Gravitational-wave Astrophysics
Summer term 2021

Due Tuesday **Jul 20, 2021 (before 8am)** via email to **nikolas.wittek@aei.mpg.de**

PROBLEM SET 7

7.1. The first LIGO detection [10 pts.]

In this exercise we look at the original LIGO data from the first detection of gravitational waves on September 14th, 2015. This event has been (very creatively) named GW150914. *The data from this and other GW detections is publicly available online in the* LIGO/Virgo Gravitational Wave Open Science Center [1]*, so we can analyse it and convince ourselves that we have indeed detected the GW signal of two merging black holes. Here we consider data up to O2, the second LIGO/Virgo observing run.*

(a) Open the Python notebook and run the code to plot the GW150914 data in Fig. 1 for both the Livingston (`L1`) and Hanford (`H1`) LIGO detectors. Give a rough estimate of the dominant frequency in the data. What could be the source of this noise?

Note: This notebook makes use of a LIGO Python package called `pycbc` to access and analyse the data. This package can simply be installed via pip or conda. Write to me if you have problems installing it!

(b) Compute the amplitude spectral density for both detectors (Fig. 2 in the notebook). Briefly answer the following questions by referring to the plot and the lecture:

   (a) What is the frequency range where the LIGO detectors are most sensitive in, and what source of noise limits the sensitivity in this region?

   (b) What are the dominant sources of noise at low and high frequencies?

   (c) Estimate the frequencies of a few prominent peaks in the sensitivity curve. Can you match them to known instrumental lines (see e.g. [2])?

(c) *Whiten* the data by weighting it with the sensitivity curve in frequency space (Fig. 3), then *bandpass* it by removing very low and high frequency content that the detectors are not sensitive to anyway (Fig. 4). Note that the GW150914 signal was so loud that we can already see it in the data! You see the signals arrived with a time difference of ≈ 7ms between the two detectors in Livingston, Louisiana and Hanford, Washington. Estimate the angle from the axis connecting the detectors, where you would expect the GW signal to come from.

(d) To extract the signal from the data more systematically we perform *matched-filtering* with a *template* of the expected gravitational waveform. First, prepare the data (Fig. 5). Then, generate a waveform from the `SEOBNRv4` model for an equal-mass black-hole binary with component masses M. Try different component masses around $M \sim 30 M_\odot$ and briefly describe how the waveform changes (you can get some help by plotting multiple waveforms in the same figure with Python). You can also try varying the mass-ratio.

Note: `SEOBNRv4` is an *effective-one-body (EOB)* waveform model for *spinning* black-hole binaries, calibrated to *numeric relativity (NR)* simulations, and co-developed at the AEI in Potsdam [3].

(e) Perform the matched-filtering (Fig. 7) with one of the templates you generated and align the template with the data (Fig. 8). What information can you extract from Fig. 7?

(f) Finally, perform matched-filtering for a range of component masses between $25 M_\odot$ and $45 M_\odot$ (Fig. 9 and 10) and estimate the mass of the black-hole binary that emitted GW150914. How well does your estimate agree with the published LIGO results?

(g) How can you gain more confidence that the matched signal is not a glitch but a real GW signal?

(h) Use the methods demonstrated above to see if you can calculate the SNR time series of the five data sets provided. What is the SNR of each signal? Which template matched which data?

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[1] https://www.gw-openscience.org
[2] https://www.gw-openscience.org/speclines/
[3] https://arxiv.org/abs/1611.03703

7.2. Deriving the strain of gravitational wave data in the frequency domain [4 pts.]

In the lecture you were shown a plot of the gravitational wave strain and the detector noise sensitivity using a post-Newtonian approximation. In the jupyter notebook, follow the instructions to derive this plot with a real waveform.

7.3. The rate of binary black hole mergers [6 pts.]

During the first and second LIGO/Virgo Observing runs, ten binary black holes were observed. Let's use these observations to estimate the rate of binary black-hole (BBH) mergers, i.e. the number of BBH mergers per Gpc^3 *per year. The overall idea is simple: For each BBH* $i = 1, \dots 10$, *compute the volume of space* V_i *in which it* could have been *detected. We know that within this volume* V_i *and the observation time* T, *there was exactly one such BBH event. Therefore, the overall rate is going to be*

$$R_{BBH} = \frac{1}{T}\sum_i \frac{1}{V_i}. \tag{1}$$

The important paper is nicknamed 'GWTC-1', which stands for the first Gravitational Wave Transient Catalog. You can access it at https://arxiv.org/abs/1811.12907.

(a) Table I of GWTC-1 lists the ten BBH detections, as well as the one BNS detection. It also gives the network SNR for each signal, for the two matched filter searches 'PyCBC' and 'GstLAL'[4]. By averaging the PyCBC and GstLAL SNR's, compute one SNR-value ρ_i for each BBH event.

(b) To compute the maximum distance $D_{\max,i}$, to which each event could have been detected, we need to know how SNR changes with distance. Argue how the observed SNR ρ of a GW-event changes with distance to the event, when all other source-parameters are held constant.

Hint: trace the distance-depencence from the Quadrupole-formula through the formula for SNR, $\rho_{\rm opt} = \sqrt{(h,h)}$, where h is the waveform of the GW-event.

(c) Next, we need to figure out how far away these ten events could have been detected. In reality, this is measured by adding thousands of 'artificial signals' into LIGO detector noise, and counting how many of them are recovered by the search-pipelines. For a simpler estimate, we look at Table II. We see that signals with SNR$\sim$ 10 are classified as marginal triggers, so they are just below the threshold for confident detection. Therefore, let's take as a concrete threshold $\rho_{\rm th} = 10$. [5]

Given your results from part (a) and (b), and the actual luminosity distance of each event reported in Table III, estimate the volume of space V_i in which each event could have been observed. For simplicity, assume Euclidean geometry.

(d) The second observing run O2 resulted in about 118 days of usable data to analyse, as explained toward the end of section II.C. of GWTC-1. The first observing run O1 resulted in about 48 days of usable data (end of Section II of `https://arxiv.org/pdf/1606.04856.pdf`).

Using these times, and the results from above, estimate $R_{\rm BBH}$. Express your result in units of Gpc^{-3}yr^{-1}.

(e) Let's try to make some sense of the rather abstract number $R_{\rm BBH}$. Compute: (i) The number of BBH mergers per galaxy per million years and (ii) the rate of GWs originating from BBH mergers that pass through Earth in units of day^{-1}.

Note: for (i), look up the galaxy number-density in the universe in your favorite astronomy book. For (ii), assume that BBH started to merge 13 billion years ago (i.e. a few 100 Million years after the formation of the first galaxies), assume that the BBH merger rate is constant all the way back to that time, and assume that Euclidean geometry is valid.

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[4] Table I also lists data for the search without templates 'cWB'. As discussed in class this search is less sensitive to known signals, so we do not use it here.

[5] This $\rho_{\rm th} = 10$ differs from the value derived in class in two important aspects: First, it relates to the combined SNR of multiple detectors. Second, it is actually measured from detector data, rather than assuming Gaussian noise.