

# Homework 1

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**Abstract—** 3 Scenarios as listed in the homework were performed in MATLAB and numerical results were recorded and plotted.

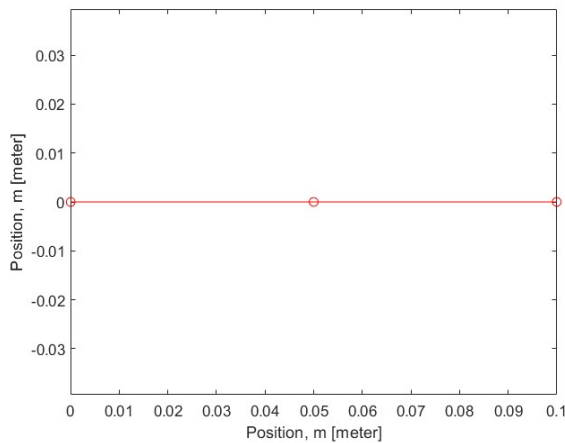
## I. SIMULATION 1

1. Simulation of the motion of three connected spheres falling inside viscous fluid (see Section 4.2)

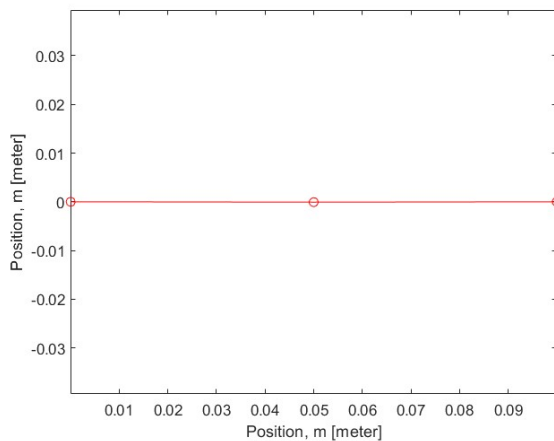
A. Show the shape of the structure at  $t = \{0, 0.01, 0.05, 0.10, 1.0, 10.0\}$  s. Plot the position and velocity (along y-axis) of R2 as a function of time.

Plots are as follows:

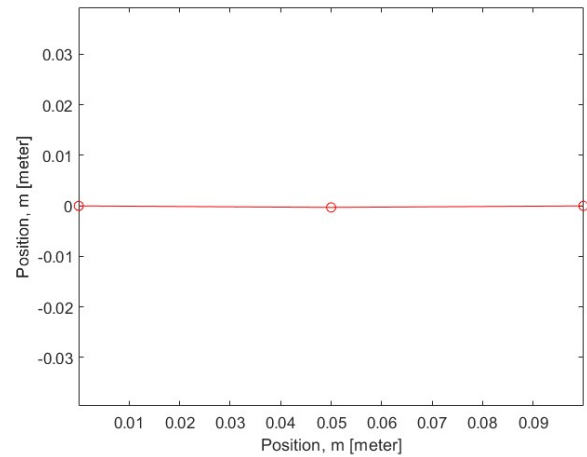
i) Time: 0 sec



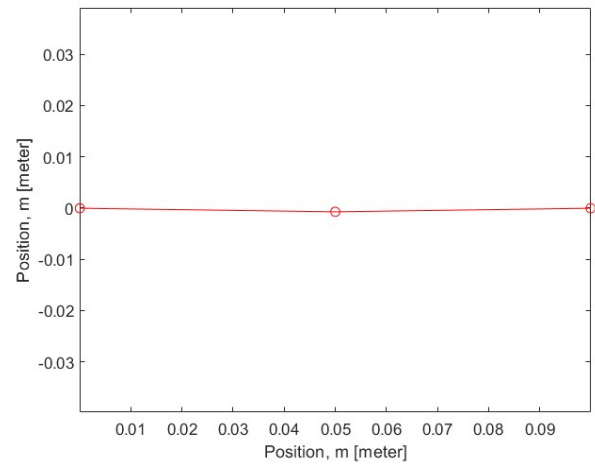
ii) Time :0.01 sec



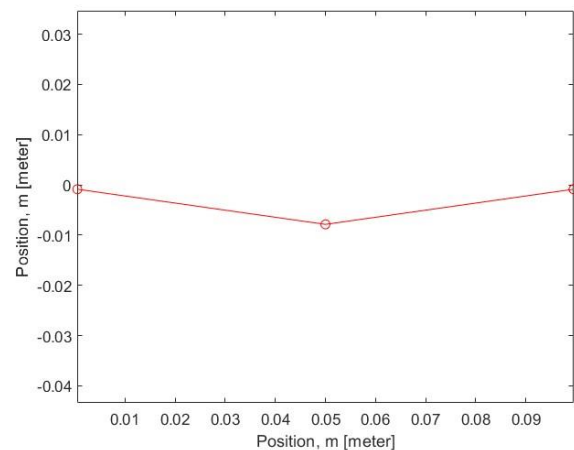
iii) Time:0.05 sec



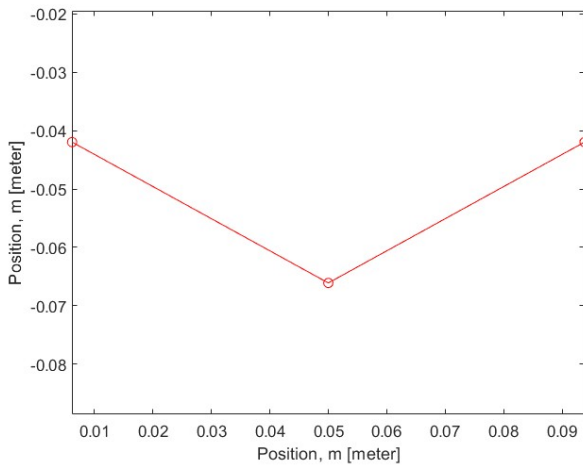
iv) Time: 0.1 sec



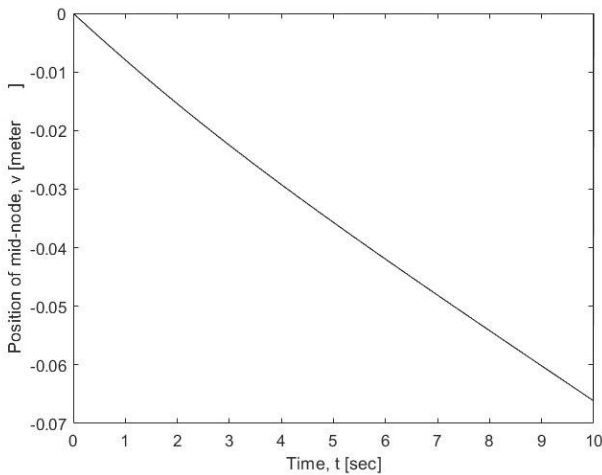
v) Time: 1 sec



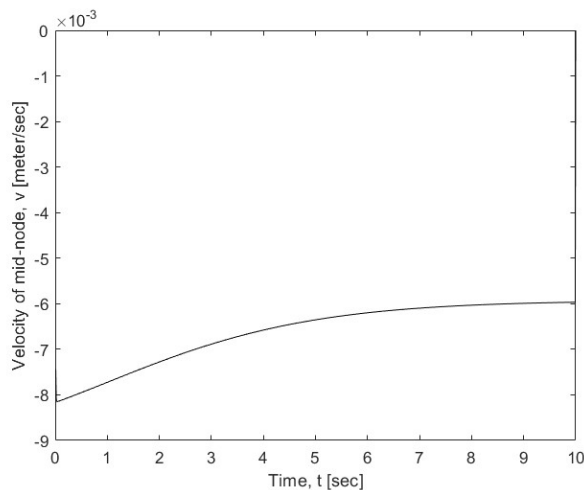
vi) Time: 10 sec



vii) Position vs Time



viii) Velocity vs Time



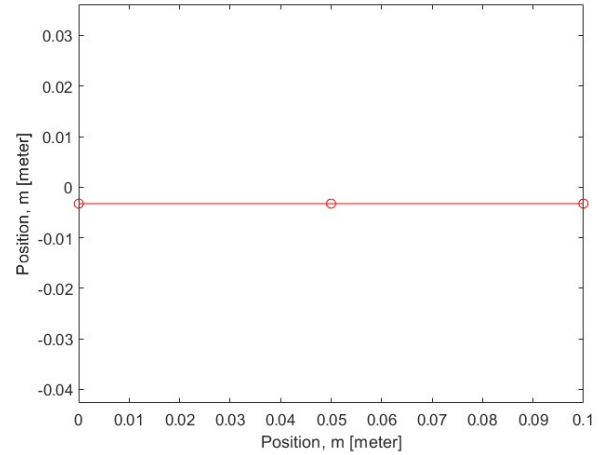
B. What is the terminal velocity (along y-axis) of this system?

The terminal velocity of the system is -0.006m/sec.

C. What happens to the turning angle if all the radii ( $R_1, R_2, R_3$ ) are the same?

Does your simulation agree with your intuition?

If all the three balls are of the same radii ( $R_1=R_2=R_3$ ) then they fall with the same velocity downwards as visible from the plot given below. There is no relative deformation between two nodes, all nodes maintain the same y-coordinates in the simulation.



D. Try changing the time step size ( $\Delta t$ ), particularly for your explicit simulation, and use the observation to elaborate the benefits and drawbacks of the explicit and implicit approach?

Explicit simulation does not converge if the step size is not small enough, that is not the case with implicit simulation it was proven again by altering the step size with a wide range of values in Question 2.

E. Plot Discussion

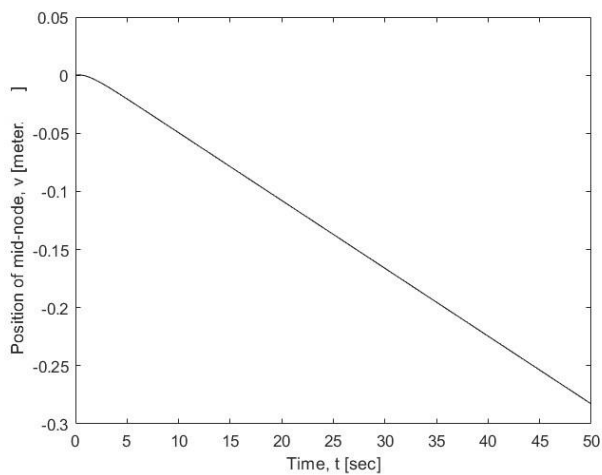
Explicit simulation does not converge if the step size is not small enough, that is not the case with implicit simulation it was proven again by altering the step size with a wide range of values in Question 2. It is understandable that the deviation/displacement with respect to y co-ordinates is minimal in plots of 0, 0.01, 0.05 and 0.1 second. As the simulation progresses plots of 1 and 10 seconds show the relative displacement of the middle node with respect to the other corner nodes. Position of the middle nodes keep on decreasing with time whereas velocity reaches a terminal velocity of -0.006m/sec and remains that way.

## II. SIMULATION 2

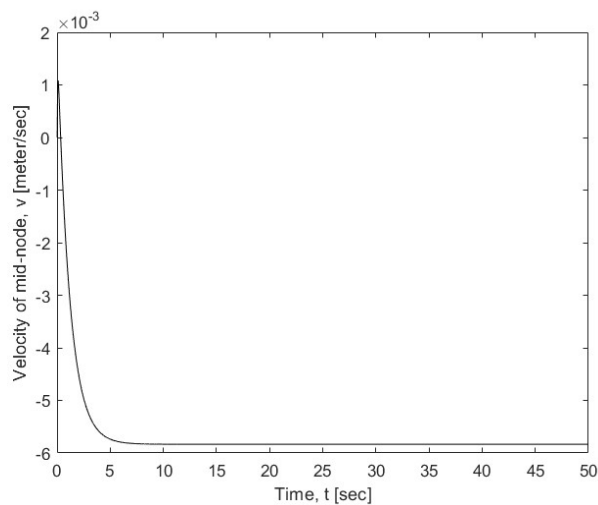
2. Simulation of the motion of N-connected spheres falling inside viscous fluid (see Section 4.3)

A. Include two plots showing the vertical position and velocity of the middle node with time. What is the terminal velocity?

### i) Vertical Position of Mid Node vs Time

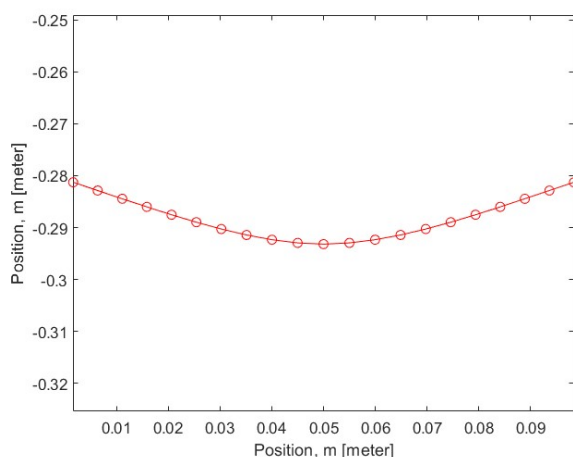


### i) Velocity of Mid Node vs Time



Terminal Velocity is -0.0058m/sec.

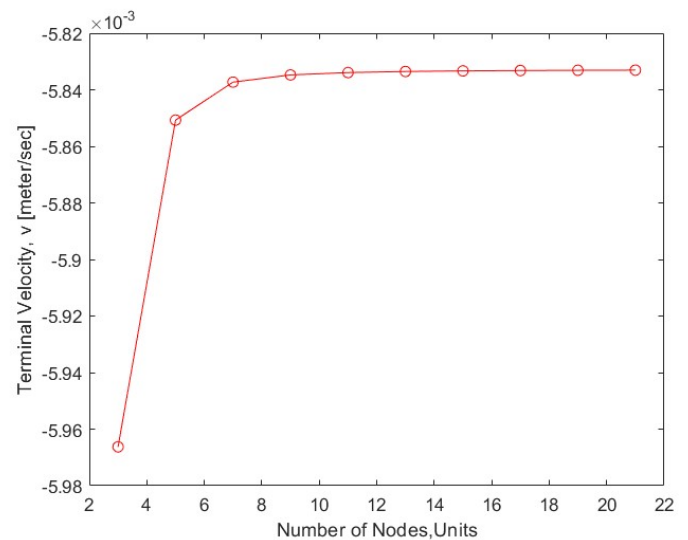
B. Include the final deformed shape of the beam.



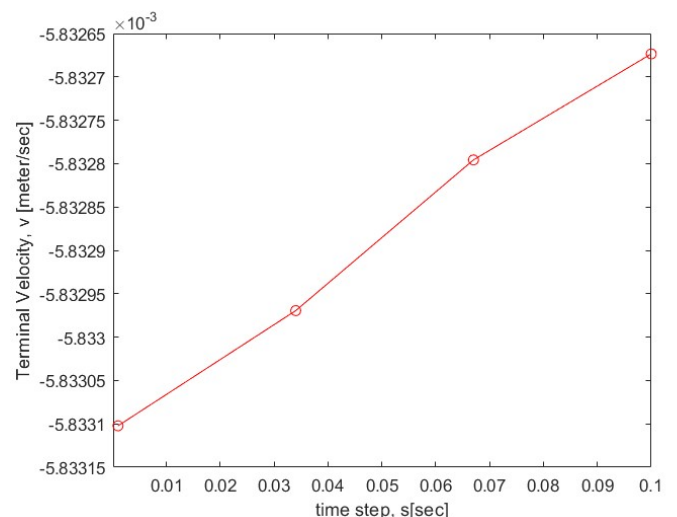
C. Discuss the significance of spatial discretization (i.e. the number of nodes,  $N$ ) and temporal discretization (i.e. time step size,  $\Delta t$ ). Any simulation should be sufficiently discretized such that the quantifiable metrics, e.g. terminal velocity, do not vary much if  $N$  is increased and  $\Delta t$  is decreased. Include plots of terminal velocity vs. the number of nodes and terminal velocity vs. the time step size.

Significance can be visualized by the plots:

### i) Terminal Velocity of Mid Node vs Number of Nodes



### ii) Terminal Velocity vs Time Step size $\Delta t$



Time

### D. Plot Discussion

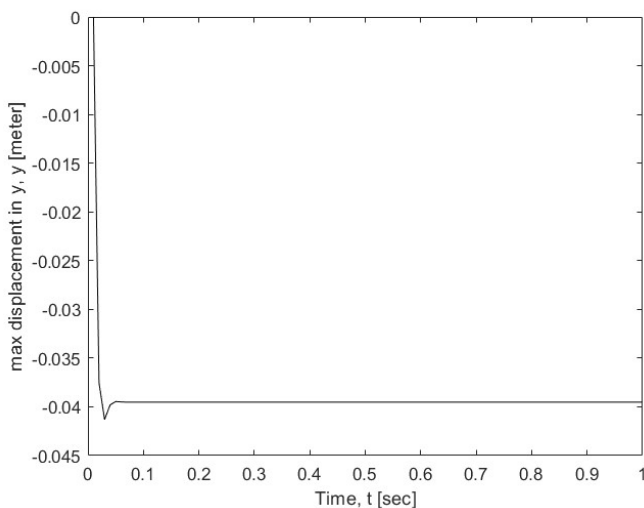
With respect to temporal discretization it can be observed that for a change from  $10^{-4}$  to  $10^{-1}$  the terminal velocity did not vary significantly and stayed very close of  $-5.8 \times 10^{-3}$  m/sec only changes happen to significant digits after 5.8, so the changes were very low in magnitude. Spatial distribution altered the graph more in comparison to temporal discretization. Terminal velocity was  $5.96 \times 10^{-3}$  m/sec nodes which also was the result for Simulation 1. If the nodes are

above 7 the terminal velocity does not vary significantly and remains near  $5.85 \times 10^{-3}$  m/sec.

### III. SIMULATION 3

3. Simulation of the deformation of elastic beams and comparison with Euler-Bernoulli beam theory (see Section 4.4)

A. Plot the maximum vertical displacement,  $y_{\max}$ , of the beam as a function of time. Depending on your coordinate system,  $y_{\max}$  may be negative. Does  $y_{\max}$  eventually reach a steady value? Examine the accuracy of your simulation against the theoretical prediction from Euler beam theory:



$y_{\max}$  does reach a steady value. Deflections predicted by the theoretical model and simulation are within 0.001m thus the results can be held accurate.

B. What is the benefit of your simulation over the predictions from beam theory?

To address this, consider a higher load  $P = 20000$  such that the beam undergoes large deformation. Compare the simulated result against the prediction from beam theory in Eq. 4.21. Euler beam theory is only valid for small deformation whereas your simulation, if done correctly, should be able to handle large deformation. Optionally, you can make a plot of  $P$  vs.  $y_{\max}$  using data from both simulation and beam theory and quantify the value of  $P$  where the two solutions begin to diverge.

Simulation Result at 2000 N = -0.0395 m

Euler Theory Result at 2000 N = -0.038045 m

Simulation Result at 20000 N = -0.2456 m

Euler Theory Result at 2000 N = -0.380449 m

For a small load applied  $P=2000$ N, Euler theory has given accurate results. If there is a large force involved  $P=20000$ N and the deformation is significant the error given by the Euler Theory exceeds a generally accepted margin of  $\pm 5\%$ .

### IV. CONCLUSION

### APPENDIX.

### ACKNOWLEDGMENT

The homework makes use of gradEb.m, gradEs.m, hesseEb.m, hessEs.m, etc written by Prof. Khalid Jawed, UCLA, Sameuli School of Engineering.

### REFERENCES