## THE INCREDIBLE SHRINKING NEURAL NETWORK: PRUNING TO OPERATE IN CONSTRAINED MEMORY ENVIRONMENTS

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## 1 2ND DERIVATIVE BACKPROP WEIGHT UPDATE RULE

Weight error (1st order approx):

$$E(w) = E(w_0) + \frac{\partial E}{\partial w}(w - w_0)$$
(1)

We want:

$$E\left(w\right) = 0\tag{2}$$

$$0 = E(w_0) + \frac{\partial E}{\partial w}(w - w_0)$$
(3)

$$0 = E(w_0) + \frac{\partial E}{\partial w}w - \frac{\partial E}{\partial w}w_0 \tag{4}$$

$$\frac{\partial E}{\partial w}w = \frac{\partial E}{\partial w}w_0 - E\left(w_0\right) \tag{5}$$

$$w = w_0 - \frac{E(w_0)}{\frac{\partial E}{\partial w}} \tag{6}$$

Weight error (2nd order approx):

$$E(w) = E(w_0) + \frac{\partial E}{\partial w}(w - w_0) + \frac{1}{2}\frac{\partial^2 E}{\partial w^2}(w - w_0)^2$$
 (7)

We want:

$$E\left(w\right) = 0\tag{8}$$

$$0 = \underbrace{E(w_0)}_{c} + \underbrace{\frac{\partial E}{\partial w}}_{b} (w - w_0) + \underbrace{\frac{1}{2} \frac{\partial^2 E}{\partial w^2}}_{a} (w - w_0)^2$$

$$\tag{9}$$

$$0 = ax^2 + bx + c \tag{10}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{11}$$

$$(w - w_0) = \frac{-\left(\frac{\partial E}{\partial w}\right) \pm \sqrt{\left(\frac{\partial E}{\partial w}\right)^2 - 4\left(\frac{1}{2}\frac{\partial^2 E}{\partial w^2}\right)E\left(w_0\right)}}{2\left(\frac{1}{2}\frac{\partial^2 E}{\partial w^2}\right)}$$
(12)

$$w = w_0 + \frac{-\left(\frac{\partial E}{\partial w}\right) \pm \sqrt{\left(\frac{\partial E}{\partial w}\right)^2 - 2\left(\frac{\partial^2 E}{\partial w^2}\right) E\left(w_0\right)}}{\left(\frac{\partial^2 E}{\partial w^2}\right)}$$
(13)

We only need one of the zeros so...

$$w = w_0 + \frac{\sqrt{\left(\frac{\partial E}{\partial w}\right)^2 - 2\left(\frac{\partial^2 E}{\partial w^2}\right)E\left(w_0\right) + \left(\frac{\partial E}{\partial w}\right)}}{\left(\frac{\partial^2 E}{\partial w^2}\right)}$$
(14)