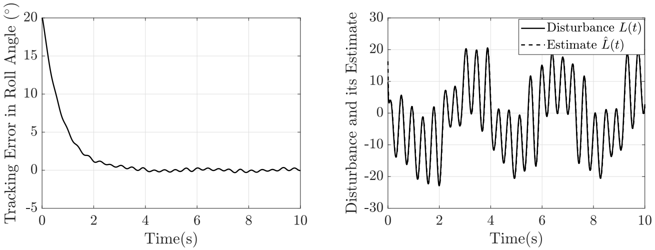
Results for Example 2 (3)



(a) Tracking for ϕ (b) Roll rate p (c) Aileron Deflection (input)

Figure 6: Tracking for roll angle (1) ($r(t) = 3\sin(2\pi 0.3t)$)

Results for Example 2 (4)



(a) Tracking error in ϕ (b) Disturbance Estimation

Figure 7: Tracking for roll angle (2) $(r(t) = 3\sin(2\pi 0.3t))$

Tracking control and accurate disturbance estimation achieved with DT-UDE + ZOH.

Part 7: Concluding Remarks

Summary

- For CT-UDE: A new time-domain, error-based control law.
  - Contrast with the original frequency-domain approach in [5].
- A complete investigation of DT-UDE:
  - Use of a novel digital filter.
  - Design of control law.
  - Stability analysis.
- Controlling LTI CT systems with DT-UDE, using sampling and ZOH.
- Controlling a class of nonlinear systems: wing-rock motion.
- Simulation results demonstrate the efficacy of proposed strategy.

Avenues for Future Work

All control laws designed require all states to be available for feedback.

- Investigate the design of a robust, DT observer to address this.

MIMO systems in more detail:

- Applications such as robot manipulators, missile guidance systems.

Asymptotic stability depends on slowly varying $L(t)$ or $L_d(k)$:

- The use of higher-order and modified filters for disturbance estimation can mitigate this.
- See [7, 13] for CT examples.

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