

a)

We know that,

$$Y = \begin{bmatrix} \theta_1'' & \cos(2\theta_1'' + \theta_2'')-2 & \theta_2'' & -g\sin(\theta_1) & -g\sin(\theta_1+\theta_2) \\ 0 & \sin(\theta_2)(\theta_2''^2) + \cos(\theta_2)*\theta_1'' & \theta_1''+\theta_2' & 0 & -g\sin(\theta_1+\theta_2) \end{bmatrix}$$

$$\alpha = \begin{bmatrix} m_2 l_1^2 + m_1 r_1^2 + m_2 r_2^2 + l_1 + l_2 \\ m_2 l_1 r_2 \\ m_2 r_2^2 + l_2 \\ m_1 r_1 + m_2 l_1 \\ m_2 r_2 \end{bmatrix}$$

$$Y^* \alpha = [\theta_1''(m_2 l_1^2 + m_1 r_1^2 + m_2 r_2^2 + l_1 + l_2 + 2\cos(\theta_1)m_2 l_2 r_2) + \theta_2''(m_2 l_2 + 2\sin(\theta_2)(2\theta_1' \theta_2 + \theta_2'^2) - g \sin(\theta_1)(m_1 r_1 + m_2 l_2) - g \sin(\theta_1 + \theta_2) m_2 r_2$$

$$\theta_1''(l_2 + m_2 r_2^2 + m_2 l_1 r_2 \cos(\theta_2) + \theta_2''(m_2 r_2^2 + l_2) + m_2 l_1 r_2 (\sin(\theta_2)) \theta_1'^2 - m_2 r_2 g \sin(\theta_1 + \theta_2)]$$

Manipulator form

$$M(q)*q'' + C(q,q') q' + g(q) = \tau$$

By comparing $\tau = [\tau_1; \tau_2]$ using MATLAB we can safely say that,

$$Y^* \alpha = \tau$$

b)

$$A = [0 \ 0 \ 1 \ 0; 0 \ 0 \ 0 \ 1; 0 \ 0 \ 0 \ 0; 0 \ 0 \ 0 \ 0];$$

$$B = [0 \ 0; 0 \ 0; 1 \ 0; 0 \ 1];$$

$$\lambda = [-3 \ -3 \ -4 \ -4];$$

$$K = \text{place}(A, B, \lambda);$$

$$Acl = A - B*K;$$

$$Q = \text{eye}(4).*10;$$

$$P = \text{lyap}(Acl', Q);$$

Adaptive controller design

$\Theta_{ddot{1}d}, \theta_{ddot{2}d} \rightarrow$ desired acceleration

$\Theta_{1d}, \theta_{2d} \rightarrow$ desired angles

$\Theta_{dot{1}d}, \theta_{dot{2}d} \rightarrow$ desired velocity

$$error = [\theta_1; \theta_2] - [\theta_{1d}; \theta_{2d}];$$

$$error_dot = [\dot{\theta}_1; \dot{\theta}_2] - [\dot{\theta}_{1d}; \dot{\theta}_{2d}];$$

$$v = [\ddot{\theta}_{1d}; \ddot{\theta}_{2d}] - K[error; error_dot];$$

Designing control input 'U'

$$U = \hat{M} * v + \hat{C} * [\dot{\theta}_1; \dot{\theta}_2] + \hat{G};$$

$$\ddot{\theta} = M \backslash (U - C * [\dot{\theta}_1; \dot{\theta}_2] - G);$$

Where M, C, and G are original matrices of the manipulator form

$$\phi = M \backslash Y;$$

$$\gamma = eye(5) * 0.3;$$

$$\dot{\alpha}_{est} = -\gamma \backslash (\phi' * B' * P * [error; error_dot]);$$

d)

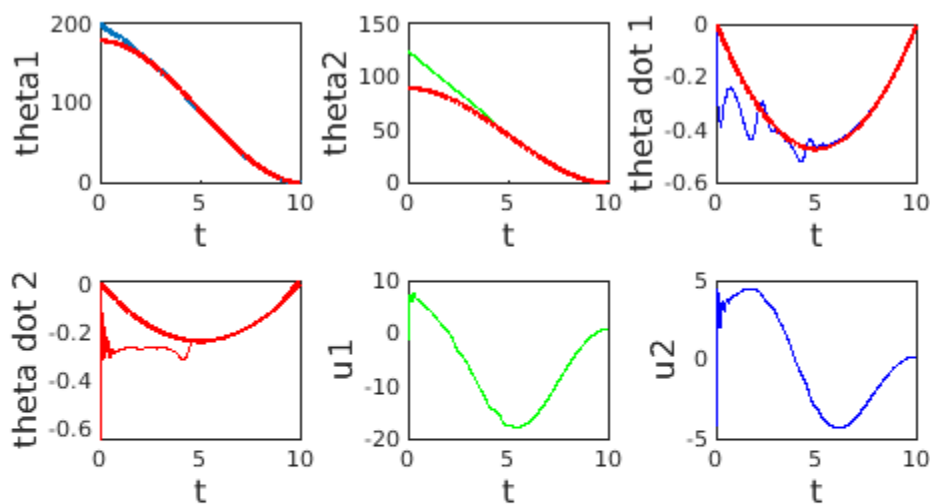


Figure 1: Adaptive control with P not equal to Zero

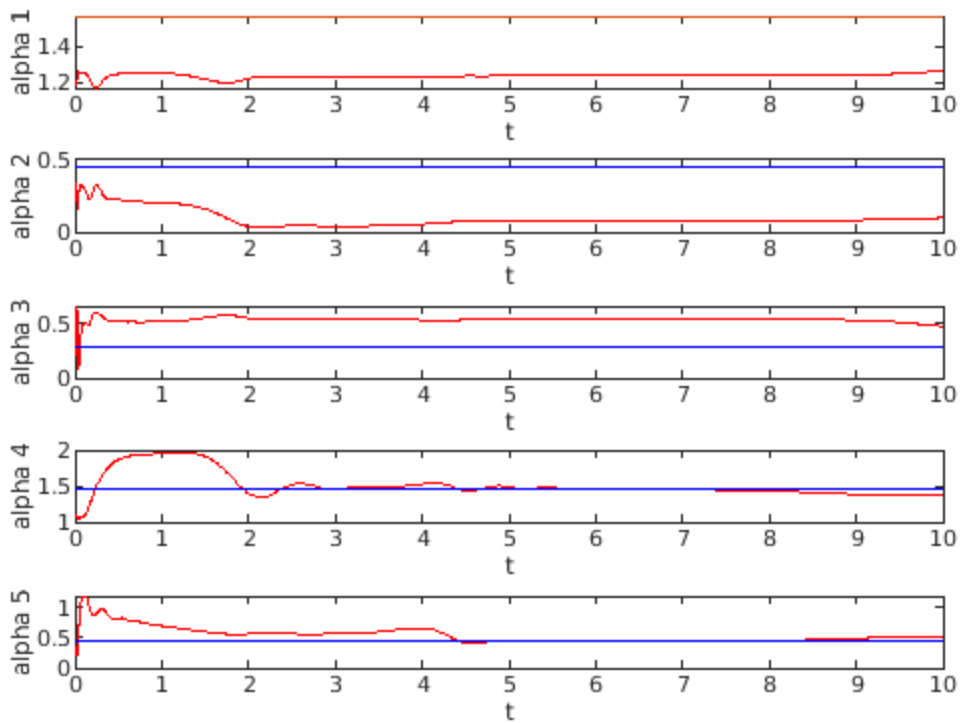


Figure 2: convergence using adaptive control (P non zero)

e)

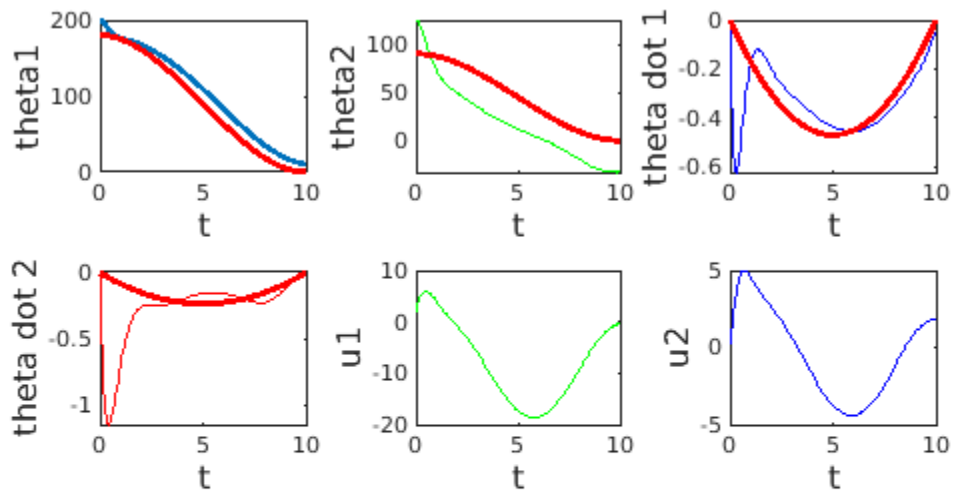


Figure 3: Adaptive control with P equal to Zero

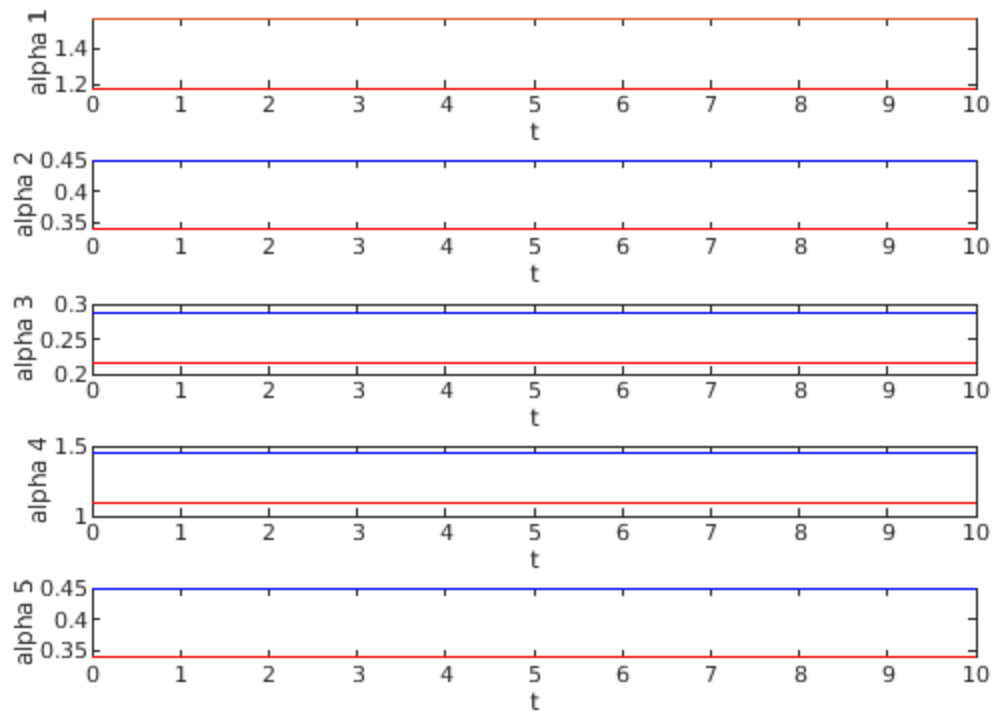


Figure 4: convergence using adaptive control (P zero)

As you can see from the Figure 4 that when $P = 0$, the adaptive alpha wouldn't converge to the desired value.

f)

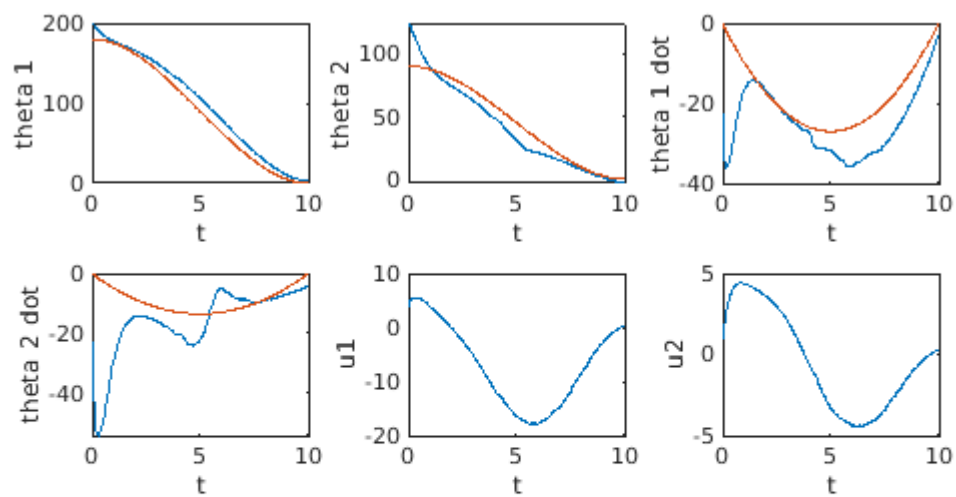


Figure 5: Plot from gazebo simulation