Algorithm describing current setup for Distributed IVA

1. Notation

Here we will describe notation as well as assumptions.

There are K total sites

There are N subjects per site

There are C components per subject

W_l^k denotes the unmixing matrix for subject l at site k

X_l^k denotes the l^{th} subject at site k

Y_l^k denotes the approximation to subject l at site k

$\hat{Y}^k$ denotes the site source for site k. Explained further in algorithm section

W^{Gk} denotes the master node unmixing matrix for the i^{th} site. Note that this starts out as the identity

S_l^k denotes the master node approximation to the true source of site k

At each site, $k = 1 \dots K$ there is a sub IVA (or ICA / group ICA) problem of finding statistically independent sources Y_l^k for each subject

$$(1) \quad \begin{pmatrix} W^{Gk} & 0 & \dots & 0 \\ 0 & W^{Gk} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & W^{Gk} \end{pmatrix} \begin{pmatrix} W_1^k & 0 & \dots & 0 \\ 0 & W_2^k & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & W_N^k \end{pmatrix} \begin{pmatrix} X_l^k \\ X_2^k \\ \vdots \\ X_N^k \end{pmatrix} = \begin{pmatrix} Y_1^k \\ Y_2^k \\ \vdots \\ Y_N^k \end{pmatrix}$$

At the master node, there is a main IVA (or ICA / group ICA) problem of finding statistically independent sources for the site sources $\hat{Y}$

$$(2) \quad \begin{pmatrix} W^{G1} & 0 & \dots & 0 \\ 0 & W^{G2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & W^{GK} \end{pmatrix} \begin{pmatrix} \hat{Y}^1 \\ \hat{Y}^2 \\ \vdots \\ \hat{Y}^K \end{pmatrix} = \begin{pmatrix} S^1 \\ S^2 \\ \vdots \\ S^K \end{pmatrix}$$

2. Algorithm

We will now describe the current algorithm

while not all sites and master node have converged **do**
  for all sites k = 1..K **do**
    Site k optimizes W^k based on $Y^k = W^{Gk} W^k X^k$ as seen in **??**
    Create $\hat{Y}^k$ out of components of Y_l^k
    Send the $\hat{Y}^k$ to the master node
  end for
  Master node computes Global IVA, as seen in **??**
  Master node sends W^{Gk} back to site k
end while

3. Issues / Problems

There are a couple of issues:

(1) How do we know that this will actually ever converge, or at least, how do we know that all sites and master node will eventually converge at same time? Even if we only require that the master node converges, how do we know that that will converge?
(2) Computing Y = W X is simple enough in normal IVA. When the Global unmixing matrix is added, it is not clear (to me at least) how to optimize W.