Untitled

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### Loading Libraries

library(astsa)

## 3.1

### Part a)

trend = time(jj) - 1970  
q = factor(cycle(jj))  
req = lm(log(jj) ~ 0 + trend + q, na.action = NULL)  
model.matrix(req)

## trend q1 q2 q3 q4  
## 1 -10.00 1 0 0 0  
## 2 -9.75 0 1 0 0  
## 3 -9.50 0 0 1 0  
## 4 -9.25 0 0 0 1  
## 5 -9.00 1 0 0 0  
## 6 -8.75 0 1 0 0  
## 7 -8.50 0 0 1 0  
## 8 -8.25 0 0 0 1  
## 9 -8.00 1 0 0 0  
## 10 -7.75 0 1 0 0  
## 11 -7.50 0 0 1 0  
## 12 -7.25 0 0 0 1  
## 13 -7.00 1 0 0 0  
## 14 -6.75 0 1 0 0  
## 15 -6.50 0 0 1 0  
## 16 -6.25 0 0 0 1  
## 17 -6.00 1 0 0 0  
## 18 -5.75 0 1 0 0  
## 19 -5.50 0 0 1 0  
## 20 -5.25 0 0 0 1  
## 21 -5.00 1 0 0 0  
## 22 -4.75 0 1 0 0  
## 23 -4.50 0 0 1 0  
## 24 -4.25 0 0 0 1  
## 25 -4.00 1 0 0 0  
## 26 -3.75 0 1 0 0  
## 27 -3.50 0 0 1 0  
## 28 -3.25 0 0 0 1  
## 29 -3.00 1 0 0 0  
## 30 -2.75 0 1 0 0  
## 31 -2.50 0 0 1 0  
## 32 -2.25 0 0 0 1  
## 33 -2.00 1 0 0 0  
## 34 -1.75 0 1 0 0  
## 35 -1.50 0 0 1 0  
## 36 -1.25 0 0 0 1  
## 37 -1.00 1 0 0 0  
## 38 -0.75 0 1 0 0  
## 39 -0.50 0 0 1 0  
## 40 -0.25 0 0 0 1  
## 41 0.00 1 0 0 0  
## 42 0.25 0 1 0 0  
## 43 0.50 0 0 1 0  
## 44 0.75 0 0 0 1  
## 45 1.00 1 0 0 0  
## 46 1.25 0 1 0 0  
## 47 1.50 0 0 1 0  
## 48 1.75 0 0 0 1  
## 49 2.00 1 0 0 0  
## 50 2.25 0 1 0 0  
## 51 2.50 0 0 1 0  
## 52 2.75 0 0 0 1  
## 53 3.00 1 0 0 0  
## 54 3.25 0 1 0 0  
## 55 3.50 0 0 1 0  
## 56 3.75 0 0 0 1  
## 57 4.00 1 0 0 0  
## 58 4.25 0 1 0 0  
## 59 4.50 0 0 1 0  
## 60 4.75 0 0 0 1  
## 61 5.00 1 0 0 0  
## 62 5.25 0 1 0 0  
## 63 5.50 0 0 1 0  
## 64 5.75 0 0 0 1  
## 65 6.00 1 0 0 0  
## 66 6.25 0 1 0 0  
## 67 6.50 0 0 1 0  
## 68 6.75 0 0 0 1  
## 69 7.00 1 0 0 0  
## 70 7.25 0 1 0 0  
## 71 7.50 0 0 1 0  
## 72 7.75 0 0 0 1  
## 73 8.00 1 0 0 0  
## 74 8.25 0 1 0 0  
## 75 8.50 0 0 1 0  
## 76 8.75 0 0 0 1  
## 77 9.00 1 0 0 0  
## 78 9.25 0 1 0 0  
## 79 9.50 0 0 1 0  
## 80 9.75 0 0 0 1  
## 81 10.00 1 0 0 0  
## 82 10.25 0 1 0 0  
## 83 10.50 0 0 1 0  
## 84 10.75 0 0 0 1  
## attr(,"assign")  
## [1] 1 2 2 2 2  
## attr(,"contrasts")  
## attr(,"contrasts")$q  
## [1] "contr.treatment"

summary(req)

##   
## Call:  
## lm(formula = log(jj) ~ 0 + trend + q, na.action = NULL)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.29318 -0.09062 -0.01180 0.08460 0.27644   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## trend 0.167172 0.002259 74.00 <2e-16 \*\*\*  
## q1 1.052793 0.027359 38.48 <2e-16 \*\*\*  
## q2 1.080916 0.027365 39.50 <2e-16 \*\*\*  
## q3 1.151024 0.027383 42.03 <2e-16 \*\*\*  
## q4 0.882266 0.027412 32.19 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1254 on 79 degrees of freedom  
## Multiple R-squared: 0.9935, Adjusted R-squared: 0.9931   
## F-statistic: 2407 on 5 and 79 DF, p-value: < 2.2e-16

### Part b)

The estimated average annual increase in the logged earnings per share is

which can be extracted from the summary table

1.052793 + 1.080916 + 1.151024 + 0.882266

## [1] 4.166999

### Part c)

Average logged earning rate - decreases.

0.882266 - 1.151024

## [1] -0.268758

The percentage of the decrease is

(0.269/1.151024) \* 100

## [1] 23.37049

### Part d)

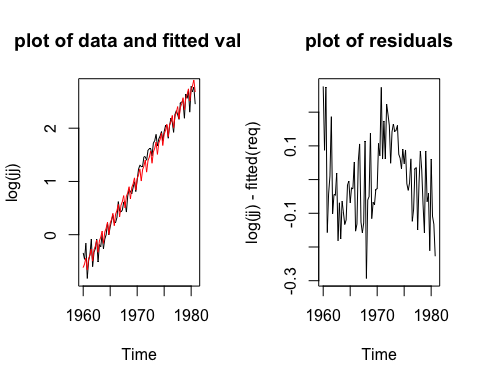
trend = time(jj) - 1970  
q = factor(cycle(jj))  
req = lm(log(jj) ~ trend + q, na.action = NULL)  
summary(req)

##   
## Call:  
## lm(formula = log(jj) ~ trend + q, na.action = NULL)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.29318 -0.09062 -0.01180 0.08460 0.27644   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## (Intercept) 1.052793 0.027359 38.480 < 2e-16 \*\*\*  
## trend 0.167172 0.002259 73.999 < 2e-16 \*\*\*  
## q2 0.028123 0.038696 0.727 0.4695   
## q3 0.098231 0.038708 2.538 0.0131 \*   
## q4 -0.170527 0.038729 -4.403 3.31e-05 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1254 on 79 degrees of freedom  
## Multiple R-squared: 0.9859, Adjusted R-squared: 0.9852   
## F-statistic: 1379 on 4 and 79 DF, p-value: < 2.2e-16

Having an intercept removes the first quarter away. Also the intercept appears in all the quarters. It is of no use because we want to analyze all the quarters separately.

### Part e)

par(mfrow=c(1,2))  
plot(log(jj), main="plot of data and fitted value")  
lines(fitted(req), col="red")   
plot(log(jj) - fitted(req), main="plot of residuals")



The residuals are not following any pattern hence it looks fairly white. The fit of the model is decent.

## 3.2

### Part a)

temp = tempr - mean(tempr)  
#ded = ts.intersect(trend=time(cmort),temp,temp2=temp^2,part,partL4=lag(part,-4))  
temp2 = temp^2  
trend = time(cmort)  
fit = lm(cmort ~ trend + temp + temp2 + part + lag(part,-4), na.action = NULL)  
#fit = lm(cmort ~ ded, na.action=NULL)  
summary(fit)

##   
## Call:  
## lm(formula = cmort ~ trend + temp + temp2 + part + lag(part,   
## -4), na.action = NULL)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -19.0760 -4.2153 -0.4878 3.7435 29.2448   
##   
## Coefficients: (1 not defined because of singularities)  
## Estimate Std. Error t value Pr(>|t|)   
## (Intercept) 2.831e+03 1.996e+02 14.19 < 2e-16 \*\*\*  
## trend -1.396e+00 1.010e-01 -13.82 < 2e-16 \*\*\*  
## temp -4.725e-01 3.162e-02 -14.94 < 2e-16 \*\*\*  
## temp2 2.259e-02 2.827e-03 7.99 9.26e-15 \*\*\*  
## part 2.554e-01 1.886e-02 13.54 < 2e-16 \*\*\*  
## lag(part, -4) NA NA NA NA   
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 6.385 on 503 degrees of freedom  
## Multiple R-squared: 0.5954, Adjusted R-squared: 0.5922   
## F-statistic: 185 on 4 and 503 DF, p-value: < 2.2e-16

**Some conclusion**

### Part b)

AIC(fit)/length(cmort) - log(2\*pi)

## [1] 4.721732

BIC(fit)/length(cmort) - log(2\*pi)

## [1] 4.771699

**Some conclusion on this.**

## 3.4

### Part a)

The equation is

Mean of

It is dependent on time t, hence the series is not stationary.

### Part b)

The first order difference can be simplified into

$$
\Delta x\_{t} = x\_{t} - x\_{t-1}
\\
= \beta\_{0} + \beta\_{1}t + w\_{t} - [\beta\_{0} + \beta\_{1}(t-1) + w\_{t-1}]
\\
= w\_{t} - w\_{t-1} + \beta\_{1}
$$

Mean of the function would be

$$
E[\Delta x\_{t}] = E[w\_{t} - w\_{t-1} + \beta\_{1}]
\\
=\beta\_{1} + E[w\_{t}] - E[w\_{t-1}]
\\
=\beta\_{1}
$$

Independent of time

and now Auto Covariance Function is:

$$
\gamma \Delta x\_{t}(t+h,h) = Cov(x\_{t+h},x\_{t})
\\
= Cov(w\_{t+h} - w\_{t+h-1} + \beta\_{1},w\_{t} - w\_{t-1} + \beta\_{1})
\\
= Cov(w\_{t+h} - w\_{t+h-1},w\_{t} - w\_{t-1})
\\
\begin{cases}
2 \sigma^2\_{w}, & \text{h=0}
\\
-\sigma^2\_{w}, & \text{|h| = 1}
\\
0, & \text{|h|>1}
\end{cases}
$$

which is also free from t, hence is stationary.