

$$x = \begin{bmatrix} z \\ \phi \\ \theta \\ \psi \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{matrix} \Rightarrow \dot{x} = \begin{bmatrix} \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \\ \frac{1}{m}(\cos(\alpha_2) \cdot \cos(\alpha_3)) \cdot u_1 - g \\ \frac{I_x}{I_x} \{ x_7 x_8 (I_y - I_z) - I_P \Omega x_7 + u_2 \} \\ \frac{I_x}{I_y} \{ x_6 x_8 (I_z - I_x) + I_P \Omega x_6 + u_3 \} \\ \frac{I}{I_z} \{ x_6 x_7 (I_x - I_y) + u_4 \} \end{bmatrix}$$

① \* Designing  $z$  :-

$$s_1 = \dot{e} + \lambda_1 e = (\dot{z} - \dot{z}_d) + \lambda_1 (z - z_d) \\ = (x_5 - \dot{z}_d) + \lambda_1 (x_1 - z_d) \\ \dot{s}_1 = x_5 - \dot{z}_d + \lambda_1 (x_1 - z_d)$$

$$= \frac{1}{m} \{ \cos(\alpha_2) \cdot \cos(\alpha_3) u_1 - g \} - \ddot{z}_d + \lambda_1 (x_5 - \dot{z}_d)$$

$$s_1 \dot{s}_1 = s_1 \cdot \left\{ \frac{\cos(\alpha_2) \cdot \cos(\alpha_3)}{m} \left\{ m \left\{ \lambda_1 (x_5 - \dot{z}_d) - \ddot{z}_d \right\} - \frac{g}{\cos(\alpha_2) \cdot \cos(\alpha_3)} + u_1 \right\} \right\}$$

$$s(x) \geq \frac{m(\lambda_1(x_5 - \dot{z}_d) - \ddot{z}_d) - g}{\cos(\alpha_2) \cdot \cos(\alpha_3)}$$

$$u_1 = - \left\{ \frac{m(\lambda_1(x_5 - \dot{z}_d) - \ddot{z}_d) - g}{\cos(\alpha_2) \cos(\alpha_3)} + K_I \right\} \cdot \text{sat}(s_1)$$

② \* Designing  $\phi$  :-

$$s_2 = \dot{e} + \lambda_2 e = (\dot{\phi} - \dot{\phi}_d) + \lambda_2 (\phi - \phi_d)$$

$$\dot{s}_2 = (\ddot{\phi} - \ddot{\phi}_d) + \lambda_2 (\dot{\phi} - \dot{\phi}_d)$$

$$= \left\{ \frac{1}{I_x} \left\{ (x_7 x_8 (I_y - I_z) - I_P \Omega x_7 + u_2) + \lambda_2 I_x x_6 \right\} \right\}$$

$$s_2 \dot{s}_2 = \frac{s_2}{I_x} \left\{ (x_7 x_8 (I_y - I_z) - I_P \Omega x_7 + \lambda_2 I_x x_6) + u_2 \right\}$$

$$u_2 = - \left\{ x_7 x_8 (I_y - I_z) - I_P \Omega x_7 + \lambda_2 I_x x_6 + K_2 \right\} \cdot \text{sat}(s_2)$$

③ Designing  $\theta$  :-

$$s_3 = \dot{e} + \lambda_3 e = (\dot{\theta} - \dot{\theta}_d) + \lambda_3 (\theta - \theta_d)$$

$$\dot{s}_3 = (\ddot{\theta} - \ddot{\theta}_d) + \lambda_3 (\dot{\theta} - \dot{\theta}_d)$$

$$= \frac{1}{I_y} \{ x_6 x_8 (I_z - I_x) + I_P \Omega x_6 + u_3 \} + \lambda_3 x_7$$

$$s_3 \dot{s}_3 = \frac{s_3}{I_y} \left\{ (x_6 x_8 (I_z - I_x) + I_P \Omega x_6 + \lambda_3 I_y x_7) + u_3 \right\}$$

$$u_3 = - (x_6 x_8 (I_z - I_x) + I_P \Omega x_6 + \lambda_3 I_y x_7 + K_3) \cdot \text{sat}(s_3)$$

$$\text{can we choose } \Omega = \omega_1 - \omega_2 + \omega_3 - \omega_4$$

$$\Downarrow = \omega_1 + \omega_3 - (\omega_2 + \omega_4)$$

$$\{ \text{Max}(\Omega) = \text{max}(\omega_1 + \omega_3) - \text{min}(\omega_2 + \omega_4) \}$$

$$\text{Max}(\Omega) = 2 \times \text{max}(\omega)$$

④ Designing  $\psi$  :-

$$s_4 = \dot{e} + \lambda_4 e = (\dot{\psi} - \dot{\psi}_d) + \lambda_4 (\psi - \psi_d)$$

$$\dot{s}_4 = (\ddot{\psi} - \ddot{\psi}_d) + \lambda_4 (\dot{\psi} - \dot{\psi}_d)$$

$$\dot{s}_4 = \frac{I}{I_z} \{ x_6 x_7 (I_x - I_y) + u_4 \} + \lambda_4 x_8$$

$$s_4 \dot{s}_4 = s_4 \cdot \frac{I}{I_z} \{ x_6 x_7 (I_x - I_y) + \lambda_4 I_z x_8 + u_4 \}$$

$$u_4 = - (x_6 x_7 (I_x - I_y) + \lambda_4 I_z x_8 + K_4) \cdot \text{sat}(s_4)$$